European Laboratory for Particle Physics

Large Hadron Collider Project

LHC Project Report 397

Revision of the closed orbit corrector system of the LHC

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Abstract

The closed orbit corrector system of the LHC has been revisited in accordance with the project progress. The magnet measurement procedures are now well defined. This makes the error on the magnet strength and positioning better known than at the time the orbit correctors were specified.

The LHC is in a favourable context in the sense that we have a precise knowledge of the vertical movement of the LEP tunnel and that an efficient code to detect field errors is available. Under these conditions the strength of the closed orbit correctors in the arc and in the dispersion suppressors is sufficient. The demand on closed orbit correctors in the insertions can be somewhat relaxed.

The tolerances on the longitudinal positions of the dipoles and quadrupoles are easy to satisfy and do not impose further constraints on the closed orbit corrector system.

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Geneva, August 1, 2000

1 Introduction.

The closed orbit correctors are basic elements to make machine operation possible. Their strength is determined mainly by the transverse positioning tolerances of the quadrupoles and the dipole field errors in the dipoles. The longitudinal positioning is less critical. Its importance is evaluated below.

An analytical estimate of ideal corrector strengths is presented in section 2 to establish what is needed to compensate both random and systematic field errors. For the case of random errors it is shown that this agrees with the previous estimates. The calculation done with the new table of alignment errors is done for the case of the arc correctors in section 4.

The LHC insertions were modified some years ago, which led to a request of new orbit correctors according to the quadrupole changes. The relevance of this change is evaluated in sections 5 and 6. A summary of our knowledge of the ground motion in the LHC tunnel is recalled in section 7 in order to support the arguments concerning the insertion correctors. The characteristics of an existing orbit analysis code which could help a lot in the first years of the LHC operation is equally recalled in section 8. Finally it is shown in section 9 that the longitudinal positioning and tilt do not introduce further constraints on the closed orbit corrector system.

2 Closed orbit correction

2.1 Corrector strengths for the random field errors

Conceptually the closed orbit correctors are aimed at correcting locally the magnet field errors in order to minimise the closed orbit excursion. In the LHC a closed orbit corrector acting in the relevant plane will be installed close to each quadrupole. In this context, the best correction procedure is to make local orbit bumps by means of two correctors and the field error kicks over a piece of machine with a π phase advance at most (in the case of the LHC this represents two arc cells). This makes it possible to obtain directly the expectation value of the orbit corrector kick angle as a function of the error distribution parameters. This procedure cannot be applied for the practical orbit correction merely because the field errors are unknown.

It is important to point out that the beam position monitors are not considered here since they should not intervene in the determination of the strength of the orbit correctors. This implies that the perturbation of the closed orbit distortion they make can be neglected.

For a piece of machine extending over a phase advance of π, there must be in each plane two correctors with kick angles θ_1 and θ_2, installed at places where the betatron function has values β_1 and β_2 and the phases are μ_1 and μ_2. The conditions to make a local closed orbit bump by means of the two correctors and the field error kicks are obtained by specifying that the trajectory due to this ensemble of kicks of index k is zero everywhere downstream. The transverse amplitude of this trajectory is given by

$$X(s) = \Sigma_k \theta_k \sqrt{\beta(s)\beta_k} \sin[\mu(s) - \mu_k]. \tag{1}$$

The index k refers to all kicks including the correctors and s is the longitudinal coordinate. Expressing that $X(s)$ is zero for any s downstream of the set of kicks, gives two equations :

$$\theta_1 \sqrt{\beta_1} \sin \mu_1 + \theta_2 \sqrt{\beta_2} \sin \mu_2 + \Sigma_{kicks} \theta_i \sqrt{\beta_i} \sin \mu_i = 0,$$

$$\theta_1 \sqrt{\beta_1} \cos \mu_1 + \theta_2 \sqrt{\beta_2} \cos \mu_2 + \Sigma_{kicks} \theta_i \sqrt{\beta_i} \cos \mu_i = 0,$$

which can be solved to obtain the corrector strengths

$$\theta_1 = \frac{1}{\sqrt{\beta_1}\sin(\mu_1-\mu_2)}\Sigma_{kicks}\theta_i\sqrt{\beta_i}\sin(\mu_2-\mu_i),$$

$$\theta_2 = \frac{1}{\sqrt{\beta_2}\sin(\mu_2-\mu_1)}\Sigma_{kicks}\theta_i\sqrt{\beta_i}\sin(\mu_1-\mu_i).$$

The kicks associated with errors belong to a series of random distributions with zero average. The only relevant quantity we can compute is the expectation value of the corrector kick angles. As the optics functions and the kick angles are uncorrelated, we can do their quadratic sums independently and we obtain

$$<\theta_{1,2}>= \frac{1}{|\sin(\mu_1-\mu_2)|}\sqrt{\frac{\Sigma_k\beta_k\theta_k^2}{2\beta_{1,2}}}. \tag{2}$$

Here we have replaced the expectation value of the product $\sqrt{\beta_i}\sin\mu_i$ by the product of the square root of the average β-function and 1/2 which is the average of the square of the sine function. It is important to note that the sum in formula (2) has to be done over all possible kicks for each error distribution. For instance if only the quadrupoles are considered in the case of the LHC, the expression under the square root becomes $\theta_{QF}^2 + \frac{\beta_{QD}}{\beta_{QF}}\theta_{QD}^2$ for the horizontal plane. The factor 2 at the denominator has disappeared because there are two QF, two QD and two correctors in two cells which extend over a phase advance of about π. The formula expresses that the corrector at QF has to compensate not only the misalignment of its neighbouring quadrupole QF but also the QD misalignment. With the present β ratio of 32/180, the strength which would be necessary for the correction of QF alone has to be increased by 8%, which is non negligible.

It is interesting to compare the respective impacts of quadrupole misalignments and b_1 error in the dipoles on the orbit correction system. To this end we use equation (2) either for the case of quadrupoles only or for the case of dipoles only. Equating the two values, we obtain the relation $b_1(10^{-4}) = 33.5\,\Delta x(\mathrm{mm})$. An r.m.s. misalignment of the arc quadrupoles by 0.1 mm has as much impact on average on the orbit correction system as a random b_1 of a little more than 3 units in the dipoles.

We examine now the maximum orbit excursion which is a fundamental parameter for aperture calculations. In the context of the above correction procedure, the orbit excursion is zero at the beginning of the orbit bump and grows up as a random walk up to the first corrector, due to the kicks associated with the random error. The quadratic sum of the amplitudes obtained from formula (1) is

$$<x_{co}>= \sqrt{\frac{\hat{\beta}}{2}\Sigma_k\beta_k\theta_k^2}, \tag{3}$$

where $\hat{\beta}$ is the peak value of the betatron function and the sum is done up to the first corrector.

2.2 Corrector strengths for the systematic field errors

These strengths are evaluated separately as the correction can be computed exactly. The field errors considered here are only those of the dipoles.

A simple minded correction scheme consists of kicking the beam only at the focusing correctors with the same angle as that due to the error. This makes a periodic orbit like

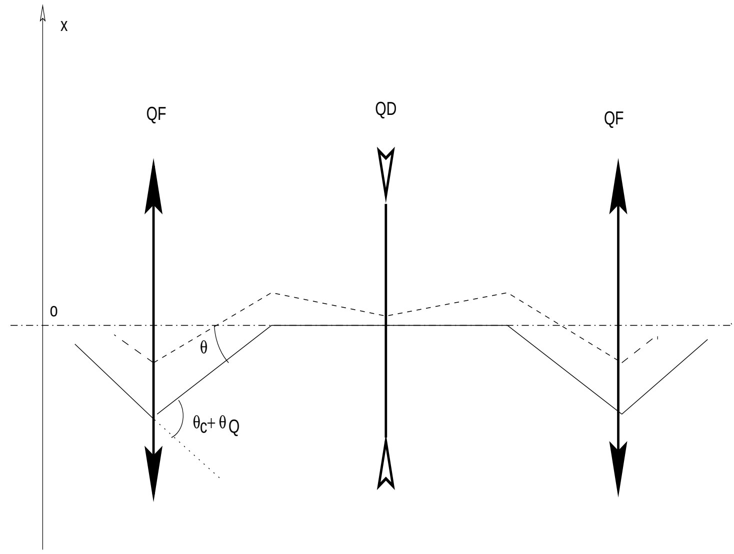

Figure 1: Trajectories with a compensated systematic field error in the dipoles. θ is the kick angle due to this field error. The kick θ due to all systematic dipole field errors occurs at the middle of the distance between QF and QD. θ_C and θ_Q are the kick angles due to the corrector and to the quadrupole, respectively.

the full line shown in figure 1 for the horizontal case. The corrector kick angle θ_c is computed as follows. The sum of this angle and the kick angle due to the quadrupole θ_Q is equal to twice the kick angle due to the field error θ. The orbit excursion in the quadrupole x_Q and the kick angle due to the quadrupole are related by : $\theta_Q = x_Q k$, k being the integrated normalised strength of the quadrupole. x_Q is also equal to $\theta L/4$ where L is the cell length. Remembering that the cell phase advance μ_c is obtained from $\sin \mu_c/2 = kL/4$, we obtain by combining these equations : $\theta_c = \theta(2 - \sin \mu_c/2)$. The maximum orbit excursion associated with this scheme is equal to $\theta L/4$. For LHC, L is equal to 107 m and the dipole angle is 5.1 mrad. Thus for a systematic relative field error of 0.001, the value of θ is 15.3 μrad (3 dipoles between two quadrupoles) and the maximum orbit excursion is 0.41 mm. As this is non negligible, a possible reduction of the excursion by a factor 2 can be achieved by making an oscillation as the dashed line on figure 1. The corrector strength is a little larger. The computation is a little more complicated than the above one. It gives : $\theta_c = \theta\frac{(4-\sin\mu_c)(2+\sin\mu_c)}{4+3\sin\mu_c}$. For a phase advance per cell of $\pi/2$, it become equal to $1.46\,\theta$ instead of $1.29\,\theta$ for the simple scheme.

2.3 Maximum corrector strengths

In order to determine the maximum corrector strength in the arcs, we accept that one corrector of the 352 can exceed the maximum strength. This means that we accept to realign one machine section in order to reduce this corrector strength. Assuming a Gaussian distribution of the corrector strengths, this distribution is cut accordingly at n_σ such that $exp(-n_\sigma^2/2) = 1/352$. This gives n_σ=3.42. Then the strength necessary for the correction of the systematic field errors has to be added as it is uniformly distributed.

For the dispersion suppressors and the insertions the number of correctors is much smaller and we will request merely that the probability of realignment is 0.01, i.e. the maximum strength is 3 times the r.m.s. Again, the strength necessary for the correction of the systematic field errors has to be added.

The closed orbit correctors can be used to scan the machine aperture. The kick angle needed to this end is equal to the vacuum chamber radius divided by the maximum β-function. For the arcs it is about $120\,\mu$rad. It would be unreasonable to design orbit correctors to fulfil this additional constraint at nominal energy. Therefore the aperture scans will only done at injection and up to about 2.8 TeV. This assumes that no deformation of the vacuum chamber, which could change the location of the aperture restrictions, due to the setting of the nominal magnetic field is expected.

3 Comparison with previous results concerning the LHC arcs

This section is aimed at a comparison with the results in [1]. In the case of the LHC we consider a local orbit correction over two arc cells, as the phase advance per cell is close to $90°$.

In the horizontal plane we have to consider only the b_1 error and the misalignment of the quadrupoles. The alignment of the dipoles introduces only second order field errors. In the vertical plane, the field errors originate mainly from the dipole roll, the a_1 error and the vertical misalignment of the quadrupoles.

It is clear that the feed-down effect of the multipoles is completely negligible. This can be illustrated by the lower order case, that of the skew quadrupole component, which corresponds to the largest perturbation. The kick due to a closed orbit distortion of amplitude x_{co} combined with a_2 in the dipoles of bending angle $\theta_D = 5.1\,\mathrm{mrad}$, is given by $\frac{a_2}{R}\theta_D x_{co}$, where $R = 0.017\,\mathrm{m}$ is the reference radius for the errors. For a closed orbit amplitude of 4 mm, the kick is $(0.12 \times a_2)\,\mu$rad. For a value of a_2 of less than 2 units, the kick is smaller than that associated with a quadrupole displacement of 0.1 mm by one order of magnitude.

We first recompute the r.m.s. corrector strength for the vertical plane with the errors specifications already taken in [1]. The r.m.s. roll of the dipoles is 1.6 mrad (a large value, apparently based on HERA measurements [1]). The a_1 uncertainty is not considered. The associated kick is $8.2\,\mu$rad, there are twelve dipoles for two correctors, their average β-function is 106 m. The r.m.s. quadrupole misalignment is 0.6 mm. The associated kick is $17.8\,\mu$rad for the nominal strength of $0.00871\,\mathrm{m}^{-2}$ at 7 TeV (gradient of 203 T/m). The value of the β-function at the QF and QD quadrupoles is 180 m and 32 m, respectively. Putting all these numbers in formula (2), we obtain an r.m.s. value of the corrector kick angle of $22.3\,\mu$rad. This is about the value obtained in [1]. The maximum corrector strength is $76.3\,\mu$rad according to the above specifications. This is close to the maximum of $80.8\,\mu$rad. We arrive then at a conclusion similar to that given in [1], i.e. that one local realignment has to be expected.

In the horizontal plane, the random b_1 in the dipoles is 0.001 and the horizontal r.m.s. misalignment of the quadrupoles is 0.5 mm [1]. The r.m.s. corrector strength obtained with these numbers is $16.4\,\mu$rad. This is again about the value obtained in [1]. The maximum corrector strength is $56.1\,\mu$rad according to the above specifications. This is far from the maximum strength but the contribution of the systematic field errors has not been considered.

So far we arrive at about the same conclusions as those of the previous study. We consider now the situation with the present estimates of the misalignments and field errors.

4 Revision of the arc correctors

The errors assumed in [1] were somewhat overestimated. An inventory of field errors has been done in 1999. For the quadrupole transverse alignment, the new r.m.s. values are 0.37 mm in both planes after one year (see presentation by W. Scandale at the LHC Parameter and Layout Committee on the 12^{th} of April 1999). A random drift of 0.1 mm which is supposed to occur over one year has been included. The associated kick is, for the nominal quadrupole excitation of $0.00956\,\mathrm{m}^{-2}$ (223 T/m at 7 TeV), 11 μrad in both planes.

For the dipoles, the random b_1 error is $7{\times}10^{-4}$ (kick of 3.57 μrad) and the b_1 uncertainty due to geometrical errors is $10{\times}10^{-4}$ (kick of 5.1 μrad). The effect of the persistent currents has not been considered, as we deal with the nominal energy where they do not have any effect.

The dipole alignment is not important for the optics except their transverse tilt. All contributions to the horizontal field errors including coil tilt (500 μrad of r.m.s. error on the measurement of the horizontal plane and 200 μrad of machine alignment error) and ramp effects add-up to an equivalent random a_1 of $5.4{\times}10^{-4}$ (kick of 2.75 μrad). The systematic error comes mainly from the uncertainty (geometrical error) and has a value of $5.8{\times}10^{-4}$ (kick of 2.96 μrad).

The r.m.s. corrector kick angles obtained by combining the random field errors in the dipoles by means of formula (2) are : 13.7 μrad in the horizontal plane and 13.0 μrad in the vertical plane. The difference between these two numbers comes from the dipoles.

The maximum corrector strengths are obtained by multiplying these numbers by 3.42 and adding the correction of the systematic part, i.e. 1.46 times the value of the systematic kick error associated with three dipoles (this makes 22.3 μrad for an uncertainty of 10 units). In the horizontal plane we obtain 69.2 μrad and in the vertical plane 57.5 μrad. The maximum corrector strength of 80.8 μrad is never exceeded.

In a real machine the alignment of the quadrupoles is done with a smooth curve as a reference. Such a realignment is not innocent in that sense that some corrector strength is needed to correct the orbit distortion due to “smooth curve alignment”. In the case of LEP, the maximum value of this corrector strength is of the order of 10 μrad [2]. With the present alignment errors there is a sufficient margin in both planes. If we forget the last item, there is a further provision for an r.m.s. drift of the alignment of 0.24 mm in the horizontal plane and 0.38 mm in the vertical plane.

The r.m.s. closed orbit excursion computed by means of equation (3) in the horizontal plane has a value of 0.84 mm (there is a contribution of one QD offset by 0.37 mm and three dipoles with a random b_1 of 7 units). As there are 200 cells in the machine, the probability that the maximum excursion is exceeded at one location corresponds to a maximum of 3.26 r.m.s., i.e. 2.73 mm. An additional excursion of 0.41 mm due to correction of the systematic error has to be added. Eventually the maximum excursion is 3.14 mm. In the aperture studies a maximum excursion of 4 mm has been assumed. This is compatible with the present estimate as our estimate is associated with an ideal correction scheme, and it is not proved that the closed orbit excursion can be reduced to this level in the real machine.

5 Closed orbit correctors in the dispersion suppressors

The quadrupoles are stronger than those of the arc, their maximum length is 6.8 m and their nominal gradient is 200 T/m. We take these parameters for all quadrupoles. There are 8 dipoles for two correctors instead of 12. The optics is about the same as in the arcs.

With the present alignment values, the r.m.s. corrector kick angle is 23.5 μrad vertically and 24.5 μrad horizontally. This is valid for both polarities of the dispersion suppressor. The maximum corrector strength is obtained by multiplying the r.m.s. value by 3 and adding the correction for the systematics. We obtain 88.4 μrad in the horizontal plane and 78.5 μrad in the vertical plane.

This is well below the maximum corrector strength of 120 μrad (3.11 T, length of 0.9 m). Using arc correctors of maximum kick angle 80.8 μrad would increase the probability of realignment from 0.01 to 0.027 in the horizontal plane and would not change the situation in the vertical plane.

6 Closed orbit correctors in the insertions

In the insertions the situation is a little complicated by the existence of the crossing angle scheme. In fact dedicated closed orbit correctors are reserved to this end so that there is little interference between this scheme and the closed orbit correction [3, 4]. What is needed in the very worst case is 14% of the corrector strengths close to Q6 and Q7, i.e. 23 μrad. These correctors are special, their length is 1.25 m and their nominal field is 3.11 T.

There are no dipoles and the correctors have to take care of the quadrupoles only. As in the dispersion suppressors, their maximum length is 6.8 m and their nominal gradient is 200 T/m.

The largest β ratio between quadrupoles is 0.44, in the physics insertions. Taking the maximum length of 6.8 m for all quadrupoles, the kick associated with the displacement of 0.37 mm is 21.6 μrad. The expectation value of the corrector kick is then 25.9 μrad for both planes. The maximum corrector strength is 77.8 μrad. This value is well below the present maximum strength of the MCBC's of 120 μrad.

The separator scheme needs 23 μrad so the maximum useful corrector strength is 97 μrad. This strength can cope with an r.m.s. displacement of 0.46 mm. Consequently, the random drift can become 0.29 mm instead of the 0.1 mm assumed above.

7 The ground motion in the LHC tunnel

Once the machine has been aligned, the ground motion of the tunnel destroys the alignment with time. For the case of the LHC the situation is well known as the vertical alignment of the LEP quadrupoles has been measured yearly from 1993. Consequently no exotic assumption about the deformation of the LHC tunnel is justified.

An exhaustive analysis of the data can be found in [5]. The main aspects of the vertical motion of the LHC tunnel can be summarised as follows according to measurements taken in 1993 and corrected from the measurements done after the installation :

- between IR8 and IR4 < 1.3 mm/year (the maximum occurs in fact at about the middle of the arc between IR2 and IR3 and never in the insertions),
- between IR4 and IR7 < 0.5 mm/year,
- between IR7 and IR8 $\simeq 3$ mm/year.

It is important to note that the two cases of large motion concerns middle of arcs and consequently do not interfere with the questions raised in section 6. In any case, it must be clear that a provision has to be made at the places where large vertical displacements occur to realign the LHC (about three centimetres in 10 years between IR7 and IR8).

Thus the situation between IR4 and IR7 represents quite well that of the LHC insertions. If only one element over 700 has a misalignment of 0.5 mm, this points to an r.m.s. misalignment of 0.14 mm. This estimate is consistent with the r.m.s. annual drift of 0.1 mm quoted at the LHC Parameter and Layout Committee. The latter is based on the LEP survey measurements done later on after 1995 [6]. The algorithm to establish the misalignment figures was kept the same from 1995 to 2000, and was different from the algorithms used between 1993 and 1995. It was shown by the author that the new algorithm used after 1995 provides results as good as the previous ones from the point of view of closed orbit correction [2].

For the horizontal plane there is much less data than for the vertical plane. One source of drift could be the change of the magnet tilt which can make an horizontal displacement because of the distance of about 1 m between the magnets and the ground. The maximum tilt observed in LEP after 4 years is 3 mrad at a single place and 2 mrad at about 10 places. Therefore we can assume a maximum tilt change of 0.5 mrad per year. This induces an horizontal displacement of 0.5 mm at 1 m, i.e. about the same as the old misalignment value.

Partial informations concerning the measured horizontal misalignment can be found in [7]. The maximum realignment in 1994 was 1.7 mm over a machine length of 8.5 km. The maximum realignment in 1995 was 1.0 mm over a machine length of 12 km (about 150 quadrupoles). The latter value points to an r.m.s. misalignment of 0.32 mm if we assume that there was only one quadrupole with a misalignment above 1 mm. Thus it seems that the position drifts in the horizontal plane are a little larger than the vertical one if we except the octant IR7/IR8.

For the worst case of the insertion correctors, we have seen that the strength margin can cope with a local displacement of three times the r.m.s. value, i.e. 0.87 mm. As the maximum measured displacement is about 0.5 mm in the vertical plane, this means that the situation is quite safe and no realignment is needed during the first year of operation provided the LHC alignment has the same quality as that of LEP. In the horizontal plane there is a probability of 0.13 of a realignment during the first year of operation if we assume a maximum horizontal drift of 1 mm.

8 Orbit analysis

An efficient code for analysing the closed orbit measurements has been written in 1993 [8] and unfortunately little used for LEP. It is based on fitting a closed orbit measurement with a betatron oscillation together with checking the relevance of the BPM's readings. A bad fit reveals a discontinuity in the orbit or a bad orbit reading.

In principle, by using a sliding window to do the fits, it is possible to distinguish a wrong reading from a kick by looking at the fits residues associated with different numbers of measurements per fit. In practice this is not easy.

Once an orbit discontinuity is localised from the inspection of the fit residues, the associated kick is computed by means of two fits, one upstream and one downstream, and by seeking the machine element which makes the best matching between the two fits. The automatic system makes it possible to check the measurements so that the orbit discontinuities can be identified with confidence. Once kicks have been computed, their relevance is checked and their contribution to the closed orbit distortion is subtracted. When the whole machine has been inspected, the measurements, corrected from the contribution of the kicks, are all tested to check that good BPM readings have not been discarded. Then the kicks are tested again in order to avoid to compute a kick at a place where a wrong reading had not been detected. The complete procedure takes some seconds in the LEP control system. An interesting outcome is the evaluation of BPM offsets.

The code has been reactivated in 1999 by J. Wenninger. It made it possible to rule out field errors suspected in the octant 3 of LEP. This code, which is written in C, will be conserved with the rest of the LEP control system. It is important to use it from the beginning of the LHC running, when the machine is not too badly misaligned. We have noticed that it becomes almost useless if the misalignments are too important since there is not clear reference curve for the alignment in this case. This is what happened in 1992 when LEP was badly misaligned. Using this code from the beginning of operation can help to overcome the impossibility of mechanical measurements along the beam trajectory once the LHC machine is mounted.

9 Tolerance on longitudinal alignment

The longitudinal alignment tolerances are seldomly addressed as they are easily fulfilled. In this section the longitudinal positioning and the longitudinal tilt are examined.

9.1 Longitudinal positioning of the dipoles

The longitudinal tilt of a dipole does not have any effect up to second order as it reduces its field and increases its length by the same factor if the end effect are neglected.

If a rectangular dipole is displaced longitudinally by a positive quantity Δs small compared with its length, a positive horizontal kick of value $-\frac{\Delta s}{\rho}$ appears at its entrance where the horizontal β-function has the value β_1 and the horizontal phase the value μ_1, and a negative kick of value $\frac{\Delta s}{\rho}$ appears at its exit where the horizontal β-function has the value β_2 and the horizontal phase the value μ_2. The combination of these two kicks produces a closed orbit distortion x given, downstream of the dipole, by :

$$x(s) = \frac{\Delta s\sqrt{\beta(s)}}{2\rho\sin\pi Q}\left[\sqrt{\beta_1}\cos(\pi Q + \mu(s) - \mu_1) - \sqrt{\beta_2}\cos(\pi Q + \mu(s) - \mu_2)\right].$$

After expanding and rearranging the cosine terms, the amplitude of the closed orbit distortion can be written :

$$\hat{x}(s) = \frac{\Delta s\sqrt{\beta(s)}}{2\rho\sin\pi Q}\left[\beta_1 + \beta_2 - 2\sqrt{\beta_1\beta_2}\cos(\mu_1 - \mu_2)\right]. \qquad (4)$$

This formula can be expressed by means of the β-functions only. Indeed in a gradient free region (which means that we neglect the focusing effect of the dipole), the trajectory of equation $x(s) = s$ can be written :

$$x(s) = s = \sqrt{\beta(s)\beta_1}\sin[\mu(s) - \mu_1].$$

The index 1 is associated with the optics functions at $s = 0$. For our case, if $s = L$, where L is the length of the dipole, we obtain :

$$L = \sqrt{\beta_2\beta_1}\sin(\mu_2 - \mu_1). \qquad (5)$$

The cosine term in equation (4) can be re-expressed by means of this expression. Eventually equation (4) can be rewritten using the average $2\bar{\beta} = \beta_1 + \beta_2$ and the difference $\Delta\beta = \beta_1 - \beta_2$. Assuming that both L and $\Delta\beta$ are small compared with $\bar{\beta}$, the amplitude of the closed orbit distortion is approximately :

$$\hat{x} = \frac{\Delta s\sqrt{\beta(s)}}{2\rho\sin\pi Q}\sqrt{\frac{L^2 + \Delta\beta^2}{\bar{\beta}}}. \qquad (6)$$

It is quite helpful to compare the effect of this error with the effect of a random b_1 error. The kick associated with b_1 is $b_1 L/\rho$. Comparing the amplitude of the closed orbit distortion associated with this kick with equation (6), we can define an equivalent $b_1(\Delta s)$ error :

$$b_1(\Delta s) = \frac{\Delta s \sqrt{L^2 + \Delta\beta^2}}{L\bar{\beta}}.$$

For a dipole of length 14.3 m in the middle of the half cell, $\bar{\beta} = 80$ m and $\Delta\beta = 40$ m. The equivalent b_1 in units of 10^{-4} is given by

$$b_1(\Delta s) = 0.37\,\Delta s(\text{mm}).$$

If we consider a random b_1 equal to 5 units, an r.m.s. error of longitudinal position of the dipoles of 6 mm is equivalent to increasing this random b_1 by 10%.

9.2 Longitudinal positioning of the quadrupoles

The most important effect due to random gradient errors is the β-beating. Using the same notations as for the dipole, this beating is given by

$$\frac{K\Delta s}{\sin 2\pi Q}\left[-\beta_1 \cos\phi + \beta_2 \cos(\phi + \mu_Q)\right].$$

The phase advance across the quadrupole can be obtained from formula (5) by replacing L by $\frac{\sin\sqrt{K}L}{\sqrt{K}}$ for a focusing quadrupole. This expression is approximately equal to L when KL^2 is smaller than 1. For a defocusing quadrupole, the sine function has to be replaced with the hyperbolic sine.

Expanding the cosines, we can compute the maximum amplitude amplitude of the beating. Furthermore, assuming that $\frac{\sin\sqrt{K}L}{K\beta_1\beta_2}$ is smaller than 1, we obtain :

$$\frac{K\Delta s}{\sin 2\pi Q}\sqrt{\frac{\sin^2\sqrt{K}L}{K} + \Delta\beta^2}.$$

Comparing this expression with that due to a single gradient error, we can define an equivalent relative gradient error :

$$b_2(\Delta s) = \frac{\Delta s}{L\bar{\beta}}\sqrt{\frac{\sin^2\sqrt{K}L}{K} + \Delta\beta^2}.$$

For an arc quadrupole, $\Delta\beta = 0$ and the equivalent gradient error is given merely by $\Delta s/\bar{\beta}$. For a β value of 180 m, we see that one random b_2 unit is obtained for a random longitudinal misalignment of 18 mm. This explains why the longitudinal misalignment is rarely considered for ensembles of FODO cells. The situation is different for low-β insertions where β_1 can be very different from β_2. For instance for the quadrupole Q3 of the low-β inner triplet, $\Delta\beta, \bar{\beta}$ and L are respectively equal to 1200 m, 4200 m and 6.3 m. One random b_2 unit is now obtained for a random longitudinal misalignment of 2 mm.

9.3 Longitudinal tilt of the quadrupoles

For this case we integrate the expression of the closed orbit distortion across the quadrupole. Taking the origin of the longitudinal coordinate at the quadrupole centre, the elementary kick angle due to the field error associated with a longitudinal tilt of angle θ is

$$K\theta s ds,$$

s being the longitudinal coordinate. The closed orbit distortion due to misaligned quadrupole is obtained by integrating the effect of this kick :

$$x(s) = \frac{\sqrt{\beta(s)}}{2\sin\pi Q}\int_{-\frac{L}{2}}^{+\frac{L}{2}} K\theta t dt\sqrt{\beta(t)}\cos[\phi+\mu(t)].$$

This integral can be computed by expanding the cosine and integrating each term by parts, using the same substitution as in the preceding section, plus another one for the $\sqrt{\beta(t)}\cos[\mu(t)]$. After about one page of algebra, we obtain the amplitude of the closed orbit distortion for a focusing quadrupole :

$$x(s) = \frac{\theta\sqrt{\beta(s)\gamma_Q}}{2\sin\pi Q}\left[\frac{2}{\sqrt{K}}\sin(\sqrt{K}L/2) - L\cos(\sqrt{K}L/2)\right],$$

where γ_Q is the TWISS parameter at the quadrupole centre. After expansion of the trigonometric functions, we obtain

$$x(s) = \frac{KL^3\theta}{12\sin\pi Q}\sqrt{\beta(s)\gamma_Q}.$$

By comparing this expression with that associated with a transverse displacement Δx, we obtain the relation

$$\theta(\Delta x) = \frac{6\Delta x\beta_Q}{L^2\sqrt{1+\alpha_Q^2}},$$

where α_Q and β_Q are the TWISS parameters at the quadrupole centre. For an arc quadrupole of length 3.1 m, where β_Q is equal to 180 m and α_Q almost zero, a longitudinal tilt of 11 mrad makes the same distortion as a transverse displacement of 0.1 mm. Practically, the longitudinal tilt of a quadrupole can be adjusted within better than 1 mrad, so its effect is completely negligible compared with that of the transverse alignment. This is also true for the low-β quadrupoles as long as α_Q is smaller than 1. It is clear that the mechanical tolerances associated with a longitudinal tilt are more important than the optics tolerances.

10 Conclusion

The specification of closed orbit correctors is intimately linked with the realignment probability compatible with the machine operation.

The present alignment tolerances are much better than previously assumed, with the obvious consequence that the closed orbit corrector strengths are reduced. Furthermore the situation of the LHC is favourable as :

- the behaviour of its tunnel is well known and documented,
- there is a powerful code for analysing the orbits available from the beginning of the machine operation.

It is worth recalling that a random field error of r.m.s. value one per mil in the dipoles is statistically equivalent to a random transverse misalignment of r.m.s. 0.24 mm of the arc quadrupoles or 0.13 mm of the large quadrupoles of the dispersion suppressor.

It appears that the closed orbit correctors of length 1.25 m close to the quadrupoles Q6 and Q7 could be replaced with correctors of length 0.9 m within the present alignment budget.

As in the other A.G. machines, the longitudinal alignment of the magnets is not critical from the optics point of view. It is likely that those tolerances will be determined by the mechanical assembly constraints.

11 Acknowledgements

I thank my colleagues of the AP group who contributed to this report by constructive criticisms, namely O. Brüning, J.B. Jeanneret, W. Herr and F. Ruggiero.

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