

Electroweak Sudakov at two loop level

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Abstract

We investigate the Sudakov double logarithmic corrections to the form factor of fermion in the $SU(2)\otimes U(1)$ electroweak theory. We adopt the familiar Feynman gauge and present explicit calculations at the two loop level. We show that the leading logarithmic corrections coming from the infrared singularities are consistent with the “postulated” exponentiated electroweak Sudakov form factor. The similarities and differences in the “soft” physics between the electroweak theory and the unbroken non-abelian gauge theory (QCD) will be clarified.

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1 Introduction

Recently the high energy behavior of the Standard Model (SM) electroweak theory receives much attention from both theoretical and phenomenological viewpoints. At future high energy colliders, the total energy is much bigger than the masses of the SM gauge bosons and large double logarithmic (DL) corrections originating from the infrared behavior of the gauge theory [1] can not be neglected for the exclusive [2] and also for the inclusive [3] processes.

The infrared behavior of the gauge theory has been one of the main subjects of the particle physics and many investigations have been made mainly for the QED and the unbroken non-abelian gauge theory (QCD). For the form factor of fermion in QED and QCD, it is known that the leading singularities can be exponentiated resulting in the Sudakov form factor [4]. In the case of the SM electroweak theory, the situation is much more complicated than QCD in two aspects. The first is that the symmetry is spontaneously broken. Secondly, the pattern of the symmetry breaking is not diagonal, namely a particular combination of the direct product of the gauge group $SU(2)\otimes U(1)$ is survived as an unbroken gauge group $U(1)_{\text{em}}$. These mean that we must carefully examine the non-abelian structure and the mixing effect which leads to “mass gap” between the gauge bosons in the electroweak theory. Therefore it is never trivial that the infrared singularities which appear in the form factor can be exponentiated as in QED and QCD. Several authors have addressed this problem [5, 6, 7, 8, 9, 10]. The authors in Ref.[5] used the formalism which has been developed in QCD [11]. The infrared evolution equation [12] has been adopted in Refs.[8, 9]. The explicit two loop calculations have been done by the authors in Refs.[6, 10] in the Coulomb or axial gauge. Unfortunately the situation is still somehow ambiguous concerning the possibility of the exponentiation of the infrared singularity in the electroweak theory.

Recently, a part of the mixing effect (mass gap effect) mentioned above is investigated by Melles [13] in the Feynman gauge. In this paper, we extend the analysis by Melles to the general case which includes also the information on the non-abelian structure of the electroweak theory. Since there are many investigations in the Feynman gauge for QCD, we believe that our explicit two loop calculation of fermion’s form factor in the Feynman gauge is useful and instructive. Furthermore we will be able to clarify the similarities and differences in the “soft” physics between the electroweak theory and QCD. The process we consider is the fermion pair production from the $SU(2)\otimes U(1)$ singlet source. To the accuracy of the DL corrections, the chirality of the

fermion is conserved and we can discuss the left and right handed fermion separately. Therefore we consider, in this paper, the left handed fermion and the right handed antifermion production*. The masses of the W and Z bosons will be approximated to be equal $M_W \sim M_Z \equiv M$. We give a fictitious small mass λ to the photon to regularize the “real” infrared divergence and the fermion is assumed to be massless. We assume the situation, $s \gg M^2 \gg \lambda^2$ with s the total energy of the produced fermions.

2 One loop DL contribution

To fix our convention and the calculational framework, we present the Feynman rule (in the Feynman gauge) and the one loop calculation. The Feynman rules for the gauge boson propagators and the fermion gauge boson couplings read †,

$$\gamma \quad : \quad \frac{-ig_{\mu\nu}}{q^2 - \lambda^2} \quad , \quad W^\pm, Z \quad : \quad \frac{-ig_{\mu\nu}}{q^2 - M^2} \quad ,$$

and

$$\begin{aligned} \gamma ff & : \quad ieQ(\gamma_\mu\omega_- + \gamma_\mu\omega_+) \quad , \\ W^\pm ff & : \quad \frac{ig}{\sqrt{2}}(T^1 \pm iT^2)\gamma_\mu\omega_- \quad , \\ Zff & : \quad \frac{ig}{\cos\theta_W} \left[(T^3 - \sin^2\theta_W Q)\gamma_\mu\omega_- - \sin^2\theta_W Q\gamma_\mu\omega_+ \right] \quad , \end{aligned} \tag{1}$$

where T^a ($a = 1, 2, 3$) is the SU(2) generator, Q is the charge of fermion given by $Q = T^3 + Y$ with Y the hypercharge and θ_W is the Weinberg angle. $\omega_\pm \equiv \frac{1 \pm \gamma_5}{2}$ is the projection operator. The coupling constants of the SU(2) and U(1) gauge groups are g and $g' = g \tan\theta_W$ respectively and the electric charge e (> 0) is related to g as $e = g \sin\theta_W$. As explained in the Introduction, we concentrate on only the left chiral part in Eq.(1) in what follows.

Let us present the group factor of SU(2)⊗U(1) and the “kinematical” factor from loop integration separately. The group factors become:

$$\begin{aligned} \gamma \text{ exchange} & : \quad e^2 Q^2 \quad , \\ W \text{ exchange} & : \quad g^2 \sum_{a=1,2} T^a T^a \quad , \end{aligned}$$

*The case of the right handed fermion and the left handed antifermion production is discussed by Melles [13]

†Since we assume fermions to be massless, the ghost and Higgs particles do not contribute.

$$\begin{aligned}
Z \text{ exchange} & : \frac{g^2}{\cos^2 \theta_W} (T^3 - \sin^2 \theta_W Q)(T^3 - \sin^2 \theta_W Q) \\
& = g^2 T^3 T^3 + g'^2 Y^2 - e^2 Q^2 .
\end{aligned}$$

The loop integrations in which the weak bosons and photon are exchanged produce the double logarithmic corrections,

$$-\frac{1}{16\pi^2} \ln^2 \frac{s}{M^2} \quad , \quad -\frac{1}{16\pi^2} \ln^2 \frac{s}{\lambda^2} \quad ,$$

respectively. Therefore, the final result up to the one loop level for the fermion pair production reads,

$$\Gamma^{(1)} = 1 - \frac{1}{16\pi^2} \left(g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2 \right) \ln^2 \frac{s}{M^2} - \frac{1}{16\pi^2} e^2 Q^2 \ln^2 \frac{s}{\lambda^2} \quad ,$$

where $C_2(R)$ is the SU(2) Casimir operator for the fundamental representation.

3 Two loop DL contribution

We classify the two loop diagrams into three groups. The first group is composed from the ladder and crossed ladder diagrams shown in Fig.1. The second includes the diagrams which contain the triple gauge boson coupling shown in Fig.2. The symmetric graphs are not shown in Figs.1 and 2. The diagrams with the vertex correction and the self energy insertion make the third group (graphs are not shown).

For the evaluation of diagrams in the second group, it is known that the application of the pinch technique [14] to the triple gauge boson coupling makes the situation simple [15]. Omitting the coupling constant, the triple gauge boson vertex will be decomposed into two parts.

$$\begin{aligned}
\Gamma_{\alpha\mu\nu}(k, q) & = (k - q)_\nu g_{\alpha\mu} + (2q + k)_\alpha g_{\mu\nu} - (q + 2k)_\mu g_{\nu\alpha} \\
& = \Gamma_{\alpha\mu\nu}^P + \Gamma_{\alpha\mu\nu}^F \quad ,
\end{aligned}$$

with

$$\Gamma_{\alpha\mu\nu}^P \equiv -(k + q)_\nu g_{\alpha\mu} - q_\mu g_{\nu\alpha} \quad , \quad (2)$$

$$\Gamma_{\alpha\mu\nu}^F \equiv (2q + k)_\alpha g_{\mu\nu} + 2k_\nu g_{\alpha\mu} - 2k_\mu g_{\nu\alpha} \quad . \quad (3)$$

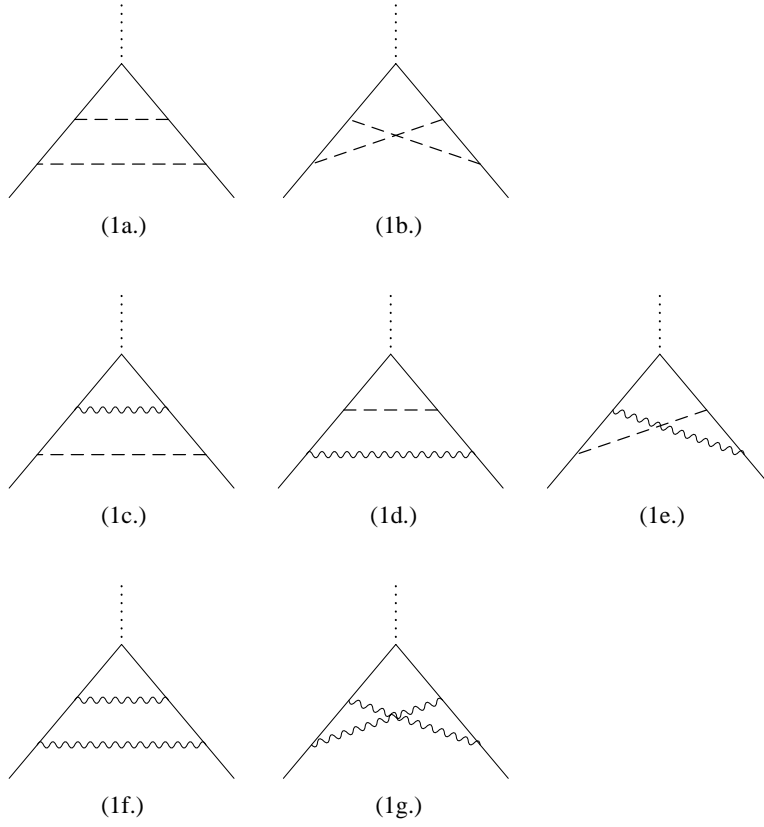


Figure 1: The ladder and crossed ladder diagrams. The dashed (wavy) line represents the photon (W and/or Z) with the mass λ (M).

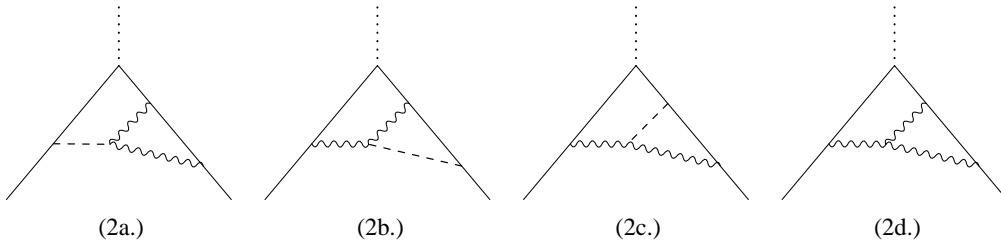


Figure 2: The diagrams which have the triple point couplings. The meaning of lines is the same as in Fig.1.

Eq.(2) gives rise to pinch parts when contracted with γ matrices and it is easily seen that the contributions from this term are reduced to the effective diagrams shown in Figs.3b and 3c. The contribution from Eq.(3) is depicted in Fig.3a with the blob vertex.

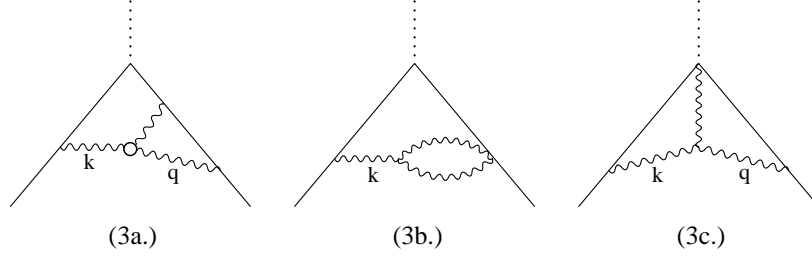


Figure 3: The reduced Feynman diagrams for Fig.2. The blob stands for the vertex Γ^F defined in Eq.3. The wavy lines represent both the photon and W, Z .

By combining the contributions from Fig.3a (Fig.3b) with those from the diagrams with the vertex correction (self energy insertion), one will get the “gauge invariant” vertex (propagator). The important fact for our purpose is that these contributions are less singular at least by one power of $\ln s$ compared to diagrams Fig.1 and Fig.3c [15]. Therefore, at the DL level, it is sufficient to calculate diagrams Fig.1 and Fig.3c configurations of Fig.2.

We present again the contributions from the group factor of $SU(2) \otimes U(1)$ and the “kinematical” factor from loop integration separately because such presentation is very useful to clarify the similarities and differences between the electroweak theory and QCD.

The group factors for the ladder and crossed ladder diagrams in Fig.1 become,

$$(1a.) = (1b.) = (e^2 Q^2)^2, \quad (4)$$

$$(1c.) = e^2 Q^2 (g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2), \quad (5)$$

$$(1d.) = e^2 Q^2 (g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2) - 2g^2 e^2 Y T^3, \quad (6)$$

$$(1e.) = 2 \times [e^2 Q^2 (g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2) - g^2 e^2 Q T^3], \quad (7)$$

$$(1f.) = (g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2)^2 + 2g^2 e^2 Y T^3, \quad (8)$$

$$(1g.) = (g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2)^2 - g^4 C_2(R) + 2g^2 e^2 Q T^3. \quad (9)$$

Those for the diagrams having the triple gauge boson couplings (Fig.2) read,

$$(2a.) = (2b.) = 2 \times [g^2 e^2 Q T^3], \quad (10)$$

$$(2c.) = 2 \times [-g^2 e^2 Y T^3 + g^2 e^2 T^3 T^3], \quad (11)$$

$$(2d.) = 2 \times [g^4 C_2(R) - 2g^2 e^2 Q T^3 + g^2 e^2 Y T^3 - g^2 e^2 T^3 T^3]. \quad (12)$$

The factor 2 in Eqs.(7,10,11,12) comes from the symmetric diagram. One can see that the structure of the group factors is slightly more complicated than that of QCD. In QCD, only the Casimir operator $C_2(R)^2$ appears in the ladder diagram. The crossed ladder diagram is proportional to $C_2(R)^2 - \frac{1}{2}C_2(R)C_2(G)$ ($C_2(G)$ is the Casimir for the adjoint representation) and the second term is a non-exponentiating term. However it is known that this term is canceled by the contribution from Fig.2. On the other hand, the situation is different in the electroweak theory. The non-abelian nature of SU(2) and the mixing effect between SU(2) and U(1) lead to new contributions compared to QCD. Since the propagation of the electroweak bosons is not proportional to the Casimir operator, there are non-exponentiating terms even in the ladder diagrams. See Eqs.(6,8) (Figs.1d, 1f). The crossed ladder diagrams and Fig.2 receive the contributions which originate from both the mixing effect and the non-abelian nature of SU(2). The latter is the same as in QCD.

To evaluate the loop integrals, we follow the method explained in Ref.[13] for the ladder and crossed ladder diagrams. We apply the standard method of Feynman parametrization in evaluating Fig.2. The result for each diagram turns out to be,

$$(1a.) = \frac{1}{(8\pi^2)^2} \frac{1}{24} \ln^4 \frac{s}{\lambda^2}, \quad (13)$$

$$(1b.) = \frac{1}{(8\pi^2)^2} \frac{1}{12} \ln^4 \frac{s}{\lambda^2}, \quad (14)$$

$$(1c.) = \frac{1}{(8\pi^2)^2} \left[\frac{1}{8} \ln^4 \frac{s}{M^2} - \frac{1}{3} \ln^3 \frac{s}{M^2} \ln \frac{s}{\lambda^2} + \frac{1}{4} \ln^2 \frac{s}{M^2} \ln^2 \frac{s}{\lambda^2} \right], \quad (15)$$

$$(1d.) = (1f.) = \frac{1}{(8\pi^2)^2} \frac{1}{24} \ln^4 \frac{s}{M^2}, \quad (16)$$

$$(1e.) = 2 \times (2a.) = 2 \times (2b.) = \frac{1}{(8\pi^2)^2} \left[-\frac{1}{12} \ln^4 \frac{s}{M^2} + \frac{1}{6} \ln^3 \frac{s}{M^2} \ln \frac{s}{\lambda^2} \right], \quad (17)$$

$$(1g.) = 2 \times (2c.) = 2 \times (2d.) = \frac{1}{(8\pi^2)^2} \frac{1}{12} \ln^4 \frac{s}{M^2}. \quad (18)$$

A comment is in order concerning the above results. The diagrams Fig.1d and 1f (Fig.2c and 2d) lead to the same result Eq.(16)(Eq.(18)). As already pointed out by Melles [13], the physical reason is that the virtuality of the momentum circulating the loop is determined by the singularity of the most external propagator in the diagram in order to produce the DL leading contribution. Therefore the singularity from the photon which propagates inside the W and/or Z loop in Figs.1d and 2c is already regulated by the W and/or Z mass.

By combining the group factors and the loop integrals, we find that all non-

exponentiating terms are canceled out completely and obtain the two loop result which is the second term of the expansion of the exponentiated Sudakov form factor.

$$\Gamma^{(2)} = 1 - \frac{1}{16\pi^2} (g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2) \ln^2 \frac{s}{M^2} - \frac{1}{16\pi^2} e^2 Q^2 \ln^2 \frac{s}{\lambda^2},$$

$$+ \frac{1}{2!} \left[\frac{1}{16\pi^2} (g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2) \ln^2 \frac{s}{M^2} + \frac{1}{16\pi^2} e^2 Q^2 \ln^2 \frac{s}{\lambda^2} \right]^2.$$

The cancellation of the non-exponentiating terms from each diagram occurs as follows. The terms coming from the $SU(2) \otimes U(1)$ mixing effect are canceled by the fact that the accompanying integrals turn out to be the same. The mechanism of cancellation of other terms are the same as in QCD.

4 Summary

We have considered the electroweak form factor at two loop level in the DL approximation. We have used the standard Feynman gauge. Our results have shown the exponentiation of the electroweak Sudakov form factor at two loop level. The cancellation of the non-exponentiating contributions is never trivial. The typical aspects of electroweak theory, the non-abelian nature of $SU(2)$ and the mixing effect between $SU(2)$ and $U(1)$, produce a new situation compared to QCD. We have shown that there appear non-exponentiating terms not only in the crossed ladder diagrams and diagrams with triple gauge boson coupling but also in the ladder diagrams. The reason is that the propagation of the electroweak bosons has the contribution which is not proportional to the Casimir operator. However these new non-exponentiating terms are canceled out each other by the dynamical reason that the virtuality of the momentum circulating the loop is determined by the singularity of the most external propagator to the DL accuracy. The cancellation of the non-exponentiating terms coming from the non-abelian nature of $SU(2)$ occurs in the same way as in QCD.

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