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# Testing the Randall-Sundrum Model at a High Energy $e^-e^-$ Collider

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## ABSTRACT

We study the process  $e^-e^- \rightarrow e^-e^-$  at a high energy  $e^-e^-$  collider including the effect of graviton exchanges in the warped gravity model of Randall and Sundrum. Discovery limits for gravitons are established and the effects of polarization are discussed.

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A great deal of recent interest centres around the physics possibilities of a high energy linear collider with  $e^\pm$  beams[1]. Such a machine can be run in  $e^+e^-$  or  $e^-e^-$  collision modes. The principal scattering processes at these are, respectively, Bhabha scattering  $e^+e^- \rightarrow e^+e^-$  and Møller scattering  $e^-e^- \rightarrow e^-e^-$ .

One of the useful features of Møller scattering  $e^-e^- \rightarrow e^-e^-$  at a high energy  $e^-e^-$  collider is that it can receive only a limited number of contributions from physics options[2] which go beyond the Standard Model (SM). Among the interesting beyond-Standard-Model (BSM) options are the exchange of multiple gravitons in models with low-scale quantum gravity. The exchange of multiple gravitons, in the  $t$ -channel as well as the  $u$ -channel, can affect the process  $e^-e^- \rightarrow e^-e^-$  in two ways:

- by changing (increasing or decreasing) the total cross-section from the SM value — this being the usual effect of BSM physics;
- by changing the kinematic distributions of the final state electrons — this being the effect of exchanging particles with higher spin.

Two different scenarios of low-scale quantum gravity have attracted a great deal of recent attention. In one of these, due to Arkani-Hamed, Dimopoulos and Dvali (ADD)[3], one envisages a spacetime with  $4+d$  dimensions, where the extra  $d$  dimensions are compactified with radii  $R_c$  as large as a millimetre. In the ADD scenario, in four dimensions there is a tower of massive Kaluza-Klein modes of the graviton, whose masses are so densely-spaced (by as little as  $10^{-13}$  GeV) as to form a quasi-continuum. Though each graviton mode couples to electrons with the feeble strength of Newtonian gravity, the collective effect of all the gravitons contributes to interactions of almost electroweak strength[4]. Effects of multiple exchange of gravitons in Møller scattering, within the ADD scenario, have been studied in Ref.[5].

The other popular scenario of low-scale quantum gravity is that due to Randall and Sundrum[6], who write a non-factorizable spacetime metric

$$ds^2 = e^{-\mathcal{K}R_c\phi} \eta_{\mu\nu} dx^\mu dx^\nu + R_c^2 d\phi^2 \quad (1)$$

involving one extra dimension  $\phi$  compactified with a radius  $R_c$ , which is assumed to be marginally greater than the Planck length  $10^{-33}$  cm, and an extra mass scale  $\mathcal{K}$ , which is related to the Planck scale  $M_P^{(5)}$  in the five-dimensional bulk by  $\mathcal{K} [M_P^{(4)}]^2 \simeq [M_P^{(5)}]^3$ . Such a ‘warped’ geometry is motivated by compactifying the extra dimension on a  $\mathbf{S}^1/\mathbf{Z}_2$  orbifold, with two  $D$ -branes at the orbifold fixed points, *viz.*, one at  $\phi = 0$  (‘Planck brane’ or ‘invisible brane’), and one at  $\phi = \pi$  (‘TeV brane’ or ‘visible brane’). The interesting physical consequence of this geometry is that any mass scale on the Planck brane gets scaled by the ‘warp factor’  $e^{-\pi\mathcal{K}R_c}$  on the TeV brane. It now requires  $\mathcal{K}R_c \simeq 12$  — which is hardly unnatural — to obtain the hierarchy between the Planck scale and the electroweak scale. There still remains a minor problem: that of stabilizing the radius  $R_c$  (which is marginally smaller than the Planck scale)

against quantum fluctuations. A simple extension of the RS construction involving an extra bulk scalar field has been proposed [7] to stabilize  $R_c$  and this predicts light radion excitations with possible collider signatures [8]. Alternatively, supersymmetry on the branes can also act as a stabilizing effect[9]. Models with SM gauge bosons and fermions in the bulk have also been considered[10], but will not be discussed in this work.

The mass spectrum and couplings of the graviton in the RS model have been worked out, in Refs. [11, 12]. We do not describe the details of this calculation, but refer the reader to the original literature. It suffices here to note the following points.

1. There is a tower of massive Kaluza-Klein modes of the graviton, with masses

$$M_n = x_n \mathcal{K} e^{-\pi \mathcal{K} R_c} \equiv x_n m_0 \quad (2)$$

where  $m_0 = \mathcal{K} e^{-\pi \mathcal{K} R_c}$  sets the scale of graviton masses and is essentially a free parameter of the theory. The  $x_n$  are the zeros of the Bessel function  $J_1(x)$  of order unity.

2. The massless Kaluza-Klein mode couples to matter with gravitational strength; consequently its effects can be ignored for all practical purposes.
3. Couplings of the massive Kaluza-Klein modes are gravitational, scaled by the warp factor  $e^{\pi \mathcal{K} R_c}$  and are consequently of electroweak strength.

Feynman rules (to the lowest order) for these modes have been worked out in Refs. [13] and [14] in the context of ADD-like scenarios. Each graviton couples to matter with strength  $\kappa = \sqrt{16\pi G_N}$ . All that we need to do to get the corresponding Feynman rules in the RS model is to multiply the coupling constant  $\kappa$  by the warp factor  $e^{\pi \mathcal{K} R_c}$  wherever necessary. It is convenient to write

$$\kappa e^{\pi \mathcal{K} R_c} = \sqrt{32\pi} \frac{c_0}{m_0} \quad (3)$$

where  $\kappa = \sqrt{16\pi G_N}$ , using Eqn. (2) and introducing another undetermined parameter  $c_0 \equiv \mathcal{K}/M_P^{(4)}$ . Thus  $(c_0, m_0)$  may conveniently be taken as the free parameters of the theory<sup>2</sup>. Though  $c_0$  and  $m_0$  are not precisely known, one can make estimates of their magnitude. The RS construction requires  $\mathcal{K}$  to be at least an order of magnitude less than  $M_P^{(4)}$ , which means that the range of interest for  $c_0$  is about 0.01 to 0.1 (the lower value being determined by naturalness considerations).  $m_0$ , which is of electroweak scale, may be considered in the range of a few tens of GeV to a few TeV. Eq. (2) tells us that the first massive graviton lies at  $M_1 = x_1 m_0 \simeq 3.83 m_0$ . Since no graviton

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<sup>2</sup>Though we differ from the exact choice of parameters in Ref. [12], a translation is easily made using the formulae  $c_0 = \frac{1}{8\pi} (\mathcal{K}/\overline{M}_P)$  and  $m_0 = \Lambda_\pi (\mathcal{K}/\overline{M}_P)$ . It follows that  $c_0$  is roughly an order of magnitude less than  $\mathcal{K}/\overline{M}_P$  and  $m_0$  can be one or two orders of magnitude smaller than  $\Lambda_\pi$ .

resonances have been seen at LEP-2, running at energies around 200 GeV, it is clear that we should expect  $m_0 > 50$  GeV at least.

In this letter, we examine the effects of multiple graviton exchange in Møller scattering in the RS scenario. We focus on the possibility of observing an excess in  $e^-e^-$  events over the SM prediction, and comment on possible refinements using the kinematic distributions of the final-state electrons. As earlier calculations[12] have shown, in the case when  $c_0$  is large, the resonance structure in Bhabha scattering is lost and there is not much difference, qualitatively speaking, between Bhabha and Møller scattering in the RS model. In other words, Møller scattering is as good a probe of this model as Bhabha scattering in this case. It is on this option that our interest is focussed.

The calculation of the Feynman amplitude involves, for the diagrams with graviton exchange, a sum over graviton propagators of the form

$$\sum_n \frac{1}{t - M_n^2} \equiv - \frac{1}{m_0^2} \Lambda \left( \frac{\sqrt{-t}}{m_0} \right) \quad (4)$$

and a similar sum with  $t \leftrightarrow u$ . Using the properties of the zeros of Bessel functions, the function  $\Lambda(x_t)$  can be written, to a very good approximation, as[15]

$$\Lambda(x_t) = \frac{1}{\pi x_t} \text{Im} \psi \left( 1.2331 + i \frac{x_t}{\pi} \right) + \frac{0.32586}{220.345 + 29.6898 x_t^2 + x_t^4} \quad (5)$$

where  $\psi(z)$  is the well-known digamma function. The variation of  $\Lambda(x_t)$  with  $x_t$  is illustrated in Figure 1. It is immediately obvious that the effective coupling of the gravitons varies according to the scattering angle, except in the case when  $\sqrt{-t} \ll m_0$ , i.e.  $x_t \rightarrow 0$ . This is a feature quite different from that observed in the related ADD model, where it is possible to take a limit in which a similarly-defined  $\lambda(x_t)$  is either constant or a slowly-varying function. This is also a feature which can potentially change the angular distribution of the final-state electrons.

There are six Feynman diagrams corresponding to Møller scattering

$$e^-(p_1, \lambda_1) + e^-(p_2, \lambda_2) \longrightarrow e^-(p_3, \lambda_3) + e^-(p_4, \lambda_4) \quad (6)$$

including the Standard Model as well as graviton-exchange diagrams. Evaluation of these, using the Feynman rules for the RS model, and summing over the final-state helicities  $\lambda_3, \lambda_4$ , is straightforward and leads to a squared matrix element  $|\mathcal{M}(\lambda_1, \lambda_2)|^2$ , whose explicit form is not given here in the interests of brevity. If the initial-state electrons have a left-handed longitudinal polarization  $P$ , the differential cross-section is given by

$$\begin{aligned} \frac{d\sigma}{dt} = \frac{1}{64\pi s^2} & \left[ (1 - P)^2 |\mathcal{M}(+, +)|^2 + (1 + P)^2 |\mathcal{M}(-, -)|^2 \right. \\ & \left. + (1 - P^2) \{ |\mathcal{M}(+, -)|^2 + |\mathcal{M}(-, +)|^2 \} \right] \quad (7) \end{aligned}$$

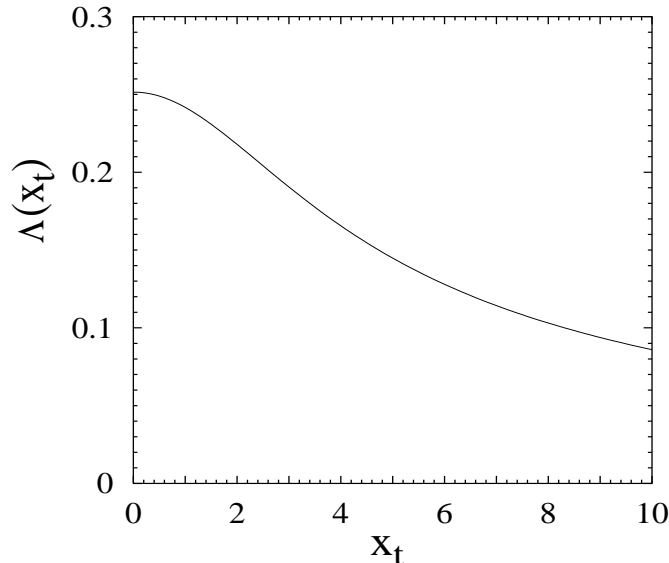


Figure 1: *Illustrating the variation of the effective coupling  $\Lambda$  of RS graviton towers to electrons as a function of  $x_t = \sqrt{-t}/m_0$ .*

assuming that both the beams are identically polarized. The importance of the polarization factor  $P$  is considerable, since it can be used, among other things, to enhance or decrease the SM contribution to the cross-section. In fact, polarization studies form an important part of the physics program at a linear collider[16].

In order to make a numerical estimate of the cross-section, we have incorporated the calculated cross-section into a Monte Carlo event generator, by means of which we calculate the cross-section for  $e^-e^- \rightarrow e^-e^-$  subject to the following kinematic cuts.

- The scattering angle of the electron(s) should not lie within  $10^0$  of the beam pipe.
- The transverse momentum of the electron(s) should not be less than 10 GeV.

These ‘acceptance’ cuts are more-or-less basic ones for any process at a high-energy collider with electron and/or positrons. Though further selection cuts will become appropriate when a more detailed analysis is done, it suffices for our analysis, which is no more than a preliminary study, to take the above cuts. We then calculate the cross-section in the SM and in the RS Model (including interference effects) for a fixed polarization  $P$  and given input parameters  $c_0$  and  $m_0$  of the RS Model. Our results are given in Fig. 2.

In Fig. 2(a), we present the total cross-section for the unpolarized case  $P = 0$  as a function of machine energy for three different values of the RS mass scale  $m_0 = 150, 250$  and  $500$  GeV. The dashed line represents the SM prediction and this exhibits the expected falling-off with machine energy. For large values of the graviton mass

$m_0$ , this behaviour is preserved, since the graviton contribution is very small anyway. However, when the graviton mass is smaller, the cross-sections show a marked increase with energy, which reflects the well-known behaviour of gravity. Obviously, at energies of 3–4 TeV, the gravitational contribution is huge if the mass scale  $m_0$  is small; however, a discernible difference exists even when  $m_0 = 500$  GeV. Thus, we can expect larger effects — or, conversely, stronger bounds — on the RS Model as the machine energy increases.

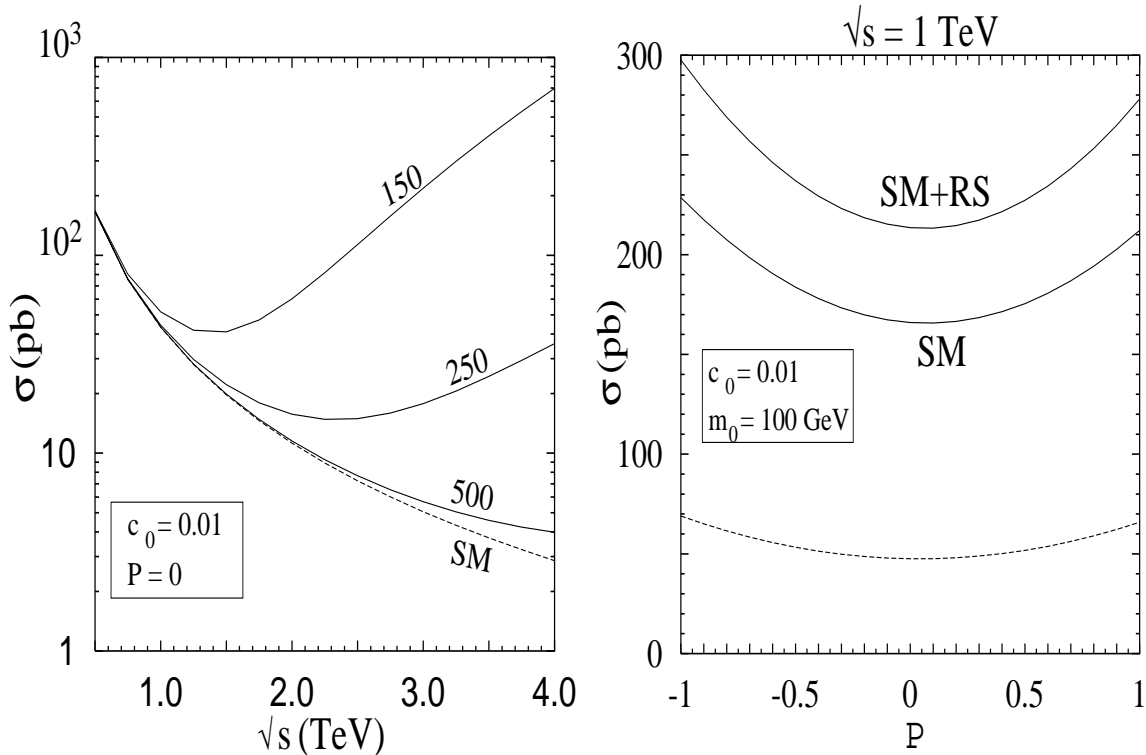


Figure 2: Variation of the cross-section for Moller scattering with (a) machine energy and (b) polarization of the electron beams. In (a) the solid curves correspond to the RS Model predictions for  $m_0 = 150, 250$  and  $500$  GeV, while the dashed line represents the SM contribution. In (b), the solid lines represent the SM and RS model contributions, while the dashed line represents their difference. Other parameters are marked (in the boxes).

In Fig. 2(b), we present the variation of the cross-section with the polarization  $P$ , at a 1 TeV machine, for the parameter set marked in the inset box. The solid curves correspond to the SM and the RS model predictions, for a fixed set of parameters  $(c_0, m_0)$ , while the dashed line represents the difference between the two. It is obvious that there is a modest advantage to be gained from polarizing the beams, and there is little difference between the cases when the beam is dominantly left- or right-handed. This is also expected, since graviton exchanges are non-chiral; in fact, the small difference arises from the interference between diagrams with graviton and  $Z$ -exchange.

In order to estimate the discovery reach of a linear collider, we adopt the following

strategy. Discovery limits will be reached if the total experimental cross-section agrees — within the experimental precision — with the SM. Any excess or deficit must be attributed to BSM physics. Thus, for a given energy  $\sqrt{s}$ , a given polarization  $P$  and a fixed set of parameters  $(c_0, m_0)$ , we calculate the total cross-section in the RS model. A corresponding calculation of the SM cross-section, multiplied by the luminosity, would lead to a predicted number of events. We then estimate the errors assuming that the statistical errors are Gaussian and that there are no systematic errors. While this certainly makes our estimates of the discovery limits over-optimistic, we can argue that electron detection efficiencies are generally high enough to allow us to make a reasonable estimate in this approximation. In any case, before more detailed studies of the detector design and systematic effects are undertaken, any estimate of systematic errors must be pure guesswork. We choose, therefore, to neglect such effects. Finally the search reach of the collider is given in terms of  $3\sigma$  discovery limits.

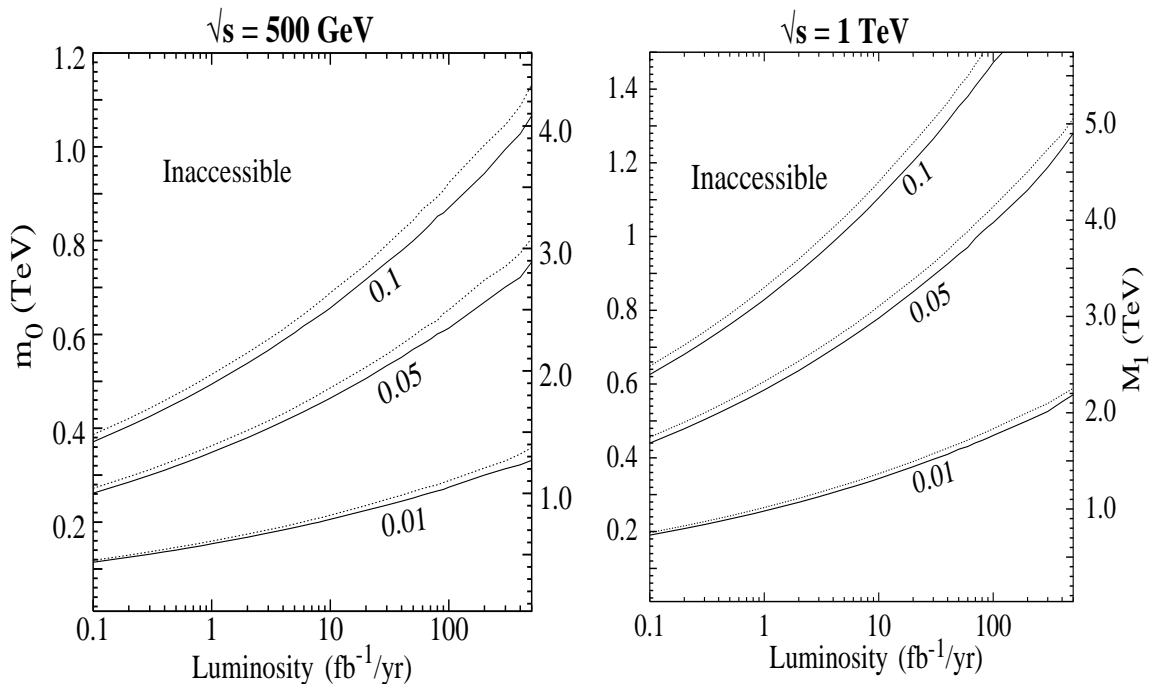


Figure 3: *Discovery limits as a function of the integrated luminosity for Randall-Sundrum graviton modes at an  $e^-e^-$  collider at centre-of-mass energies of 500 GeV and 1 TeV. Solid (dotted) lines correspond to unpolarized (80% left-polarized) electron beams. The value of  $c_0$  is written alongside the relevant curve. The ordinate is labelled as a function of the mass scale  $m_0$  on the left and the mass of the lightest resonance  $M_1$  on the right.*

Fig. 3 shows the search reach for the RS model at linear colliders running at 500 GeV and 1 TeV respectively, as a function of the integrated luminosity, for three different values of the coupling constant  $c_0$  (marked along the curves). It may be seen that a linear collider could easily probe  $m_0$  up to at least 300 GeV — which corresponds to a lightest graviton mass of around 1.3 TeV — if  $500 \text{ pb}^{-1}$  of data are

collected. A slight improvement is possible with polarized beams, as the dotted lines show. If the energy of the collider be increased to 1 TeV, the reach goes up almost by a factor of 2. It may, then, be possible to discover or exclude graviton resonances of mass 2.2 TeV or more.

While a 500 GeV or a 1 TeV collider will almost certainly be built, there has been much interest in having a collider which probes the high energy frontier[17]. In particular, it is possible that the CLIC machine at CERN will be able to achieve a centre-of-mass energy as high as 3 TeV. Moreover, the possibility of a muon collider operating at a centre-of-mass energy of 3–4 TeV has also received serious consideration. For these machines, luminosities as high as  $10^3 \text{ fb}^{-1}$  per year have also been considered. Gravitational effects in  $e^-e^- \rightarrow e^-e^-$  are, of course, identical to those in  $\mu^-\mu^- \rightarrow \mu^-\mu^-$ . In view of these possibilities, we have explored the discovery reach of a 3 TeV machine for the RS model. Our results are exhibited in Fig. 4. It may be seen that this can easily probe  $m_0$  as high as 1 TeV, which corresponds to gravitons of mass nearly 4 TeV or more.

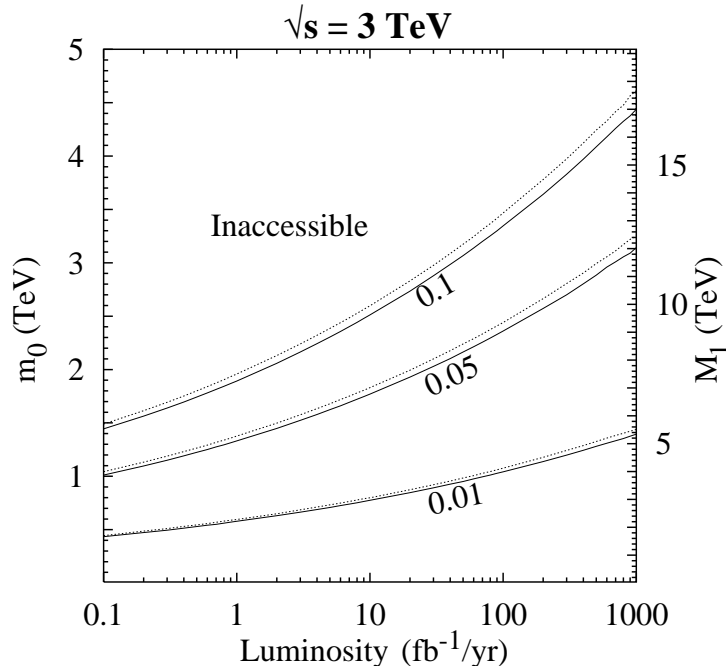


Figure 4: *Discovery limits as a function of the integrated luminosity for Randall-Sundrum graviton modes at an  $e^-e^-$  collider at a centre-of-mass energy of 3 TeV. The labelling is the same as in Fig. 2.*

It is worth noting that if graviton masses are pushed up to 5 TeV or more, then, given that the scale  $\mathcal{K}$  must be roughly an order of magnitude smaller than  $M_P^{(4)}$ , it follows that the warp factor  $e^{-\pi\mathcal{K}R_c}$  must be somewhat larger than is possible now. This would either push up the Higgs boson mass to unacceptable values, or require some mechanism to have a smaller mass scale origin for the Higgs boson mass on the Planck brane. This would be a somewhat uncomfortable situation for the RS model,



since the original simplicity — and therefore elegance — will be lost.

Finally we comment on the possibility of observing/constraining graviton effects using the angular distribution of the final state electrons. Since this form of BSM physics involves exchange of spin-2 particles, rather than spin-1 particles, as in the SM, one can, in principle, expect a rather different angular distribution for the electrons in the final state. In order to test this prediction, we have made a  $\chi^2$ -analysis of the electron angular distribution in the cases when there is graviton exchange and when there is no graviton exchange. It turns out that the difference in the distributions is rather small and confined to the central region. We find that one cannot get better discovery limits by considering the angular distributions than those which can be obtained by simply considering the total cross-section. If indeed an excess or deficit over the SM prediction is found, angular distributions might then become useful in determining the type of BSM physics responsible, *e.g.* in distinguishing between spin-1 and spin-0 exchanges. However, this would require high statistics and fine resolutions. Accordingly, in this preliminary study, we do not pursue the question of angular distributions any further.

In conclusion, therefore, an  $e^-e^-$  collider would be a useful laboratory to look for graviton exchange mechanisms, since there are very few competing BSM processes. We find that a simple study of the total cross-section for  $e^-e^- \rightarrow e^-e^-$ , subject to some minimal acceptance cuts, leads to a prediction of rather optimistic discovery limits. It is more useful to consider the total cross-section than the angular distribution, which is rather similar to that in the SM. Polarization of the beams can improve the search reach by a few percent, irrespective of whether the beams are left- or right-polarized. At a high energy collider, running at 3 or 4 TeV, the search limits can be taken as far as graviton masses of 5 TeV or more, which is more-or-less the frontier as far as the simplest version of the RS Model is concerned.

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