# EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH ORGANISATION EUROPEENE POUR LA RECHERCHE NUCLEAIRE 

## CERN - PS DIVISION

## PS/DR/Note 2000-002 (Tech.)

## STUDY OF THE BEAM POSITION ACCURACY IN THE "RIESENRAD"

## ION GANTRY

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Geneva, Switzerland
31 January 2000

## 1 INTRODUCTION

Most of the currently proposed ion cancer therapy facilities in Europe plan to install an ion gantry equipped with a 'pencil-beam' scanning system [1-4]. A gantry can improve the dose-to-target conformity, but it also increases considerably the requirements imposed on the beam transport accuracy. Usually, sub-millimetre accuracy at the gantry iso-centre is specified when treating tumours in the vicinity of critical organs, which is one of the main domains of ion therapy. So far, experience exists only for proton gantries using passive [5-7] or hybrid [8] beam delivery systems, or for the fixed beam line at GSI using the pencil-beam scanning [9, 10]. Because of the higher magnetic rigidity of ion therapy beams (three times higher than protons), the construction principles applied to existing proton gantries cannot be directly 'scaled' to ion gantries and, for this reason, several novel ion-gantry concepts have recently been proposed [11-13]. One of these proposals, the so-called 'Riesenrad' gantry has been investigated from the point of view of sensitivity to various beam transport errors and the results are reported in this paper.

## 2 RIESENRAD GANTRY

### 2.1 Principle and mechanical concept of the Riesenrad gantry

The principle of the Riesenrad gantry is shown in Figure 1. A $90^{\circ}$ dipole magnet is placed at the end of the transfer line and rotated mechanically around the horizontal axis. Consequently, the beam deflected by the dipole magnet is directed to any angle corresponding to the angle of the dipole rotation. The patient must be in this case placed eccentrically and must follow the dipole rotation. One could say that the Riesenrad gantry is an extreme case of the eccentric gantry concept [14]. There are several technical concepts of this principle [15]. The one, which is currently considered for the Med-AUSTRON facility [3], is based on independent support systems for the dipole magnet and the patient. The patient positioning system is mounted on a platform capable of vertical and horizontal translations. A desirable patient position is achieved as a combination of these two translations as it is shown in Figure 2. This approach reduces the overall gantry weight, makes it possible to increase the rigidity of the dipole magnet support structure and provides essentially more space around the patient compared to the situation when the patient cabin and the dipole magnet are integrated into a common support structure. In addition, routine and emergency access to the patient becomes easier and faster.

### 2.2 Beam transport system of the Riesenrad gantry

The beam transport to and in the Riesenrad gantry is rather special since the beams extracted from a synchrotron are expected to have significantly different emittances in the two transverse planes [16]. In order to match such non-symmetric beams to a rotating gantry, a special matching section called 'rotator' must be used [17]. The rotator is a bending free quadrupole lattice with a special transfer matrix, which is positioned upstream of the gantry and which is rotated by half the gantry angle around the horizontal axis [18, 19]. The beam transport system of the Riesenrad gantry including the rotator is shown in Figure 3 and the betatron amplitudes corresponding to the horizontal position of the gantry $\left(\alpha=0^{\circ}\right)$ are shown in Figure. 4. The betatron amplitudes in the iso-centre in two transverse planes are, in general, different, because of different beam emittances in the transverse planes. Neither the gantry nor the rotator is involved in the control of the beam size at the iso-centre. This task is accomplished by dedicated modules in the transfer line upstream of the rotator [20]. This means that the optical settings of the rotator and gantry are independent of the beam size
required at the iso-centre. However, the overall transfer matrix from the rotator entrance to the iso-centre becomes a function of the gantry angle $\alpha$, because an angle $\alpha / 2$ appears between the exit of the rotator and the entrance of the gantry.


Figure 1 Principle of the Riesenrad gantry
[(a) A $90^{\circ}$ dipole is placed at the end of the transfer line and rotated mechanically around the horizontal axis. An eccentrically located patient follows the dipole rotation and can be irradiated from any direction. (b) For comparison, a 'classical' isocentric gantry is schematically shown as well.]


Figure 2 The Riesenrad gantry with movable treatment platform.
[The patient position corresponding to a particular angle of the dipole magnet is achieved by a combination of vertical and horizontal translations of the treatment platform. In this case, the dipole is rotated from $-90^{\circ}$ to $+90^{\circ}$ ( $0^{\circ}$ is defined to be the horizontal position of the magnet) and the patient table can be rotated $360^{\circ}$ around its vertical axis in order to achieve effectively any treatment angle


Figure 3 Beam transport system of the Riesenrad gantry including the rotator.
[The rotator consists of seven quadrupoles (Q1-Q7), the gantry consists of four quadrupoles (Q8-Q12), scanning magnets (S1, S2, S3) and the $90^{\circ}$ dipole magnet. Global (room) and local (beam-transport) coordinate systems are indicated as well.]


Figure 4 Betatron amplitude functions in the rotator and gantry beam transport system for $\alpha=0^{\circ}$
[Solid line: bending plane of the dipole, dashed line: non-bending plane of the dipole]

## 3 ERROR ANYLYSIS

### 3.1 General considerations

The present analysis has been restricted to errors leading to a wrong beam position at the gantry iso-centre, which is the most critical aspect of the gantry beam transport system. The effects causing focusing errors like deviations from an exact beam size or deformations of an ideally round beam spot have been neglected. Only the misalignment of beam transport elements that causes a deviation of the beam from the optical axis has been considered. The
beam transport elements are assumed to be perfectly manufactured, correctly powered, having an ideal field quality, but displaced along and/or rotated about each of the local coordinate system axes $x, y$, and $z$ (see Figure 3).

The misalignments have been classified into two categories: systematic and random. Systematic misalignments are caused by deformations of the gantry and rotator support structure. The main feature of systematic misalignments is their short-term reproducibility as a function of the gantry angle. Long-term effects like building or ground movements are not considered and must be compensated by periodic re-alignment of the whole system. Random misalignments represent all possible effects with no reproducibility as a function of the gantry angle. A source of random misalignments could be temperature fluctuations, fabrication imperfections of the gantry supporting ring*, bearings, final precision of an original alignment, etc. These misalignments are expected to have a gaussian distribution which is superimposed on the systematic misalignments. The position of each beam transport element is therefore characterised by a particular value of the systematic misalignment (element-specific) and a standard deviation of the random misalignment distribution. The situation is illustrated in Figure 5.


Figure 5 Position probability distribution of a beam transport element showing the systematic and random misalignment components

## 3. 2 EFFECTS OF MISALIGNMENTS

A misaligned quadrupole causes a transverse 'kick' to the beam, which can be calculated from the transfer matrix of the quadrupole:

$$
\left(\begin{array}{l}
x_{1}  \tag{1}\\
x_{1}^{\prime} \\
y_{1} \\
y_{1}^{\prime}
\end{array}\right)=\left(\begin{array}{cccc}
\cos \sqrt{k} L & \frac{1}{\sqrt{k}} \sin \sqrt{k} L & 0 & 0 \\
-\sqrt{k} \sin \sqrt{k} L & \cos \sqrt{k} L & 0 & 0 \\
0 & 0 & \cosh \sqrt{k} L & \frac{1}{\sqrt{k}} \sin \sqrt{k} L \\
0 & 0 & \sqrt{k} \sin \sqrt{k} L & \cosh \sqrt{k} L
\end{array}\right) \cdot\left(\begin{array}{l}
x_{0} \\
x_{0}^{\prime} \\
y_{0} \\
y_{0}^{\prime}
\end{array}\right)
$$

for a quadrupole which focuses in the $(x, z)$ plane, where $x_{0}, x_{0}{ }^{\prime}, y_{0}, y_{0}{ }^{\prime}$ are the particle

[^0]coordinates at the entrance, $x_{1}, x_{1}{ }^{\prime}, y_{1}, y_{1}{ }^{\prime}$ are the coordinates at the exit, $L$ is the quadrupole effective length $[\mathrm{m}]$ and $k$ is the strength $\left[\mathrm{m}^{-2}\right]$ defined as $k=g /(B \rho)$ where $g$ is the gradient $[\mathrm{T} / \mathrm{m}]$ and $B \rho$ is the magnetic beam rigidity $[\mathrm{Tm}]$. For a transverse misalignment $\Delta x, \Delta y$ one gets for the kick, by putting $x_{0}=-\Delta x, x_{0}{ }^{\prime}=0, y_{0}=-\Delta y$ and $y_{0}{ }^{\prime}=0$ (see Figure 6a):
\[

$$
\begin{equation*}
x_{1}^{\prime}-x_{0}^{\prime}=(\sqrt{k} \sin \sqrt{k} L) \cdot \Delta x \tag{2}
\end{equation*}
$$

\]

for the focusing plane and

$$
\begin{equation*}
y_{1}^{\prime}-y_{0}^{\prime}=(-\sqrt{k} \sinh \sqrt{k} L) \cdot \Delta y \tag{2}
\end{equation*}
$$

for the defocusing plane of the quadrupole. Note that the misalignment of a magnet which is positive in the local beam coordinate system causes the reference particle to be negatively displaced with respect to the optical axis of the misaligned magnet, hence $x_{0}=-\Delta x$ and $y_{0}=-\Delta y$.

When tilting the magnet by angles $R_{\mathrm{x}}$ (about $x$-axis) and $R_{\mathrm{y}}$ (about $y$-axis), then $x_{0}=0, x_{0}{ }^{\prime}=-$ $R_{y}, y_{0}=0$ and $y_{0}{ }^{\prime}=+R_{\mathrm{x}}$ (see Fig. 6b). The kicks are given:

$$
\begin{equation*}
x_{1}^{\prime}-x_{0}^{\prime}=(-\cos \sqrt{k} L) \cdot R_{y}-\left(-R_{y}\right)=(1-\cos \sqrt{k} L) \cdot R_{y} \tag{3}
\end{equation*}
$$

for the focusing plane and

$$
\begin{equation*}
y_{1}^{\prime}-y_{0}^{\prime}=(\cosh \sqrt{k} L) \cdot R_{x}-R_{x}=(\cosh \sqrt{k} L-1) \cdot R_{x} \tag{3}
\end{equation*}
$$

for the defocusing plane (3). Similarly, the effects of dipole misalignments, which are basically geometrical focusing and/or trigonometric transformations between the local coordinate systems at the dipole entrance and the dipole exit, can be calculated.

Systematic and random misalignments must be treated differently. The systematic misalignments represent the situation when all elements are misaligned by a known amount. For each gantry angle, the elements have definite positions different from the ideal design positions and the whole beam line represents a certain particular combination of element misalignments. The position of the beam in the gantry iso-centre is obtained by tracing the beam through this misaligned beam line by a dedicated computer code.

The random misalignments are interpreted as an uncertainty of the actual element position. In other words, the element position is given a certain probability distribution, which is assumed to be gaussian. All misalignments in all elements are assumed to be independent and their individual contributions to the beam displacement are added quadratically. If the parameters of the element misalignment are taken as representing one standard deviation of the misalignment distribution, then the calculated beam position represents one standard deviation of a beam position probability distribution.


Figure. 6 Effects of a misaligned quadrupole lens
[Note that the outputs given by the transfer matrix of the quadrupole are in fact related to the optical axis of the misaligned element and have to be properly converted to the coordinate system following a design trajectory.]

## 4 BEAM TRANSPORT CALCULATIONS

The beam transport calculations were performed independently with the computer codes TRANSPORT [21] and WinAGILE [22] and the results were consist.

### 4.1 Systematic misalignments

Typical and probably the dominating component of the systematic misalignments are the elastic deformations of the gantry support structure. The elastic deformations were calculated by the dedicated computer code CUBUS [23] and converted from the global (room) coordinate system to the local (beam-transport) coordinate system that follows the bends and rotations of the beam line. In the local coordinate system, the $z$ axis always points in the beam direction and the $[x, z]$ and $[y, z]$ planes at the dipole exit are identical with the bending and non-bending planes, respectively, independent of the angle of gantry rotation (see Figure3). The global (room) coordinate system is fixed and does not follow the gantry rotation.

The results of the beam transport calculations showing the response of the system to the elastic deformation misalignments are shown in Figure 7. The position of the beam-centre in the gantry iso-centre is given in the local (beam-transport) co-ordinate system for different angles of gantry rotation from $-90^{\circ}$ to $+90^{\circ}$ in $10^{\circ}$ steps. Three sets of data are presented corresponding to the misalignment of the quadrupoles alone, the gantry dipole alone and all elements together.


Key Circles Only quadrupoles have been misaligned
Squares Only the dipole has been misaligned
Triangles Quadrupoles and dipole have been misaligned
Fig. 7 Beam position in the gantry isocentre for different angles of gantry rotation

### 4.2 RANDOM MISALIGNMENTS

It is difficult to assess the random misalignments in the same way as the systematic ones and a different strategy has been chosen. The sensitivity of the gantry beam transport system has been investigated thus giving the possibility to specify "backwards" the necessary tolerances on the element positions. For this purpose, some approximations have been introduced into the model. The first approximation is to express the effect of a misaligned quadrupole as an angular kick with zero displacement at the exit to the quadrupole (thin-lens approximation). The angular kicks are given by (2) and (3) in Section 3.2. The kicks then cause beam displacements at the gantry iso-centre according to the transformation:

$$
\left(\begin{array}{c}
x  \tag{4}\\
x^{\prime} \\
y \\
y^{\prime}
\end{array}\right)=\left(\begin{array}{llll}
t_{11} & t_{12} & t_{13} & t_{14} \\
t_{21} & t_{22} & t_{23} & t_{24} \\
t_{31} & t_{32} & t_{33} & t_{34} \\
t_{41} & t_{42} & t_{43} & t_{44}
\end{array}\right) \cdot\left(\begin{array}{c}
x_{0} \\
x_{0}^{\prime} \\
y_{0} \\
y_{0}^{\prime}
\end{array}\right)
$$

where $x, x^{\prime}, y, y^{\prime}$ are parameters of the reference trajectory (beam centre) in the gantry isocentre, $x_{0}, x_{0}{ }^{\prime}, y_{0}, y_{0}{ }^{\prime}$ are parameters of the reference trajectory at the exit to the misaligned element and $t_{\mathrm{ij}}$ are elements in the transfer matrix from the exit of the misaligned element to the gantry iso-centre. The thin-lens approximation gives $x_{0}=y_{0}=0$ and $x_{0}{ }^{\prime}$ and $y_{0}{ }^{\prime}$ will be called $H_{\text {kick }}$ and $V_{\text {kick }}$ for the horizontal and vertical planes, respectively. Note that there is a coupling between the horizontal and the vertical planes due to the fact that the gantry is rotated by an angle $\alpha / 2$ with respect to the rotator, $\alpha$ being the angle of gantry rotation. The terms in the
off-diagonal sub-matrices are therefore not zero. The final beam displacement due to the quadrupole shift is:

$$
\begin{gather*}
x=t_{12} \cdot H_{\text {kick }}+t_{14} \cdot V_{\text {kick }} \text { (horizontal plane) } \\
y=t_{32} \cdot H_{\text {kick }}+t_{34} \cdot V_{\text {kick }}(\text { vertical plane }) \tag{5}
\end{gather*}
$$

After evaluating the kicks using (2), one obtains:

$$
\begin{align*}
& x=t_{12} \cdot \sqrt{k} \sin \sqrt{k} L \cdot \Delta x-t_{14} \cdot \sqrt{k} \sinh \sqrt{k} L \cdot \Delta y=C_{1} \Delta x+C_{2} \Delta y \\
& y=t_{32} \cdot \sqrt{k} \sin \sqrt{k} L \cdot \Delta x-t_{34} \cdot \sqrt{k} \sinh \sqrt{k} L \cdot \Delta y=C_{3} \Delta x+C_{4} \Delta y \tag{6}
\end{align*}
$$

where $C_{1}, C_{2}, C_{3}$ and $C_{4}$ are constants depending only on the angle of gantry rotation. For analysis of random errors, the misalignments $\Delta x$ and $\Delta y$ are taken as representing a standard deviation of the position probability distribution of a misaligned element and their effects supposing independent random misalignments in any direction - must be added quadratically:

$$
\begin{align*}
& \sigma(i)_{H}=\sqrt{(x(\Delta x, \Delta y=0))^{2}+(x(\Delta x=0, \Delta y))^{2}}=\sqrt{C_{1}(i)^{2}(\Delta x)^{2}+C_{2}(i)^{2}(\Delta y)^{2}} \\
& \sigma(i)_{V}=\sqrt{(y(\Delta x, \Delta y=0))^{2}+(y(\Delta x=0, \Delta y))^{2}}=\sqrt{C_{3}(i)^{2}(\Delta x)^{2}+C_{4}(i)^{2}(\Delta y)^{2}} \tag{7}
\end{align*}
$$

where $\sigma()_{\mathrm{H}}$ and $\sigma(\mathrm{i})_{\mathrm{V}}$ now represent a standard deviation of the beam position probability distribution corresponding to the misalignment of the i-th quadrupole and indexes H and V assign the horizontal and vertical planes, respectively.

The second approximation in the model is a physically reasonable assumption that the position uncertainty for all quadrupoles in all directions is the same, i.e. $\Delta x=\Delta y=\Delta z \equiv \Delta_{\text {shift }}$ where $\Delta_{\text {shift }}$ is now introduced as representing the random misalignment of a quadrupole in any direction. Equation (7) then looks like:

$$
\begin{align*}
& \sigma(i)_{H}=\sqrt{C_{1}(i)^{2}+C_{2}(i)^{2}} \cdot \Delta_{\text {shift }}=C(i)_{H} \Delta_{\text {shift }} \\
& \sigma(i)_{V}=\sqrt{C_{3}(i)^{2}+C_{4}(i)^{2}} \cdot \Delta_{\text {shift }}=C(i)_{V} \Delta_{\text {shift }} \tag{8}
\end{align*}
$$

Equation (8) demonstrates that the beam displacement in the gantry iso-centre caused by a random quadrupole shift is simply proportional to the shift. The proportionality constants are different in the horizontal and vertical plane $\mathrm{C}\left(\mathrm{i}_{\mathrm{H}} \neq \mathrm{C}(\mathrm{i})_{\mathrm{v}}\right.$. If all quadrupoles are independently misaligned, the standard deviation of the beam position probability distribution in each plane will be given by:

$$
\begin{equation*}
\sigma_{\text {shift }}=\sqrt{\sum_{i}(\sigma(i))^{2}}=\sqrt{\sum_{i}\left(C(i)^{2} \Delta_{\text {shift }}^{2}\right)}=\sqrt{\sum_{i}\left(C(i)^{2}\right) \cdot \Delta_{\text {shift }} \propto \Delta_{\text {shift }}} \tag{9}
\end{equation*}
$$

where indexes for horizontal and vertical planes are no longer indicated keeping in mind that equation (9) differs for the different planes by only the proportionality constant.

The same strategy can be applied for effects of quadrupole tilting, dipole shift and dipole tilt yielding the final expression for the standard deviation of the beam position probability distribution in a given plane, $\sigma_{\text {total }}$ :

$$
\begin{equation*}
\sigma_{\text {total }}=\sqrt{\left(\sigma(\text { quads })_{\text {shift }}\right)^{2}+\left(\sigma(\text { quads })_{\text {tilt }}\right)^{2}+\left(\sigma(\text { dipole })_{\text {shift }}\right)^{2}+\left(\sigma(\text { dipole })_{\text {tilt }}\right)^{2}} \tag{10}
\end{equation*}
$$

where each contributing effect is proportional with a different proportionality constant to the corresponding misalignment. This enables a proportional scaling of results and to specify "backwards" the tolerable misalignments from the requirements on the beam position accuracy. In principle, calculations have to be done for all gantry angles, because the proportionality constants depends on the angle of gantry rotation.

The input data used in the random misalignment analysis were $3 \sigma_{\text {shift }}=0.1 \mathrm{~mm}$ and $3 \sigma$ tilt $=0.1 \mathrm{mrad}$ for all quadrupole and dipole magnets. Specifying the $3 \sigma$-value means practically that the elements are expected to be kept within $-3 \sigma$ to $+3 \sigma$ tolerances. The calculations have been done for gantry angles from $-90^{\circ}$ to $+90^{\circ}$ with $10^{\circ}$ step. Figure 8 shows the results for two significant angles of gantry rotation $+90^{\circ}$ and $0^{\circ}$. The values for other gantry angles were in-between these two extreme cases. The individual contributions listed in (10) were (horizontal plane/vertical plane) $3 \sigma$ (quads) shift $=0.93 / 1.23 \mathrm{~mm}, 3 \sigma$ (quads) $_{\text {tilt }}=0.16 / 0.22 \mathrm{~mm}, 3 \sigma(\text { dipole })_{\text {shift }}=0.1 / 0.06 \mathrm{~mm}$ and $3 \sigma(\text { dipole })_{\text {tilt }}=0.14 / 0.63 \mathrm{~mm}$ for $\alpha=90^{\circ}$. The $3 \sigma$-regions of the beam position probability distribution corresponding to the above values are depicted in Figure 9. The maximum overall values $3 \sigma_{\text {total }}=0.96 / 1.4 \mathrm{~mm}$ were obtained for $\alpha=90^{\circ}$.

The misalignments due to temperature fluctuations, which are considered as random misalignments, were investigated with the aid of the CUBUS code and the following results have been obtained:

- A uniform temperature rise by $1 K$ in the gantry room lifts the centre of the gantry front ring by approximately 0.05 mm , the iso-centre rises about one third of this value.
- A temperature gradient of $2 K$ from the lower $(-1 \mathrm{~K})$ towards the upper part $(+1 \mathrm{~K})$ of the gantry room. Depending on the gantry angle the effects vary, however they are in the same order of magnitude as for the uniform temperature rise (maximum shift of 0.03 mm , maximum tilt of 0.01 mrad$)$.
- A heating of $1 K$ of the dipole relative to the gantry structure. The effect is very sensitive to the design of the dipole supports. Maximum values, $0.02 \mathrm{~mm}, 0.005 \mathrm{mrad}$.
- The above values were converted into the misalignments of the beam transport elements and the response of the beam position in the gantry iso-centre was calculated. The results are shown in Figure 10. It can be seen that the contribution of the temperature fluctuations to the beam position uncertainty is about $0.4 / 0.5 \mathrm{~mm}(3 \sigma$, horizontally/vertically).


Figure 8 Uncertainty of the beam position in the gantry isocentre expressed as a $3 \sigma$-value of the beam position probability distribution for two angles of gantry rotation ( $0^{\circ}$ and $90^{\circ}$ ) and reference random misalignments of all elements 0.1 mm and 0.1 mrad


Figure. 9 Individual random misalignment contributions corresponding to shifting and tilting of the quadrupoles and the dipole for angle of gantry rotation $90^{\circ}$ and reference random misalignments of all elements 0.1 mm and 0.1 mrad


Figure 10 Beam position uncertainty due to temperature fluctuations. $3 \sigma$-regions are always indicated

## 5 SUMMARY AND DISCUSSION

The systematic misalignments caused by elastic deformations of the gantry structure lead to excursions of the beam from the gantry iso-centre less than 0.2 mm , which is within the required precision. However, there will be other components of the systematic misalignments, for example fabricating errors, which are presently not included in the calculations because their exact values are not known yet. Nevertheless, the systematic errors are considered not to be critical for routine gantry operation. By virtue of their reproducibility as a function of the gantry angle, they can be compensated by a set of fixed corrections realised, for example, as an off-set of the scanning magnets or as a kick of dedicated corrector magnets. The second alternative is likely to be preferred in order to keep the scanning and the corrections as two orthogonal functions.

The sensitivity of the beam position accuracy to random misalignments has been calculated using the 'reference' values of 0.1 mm and 0.1 mrad for misalignments in all elements independently. The response of the beam position can now be scaled proportionally to correspond to other input misalignment values. In the case of different values for different effects (quadrupole-shift, quadrupole-tilt, dipole-shift, dipole-tilt), the scaling must be done separately for each effect and the resulting contributions must be added quadratically.

The random misalignment effects have been particularly assessed for temperature fluctuations and finally a contribution from the Patient Positioning System, PPS, ( $3 \sigma=0.3 \mathrm{~mm}$ ) was added (see Figure. 11). The main results are collected in Table 1.

| Source of error | $3 \sigma$-region of the beam position <br> probability distribution for the <br> reference misalignments 0.1 mm <br> and $0.1 \mathrm{mrad}, \mathrm{H} / \mathrm{V}$ plane <br> [mm] | $3 \sigma$-region of the beam position <br> probability distribution for the <br> misalignments due to temperature <br> fluctuations H/V plane <br> [mm] |
| :--- | :---: | :---: |
| Dipole-shift | $0.10 / 0.06$ | $0.09 / 0.06$ |
| Dipole-tilt | $0.14 / 0.63$ | $0.02 / 0.11$ |
| Dipole | $0.17 / 0.64$ | $0.10 / 0.12$ |
| Quadrupole-shifts | $0.93 / 1.23$ | $0.37 / 0.49$ |
| Quadrupole-tilts | $0.16 / 0.22$ | $0.01 / 0.02$ |
| Quadrupoles | $0.94 / 1.25$ | $0.37 / 0.49$ |
| Beam transport, <br> i.e. quadrupoles + dipole | $0.95 / 1.40$ | $0.38 / 0.51$ |
| Beam transport + PPS | $1.00 / 1.43$ | $0.48 / 0.59$ |

Table 1. Summary of the effects of random misalignments


Figure 11 Beam position uncertainty due to temperature fluctuations AND resolution of the patient positioning system

The results show that the sensitivity of the beam transport to the element misalignments is rather high. In order to achieve the sub-millimetre precision, the misalignments should be kept below 0.1 mm and 0.1 mrad . However, not all misalignments are that critical. The dominating contribution comes from the shifting of quadrupoles, which is equally true for the reference situation as well as for the temperature effects. A precision of $\pm 0.08 \mathrm{~mm}$ would be required
for transversal position of the quadrupoles. The effect of quadrupole tilt is about a factor of 6 lower and can be practically neglected. For the dipole, the angular misalignments are more critical compared to the shifts, especially in the vertical plane. The tolerances 0.1 mm and 0.1 mrad are acceptable.

It would also be possible to reduce the quadrupole contribution by steering magnets located downstream of the gantry quadrupoles and upstream of the scanning magnets. These correctors would be to direct the beam into the centre of the scanning system and hence remove the position and angular errors of the incoming beam caused by the upstream misalignments. These corrector magnets could be controlled on-line by a permanent beam position monitoring system at the entrance to the scanning dipoles. A similar strategy is applied at the GSI fixed therapy beam-line, where the correction action is performed directly by the scanning system [10].

The final conclusion of the study is the fact that the angular dependence (gantry angle) of the beam position uncertainty at the iso-centre is practically negligible, which reduces drastically the amount of calculations which have to be done in the future for refinement studies of the gantry. There are two reasons for this very weak angular dependence. The first reason is that the width of the overall beam position probability distribution is given as a quadratic sum of many contributions, namely three independent shifts and tilts of each element, all elements being further independently misaligned with respect to each other. Each individual contribution has its own angular dependence which may be decreasing or decreasing with the gantry angle, so that in the quadratic sum the decrease of one contribution is well balanced by an increase of the other one. The second reason is that there are also contributions from elements that are downstream of the rotator-to-gantry coupling point. These contributions are independent of the angle of gantry rotation. The situation is illustrated in Figure 12, that shows the response of the beam position at the gantry iso-centre as a function of the gantry angle separately for horizontal and vertical misalignments $(0.1 \mathrm{~mm})$ of the first quadrupole. The quadratic sum of these two effects are indicated as "Sum I" while "Sum II" is a quadratic sum of these effects AND contributions from the elements downstream of the rotator-togantry coupling point. The overall angular dependence becomes practically negligible.

Most of the calculations were done by two computer codes using different strategies for simulating the misalignment effects. TRANSPORT [21] calculates first all individual contributions for all elements and then sums them. WinAGILE [22] generates many lattices each representing a certain particular combination of misalignments and traces the beam through each lattice. The beam positions at a specified point of interest are collected and statistically evaluated. An excellent agreement between both computer codes has been observed.


Figure 12 Demonstration of the summation of different random misalignment contributions leading to 'vanishing' of the angular dependence of the beam position accuracy

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[^0]:    * It is assumed that the support rollers and ring will not have the same relative positions each time a particular gantry angle is set.

