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AN AIR CORE QUADRUPOLE FOR THE PIMMS SYNCHROTRON

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Abstract

The Proton Ion Medical Machine Study (PIMMS) is designing an optimised synchrotron for radiation therapy, which produces a very stable extracted beam over a period of about one second. An air core quadrupole that compensates the extraction resonance movements induced by the current ripples in the synchrotron main quadrupoles is used to improve the spill stability. The present paper describes its characteristics.

* TERA Foundation

1. INTRODUCTION

Medical synchrotrons require a long (about one second) and stable extracted spill to uniformly distribute the dose on the tumour volume [l]. To improve the spill stability an air core quadrupole, that compensates the extraction resonance movements induced by the current ripples in the synchrotron main quadrupoles, is used. The use of an air core quadrupole is justified to correct the high frequency ripple (few tens of kHz) for which a *magnetic core* is penalising.

This note describes the characteristics of the air core quadrupole for the PIMM synchrotron. The magnetic field and gradient have been calculated starting from the Laplace equation as detailed in Appendix A. The calculation is simplified by the absence of ferromagnetic materials and the results have been tested using a 3D magnetic code [2]. Appendix B shows the calculation of the air core quadrupole time constant.

2. DESIGN CHARACTERISTICS

To define the characteristics of the air core quadrupole, the horizontal plane parameters are of interest since this is the plane concerned with the slow-extraction process. The current ripples in the power supplies of normal quadrupoles produce movements of the extraction resonance and, as a consequence, fluctuations in the extracted spill current.

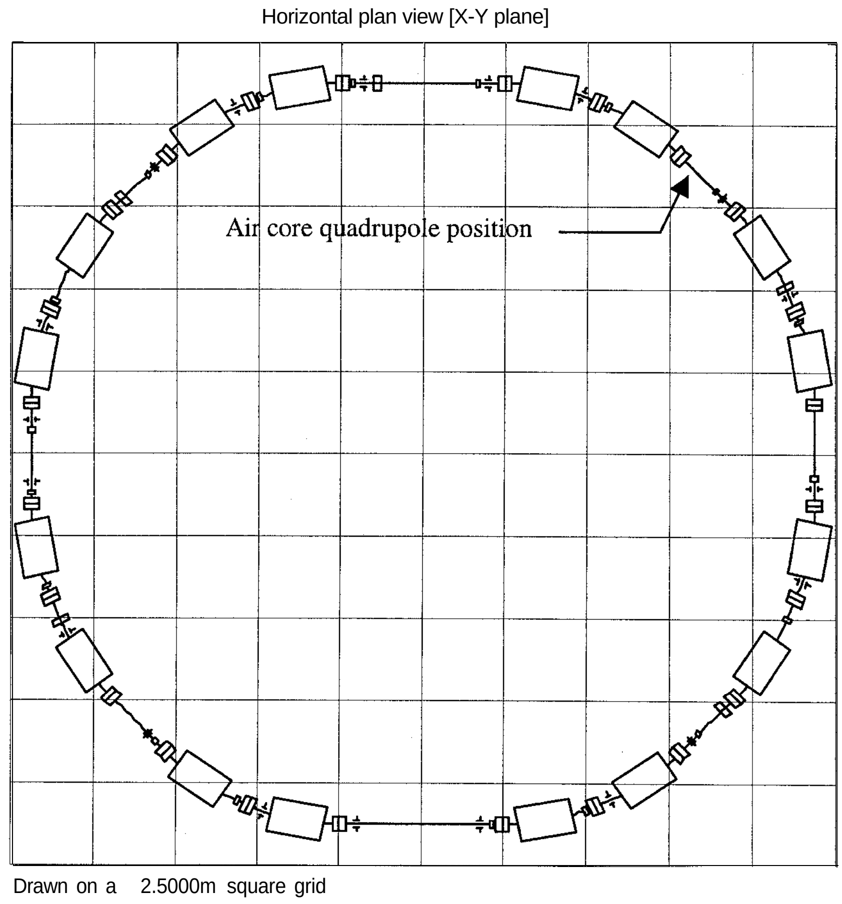

Fig. 1. *Position of the air core quadrupole in the PIMM lattice*

Taking into account the position, the strength and the length of the lattice quadrupoles, it is possible to calculate the tune shift produced by a current ripple (dI/I) in the quadrupole power supplies:

$$\Delta Q_H \approx 1.5 \frac{dI}{I}$$

Assuming a current stability of the order of $dI/I = 5 \cdot 10^{-4}$, we can reasonably estimate that the PIMM air core quadrupole should be able to produce a tune variation of the order of:

$$\Delta Q_H \approx 7.5 \cdot 10^{-4}$$

Fig. 1 shows the location of the air core quadrupole in the lattice of the PIMM synchrotron. Summarised in Table 1 are the characteristics of the air core quadrupole and the calculated tune shift that produces considering a beam of carbon ions with kinetic energy of 400 MeV/u, corresponding to the maximum extraction energy.

Table 1 *Characteristics of the air core quadrupole*

Air core quadrupole length (*lq*) [mm]	400
Number of windings per coil	1
Cross section of the winding [mm^2]	4×4
Current [A]	80
Current density [A mm^{-2}]	5
Air core quadrupole inductance [µH]	0.11
Air core quadrupole resistance (20 °C) [mΩ]	4.21
Time constant (τ) [ms]	0.03
“Cut-off” frequency (f) [kHz]	38.48
Good field region H × V [mm^2]	120×60
β_H at the air core quadrupole position [m]	15.6
Horizontal tune shift	$9.3 \cdot 10^{-4}$

Detailed in Appendix B is the calculation of the air core quadrupole cut-off frequency, starting from the evaluation of its total inductance and total resistance. Its value, f = 38.48 kHz, shows the suitability of this device to act as a fast feedback system to compensate high frequency ripples occurring in the main quadrupoles power supplies.

Fig. 2 shows a cross-section of the upper half of the air core quadrupole. Also indicated are the aperture of the vacuum chamber and the good field region. The *xy* co-ordinates of the wire indicated by the arrow are: x_{min} = 85 mm, x_{max} = 89 mm, y_{min} = 22 mm and y_{max} = 26 mm. The other wires are symmetrically placed with respect to the beam axis.

Views of the air core quadrupole are shown in Figs. 3 to 5. One can also see the orientation of the co-ordinate system xyz, where the z-axis is tangent to the particle trajectory, the *x*-axis is contained in the medial plane and the y-axis is perpendicular to it.

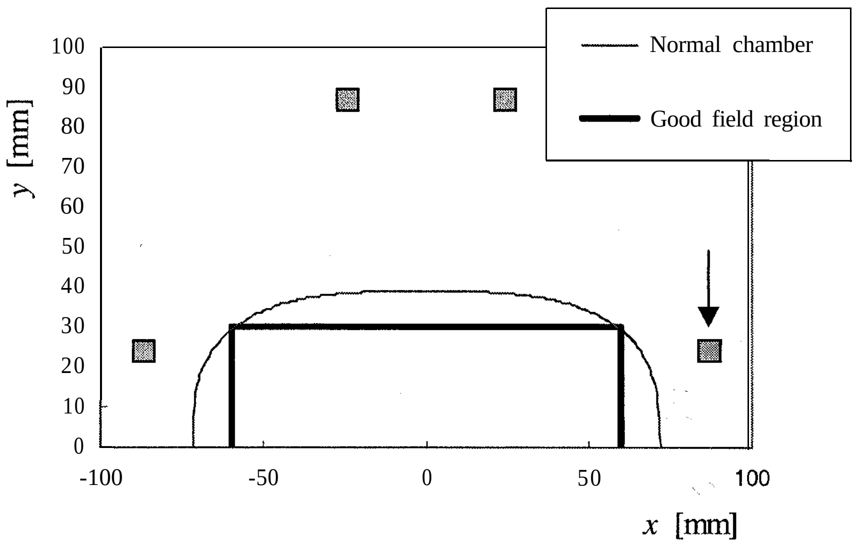

Fig. 2. Cross-section of the vacuum chamber and of the air core quadrupole at z = 0

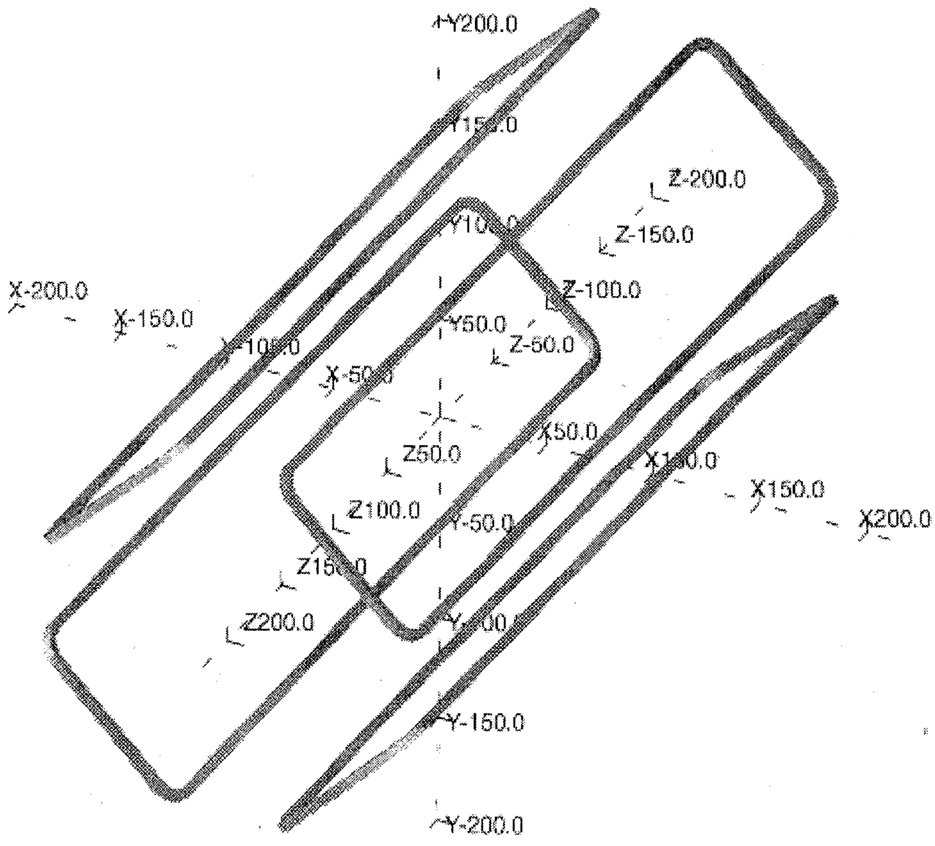

Fig. 3. 3D view of the air core quadrupole

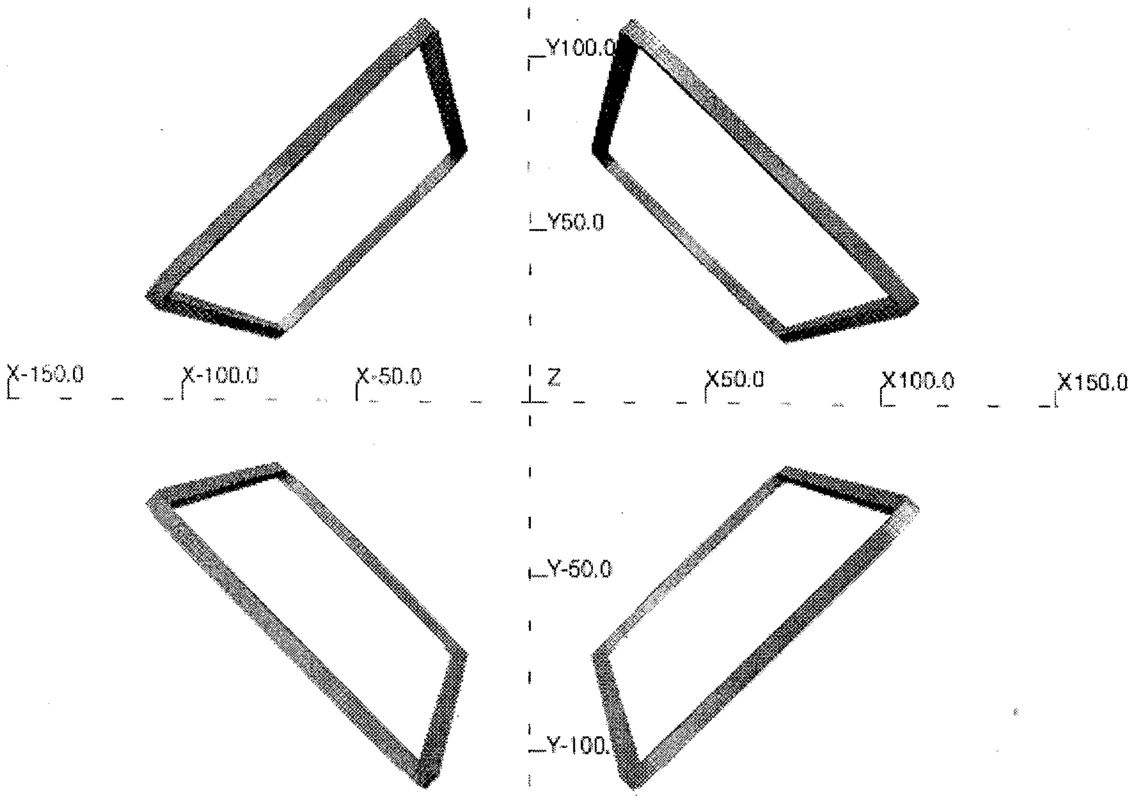

Fig. 4. *Front view of the air core quadrupole*

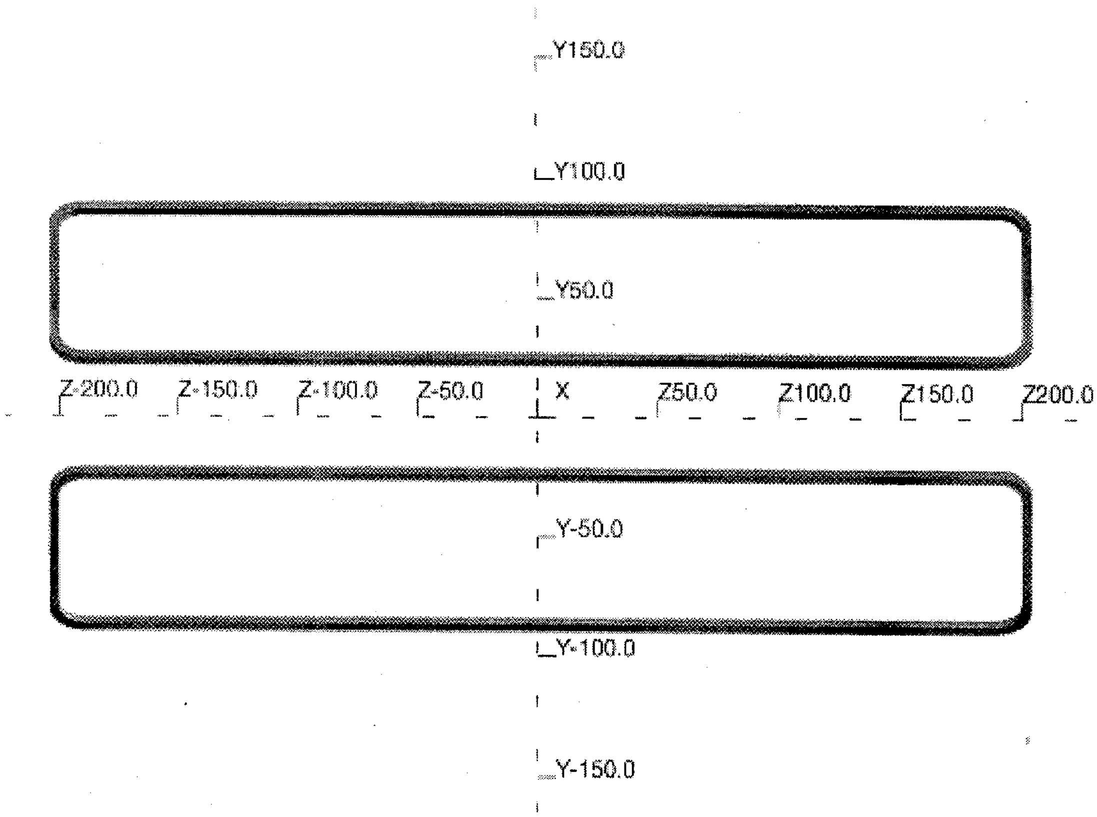

Fig. 5. *Lateral view of the air core quadrupole*

3. TUNE SHIFT EVALUATION: METHOD AND RESULTS

The field generated by the air core quadrupole has been calculated using the Laplace equation for the magnetic field generated by a current density $\vec{J}$:

$$\vec{B}_K = \frac{\mu_0}{4\pi} \iiint \frac{\vec{J} \times \vec{v}}{r^2} d\tau$$

that gives the magnetic field at the point K, located at a distance r from the wire unit volume $d\tau = dx\,dy\,dz$, and where $\vec{v} = \vec{r}/r$ is the unit vector. The integration interests the coils, i.e. the space in which the current density is non-zero. Since no other ferromagnetic materials are used in the quadrupole construction, the field calculations are exact.

As the calculation of the volume integral is quite time consuming, the problem has been simplified by dividing the air core quadrupole in three parts: the central body, composed of 8 wires of length $b = lq = 0.4$ m, and the two extremities of 4 wires of length a = 0.089 m each.

For each part the volume integral has been analytically reduced to a linear integral. Then the three contributions have been added to give the B field and the gradient in each point of the space around the air core quadrupole (see Appendix A for details on the calculations).

Fig. 6 shows the field and the gradient along the x-axis, up to the limit of the good field region, in the centre of the quadrupole, i.e. at y = z = 0. These values are obtained considering a current Z = 80 A flowing in the four coils.

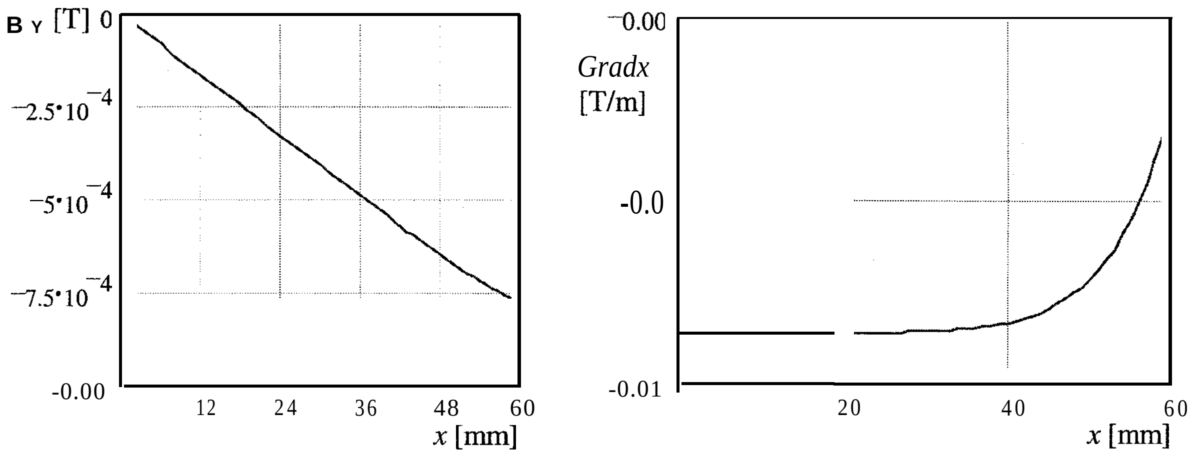

***Fig. 6.** B field and gradient on the median plane at the quadrupole centre (y = z = **0**)*

Fig. 7 shows the same quantities for $z = 0$ and $y = 30$ mm, i.e. at the vertical limit of the good field region.

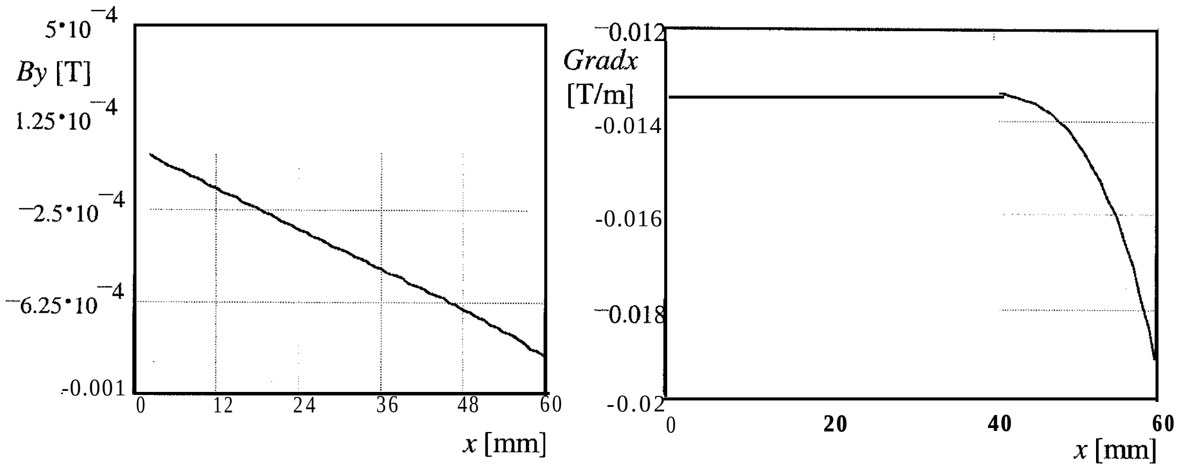

Fig. 7. *B field and gradient at y = 30 mm from the centre of the quadrupole (z = 0)*

To calculate the mean value of the tune shift, the quadrupole has been cut in several slices along its length (each slice is long *dlq = 50 mm)*. For each slice the mean gradient has been calculated averaging the values in the median plane and at the limit of the good field region. The mean quadrupole strength ($\bar{k}$) has then been derived. The tune shift is thus approximated by a discrete sum of the slices' contributions:

$$\Delta Q_H \approx \frac{1}{4\pi} \beta_H \sum_{lq} \bar{k} \cdot dlq$$

where β_H is the horizontal betatron amplitude at the quadrupole position. The mean tune shift is $9.3 \cdot 10^{-4}$, which satisfies the requirements. To test these results the air core quadrupole has been modelled with the programme OPERA 3D [3]. The gradient calculations agree within 5% with those obtained applying the simple method presently used.

5. CONCLUSIONS

The study of the PIMMS air core quadrupole has been presented. A simple programme based on the Laplace equation has been developed to calculate the tune shift produced by the air core quadrupole. The results, also tested with a magnetic programme, confirm that the characteristics of the air core quadrupole satisfy the requirements. The cut-off frequency has been evaluated to verify the suitability of the air core quadrupole for the correction of high frequency ripples effects.

6. AKNOWLEDGMENTS

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Distribution :

PIMMS Study Team

APPENDIX A: MAGNETIC FIELD AND GRADIENT

Let us calculate the B field produced by a current density $\vec{J}$ using the law of Laplace:

$$\vec{B}_K = Bx_k\vec{u}_x + By_k\vec{u}_y + Bz_k\vec{u}_z = \frac{\mu_0}{4\pi}\iiint\frac{\vec{J}\times\vec{v}}{r^2}d\tau$$

where r is the distance between the wire unit volume $d\tau = dx\ dy\ dz$ and the test point K, $\vec{v} = \vec{r}/r$ is the unit vector. The integration limits are given by the coil geometry. The calculation has been simplified by dividing the air core quadrupole in three sections: the central part and the two ends. For example, the gradient along the x–axis is calculated as follows :

Central part of the air core quadrupole: 8 wires

Let us introduce some conventions:

- s is the wire cross-section;
- lq is the air core quadrupole length;
- I is the circulating current;
- $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$; $r(k) = \begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix}$ is a sequence of test points of abscissa k along the x axis.

The central part of the air core quadrupole is composed of 8 parallel wires (numbered from 0 to 7) extending in z from $-lq/2$ to $+\ lq/2$. Fig. A1, representing a cross-section of the quadrupole at z = 0, shows the assumptions on the current flow: sign + means current vector perpendicular to the foil plane toward the reader, sign – means current vector opposite to it. The unit vector which gives the direction of the current flow in each wire is:

$$u_{j,i} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \end{pmatrix}$$

with $j = 0, 1, 2 \equiv x, y, z$; $i = 0 .. 7 \equiv$ the wires numbers indicated in Fig. A1.

The co-ordinates of the 8 wires are (wi stands for initial, wf for final):

$$wi_i = \begin{pmatrix} \overline{x} \\ \overline{y} \\ -lq/2 \end{pmatrix}_i \quad wf_i = \begin{pmatrix} \overline{x} \\ \overline{y} \\ lq/2 \end{pmatrix}_i \quad \text{with } i = 0 .. 7$$

An analytical double integration of the Laplace equation allows us to express the y component of the B field produced by the 8 wires at the points $r(k)$ as:

$$ByWtot_k = \sum_{i=0}^{7} \frac{\mu_0 I}{4\pi S} \int_{wi_{1,i}}^{wf_{1,i}} \left(-u_{0,i} Logy1(y) + u_{2,i} Logy2(y)\right) dy$$

with:

$$Logy1(y) = \ln\left[\frac{\left(r(k)_0 - wf_{0,i}\right) + \sqrt{\left(wf_{0,i} - r(k)_0\right)^2 + \left(y - r(k)_1\right)^2 + \left(wi_{2,i} - r(k)_2\right)^2}}{\left(r(k)_0 - wf_{0,i}\right) + \sqrt{\left(wf_{0,i} - r(k)_0\right)^2 + \left(y - r(k)_1\right)^2 + \left(wf_{2,i} - r(k)_2\right)^2}} \cdot \frac{\left(r(k)_0 - wi_{0,i}\right) + \sqrt{\left(wi_{0,i} - r(k)_0\right)^2 + \left(y - r(k)_1\right)^2 + \left(wf_{2,i} - r(k)_2\right)^2}}{\left(r(k)_0 - wi_{0,i}\right) + \sqrt{\left(wi_{0,i} - r(k)_0\right)^2 + \left(y - r(k)_1\right)^2 + \left(wi_{2,i} - r(k)_2\right)^2}} \right] \quad \text{(a)}$$

$$Logy2(y) = \ln\left[\frac{\left(r(k)_2 - wf_{2,i}\right) + \sqrt{\left(wf_{2,i} - r(k)_2\right)^2 + \left(y - r(k)_1\right)^2 + \left(wi_{0,i} - r(k)_0\right)^2}}{\left(r(k)_2 - wf_{2,i}\right) + \sqrt{\left(wf_{2,i} - r(k)_2\right)^2 + \left(y - r(k)_1\right)^2 + \left(wf_{0,i} - r(k)_0\right)^2}} \cdot \frac{\left(r(k)_2 - wi_{2,i}\right) + \sqrt{\left(wi_{2,i} - r(k)_2\right)^2 + \left(y - r(k)_1\right)^2 + \left(wf_{0,i} - r(k)_0\right)^2}}{\left(r(k)_2 - wi_{2,i}\right) + \sqrt{\left(wi_{2,i} - r(k)_2\right)^2 + \left(y - r(k)_1\right)^2 + \left(wi_{0,i} - r(k)_0\right)^2}} \right] \quad \text{(b)}$$

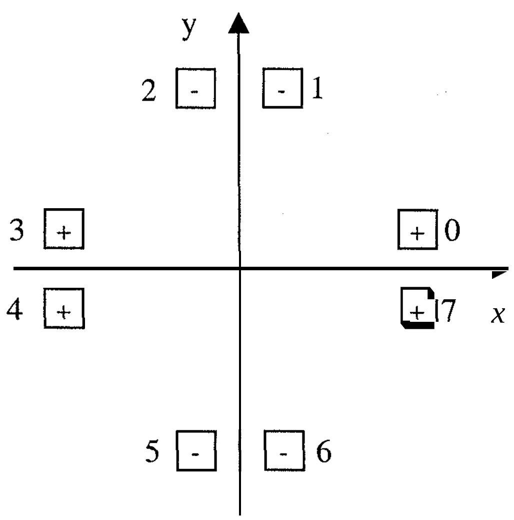

Fig. Al. *Cross-section of the quadrupole at z = 0 and conventions*

The last integration in the y variable has been performed by a mathematical programme [4] to get the contribution of the central part of the air core quadrupole to the y component of the field.

Layer 1: extreme of the quadrupole, z = *-lq/2*

Represented in Fig. A2 is the current flow in the air core quadrupole and the nomenclature of the so-called return wires, those that belong to the xy planes at $z = -lq/2$ and $z = +lq/2$.

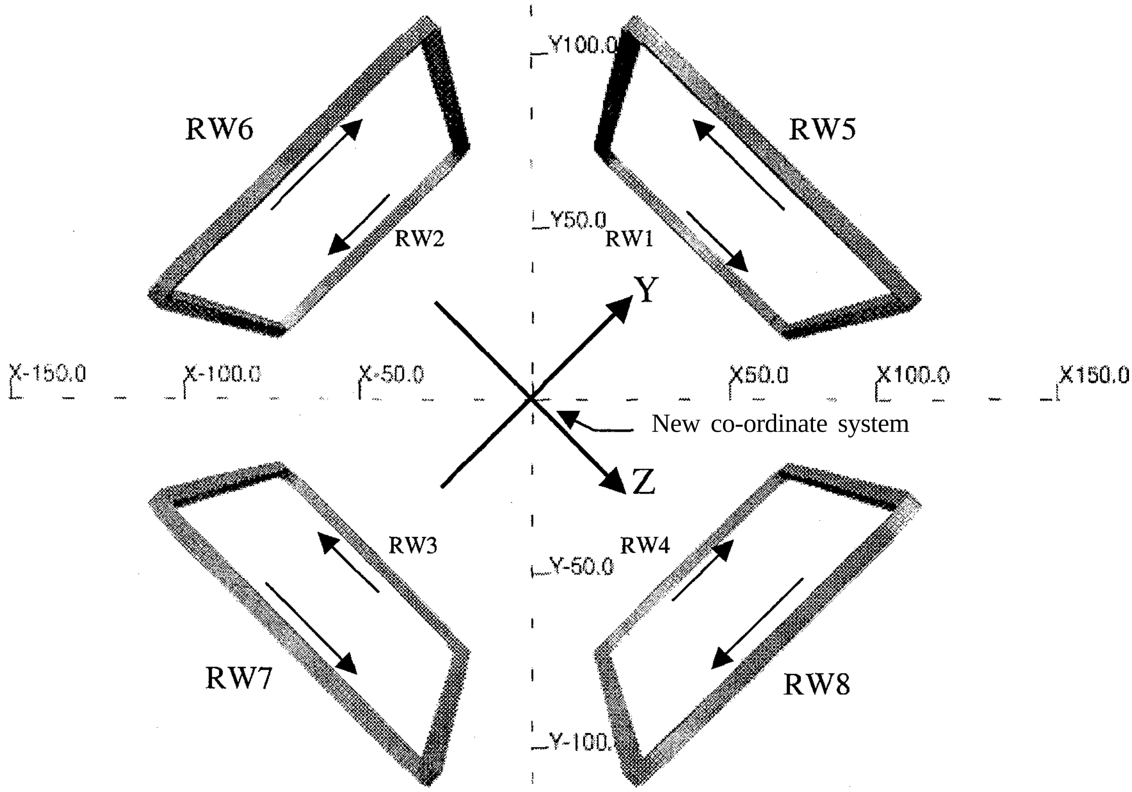

***Fig. A2.** Front view of the quadrupole with current direction in the wires*

The return wires contained in the layer at $z = -lq/2$ *are* named: RW 1, RW2, RW3 and RW4. To calculate the contribution of RW1 and RW3, let us introduce a new co-ordinate system with the origin translated at $-lq/2$ and with new axes as shown in Fig. A2 (X is pointing out of the foil). In this co-ordinates system, the co-ordinates of RWl and RW3 are:

$$RWi = \begin{pmatrix} -2 & -2 \\ Radius - 2 & -Radius - 2 \\ -a/2 & -a/2 \end{pmatrix} \qquad RWf = \begin{pmatrix} 2 & 2 \\ Radius + 2 & -Radius + 2 \\ al2 & a/2 \end{pmatrix}$$

where the first column refers to RWl, the second to RW3, RWi are initial co-ordinates of the wire and RWf the final co-ordinates to be used in the integration. *Radius* is the wire distance from the origin.

In the new system the test points have the following co-ordinates:

$$R(k) = \begin{pmatrix} 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ -1 & 0 & 0 \end{pmatrix}^{-1} \left[r(k) - \begin{pmatrix} 0 \\ 0 \\ -lq/2 \end{pmatrix} \right]$$

The unit vector for the current flowing in RW 1 and RW3 is:

$$Ru = \left(0\,0\,1 \quad -1\,0\,0 \right)$$

The axes rotation allows the adoption of the same formalism derived, by double integration, for the central part of the air core quadrupole. In this way the component of the B field produced by the two return wires is:

$$B13Ytot_k = \sum_{i=0}^{1} \frac{\mu_0 I}{4\pi s} \int_{RWi_{1,i}}^{RWf_{1,i}} \left(-Ru_{0,i} LogY1(Y) + Ru_{2,i} LogY2(Y) \right) dY$$

and expression for $LogY1(Y)$ and $LogY2(Y)$ are obtainable from (a) and (b), respectively, with *R(k)* instead of *r(k), RW* instead of *w* and Y instead of y. To go back to the original co-ordinate system a rotation is needed, to get the B field produced by the two return wires:

$$B13xyztot_k = \begin{pmatrix} 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} B13Xtot_k \\ B13Ytot_k \\ B13Ztot_k \end{pmatrix}$$

A similar approach is used in the same layer at $-lq/2$ for the return wires RW2 and RW4 and also in the symmetric layer at z = *lq/2*. Adding up the effect of all the return wires we get for the y component of the field:

$$ByRWtot_k = \left(B13xyztot_k\right)_1 + \left(B24xyztot_k\right)_1 + \left(B57xyztot_k\right)_1 + \left(B68xyztot_k\right)_1$$

where the subscript 1 represents the y component.

Total value of field and gradient

The total y component of the B field at the test point K is obtained adding the contribution of the central part to the contribution of the return wires:

$$By, = ByWtot_k + ByRWtot_k$$

and now it is possible to calculate the x component of the gradient:

$$Gradx_k = \frac{BY, - By_{k-1}}{dk}$$

where dk is the interval between two consecutive test points. Analogous expressions are valid for the other components of the B field and of the gradient.

APPENDIX B: TIME CONSTANT

Let us start calculating the inductance of the air core quadrupole. The sum of the single coil inductance's (L_i) and of the mutual inductance's between the coils (M_{ij}) give the total inductance:

$$L_{tot} = L_1 + L_2 + L_3 + L_4 + 2(M_{12} - M_{13} + M_{14} + M_{23} - M_{24} + M_{34})$$

The signs of the mutual inductance's M_{ij} are positive (negative) if the field produced by the j-th coil, at the position of the i-th coil, has the same (opposite) versus of the field generated by the current circulating in the i-th coil.

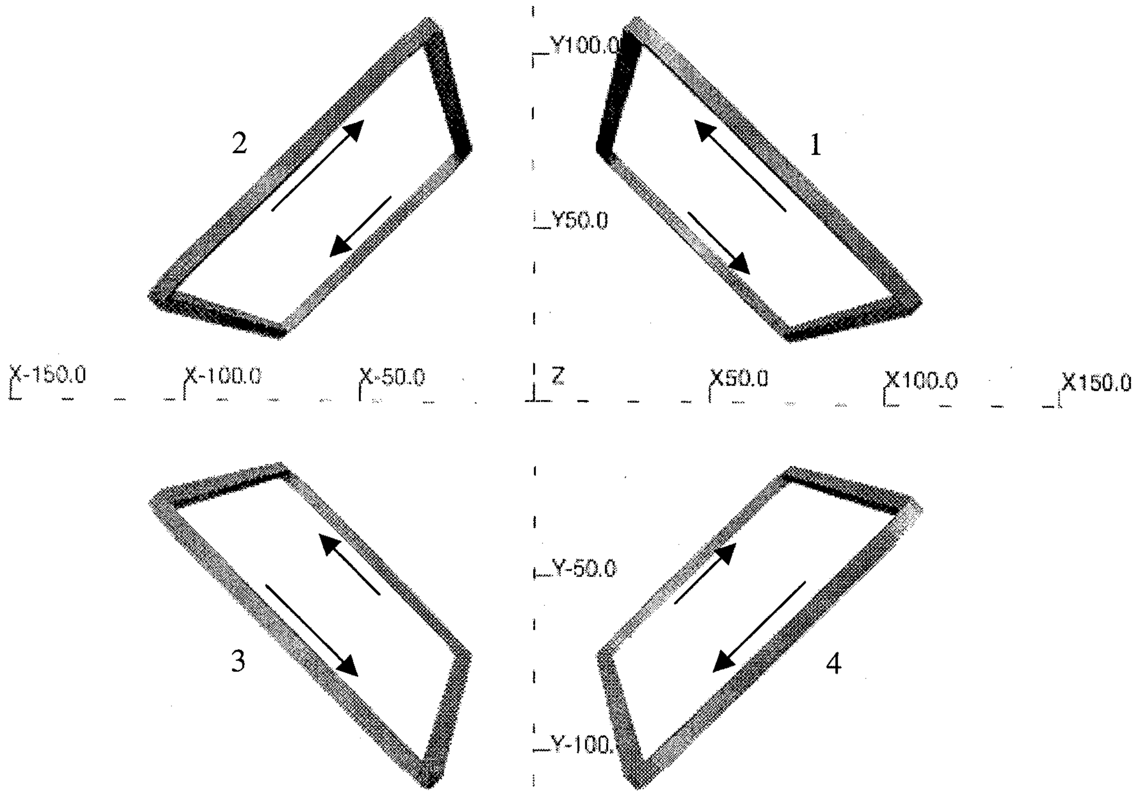

Fig. Bl. *Coil numbering and current flow conventions*

Each single coil is approximated by a rectangle and its inductance is given by [5]:

$$L_i = \frac{\mu_0}{4\pi}\left[4(a+b)\ln\left(\frac{2a2a}{d^*}\right) 4\, a\ln\left(\frac{a+d}{b}\right) - 4b\ln\left(\frac{b+d}{b}\right) - 8(a+b-d)\right]$$

where a and b *are* the sides of the rectangle, in our case a = 0.089 m and $b = lq$ = 0.4 m, d the diagonal and $d^* \cong 0.2235\ (a+b)$ the mean geometrical distance between two portions of wire (in the hypothesis of uniform current in the wire). Since the four coils are identical the following equality is valid:

$$L_1 = L_2 = L_3 = L_4$$

The mutual inductance's are given by the following law:

$$M_{ij} = \frac{\mu_0}{4\pi} \oint_i \oint_j \frac{d\vec{l}_i \cdot d\vec{l}_j}{r_e}$$

where $d\vec{l}$, directed as the current flow, represents the unit length vector of the coil and r_e is the distance between the starting points of the unit length vectors. It is clearly the symmetry of the mutual inductance expression, so that $M_{ij} = M_{ji}$. In the previous formula, due to the coils' geometry, only the parallel portions of wires give non-zero contributions to the double integral. For these contributions, the double integral can be analytically reduced to a single integration that is rapidly solved by a mathematical programme [4]. The integrals to be solved are of the type:

$$\frac{\mu_0}{4\pi} \int_0^y \ln\left[\frac{(y-x)+\sqrt{(y-x)^2+h^2}}{\sqrt{x^2+h^2}-x}\right] dx$$

where the integration limit y is either a or b (the sides of the coil) and h is the distance between the parallel portions of the i-th and j-th coils. Adding up the contributions, particular care has to be devoted to the sign: positive if the current has the same versus in the two portions, negative if the current flow has opposite direction. The total inductance of the air core quadrupole results to be: $L_{tot} = 1.09 \cdot 10^{-7}$ H.

The coils of the air core quadrupole are made out of a single wire. The total resistance is thus calculated considering a copper length equal to four times the single coil length. Introducing the copper resistivity at 20 °C (ρ_{20}) and the wire cross-section *(s)* the total resistance given by:

$$R_{tot} = \rho_{20} \frac{8(a+b)}{s}$$

is equal to: R_{tot} = 4.21 · 10" Ω.

The time constant of the air core quadrupole is given by:

$$\tau = \frac{L_{tot}}{R_{tot}} \qquad \tau = 0.03 \text{ ms}$$

and its inverse gives the "cut-off' frequency of the air core quadrupole:

$$f = \frac{1}{\tau} \qquad f = 38.48 \text{ kHz.}$$

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