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A Strategy for the Analysis of Semi-Inclusive Deep Inelastic Scattering

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Abstract

We present a strategy for the systematic extraction of a vast amount of detailed information on polarized parton densities and fragmentation functions from semi-inclusive deep inelastic scattering $l + N \to l + h + X$, in both LO and NLO QCD. A method is suggested for estimating the errors involved in the much simpler, and therefore much more attractive, LO analysis. The approach is based upon a novel interplay with data from inclusive DIS and from $e^+e^- \to hX$, and leads to a much simplified form of the NLO expressions. No assumptions are made about the equality of any parton densities and the only symmetries utilised are charge conjugation invariance and isotopic spin invariance of strong interactions.

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1 Introduction

There are two major problems in the QCD analysis of polarized *inclusive* deep inelastic scattering (DIS): i) the absence of neutrino data makes it impossible, in principle, to determine the non-strange polarized sea-quark densities $\Delta\bar{u}(x, Q^2)$ and $\Delta\bar{d}(x, Q^2)$, ii) the separate determination of the polarized strange quark density $\Delta s(x, Q^2)$ and the polarized gluon density $\Delta G(x, Q^2)$ relies heavily on the QCD evolution in Q^2 and use of the flavour $SU(3)_F$-invariance relation

$$\int_0^1 dx \left[\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} - 2\Delta s - 2\Delta\bar{s}\right] = 3F - D. \tag{1}$$

The absence of a long lever arm in Q^2, in the polarized case, and doubt concerning the reliability of $SU(3)_F$-invariance for hyperon β-decay means that Δs and ΔG are still rather poorly known [1], despite the dramatic improvement in the quality of the data in the past few years.

The direct resolution of these problems must await a series of new machine development projects, based on very high intensity neutrino beams, which are most unlikely to come into operation before the year 2015.

In the meantime there is currently a major experimental effort at CERN [2], HERA [3] and Jefferson Lab. to study semi-inclusive polarized DIS reactions

$$\overrightarrow{e} + \overrightarrow{N} \to e + h + X, \tag{2}$$

involving the detection of the produced hadron h.

The theoretical structure for the analysis of such reactions, in both leading order QCD (LO) and next-to-leading order (NLO), exists [4]. However we are critical of the type of LO analysis carried out thus far by the experimental groups.

The analysis of semi-inclusive DIS [2, 3] involves both parton densities and fragmentation functions. In the LO treatments referred to above, the fragmentation functions (FF's) are treated as known quantities from inclusive $e^+e^- \to hadrons$, and are used in constructing an auxiliary quantity called "purity". However, it is well known that in $e^+e^- \to hadrons$, both in LO and NLO, first, only the combinations $D_q^h + D_{\bar{q}}^h$ can be determined while for the analyses of semi-inclusive DIS both D_q^h and $D_{\bar{q}}^h$ are needed separately, and second, the existing analysis are rather ambiguous: a detailed study in [5] makes a 31 parameter fit to the data, and no errors are quoted, and, in a more recent study [6], the FF's differ significantly from those in [5], by 40% or more in some regions of z. Under these circumstances it is unreasonable to pretend to have an absolute knowledge of the fragmentation functions.

Two NLO analyses based upon a global analysis of the inclusive and semi-inclusive data have been attempted [7], [8]. In the more recent analysis [8] the authors relax the equality of $\Delta\bar{u}(x)$ and $\Delta\bar{d}(x)$ and find a preference for a positive $\Delta\bar{u}(x)$, but effectively no constraint on the sign of $\Delta\bar{d}(x)$. However, in both these analyses it is again assumed that the FF's are known exactly: those of [5] being used in [7], and those of [6] in [8].

In addition, in the above mentioned analysis, some simplifying assumptions are made about relations between various polarized parton densities. In the following, except where expressly indicated, we make no assumptions at all concerning the polarized or unpolarized parton densities. Indeed there are persuasive arguments from the large-N_c limit of QCD that a significant difference should exist between $\Delta\bar{u}(x)$ and $\Delta\bar{d}(x)$ [9] with $|\Delta\bar{u} - \Delta\bar{d}| > |\bar{u} - \bar{d}|$, and it has been argued that such a situation is compatible with all present day data [10]. Further, bearing in mind recent arguments [11] that $s(x) \neq \bar{s}(x)$ and $\Delta s(x) \neq \Delta\bar{s}(x)$, we even refrain from the very common assumption of the *equality of these densities*.

However, from experience gained in the analysis of inclusive polarized DIS, it appears that the parameter space is sufficiently complicated to be able to produce biases in the χ^2 analysis, which can lead to unphysical results. We thus believe it to be dangerous in either LO or NLO QCD, to put together all inclusive and semi-inclusive data in one global analysis. Rather, what is required, is a working strategy, making optimal use of selected parts of the data.

The aim of this paper is precisely to provide a strategy, in both LO and NLO QCD, for the analysis of the semi-inclusive data. We proceed as follows: Information about the FF's is obtained from *unpolarized* semi-inclusive data. Then this information is used to determine the polarized parton densities from *polarized* semi-inclusive DIS. Appropriate use of the information from DIS and $e^+e^- \to hadrons$ is made as well. We suggest, for example, that one should use as *input* not just a knowledge of the unpolarized parton densities and their errors, but rather the polarized isotopic combination

$$\Delta q_3(x, Q^2) = (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d}) \tag{3}$$

which is by now very well determined from polarized inclusive data, and which is free from any influence of Δs and ΔG. Inclusive $e^+e^- \to hadrons$ and DIS are used also to considerably simplify the NLO expressions for the cross sections of (2).

Further, in stead of dealing with each parton density and FF separately, as is often done we work with their singlet and non-singlet combinations. As both the parton densities and the FF's enter the semi-inclusive cross sections, we single out such observables that are singlet (non-singlet) combinations on both the parton densities and the FF's.

This leads to the fact that we often consider linear combinations of experimentally measured quantities, and it may be objected that thereby we are dealing with experimental observables with possibly large errors. It has to be understood that that is a reflection of the true situation and not an artifact of our approach. To take an absolutely trivial example, suppose E_1 and E_2 are two experimentally measured functions used in an attempt to determine the theoretical functions T_1 and T_2, where $E_1 = T_1 + T_2$ and $E_2 = T_1 - T_2$. Now, if it happens that $E_1 \sim E_2$ and if we write $T_{1,2} = (1/2)(E_1 \pm E_2)$ then T_2 will be very poorly determined. This is unavoidable. It does not help to do a best fit to E_1 and E_2 with some parametrisations of T_1 and T_2. *Inherently* the result for T_2 will have a large relative error.

Thus we believe that any relatively large errors occurring in our manipulation of the experimental quantities reflects a genuine and unavoidable imprecision in the determination of certain theoretical quantities.

In order to minimize systematic errors experimentalists prefer to consider asymmetries or ratios of cross sections where the detection efficiencies should roughly cancel out, e.g. ratios of polarized to unpolarized DIS or ratios of polarized to unpolarized semi-inclusive for a given detected hadron. We appreciate that this is a fact of experimental life. However we wish to stress that a large amount of information is lost in restricting oneself to only these ratios. It is vitally important to gain control of the systematic errors in detection efficiencies, and although we try as far as possible to deal with the favoured kind of ratios we will be forced also to consider other types of cross-section ratios.

Throughout the rest of this paper we assume that a kinematic separation is possible between hadrons produced in the current fragmentation region and those produced from the target remnants. We consider only the current fragmentation region so that our formulae apply only to this region and fracture functions [13] play no role in our discussion.

It has to be stressed that there is a huge difference in complexity between the

LO and NLO treatments. Thus it makes sense to utilize the LO approach, provided appropriate checks, (which we suggest) are carried out.

Our analysis proceeds in a step-by-step fashion. Firstly we describe a generic test for the reliability of a LO treatment. Assuming this to be successful we present the analysis in LO. In LO the information on the polarized valence quark densities Δu_V and Δd_V and on the breaking of SU(2) invariance of the polarized sea is obtained without any knowledge of the FF's. However, it should be clear that any information on the sea quark densities in LO cannot be reliable, since they are expected to be small and thus comparable to the NLO corrections. We consider also the experimentally difficult case of ϕ production, which seems to be the best way to get an accurate determination of $\Delta s + \Delta \bar{s}$ in LO. We discuss also how, in principle, one can test $s(x) = \bar{s}(x)$ and/or $\Delta s(x) = \Delta \bar{s}(x)$.

In the NLO treatment we show in a new way how information on $e^+e^- \to h + X$ and inclusive DIS can be incorporated directly so as to considerably simplify the NLO expressions for semi-inclusive cross sections. This key result is presented in eqs. (69) and (70). Next we discuss the fragmentation combination $D_u^{\pi^+} - D_u^{\pi^-}$ and use it to evaluate Δu_V and Δd_V in NLO. Then we obtain expressions for $D_u^{\pi^+} + D_u^{\pi^-}$, for $D_G^{\pi^+}$ and for $D_s^{\pi^+} + D_s^{\pi^-}$. With these we are able to obtain $(\Delta \bar{u} - \Delta \bar{d})$ in NLO. Finally we have a set of 2 equations involving 3 unknown functions, $(\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d})$, $(\Delta s + \Delta \bar{s})$ and ΔG. An *accurate* determination of all three functions would require data over a presently impossibly wide range of Q^2. We thus suggest using here for ΔG its determination from charm production [12]. It should then finally be possible to get an accurate assessment of $\Delta s + \Delta \bar{s}$ in NLO. Lastly we consider the evaluation of $s(x) - \bar{s}(x)$ and $\Delta s(x) - \Delta \bar{s}(x)$.

2 A strategy for semi-inclusive DIS

In NLO QCD, the expressions for semi-inclusive cross sections involve *convolutions* of parton densities and fragmentation functions with (known) Wilson coefficients . Our lack of knowledge of the errors on the FF's will make it difficult to assess the accuracy of the parton densities which we are trying to determine. In the LO QCD approximation, on the other hand there are no convolutions, but simple products only *(independent fragmentation)*, and it becomes possible to construct measurable combinations of cross sections in which the FF's completely cancel out [15, 16].

However it is not clear how reliable the LO is.

We believe it is quite safe when determining the large Δu and Δd densities, but could be quite misleading for $\Delta\bar{u}$, $\Delta\bar{d}$ and Δs. In any event it is absolutely essential to *test* independent fragmentation in order to have a feeling for the errors on parton densities obtained via the LO formalism.

In LO, the structure of the expressions is generally of the form

$$parton\ density\ \Delta q(x,Q^2) = experimental\ observable\ E(x,z,Q^2) \tag{4}$$

or

$$fragmentation\ function\ D(z,Q^2) = experimental\ observable\ E(x,z,Q^2). \tag{5}$$

In both cases the characteristic feature of the LO treatment is that the RH sides, which can in principle depend on (x,z,Q^2), should only depend on two of these, either (x,Q^2) or (z,Q^2) respectively, so that there is an independence of the third variable, which we shall call the *passive* variable.

Every expression of the form (4) or (5) should be tested for dependence on the passive variable. If a significant dependence is found it does not mean that the LO analysis must be abandoned, but it suggests that the variation with the passive variable be used as an estimate of the *theoretical* errors, $\delta_{TH}[\Delta q(x,Q^2)]$, $\delta_{TH}[D(z,Q^2)]$ on the sought for quantities.

In Section 7 we discuss a strategy for the analysis in NLO.

3 Parton densities in LO QCD

It is useful to introduce the following notation for semi-inclusive processes:

$$\tilde{\sigma}^h \equiv \frac{x(P+l)^2}{4\pi\alpha^2}\left(\frac{2y^2}{1+(1-y)^2}\right)\frac{d^3\sigma^h}{dx\,dy\,dz} \tag{6}$$

and

$$\Delta\tilde{\sigma}^h \equiv \frac{x(P+l)^2}{4\pi\alpha^2}\left(\frac{y}{2-y}\right)\left[\frac{d^3\sigma^h_{++}}{dx\,dy\,dz} - \frac{d^3\sigma^h_{+-}}{dx\,dy\,dz}\right] \tag{7}$$

where P^μ and l^μ are the nucleon and lepton four momenta, and $\sigma_{\lambda\mu}$ refer to a lepton of helicity λ and a nucleon of helicity μ. The variables x, y, z are the usual DIS kinematic variables.

Then in LO the cross sections for the semi-inclusive production of a hadron h have the simple y-independent form

$$\Delta\tilde{\sigma}^h(x,z,Q^2) = \sum_{q,\bar{q}} e_q^2\, \Delta q_i(x,Q^2)\, D_i^h(z,Q^2) \tag{8}$$

$$\tilde{\sigma}^h(x,z,Q^2) = \sum_{q,\bar{q}} e_q^2\, q_i(x,Q^2)\, D_i^h(z,Q^2), \tag{9}$$

where the sum is over quarks and aniquarks, and where D_i^h is the fragmentation function for quark or antiquark i to produce h.

We consider sum and difference cross sections for producing h and its charge conjugate $\bar{h}$ on both protons and neutrons, and define

$$\Delta A_{p,n}^{h\pm\bar{h}}(x,z,Q^2) = \frac{\Delta\tilde{\sigma}_{p,n}^h \pm \Delta\tilde{\sigma}_{p,n}^{\bar{h}}}{\tilde{\sigma}_{p,n}^h \pm \tilde{\sigma}_{p,n}^{\bar{h}}} \equiv \frac{\Delta\tilde{\sigma}_{p,n}^{h\pm\bar{h}}}{\tilde{\sigma}_{p,n}^{h\pm\bar{h}}}, \tag{10}$$

$$\Delta A_{p\pm n}^{h\pm\bar{h}}(x,z,Q^2) = \frac{\Delta\tilde{\sigma}_p^{h\pm\bar{h}} \pm \Delta\tilde{\sigma}_n^{h\pm\bar{h}}}{\tilde{\sigma}_p^{h\pm\bar{h}} \pm \tilde{\sigma}_n^{h\pm\bar{h}}}. \tag{11}$$

For inclusive unpolarized and polarized DIS cross sections we use the notation:

$$\tilde{\sigma}^{DIS} \equiv \frac{x(P+l)^2}{4\pi\alpha^2}\left(\frac{2y^2}{1+(1-y)^2}\right)\frac{d^2\sigma^{DIS}}{dx\,dy} \tag{12}$$

and

$$\Delta\tilde{\sigma}^{DIS} \equiv \frac{x(P+l)^2}{4\pi\alpha^2}\left(\frac{y}{2-y}\right)\left[\frac{d^3\sigma_{++}^{DIS}}{dx\,dy} - \frac{d^3\sigma_{+-}^{DIS}}{dx\,dy}\right]. \tag{13}$$

Then in LO we have the y-independent expressions:

$$\tilde{\sigma}^{DIS}(x,Q^2) = 2F_1^N(x,Q^2)|_{LO} = \sum_{q,\bar{q}} e_q^2\, q_i(x,Q^2) \tag{14}$$

$$\Delta\tilde{\sigma}^{DIS}(x,Q^2) = 2g_1^N(x,Q^2)|_{LO} = \sum_{q,\bar{q}} e_q^2\, \Delta q_i(x,Q^2)\,. \tag{15}$$

In addition to (10) and (11) we consider the ratios of sum and difference hadron yields for the unpolarized semi-inclusive and inclusive processes:

$$R_{p,n}^{h\pm\bar{h}}(x,z,Q^2) = \frac{\tilde{\sigma}_{p,n}^h \pm \tilde{\sigma}_{p,n}^{\bar{h}}}{\tilde{\sigma}_{p,n}^{DIS}}, \qquad R_{p\pm n}^{h\pm\bar{h}}(x,z,Q^2) = \frac{\tilde{\sigma}_p^{h\pm\bar{h}} \pm \tilde{\sigma}_n^{h\pm\bar{h}}}{\tilde{\sigma}_p^{DIS} \pm \tilde{\sigma}_n^{DIS}}. \tag{16}$$

(It is equally good to use a sum over any set of hadrons h and their charge conjugate $\bar{h}$.)

3.1 Testing LO QCD

Using only charge conjugation invariance it is easy to show that [16]

$$\Delta A_{p-n}^{h+\bar{h}}(x,z,Q^2) = \frac{\Delta\tilde{\sigma}_p^{DIS} - \Delta\tilde{\sigma}_n^{DIS}}{\tilde{\sigma}_p^{DIS} - \tilde{\sigma}_n^{DIS}}(x,Q^2) = \frac{(g_1^p - g_1^n)|_{LO}}{(F_1^p - F_1^n)|_{LO}}(x,Q^2). \tag{17}$$

This is a key relation for testing the reliability of the LO. The RHS is completely known from inclusive DIS and is independent of z. The LHS, in principle, depends also upon z and upon the hadron h. Only in LO (or in the simple parton model) should it be independent of z and of h. It is thus crucial to test this feature.

To help with statistics it is also possible to formulate an integrated version of (17). This is given in [16]. For the rest of this section we assume that the test (17) has been successful and proceed with the analysis in LO.

3.2 The valence quark densities in LO

The polarized valence quark densities can be obtained from $\pi^\pm$ production, assuming only isospin invariance:

$$\begin{aligned} \Delta u_V &= \frac{1}{15}\left\{4(4u_V - d_V)\Delta A_p^{\pi^+-\pi^-} + (4d_V - u_V)\Delta A_n^{\pi^+-\pi^-}\right\} \\ \Delta d_V &= \frac{1}{15}\left\{4(4d_V - u_V)\Delta A_n^{\pi^+-\pi^-} + (4u_V - d_V)\Delta A_p^{\pi^+-\pi^-}\right\}. \end{aligned} \tag{18}$$

For the case of $K^\pm$ or $\Lambda, \bar{\Lambda}$ production, if one makes also the conventional assumption that $s = \bar{s}$ and $\Delta s = \Delta\bar{s}$ one has in addition:

$$\begin{aligned} \Delta u_V &= \frac{1}{2}\left\{(u_V + d_V)\Delta A_{p+n}^{K^+-K^-} + (u_V - d_V)\Delta A_{p-n}^{K^+-K^-}\right\} \\ \Delta d_V &= \frac{1}{2}\left\{(u_V + d_V)\Delta A_{p+n}^{K^+-K^-} - (u_V - d_V)\Delta A_{p-n}^{K^+-K^-}\right\}. \end{aligned} \tag{19}$$

Note that we have not assumed $D_d^{K^+} = D_d^{K^-}$, although that is suggested by the absence of a d quark in the leading Fock state of $K^\pm$. Indeed the above equality can be tested (see Section 5.2).

For isoscalar hadrons like $\Lambda, \bar{\Lambda}$ again assuming $s = \bar{s}$ and $\Delta s = \Delta\bar{s}$, one finds

$$\begin{aligned} \Delta u_V &= \frac{1}{15}\left\{4(4u_V + d_V)\Delta A_p^{\Lambda-\bar{\Lambda}} - (4d_V + u_V)\Delta A_n^{\Lambda-\bar{\Lambda}}\right\} \\ \Delta d_V &= \frac{1}{15}\left\{4(4d_V + u_V)\Delta A_n^{\Lambda-\bar{\Lambda}} - (4u_V + d_V)\Delta A_p^{\Lambda-\bar{\Lambda}}\right\}. \end{aligned} \tag{20}$$

We shall comment in Section 5.5 on the situation if one does not assume $s = \bar{s}$ and $\Delta s = \Delta\bar{s}$, where we suggest a method for estimating if the failure of these equalities is serious or not. In any event, the safe way to obtain Δu_V and Δd_V is via $\pi^\pm$ production, eq. (18).

Once again the reliability of these LO equations can be tested by checking that the RH sides of (18), (19) and (20) do not depend on z. If it is found that for a given x-bin the RH sides vary with z by some amount $\delta_{TH}[\Delta u_V]$, $\delta_{TH}[\Delta d_V]$, then these could be regarded as an estimate of the theoretical error at this x value.

4 Use of $\Delta q_3(x, Q^2)$ from polarized inclusive DIS

At NLO the spin dependent structure functions for protons and neutrons are given by:

$$g_1^p(x,Q^2) = \frac{1}{2}\sum_{q=u,d,s} e_q^2 \left[(\Delta q + \Delta\bar{q}) \otimes \left(1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q\right) + \right. \\ \left. + \frac{\alpha_s(Q^2)}{2\pi}\Delta G \otimes \delta C_G \right] \qquad (21)$$

involving a convolution of polarized parton densities with known Wilson coefficients. For the neutron, g_1^n is obtained by the replacement

$$\Delta u \Longleftrightarrow \Delta d \qquad (22)$$

in g_1^p.

Now it is clear that one can obtain information only on the combinations:

$$\Delta u + \Delta\bar{u}, \quad \Delta d + \Delta\bar{d}, \quad \Delta s + \Delta\bar{s}, \quad \Delta G. \qquad (23)$$

and that it is impossible to obtain separate information on the valence and non-strange sea quark polarizations from inclusive, neutral current, polarized DIS. [3]

In our semi-inclusive analysis we shall use as a known qauntity *only* the non-singlet combination of the polarized parton densities Δq_3, eq.(3). Now that there

[3]However, sometimes it is convenient to parametrize separately the valence and sea quarks and to assume, for example, $\Delta\bar{u} = \Delta\bar{d} = \lambda\Delta s$, where λ is a free parameter. It should be obvious then, that any claim that the χ^2 analysis favors some particular value of λ must be fictitious and a consequence of some hidden bias in the minimization procedure. Yet such claims have been made.

is such an improvement in the quality of the neutron data we believe this quantity is very well constrained directly by the inclusive data. For one has, from (21)

$$g_1^p(x,Q^2) - g_1^n(x,Q^2) = \frac{1}{6}\Delta q_3 \otimes \left(1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q\right) \tag{24}$$

and $\Delta q_3(x,Q^2)$ is determined without any influence from the less well known quantities Δs and ΔG, either in LO or in NLO. Of course if the semi-inclusive analysis is done in LO one must use $\Delta q_3(x,Q^2)|_{LO}$. Note that we do *not* use information on $(\Delta u + \Delta\bar{u})$ or $(\Delta d + \Delta\bar{d})$ from inclusive DIS, since these are subject to the less known strange quark and gluon effects.

4.1 SU(2) symmetry of the sea quark densities in LO

One has

$$[\Delta\bar{u}(x,Q^2) - \Delta\bar{d}(x,Q^2)]_{LO} = \frac{1}{2}\left[\Delta q_3(x,Q^2) + \Delta d_V(x,Q^2) - \Delta u_V(x,Q^2)\right]_{LO}. \tag{25}$$

Eq. (25) determines the SU(2) symmetry breaking of the polarized sea without requiring any knowledge of the unknown $\Delta\bar{q}$ and ΔG. A possible test for SU(2) breaking for the polarized sea densities that does not require any knowledge of the polarized densities was given in [16].

In order to determine the polarized sea quark densities separately we need one more relation, namely the value of

$$\Delta q_+(x,Q^2) \equiv \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d}. \tag{26}$$

For then

$$(\Delta\bar{u} + \Delta\bar{d})_{LO} = \frac{1}{2}(\Delta q_+ - \Delta u_V - \Delta d_V)_{LO} \tag{27}$$

which combined with (25) yields $\Delta\bar{u}$ and $\Delta\bar{d}$ separately.

Although each term on the RHS of (25) and (27) should be well determined in LO, the corresponding linear combinations are expected to be small and may thus be very sensitive to NLO corrections. An indication of the sensitivity may be inferred from the fact that in inclusive polarized DIS, $\Delta s(x,Q^2)$ changes by roughly a factor of 2 in going from LO to NLO or when one changes factorisation schemes from $\overline{MS}$ to AB or JET.

Note that determining, say, $\Delta\bar{u}$ via $\Delta\bar{u} = \frac{1}{2}[(\Delta u + \Delta\bar{u}) - \Delta u_V]$ is unreliable since determination of $(\Delta u + \Delta\bar{u})$ from inclusive DIS requires a knowledge of Δs.

In order to determine $\Delta q_+(x, Q^2)$ it will first be necessary to extract some information on the FF's.

4.2 Fragmentation functions in LO

1. From measurements of the ratios $R_{p-n}^{h+\bar{h}}$ of the semi-inclusive to inclusive DIS cross sections on protons and neutrons for any given hadron h, it is feasible in LO to learn a great deal about the FF's $D_q^h + D_q^{\bar{h}} \equiv D_q^{h+\bar{h}}$. This combination is measured also in $e^+e^- \to hadrons$.

Analogous to the polarized case, we define

$$q_3(x, Q^2) = u(x, Q^2) + \bar{u}(x, Q^2) - d(x, Q^2) - \bar{d}(x, Q^2) \tag{28}$$

$$q_+(x, Q^2) = u(x, Q^2) + \bar{u}(x, Q^2) + d(x, Q^2) + \bar{d}(x, Q^2). \tag{29}$$

which are well determined from inclusive DIS data and which thus can be taken as known quantities in the semi-inclusive analysis.

- Using data on unpolarized semi-inclusive DIS we have, in LO,

$$R_{p-n}^{h+\bar{h}} = \frac{2}{3}\left[4D_u^{h+\bar{h}}(z, Q^2) - D_d^{h+\bar{h}}(z, Q^2)\right]. \tag{30}$$

- For the case of pions, kaons and Λ, when SU(2) invariance can be used this simplifies to

$$R_{p-n}^{\pi^+ + \pi^-} = 2D_u^{\pi^+ + \pi^-}(z, Q^2) = 2D_d^{\pi^+ + \pi^-}(z, Q^2). \tag{31}$$

- For K mesons and Λ hyperons,

$$R_{p-n}^{K,\Lambda+\bar{\Lambda}} = 2D_u^{K,\Lambda+\bar{\Lambda}}(z, Q^2) = 2D_d^{K,\Lambda+\bar{\Lambda}}(z, Q^2), \tag{32}$$

  where the superscript K stands for the sum over all produced kaons:

$$K \equiv K^+ + K^- + K^0 + \bar{K}^0. \tag{33}$$

It will also be of great interest to compare these FF's with those obtained from $e^+e^- \to hadrons$ [5, 6] and those used in Monte Carlo models.

2. Given that $u_V(x,Q^2)$ and $d_V(x,Q^2)$ are well determined from inclusive DIS one can proceed further to obtain the other combinations of FF's $D_q^h - D_q^{\bar{h}} \equiv D_q^{h-\bar{h}}$.

- One finds for $\pi^\pm$

$$\begin{aligned} D_u^{\pi^+-\pi^-} &= \frac{9\tilde{\sigma}_p^{\pi^+-\pi^-}}{4u_V - d_V} = \frac{18\,(F_1^p)_{LO}\,R_p^{\pi^+-\pi^-}}{4u_V - d_V} \\ &= \frac{9\tilde{\sigma}_n^{\pi^+-\pi^-}}{4d_V - u_V} = \frac{18\,(F_1^n)_{LO}\,R_n^{\pi^+-\pi^-}}{4d_V - u_V} \end{aligned} \tag{34}$$

  Combined with (31) we have expressions for $D_u^{\pi^+}$ and $D_u^{\pi^-}$ separately.

- For $K^\pm$ one obtains

$$\begin{aligned} 4D_u^{K^+-K^-} - D_d^{K^+-K^-} &= \frac{9\left\{\tilde{\sigma}_p^{K^+-K^-} - \tilde{\sigma}_n^{K^+-K^-}\right\}}{u_V - d_V} \\ &= \frac{18\left[(F_1^p)_{LO}\,R_p^{K^+-K^-} - (F_1^n)_{LO}\,R_n^{K^+-K^-}\right]}{u_V - d_V} \end{aligned} \tag{35}$$

  It is usually *assumed*, and this is presumably a very good approximation, that

$$D_d^{K^+} = D_d^{K^-} \tag{36}$$

  in which case (35) can be read as an expression for $D_u^{K^+-K^-}$. It is not possible to test relation (36) without taking $s = \bar{s}$ and/or $\Delta s = \Delta\bar{s}$, but such an approach is hard to justify given that any failure of (36) is presumably very small.

- For isoscalar hadrons like $\Lambda, \bar{\Lambda}$

$$D_u^{\Lambda-\bar{\Lambda}} = D_d^{\Lambda-\bar{\Lambda}} = \frac{6\left[(F_1^p)_{LO}\,R_p^{\Lambda^+-\Lambda^-} - (F_1^n)_{LO}\,R_n^{\Lambda^+-\Lambda^-}\right]}{u_V - d_V}. \tag{37}$$

Given that Δu_V and Δd_V are determined via (18) we can write analogous expression for the above D_q^h functions using the polarized data.

Of course all the expressions (30), (31), (32), (34), (35) and (37), being LO results, must be tested by demonstrating that the RH sides are essentially independent of the passive variable x.

Now that we have determined $D_u^{\pi^++\pi^-}$ we can determine $D_s^{\pi^++\pi^-}$ in LO via

$$D_s^{\pi^++\pi^-} = \frac{9\,(F_1^p + F_1^n)_{LO}\,R_{p+n}^{\pi^++\pi^-} - 5q_+ D_u^{\pi^++\pi^-}}{2\,(s+\bar{s})}. \tag{38}$$

Similarly, since $D_u^{\Lambda+\bar{\Lambda}}$ is determined via (32), we can find $D_s^{\Lambda+\bar{\Lambda}}$ from

$$D_s^{\Lambda+\bar{\Lambda}} = \frac{9\,(F_1^p + F_1^n)_{LO}\, R_{p+n}^{\Lambda+\bar{\Lambda}} - 5q_+ D_u^{\Lambda+\bar{\Lambda}}}{2\,(s+\bar{s})}. \tag{39}$$

4.3 The non-strange sea quark densities revisited, in LO

Now that we have determined $D_u^{\pi^+ + \pi^-}$ and $D_s^{\pi^+ + \pi^-}$ in LO we can, in principle, determine $\Delta q_+(x,Q^2)$ and $\Delta s(x,Q^2) + \Delta\bar{s}(x,Q^2)$ from the semi-inclusive and inclusive relations, in LO,

$$g_1^p + g_1^n = \frac{5\Delta q_+ + 2(\Delta s + \Delta\bar{s})}{18} \tag{40}$$

$$\Delta A_{p+n}^{\pi^+ + \pi^-} = \frac{5\Delta q_+ D_u^{\pi^+ + \pi^-} + 2(\Delta s + \Delta\bar{s}) D_s^{\pi^+ + \pi^-}}{5q_+ D_u^{\pi^+ + \pi^-} + 2\,(s+\bar{s}) D_s^{\pi^+ + \pi^-}}. \tag{41}$$

Such a determination of Δq_+ in LO is likely to be reliable, but $\Delta s + \Delta\bar{s}$ and the individual $\Delta\bar{u}$ and $\Delta\bar{d}$ obtained in LO via (27) may be subject to significant uncertainty.

We note that there is an alternative way to determine Δq_+, but it requires the ability to detect K^0. In that case one has, in LO,

$$\Delta q_+ = q_+ \frac{\Delta\tilde{\sigma}_p^{K^+ + K^- - (K^0 + \bar{K}^0)} + \Delta\tilde{\sigma}_n^{K^+ + K^- - (K^0 + \bar{K}^0)}}{\tilde{\sigma}_p^{K^+ + K^- - (K^0 + \bar{K}^0)} + \tilde{\sigma}_n^{K^+ + K^- - (K^0 + \bar{K}^0)}}. \tag{42}$$

4.4 The strange quark density $\Delta s + \Delta\bar{s}$ in LO

The approach to $\Delta s + \Delta\bar{s}$ in Section 5.3 and the approach discussed in [16] are unlikely to be reliable. The problem is that for production of pions the strange quark contribution is "doubly small", since e.g. one must compare $\Delta u D_u^\pi$ with $\Delta s D_s^\pi$ in which both $|\Delta s + \Delta\bar{s}| \ll |\Delta u|$ and $|D_s^\pi| \ll |D_u^\pi|$. For kaons and Λ hyperons it is somewhat better in that $|D_u^K| \approx |D_s^K|$ and $|D_u^{\Lambda+\bar{\Lambda}}| = |D_d^{\Lambda+\bar{\Lambda}}| \approx |D_s^{\Lambda+\bar{\Lambda}}|$, but this is similar to the situation in inclusive DIS where we know that the LO determination of $\Delta s + \Delta\bar{s}$ is quite unreliable.

The only possibility we can see for a reasonable determination of Δs in LO is via ϕ production. For in this case one has, $|\Delta s + \Delta\bar{s}| \ll |\Delta u|$ but presumably $|D_s^\phi| \gg |D_u^\phi|$ so that the strange and non-strange quarks are on equal footing.

One has, by charge conjugation invariance $D_s^\phi = D_{\bar{s}}^\phi$, and it should be quite safe to take $D_u^\phi = D_{\bar{u}}^\phi = D_d^\phi = D_{\bar{d}}^\phi$. One then obtains in LO

$$\frac{\Delta s + \Delta \bar{s}}{s + \bar{s}} = \frac{3\Delta\tilde{\sigma}_{p+n}^\phi - 5\left(\frac{\Delta q_+}{\Delta q_3}\right)\Delta\tilde{\sigma}_{p-n}^\phi}{3\tilde{\sigma}_{p+n}^\phi - 5\left(\frac{q_+}{q_3}\right)\tilde{\sigma}_{p-n}^\phi}. \tag{43}$$

Moreover one has expressions for the fragmentation functions as well:

$$D_s^\phi = \frac{3}{2(s+\bar{s})}\left\{3\tilde{\sigma}_{p+n}^\phi - 5\left(\frac{q_+}{q_3}\right)\tilde{\sigma}_{p-n}^\phi\right\} \tag{44}$$

$$D_u^\phi = \frac{3\tilde{\sigma}_{p-n}^\phi}{q_3}. \tag{45}$$

As always expressions (43) - (45) must be tested for non-dependence on the relevant passive variable.

Finally we note that the same eqs. (43) - (45) hold for K, $\Lambda+\bar{\Lambda}$ and π-production if the superscript ϕ is replaced by K, $\Lambda+\bar{\Lambda}$ and $\pi^+ + \pi^-$, respectively. And though the production rates are higher, the sensitivity to the strange quarks in the K, $\Lambda+\bar{\Lambda}$ and π-productions is lower.

4.5 Concerning $s = \bar{s}$ and $\Delta s = \Delta\bar{s}$ in LO [4]

In the analysis of DIS it is conventional to assume $s = \bar{s}$ and $\Delta s = \Delta\bar{s}$. However there are models and arguments [11] which suggest that these equalities might not hold.

Within the limitations of the LO we can test these relationships via (K^+, K^-) and $(\Lambda, \bar{\Lambda})$ production. One has for the unpolarized case, assuming $D_d^{K^+-K^-} = 0$ several different possibilities:

$$\begin{aligned}(s-\bar{s})D_s^{K^+-K^-} &= 18\,(F_1^p)_{LO}\,R_p^{K^+-K^-} - 4u_V D_u^{K^+-K^-}\\ &= 18\,(F_1^n)_{LO}\,R_n^{K^+-K^-} - 4d_V D_u^{K^+-K^-}\\ &= \frac{9\left\{u_V\tilde{\sigma}_n^{K^+-K^-} - d_V\tilde{\sigma}_p^{K^+-K^-}\right\}}{u_V - d_V}.\end{aligned} \tag{46}$$

Then for the polarized case, given that Δu_V and Δd_V are known from (18) and $(s-\bar{s})D_s^{K^+-K^-}$ is determined, we can proceed to determine $(\Delta s - \Delta\bar{s})D_s^{K^+-K^-}$

[4]We are grateful to M. Anselmino, M. Boglione and U.D'Alesio for drawing our attention to this issue

from $\Delta A_p^{K^+-K^-}$ or $\Delta A_n^{K^+-K^-}$:

$$\Delta A_p^{K^+-K^-} = \frac{4\Delta u_V D_u^{K^+-K^-} + (\Delta s - \Delta \bar{s}) D_s^{K^+-K^-}}{4u_V D_u^{K^+-K^-} + (s - \bar{s}) D_s^{K^+-K^-}}$$
$$\Delta A_n^{K^+-K^-} = \frac{4\Delta d_V D_u^{K^+-K^-} + (\Delta s - \Delta \bar{s}) D_s^{K^+-K^-}}{4d_V D_u^{K^+-K^-} + (s - \bar{s}) D_s^{K^+-K^-}}, \tag{47}$$

$D_u^{K^+-K^-}$ is assumed to be known through (35).

For isoscalar hadrons like $\Lambda, \bar{\Lambda}$ we have the possiblities

$$\begin{aligned}(s-\bar{s})D_s^{\Lambda-\bar{\Lambda}} &= 18\;(F_1^p)_{LO}\,R_p^{\Lambda-\bar{\Lambda}} - (4u_V + d_V)D_u^{\Lambda-\bar{\Lambda}} \\ &= 18\;(F_1^n)_{LO}\,R_n^{\Lambda-\bar{\Lambda}} - (4d_V + u_V)D_u^{\Lambda-\bar{\Lambda}} \\ &= \frac{3}{u_V - d_V}\left\{(4u_V + d_V)\,\tilde{\sigma}_n^{\Lambda-\bar{\Lambda}} - (4d_V + u_V)\,\tilde{\sigma}_p^{\Lambda-\bar{\Lambda}}\right\},\end{aligned} \tag{48}$$

where $D_u^{\Lambda-\bar{\Lambda}}$ is assumed to be determined in (37). Here and in eq. (46) it is clear that $\tilde{\sigma}_N^{K^+-K^-}$ and $\tilde{\sigma}_N^{\Lambda-\bar{\Lambda}}$ can be replaced by the corresponding experimentally measured quantities $2\left(F_1^N\right)_{LO} R_N^{K^+-K^-}$ or $2\left(F_1^N\right)_{LO} R_N^{\Lambda-\bar{\Lambda}}$. Then $(\Delta s - \Delta\bar{s})D_s^{\Lambda-\bar{\Lambda}}$ can be determined via $\Delta A_p^{\Lambda-\bar{\Lambda}}$ or $\Delta A_n^{\Lambda-\bar{\Lambda}}$:

$$\Delta A_p^{\Lambda-\bar{\Lambda}} = \frac{(4\Delta u_V + \Delta d_V)D_u^{\Lambda-\bar{\Lambda}} + (\Delta s - \Delta\bar{s})D_s^{\Lambda-\bar{\Lambda}}}{(4u_V + d_V)D_u^{\Lambda-\bar{\Lambda}} + (s-\bar{s})D_s^{\Lambda-\bar{\Lambda}}}$$
$$\Delta A_n^{\Lambda-\bar{\Lambda}} = \frac{(4\Delta d_V + \Delta u_V)D_u^{\Lambda-\bar{\Lambda}} + (\Delta s - \Delta\bar{s})D_s^{\Lambda-\bar{\Lambda}}}{(4d_V + u_V)D_u^{\Lambda-\bar{\Lambda}} + (s-\bar{s})D_s^{\Lambda-\bar{\Lambda}}} \tag{49}$$

Although (46), (47), (48) and (49), being LO expressions, cannot be expected to yield accurate values for $s - \bar{s}$ and $\Delta s - \Delta\bar{s}$, they should nonetheless enable one to say whether they are compatible with zero since $D_s^{\Lambda-\bar{\Lambda}}$ should be relatively large. Note that $s - \bar{s}$ and/or $\Delta s - \Delta\bar{s}$ different from zero would break the independence of the RH sides of (47) and (49) on the passive variable z. Of particular interest is the speculation that $\Delta s \approx -\Delta\bar{s}$ but $s \approx \bar{s}$, the consistency of which could perhaps be tested from (46), (47), (48) and (49).

5 Semi-inclusive analysis in NLO QCD

The situation in NLO [4, 17] is much more complicated than in LO, since factorization is replaced by convolution, and it is also more complicated than inclusive DIS in NLO since here one has to contend with double convolutions of the form

$q \otimes C \otimes D$ and $\Delta q \otimes \Delta C \otimes D$ for the unpolarized and polarized cases respectively, where C and ΔC are Wilson coefficients first derived in [17] and [4].

The double convolution is defined as

$$q \otimes C \otimes D = \int \frac{dx'}{x'} \int \frac{dz'}{z'} q\left(\frac{x}{x'}\right) C(x', z') D\left(\frac{z}{z'}\right) \tag{50}$$

where the range of integration is given as follows:

- $\mathbf{I_1}$: The range is

$$\frac{x}{x + (1-x)z} \leq x' \leq 1 \quad with \quad z \leq z' \leq 1 \tag{51}$$

- $\mathbf{I_2}$: In addition to (51) there is the range

$$x \leq x' \leq \frac{x}{x + (1-x)z} \quad with \quad \frac{x(1-x')}{x'(1-x)} \leq z' \leq 1. \tag{52}$$

Note, that contrary to the case of the usual DIS convolution, the double convolution $q \otimes C \otimes D$ is not commutative.

We shall frequently encounter expressions of the form

$$qD + \frac{\alpha_s}{2\pi} q \otimes C \otimes D \tag{53}$$

corresponding to the LO plus NLO corrections. The flavour structure of the results becomes much more transparent if we adopt the following symbolic notation:

$$qD + \frac{\alpha_s}{2\pi} q \otimes C \otimes D = q\left[1 + \otimes \frac{\alpha_s}{2\pi} C \otimes\right] D. \tag{54}$$

Then the semi-inclusive polarized cross section $\Delta\tilde{\sigma}_p^h$ defined in (7) is given by

$$\begin{aligned} \Delta\tilde{\sigma}_p^h \;=\; & \sum_i e_i^2 \Delta q_i \left[1 + \otimes \frac{\alpha_s}{2\pi} \Delta C_{qq} \otimes\right] D_{q_i}^h + \\ & + \left(\sum_i e_i^2 \Delta q_i\right) \otimes \frac{\alpha_s}{2\pi} \Delta C_{qg} \otimes D_G^h + \Delta G \otimes \frac{\alpha_s}{2\pi} \Delta C_{gq} \otimes \left(\sum_i e_i^2 D_{q_i}^h\right) \end{aligned} \tag{55}$$

where the sum is over quarks and antiquarks of flavour i and parton densities and fragmentation functions are to be taken in NLO.

For the unpolarized semi-inclusive cross section in NLO it is not possible to completely factor out the y-dependence. Consequently $\tilde{\sigma}^h$ defined in (6) will depend upon y in NLO, in contrast to the LO situation in (9).

In the notation of the seminal paper of Graudenz [17] the cross section is a sum of what he refers to as "metric" (M) and "longitudinal" (L) terms, with corresponding Wilson coefficients C^M and C^L. A further complication is that the Wilson coefficients are different in the two regions of integration I_1 and I_2. Thus we have coefficients C^{jM}, C^{jL} with $j = 1, 2$.

We then define the y-dependent combinations of Wilson coefficients:

$$\mathbb{C}^j_{qq} = C^{jM}_{qq} + [1 + 4\gamma(y)]\, C^{jL}_{qq} \tag{56}$$

$$\mathbb{C}^j_{qg} = C^{jM}_{qg} + [1 + 4\gamma(y)]\, C^{jL}_{qg} \tag{57}$$

$$\mathbb{C}^j_{gq} = C^{jM}_{gq} + [1 + 4\gamma(y)]\, C^{jL}_{gq} \tag{58}$$

where

$$\gamma(y) = \frac{1-y}{1+(1-y)^2}. \tag{59}$$

Then the unpolarized semi-inclusive cross section can be written in a form analogous to the polarized one:

$$\begin{aligned} \tilde{\sigma}^h_p &= \sum_i e_i^2 q_i \left[1 + \otimes \frac{\alpha_s}{2\pi} \mathbb{C}_{qq} \otimes \right] D^h_{q_i} + \\ &+ \left(\sum_i e_i^2 q_i\right) \otimes \frac{\alpha_s}{2\pi} \mathbb{C}_{qg} \otimes D^h_G + G \otimes \frac{\alpha_s}{2\pi} \mathbb{C}_{gq} \otimes \left(\sum_i e_i^2 D^h_{q_i}\right). \end{aligned} \tag{60}$$

In (60) we have used the symbolic notation:

$$q_i \otimes \frac{\alpha_s}{2\pi} \mathbb{C}_{qq} \otimes D^h_{q_i} = \int_{I_1} q_i \otimes \frac{\alpha_s}{2\pi} \mathbb{C}^1_{qq} \otimes D^h_{q_i} + \int_{I_2} q_i \otimes \frac{\alpha_s}{2\pi} \mathbb{C}^2_{qq} \otimes D^h_{q_i}\,, \tag{61}$$

and analogously for $\mathbb{C}_{qg}$ and $\mathbb{C}_{gq}$.

Note that in NLO the unpolarized inclusive cross section, $\tilde{\sigma}^{DIS}$ (12) is given by

$$\tilde{\sigma}^{DIS} = 2F_1 \left[1 + 2\gamma(y) R\right], \tag{62}$$

where R is the usual DIS ratio of longitudinal to transverse cross sections. Note that strictly speaking here and throughout the rest of this paper R should be replaced by $R \longrightarrow \left(R - \frac{4M^2x^2}{Q^2}\right) / \left(1 + \frac{4M^2x^2}{Q^2}\right)$, since the correction terms may be important for low values of Q^2.

As in the LO discussion we doubt the reliability of a global NLO analysis of inclusive and semi-inclusive data, and we suggest that one should feed into the semi-inclusive analysis as much reliable information as one can from other sources.

As we shall see there is the oft found opposition between what is simple theoretically and what is simple experimentally. However, if the systematic errors in detection efficiencies can be brought under control then we can make remarkable theoretical simplifications and we can then extract a vast amount of information from the semi-inclusive data. This is an important experimental challenge as will be seen from the power of the results given below.

5.1 Simplification of the semi-inclusive NLO results

Bearing in mind the NLO result (21) for g_1^p for polarized DIS and the analogous result for F_1 in unpolarized DIS, we see that in the second term of (60) we may make the replacement, correct to NLO,

$$\sum_i e_i^2 q_i \longrightarrow 2F_1^p \tag{63}$$

and similarly, in the analogous eq. for $\Delta\tilde{\sigma}_p$ in (55):

$$\sum_i e_i^2 \Delta q_i \longrightarrow 2g_1^p. \tag{64}$$

Note that here *and throughout the rest of this paper* $F_1^{p,n}$ and $g_1^{p,n}$ are the experimentally measured structure functions.

Further in the reaction $e^+(p_+) + e^-(p_-) \to h + X$ in the kinematic region when we can neglect Z^0-exchange effects, we have

$$\sigma^h(z,\cos\theta,Q^2) = \frac{3}{8}(1+\cos^2\theta)\sigma_T^h(z,Q^2) + \frac{3}{4}(1-\cos^2\theta)\sigma_L^h(z,Q^2) \tag{65}$$

where $Q^2 = (p_+ + p_-)^2$.

In NLO QCD one has

$$\begin{aligned}\sigma_T^h(z,Q^2) \;\; = \;\; 3\sigma_0 \Big\{ & \sum_i e_i^2 D_{q_i}^h \otimes \left(1 + \frac{\alpha_s}{2\pi} C_q^T\right) + \\ & + \sum_i e_i^2 D_G^h \otimes \frac{\alpha_s}{2\pi} C_G^T \Big\}\end{aligned} \tag{66}$$

where the sum is over quarks and antiquarks, the C^T's are Wilson coefficients, and

$$\sigma_0 = \frac{4\pi\alpha^2}{3Q^2}. \tag{67}$$

Then, correct to the required NLO accuracy, in the third term in (60), and in its polarized analogue (55), we may make the replacement

$$\sum_i e_i^2 D_{q_i}^h = \frac{\sigma_T^h(z,Q^2)}{3\sigma_0}, \tag{68}$$

where σ_T^h is the experimentally measured cross section.

Hence, for the unpolarized and polarized semi-inclusive cross sections, in NLO accuracy, we have

$$\begin{aligned} \tilde{\sigma}_p^h &= \sum_i e_i^2 q_i \left[1 + \otimes \frac{\alpha_s}{2\pi}\mathbb{C}_{qq} \otimes\right] D_{q_i}^h + \\ & +2F_1^p \otimes \frac{\alpha_s}{2\pi}\mathbb{C}_{qg} \otimes D_G^h + \frac{1}{3\sigma_0} G \otimes \frac{\alpha_s}{2\pi}\mathbb{C}_{gq} \otimes \sigma_T^h \end{aligned} \tag{69}$$

$$\begin{aligned} \Delta\tilde{\sigma}_p^h &= \sum_i e_i^2 \Delta q_i \left[1 + \otimes \frac{\alpha_s}{2\pi}\Delta C_{qq} \otimes\right] D_{q_i}^h + \\ & +2g_1^p \otimes \frac{\alpha_s}{2\pi}\Delta C_{qg} \otimes D_G^h + \frac{1}{3\sigma_0} \Delta G \otimes \frac{\alpha_s}{2\pi}\Delta C_{gq} \otimes \sigma_T^h. \end{aligned} \tag{70}$$

Given that the unpolarized gluon density is reasonably well known, the last term in (69) can be considered as a known quantity. In the following we take as known quantities the NLO values for $q_+(x,Q^2)$, $q_3(x,Q^2)$, $u_V(x,Q^2)$, $d_V(x,Q^2)$, $G(x,Q^2)$ and $\Delta q_3(x,Q^2)$.

5.2 The polarized valence densities in NLO

Using charge conjugation invariance one obtains, for semi-inclusive pion production

$$\begin{aligned} R_p^{\pi^+-\pi^-} &= \frac{[4u_V - d_V][1 + \otimes(\alpha_s/2\pi)\mathbb{C}_{qq}\otimes]D_u^{\pi^+-\pi^-}}{18F_1^p\,[1+2\gamma(y)\,R^p]} \\ R_n^{\pi^+-\pi^-} &= \frac{[4d_V - u_V][1 + \otimes(\alpha_s/2\pi)\mathbb{C}_{qq}\otimes]D_u^{\pi^+-\pi^-}}{18F_1^p\,[1+2\gamma(y)\,R^p]}. \end{aligned} \tag{71}$$

The only unknown function in these expressions is $D_u^{\pi^+-\pi^-}(z,Q^2)$, which evolves as a non-singlet and does not mix with other FF's. A χ^2 analysis of either or both of the equations (71) should thus determine $D_u^{\pi^+-\pi^-}$ in NLO without serious ambiguity.

Assuming $D_u^{\pi^+-\pi^-}$ is now known, one can then determine Δu_V and Δd_V in NLO via the equations

$$\Delta A_p^{\pi^+-\pi^-} = \frac{(4\Delta u_V - \Delta d_V)[1 + \otimes(\alpha_s/2\pi)\Delta C_{qq}\otimes]D_u^{\pi^+-\pi^-}}{(4u_V - d_V)[1 + \otimes(\alpha_s/2\pi)\mathbb{C}_{qq}\otimes]D_u^{\pi^+-\pi^-}} \tag{72}$$

$$\Delta A_n^{\pi^+-\pi^-} = \frac{(4\Delta d_V - \Delta u_V)[1 + \otimes(\alpha_s/2\pi)\Delta C_{qq}\otimes]D_u^{\pi^+-\pi^-}}{(4d_V - u_V)[1 + \otimes(\alpha_s/2\pi)\mathbb{C}_{qq}\otimes]D_u^{\pi^+-\pi^-}} \tag{73}$$

where, of course, Δu_V and Δd_V evolve as non-singlets and do not mix with other densities. Eqs. (72) and (73) determine the densities Δu_V and Δd_V in NLO without any assumptions about the less known polarized gluon and sea densities.

5.3 SU(2) symmetry of the sea quark densities in NLO

Once Δu_V and Δd_V are known in NLO we can calculate

$$[\Delta\bar{u}(x,Q^2) - \Delta\bar{d}(x,Q^2)]_{NLO} = \frac{1}{2}[\Delta q_3(x,Q^2) + \Delta d_V(x,Q^2) - \Delta u_V(x,Q^2)]_{NLO}, \tag{74}$$

Eq. (74) determines the breaking of SU(2) symmetry for the polarized sea densities in NLO without requiring any knowledge of $\Delta\bar{q}$ and ΔG. It will be interesting to compare the values obtained from (74) with information on $\Delta\bar{u}(x)$ and $\Delta\bar{d}(x)$ which will emerge from Drell-Yan and $W^\pm$ production experiments at RHIC [18].

The separate determination of $\Delta\bar{u}$ and $\Delta\bar{d}$ requires knowledge of Δq_+, defined in (26), in NLO.

Note that determining, say, $\Delta\bar{u}$ via $\Delta\bar{u} = \frac{1}{2}[(\Delta u + \Delta\bar{u}) - \Delta u_V]$ is unreliable, since the determination of $(\Delta u + \Delta\bar{u})$ from *inclusive* DIS involves knowledge of gluon and strange quark densities.

As in the LO case we can only determine Δq_+ after obtaining some information about the fragmentation functions.

5.4 Fragmentation functions in NLO

We consider the sum for the unpolarized production of π^+ and π^-. We have

$$R_{p-n}^{\pi^++\pi^-} = \frac{q_3\left\{\left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right]D_u^{\pi^++\pi^-} + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qg}\otimes D_G^{\pi^++\pi^-}\right\}}{6\left[F_1^p\left(1 + 2\gamma(y)R^p\right) - F_1^n\left(1 + 2\gamma(y)R^n\right)\right]} \tag{75}$$

$$\Delta A_{p-n}^{\pi^++\pi^-} = \frac{\Delta q_3 \left\{ \left[1 + \otimes \frac{\alpha_s}{2\pi} \Delta C_{qq} \otimes \right] D_u^{\pi^++\pi^-} + \otimes \frac{\alpha_s}{2\pi} \Delta C_{qg} \otimes D_G^{\pi^++\pi^-} \right\}}{q_3 \left\{ \left[1 + \otimes \frac{\alpha_s}{2\pi} \mathbb{C}_{qq} \otimes \right] D_u^{\pi^++\pi^-} + \otimes \frac{\alpha_s}{2\pi} \mathbb{C}_{qg} \otimes D_G^{\pi^++\pi^-} \right\}} \, . \tag{76}$$

The only unknown functions in these relations are $D_u^{\pi^++\pi^-}$ and $D_G^{\pi^++\pi^-}$, which will mix with each other under evolution. Thus it should be possible to obtain them from a χ^2 analysis of (75) and (76).

Once we know $D_u^{\pi^++\pi^-}$ and utilise $D_u^{\pi^+-\pi^-}$ from Section 6.2 we clearly have access to both $D_u^{\pi^+}$ and $D_u^{\pi^-}$.

Note that if K^0 can be detected one can obtain information on D_u^K and D_G^K, where $K = K^+ + K^- + K^0 + \bar{K}^0$. One simply replaces the labels $\pi^+ + \pi^-$ by K everywhere in (75) and (76). Analogous equations hold also if $\pi^+ + \pi^-$ is replaced by $\Lambda + \bar{\Lambda}$, if Λ is detected.

Returning to the case of $\pi^+ + \pi^-$, the ratio $R_{p+n}^{\pi^++\pi^-}$ allows the determination of the only unknown function $D_s^{\pi^++\pi^-}$. We have

$$\begin{aligned} R_{p+n}^{\pi^++\pi^-} &= \\ = \Big\{ \Big(5q_+ \left[1 + \otimes \tfrac{\alpha_s}{2\pi} \mathbb{C}_{qq} \otimes \right] D_u^{\pi^++\pi^-} &+ 2(s+\bar{s}) \left[1 + \otimes \tfrac{\alpha_s}{2\pi} \mathbb{C}_{qq} \otimes \right] D_s^{\pi^++\pi^-} \Big) + \\ +18(F_1^p + F_1^n) \otimes \tfrac{\alpha_s}{2\pi} \mathbb{C}_{gq} \otimes D_G^{\pi^++\pi^-} &+ \tfrac{6}{\sigma_0} G \otimes \tfrac{\alpha_s}{2\pi} \mathbb{C}_{gq} \otimes \sigma_T^{\pi^++\pi^-} \Big\} / \\ /6 \left[F_1^p \left(1 + 2\gamma(y) R^p\right) + F_1^n \left(1 + 2\gamma(y) R^n\right) \right] \end{aligned} \tag{77}$$

Under evolution $D_u^{\pi^++\pi^-}$ and $D_s^{\pi^++\pi^-}$ mix with $D_G^{\pi^++\pi^-}$, but this is not a problem since the latter is supposed to be known. It would be interesting to compare these results with those from $e^+e^- \to hadrons$, obtained recently in [6].

Again analogous sets of equations holds for kaon and $\Lambda, \bar{\Lambda}$ production. One simply replaces $\pi^+ + \pi^-$ by K or $\Lambda + \bar{\Lambda}$ and this allows the determination of the only unknown function D_s^K or $D_s^{\Lambda+\bar{\Lambda}}$, respectively.

5.5 The sea-quark densities in NLO

With the NLO knowledge of $D_u^{\pi^++\pi^-}$, $D_s^{\pi^++\pi^-}$ and $D_G^{\pi^++\pi^-}$ we are now in a position to try to determine $\Delta q_+(x, Q^2)$, $\Delta s(x, Q^2) + \Delta \bar{s}(x, Q^2)$ and $\Delta G(x, Q^2)$. We have, in NLO,

$$g_1^p + g_1^n = \frac{1}{18} \left[5\Delta q_+ + 2(\Delta s + \Delta \bar{s}) \right] \otimes \left(1 + \frac{\alpha_s(Q^2)}{2\pi} \delta C_q \right) + \frac{\alpha_s(Q^2)}{2\pi} \Delta G \otimes \delta C_G, \tag{78}$$

and

$$\Delta A_{p+n}^{\pi^+ + \pi^-} = \frac{\Delta \tilde{\sigma}_p^{\pi^+ + \pi^-} + \Delta \tilde{\sigma}_n^{\pi^+ + \pi^-}}{\tilde{\sigma}_p^{\pi^+ + \pi^-} + \tilde{\sigma}_n^{\pi^+ + \pi^-}} \tag{79}$$

where

$$\begin{gathered} \Delta \tilde{\sigma}_p^{\pi^+ + \pi^-} + \Delta \tilde{\sigma}_n^{\pi^+ + \pi^-} = \\ = \left(5\Delta q_+ \left[1 + \otimes \tfrac{\alpha_s}{2\pi} \Delta C_{qq} \otimes \right] D_u^{\pi^+ + \pi^-} + 2(\Delta s + \Delta \bar{s}) \left[1 + \otimes \tfrac{\alpha_s}{2\pi} \Delta C_{qq} \otimes \right] D_s^{\pi^+ + \pi^-} \right) + \\ +18 \left(g_1^p + g_1^n \right) \otimes \tfrac{\alpha_s(Q^2)}{2\pi} \Delta C_{qg} \otimes D_G^{\pi^+ + \pi^-} + \tfrac{6}{\sigma_0} \Delta G \otimes \tfrac{\alpha_s(Q^2)}{2\pi} \Delta C_{gq} \otimes \sigma_T^{\pi^+ + \pi^-} \end{gathered} \tag{80}$$

and

$$\begin{gathered} \tilde{\sigma}_p^{\pi^+ + \pi^-} + \tilde{\sigma}_n^{\pi^+ + \pi^-} = \\ = \left(5q_+ \left[1 + \otimes \tfrac{\alpha_s}{2\pi} \mathbb{C}_{qq} \otimes \right] D_u^{\pi^+ + \pi^-} + 2(s + \bar{s}) \left[1 + \otimes \tfrac{\alpha_s}{2\pi} \mathbb{C}_{qq} \otimes \right] D_s^{\pi^+ + \pi^-} \right) + \\ +18 \left(F_1^p + F_1^n \right) \otimes \tfrac{\alpha_s(Q^2)}{2\pi} \mathbb{C}_{qg} \otimes D_G^{\pi^+ + \pi^-} + \tfrac{6}{\sigma_0} G \otimes \tfrac{\alpha_s(Q^2)}{2\pi} \mathbb{C}_{gq} \otimes \sigma_T^{\pi^+ + \pi^-}. \end{gathered} \tag{81}$$

Note that an analogous set of equations hold for $\pi^+ + \pi^-$ replaced by K or $\Lambda + \bar{\Lambda}$.

Eqs. (78) to (81) contain three unknown functions Δq_+, $\Delta s + \Delta \bar{s}$ and ΔG, which nonetheless can all be determined in principle because of their different evolution in Q^2. However, to be at all efficacious such a determination would require a huge range of Q^2, far larger than is available in present day polarised DIS.

On the other hand there is a direct and superior method for obtaining ΔG, namely via $c\bar{c}$ production. This is one of the major goals of the COMPASS experiment at CERN. We shall thus assume that ΔG has been determined, so that the last term on the RHS of (70) may be taken to be known.

It should be then straightforward to determine Δq_+ and $\Delta s + \Delta \bar{s}$ in NLO from a χ^2 analysis of (78) to (81) in which the evolution of Δq_+ and $\Delta s + \Delta \bar{s}$ would involve mixing with the supposed known ΔG.

An independent determination of Δq_+ and $\Delta s + \Delta \bar{s}$ could be obtained by combining (78) with ΔA_{p+n}^{K} for kaons and $\Delta A_{p+n}^{\Lambda + \bar{\Lambda}}$ for $\Lambda, \bar{\Lambda}$ production.

Once Δq_+ is known in NLO, we can obtain the individual $\Delta \bar{u}$ and $\Delta \bar{d}$ from (74).

5.6 Concerning $s = \bar{s}$ and $\Delta s = \Delta \bar{s}$ in NLO

It is possible to get some information on $s - \bar{s}$ and $\Delta s - \Delta \bar{s}$ in NLO.

We have, assuming $D_d^{K^+} = D_d^{K^-}$,

$$R_p^{K^+-K^-} =$$
$$= \left\{4u_V \left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_u^{K^+-K^-} + (s-\bar{s})\left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_s^{K^+-K^-}\right\} /$$
$$/18F_1^p\left[1 + 2\gamma(y)R^p\right] \tag{82}$$

$$R_n^{K^+-K^-} =$$
$$= \left\{4d_V \left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_u^{K^+-K^-} + (s-\bar{s})\left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_s^{K^+-K^-}\right\} /$$
$$/18F_1^n\left[1 + 2\gamma(y)R^n\right] \tag{83}$$

These two equations, taken together with those for $\Delta A_p^{K^+-K^-}$ and $\Delta A_n^{K^+-K^-}$:

$$\Delta A_p^{K^+-K^-} =$$
$$= \left(4\Delta u_V \left[1 + \otimes\frac{\alpha_s}{2\pi}\Delta C_{qq}\otimes\right] D_u^{K^+-K^-} + (\Delta s-\Delta\bar{s})\left[1 + \otimes\frac{\alpha_s}{2\pi}\Delta C_{qq}\otimes\right] D_s^{K^+-K^-}\right) /$$
$$/\left(4u_V \left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_u^{K^+-K^-} + (s-\bar{s})\left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_s^{K^+-K^-}\right) \tag{84}$$

$$\Delta A_n^{K^+-K^-} =$$
$$= \left(4\Delta d_V \left[1 + \otimes\frac{\alpha_s}{2\pi}\Delta C_{qq}\otimes\right] D_u^{K^+-K^-} + (\Delta s-\Delta\bar{s})\left[1 + \otimes\frac{\alpha_s}{2\pi}\Delta C_{qq}\otimes\right] D_s^{K^+-K^-}\right) /$$
$$/\left(4d_V \left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_u^{K^+-K^-} + (s-\bar{s})\left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_s^{K^+-K^-}\right) \tag{85}$$

provide four equations for the three unknown functions $(s-\bar{s})\otimes D_s^{K^+-K^-}$, $(\Delta s - \Delta\bar{s})\otimes D_s^{K^+-K^-}$ and $D_u^{K^+-K^-}$, so that, in principle, all can be determined via a χ^2 analysis.

In addition one has

$$R_p^{\Lambda-\bar{\Lambda}} =$$
$$= \left\{(4u_V + d_V)\left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_u^{\Lambda-\bar{\Lambda}} + (s-\bar{s})\left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_s^{\Lambda-\bar{\Lambda}}\right\} /$$
$$/18F_1^p\left[1 + 2\gamma(y)R^p\right] \tag{86}$$

and

$$R_n^{\Lambda-\bar{\Lambda}} =$$
$$= \left\{(4d_V + u_V)\left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_u^{\Lambda-\bar{\Lambda}} + (s-\bar{s})\left[1 + \otimes\frac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_s^{\Lambda-\bar{\Lambda}}\right\} /$$
$$/18F_1^n\left[1 + 2\gamma(y)R^n\right] \tag{87}$$

together with $\Delta A_p^{\Lambda-\bar{\Lambda}}$ and $\Delta A_n^{\Lambda-\bar{\Lambda}}$:

$$\Delta A_p^{\Lambda-\bar{\Lambda}} =$$
$$= \left((4\Delta u_V + \Delta d_V)\left[1 + \otimes\tfrac{\alpha_s}{2\pi}\Delta C_{qq}\otimes\right] D_u^{\Lambda-\bar{\Lambda}} + (\Delta s - \Delta\bar{s})\left[1 + \otimes\tfrac{\alpha_s}{2\pi}\Delta C_{qq}\otimes\right] D_s^{\Lambda-\bar{\Lambda}}\right) /$$
$$/ \left((4u_V + d_V)\left[1 + \otimes\tfrac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_u^{\Lambda-\bar{\Lambda}} + (s - \bar{s})\left[1 + \otimes\tfrac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_s^{\Lambda-\bar{\Lambda}}\right) \tag{88}$$

$$\Delta A_n^{\Lambda-\bar{\Lambda}} =$$
$$= \left((4\Delta d_V + \Delta d_V)\left[1 + \otimes\tfrac{\alpha_s}{2\pi}\Delta C_{qq}\otimes\right] D_u^{\Lambda-\bar{\Lambda}} + (\Delta s - \Delta\bar{s})\left[1 + \otimes\tfrac{\alpha_s}{2\pi}\Delta C_{qq}\otimes\right] D_s^{\Lambda-\bar{\Lambda}}\right) /$$
$$/ \left((4d_V + d_V)\left[1 + \otimes\tfrac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_u^{\Lambda-\bar{\Lambda}} + (s - \bar{s})\left[1 + \otimes\tfrac{\alpha_s}{2\pi}\mathbb{C}_{qq}\otimes\right] D_s^{\Lambda-\bar{\Lambda}}\right) \tag{89}$$

These provide four more equations but only two new unknown functions $D_u^{\Lambda-\bar{\Lambda}}$ and $D_s^{\Lambda-\bar{\Lambda}}$. The system of eight equations (82) - (89) therefore over-constrains the unknown functions and might, hopefully, allow a reasonable determination of the relation between $s - \bar{s}$ and $\Delta s - \Delta\bar{s}$ and whether or not $s(x) = \bar{s}(x)$ and/or $\Delta s(x) = \Delta\bar{s}(x)$. The actual determination of $s - \bar{s}$ or $\Delta s - \Delta\bar{s}$ requires knowledge of either $D_s^{\Lambda-\bar{\Lambda}}$ or $D_s^{K^+-K^-}$. These could be taken from the study of $e^+e^- \to hX$.

This completes the determination of all the polarized densities in NLO.

6 Conclusions

We have argued that the present LO QCD method of analysing polarized semi-inclusive DIS, using the concept of purity, is quite unjustified. We have also argued against attempts at a global analysis, either in LO or in NLO QCD, based on the combined data on polarized inclusive and semi-inclusive DIS and taking as known exactly the relevant FF's.

Instead, we have presented a strategy for a step by step evaluation of the polarized parton densities and fragmentation functions from semi-inclusive data using selectively chosen information from inclusive DIS reactions.

In our approach the usually made simplifying assumptions about relations between $\Delta\bar{u}$ and $\Delta\bar{d}$, and between the strange and non-strange polarized sea densities are unnecessary and we have even considered the possibility that $s(x) \neq \bar{s}(x)$ and $\Delta s(x) \neq \Delta\bar{s}(x)$.

Given the simplicity of the LO QCD analysis, we discuss where and when it is likely to be reliable, and stress the need to test the consistency of the LO treatment at each step. In this connection we have introduced the concept of a *passive* variable in the experimentally measured observables. We have also suggested how one might estimate the errors induced in doing the LO analysis.

In the NLO treatment we have shown how the expressions for the experimental observables can be much simplified by incorporating information from the reaction $e^+e^- \to hX$ in a novel way. Determination of the polarized valence quark densities Δu_V and Δd_V is shown to be relatively straight forward, as is the difference $\Delta\bar{u} - \Delta\bar{d}$. However we argue that the determination of $\Delta\bar{u}$, $\Delta\bar{d}$, $\Delta s + \Delta\bar{s}$ and ΔG separately, from semi-inclusive DIS involving production of π, K, Λ is unlikely to be successful, because of the limited range of Q^2 available now and in the foreseeable future. It is suggested that the independent determination of ΔG from charm production is thus an essential element if $\Delta\bar{u}$, $\Delta\bar{d}$, $\Delta s + \Delta\bar{s}$ are to be determined accurately. Finally, motivated by the arguments that possibly $s(x) \neq \bar{s}(x)$ and $\Delta s(x) \neq \Delta\bar{s}(x)$, we have demonstrated how, in principle, one can learn about $s(x) - \bar{s}(x)$ and $\Delta s(x) - \Delta\bar{s}(x)$.

The procedure we have advocated poses a real challenge to the experimentalists, since it requires a control over the systematic errors involved in hadron detection efficiencies. The price paid in the current practice of considering certain ratios of cross sections in order to limit systematic errors, is enormous, and vast amounts of interesting and theoretically valuable information are thereby lost. We hope this paper will encourage efforts to proceed further.

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