HYPER VERSION

Received: *August 17, 2000,* Accepted: *September 1, 2000*
Revised: *September 1, 2000*

Supergravity and light-like non-commutativity

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Abstract: We construct dual supergravity descriptions of field theories and little string theories with light-like non-commutativity. The field theories are realized on the world-volume of Dp-branes with light-like NS B-field and M5-branes with light-like C-field. The little string theories are realized on the world-volume of NS5-branes with light-like RR A-fields. The supergravity backgrounds are closely related to the $A = 0, B = 0, C = 0$ backgrounds. We discuss the implications of these results. We also construct dual supergravity descriptions of ODp theories realized on the worldvolume of NS5-branes with RR backgrounds.

Keywords: D-branes, AdS/CFT Correspondance.

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1. Introduction

Non-commutative field theories, open string theories and open membrane theories have been extensively studied recently. These theories are realized on the world-volume of Dp-branes with a non-zero NS B-field and M5-branes with non-zero C-field.

While space non-commutativity can be accommodated within field theory [1, 2, 3], space-time non-commutativity seems to require string theory for consistency [4]–[8].[1] However, it was argued in [25] that light-like non-commutativity (that is, $[x^\mu, x^\nu] = i\theta^{\mu\nu}$, where $\theta^{\mu\nu}$ is light-like, e.g. $\theta^{+2} \neq 0$) can be realized within field theories. One of the aims of this paper is to construct dual supergravity descriptions of such field theories in various dimensions. The field theories are realized on the world-volume of Dp-branes with light-like NS B-field in a particular decoupling limit.

Of particular interest is a six-dimensional field theory with light-like non-commutativity realized on the world-volume of M5-branes with light-like C-field. This theory is conjectured to have a DLCQ matrix description as a quantum mechanics on the resolved moduli space of instantons with the light-like C-field corresponding to the resolution parameter. We will construct a dual supergravity description of this

[1] Dual supergravity descriptions of these theories have been constructed in [9, 10]. Further interesting progress in the study of theories with space-time non-commutativity has been made in [11]–[24].

theory. We will see that the supergravity descriptions of field theories with light-like non-commutativity are closely related to those without non-commutativity, and we will discuss the implications.

Another class of interesting theories are the non-commutative little string theories. These theories are realized on the world-volume of NS5-branes with light-like RR A-fields. We will construct dual supergravity descriptions of these theories.

A second aim of this paper is to construct dual supergravity descriptions of theories realized on the worldvolume of NS5-branes, whose excitations include light-open Dp-branes (ODp) [8, 23]. Such backgrounds are obtained from NS5-branes with near critical RR fields.

The paper is organized as follows. In section 2 we construct Dp-brane, M5-brane and NS5-brane solutions corresponding to theories in various dimensions with light-like non-commutativity. We also discuss some salient aspects of these solutions and what we learn from them about the corresponding field theories. In section 3 we construct supergravity solutions of NS5-branes with RR backgrounds and obtain dual supergravity descriptions of ODp theories. As an application, we compute the absorption cross section of a graviton polarized along the world-volume.

2. Branes with light-like background fields

2.1 Construction of D-brane solutions

Our aim is to construct supergravity backgrounds with Dp-brane charge and a B-field with components B_{-2}. We will start with the D3-brane case. The construction can be done by performing an infinite Lorentz boost in the x^1 direction on the known [9, 26] D3-brane background in the presence of a B_{12} field. A finite Lorentz boost gives the following background

$$ds^2 = f^{-1/2}\left[-d\tilde{x}_0^2 + dx_3^2 + \frac{f}{H}\left(d\tilde{x}_1^2 + dx_2^2\right)\right] + f^{1/2}\left(dr^2 + r^2 d\Omega_5^2\right), \tag{2.1}$$

$$\begin{aligned} f &= 1 + \frac{\alpha'^2 R^4}{r^4}, \qquad H = 1 + \frac{\alpha'^2 R^4}{r^4}\cos^2\alpha, \\ e^{2\phi} &= g_s^2 \frac{f}{H}, \qquad F_{\tilde{0}\tilde{1}23r} = \frac{1}{g_s}\cos\alpha \frac{f}{H}\partial_r f^{-1}, \\ B &= \tan\alpha H^{-1} d\tilde{x}_1 \wedge dx_2, \qquad A = \frac{1}{g_s}\sin\alpha f^{-1} d\tilde{x}_0 \wedge dx_3, \end{aligned} \tag{2.2}$$

where

$$\tilde{x}_0 = \cosh\gamma x_0 - \sinh\gamma x_1, \qquad \tilde{x}_1 = -\sinh\gamma x_0 + \cosh\gamma x_1, \tag{2.3}$$

or $\tilde{x}_+ = e^{-\gamma}x_+$, $\tilde{x}_- = e^{\gamma}x_-$, with $x_\pm = \pm x_0 + x_1$, and A is the RR 2-form. Note also that $F_{\tilde{0}\tilde{1}23r} = F_{0123r}$.

We obtain

$$
\begin{aligned}
ds^2 &= f^{-1/2}\Big[-dx_0^2+dx_1^2+dx_3^2+\frac{f}{H}dx_2^2+\frac{\alpha'^2R^4}{Hr^4}\sin^2\alpha(\cosh\gamma\;dx_1-\sinh\gamma\;dx_0)^2\Big]+\\
&\quad+f^{1/2}\left(dr^2+r^2d\Omega_5^2\right), \qquad (2.4)\\
e^{2\phi} &= g_s^2\frac{f}{H}, \qquad F_{0123r}=\frac{1}{g_s}\cos\alpha\frac{f}{H}\partial_r f^{-1},\\
B_{02} &= -\tan\alpha\sinh\gamma H^{-1}, \qquad B_{12}=\tan\alpha\cosh\gamma H^{-1}, \qquad (2.5)\\
A_{03} &= \frac{1}{g_s}\sin\alpha\cosh\gamma f^{-1}, \qquad A_{13}=-\frac{1}{g_s}\sin\alpha\sinh\gamma f^{-1}. \qquad (2.6)
\end{aligned}
$$

The fields take the following asymptotic values:

$$
\begin{aligned}
B_{02}^{\infty} &= -\tan\alpha\sinh\gamma\equiv E, \qquad B_{12}^{\infty}=\tan\alpha\cosh\gamma\equiv B, \qquad (2.7)\\
A_{03}^{\infty} &= \frac{1}{g_s}\sin\alpha\cosh\gamma=\frac{\sqrt{B^2-E^2}\cosh\gamma}{g_s\sqrt{1+B^2-E^2}},\\
A_{13}^{\infty} &= -\frac{1}{g_s}\sin\alpha\sinh\gamma=-\frac{\sqrt{B^2-E^2}\sinh\gamma}{g_s\sqrt{1+B^2-E^2}},
\end{aligned}
$$

where we have used $B^2-E^2=\tan^2\alpha$. To have only light-like B-field components, we now take the infinite boost limit, $\gamma\to\infty$. At the same time, we must take the limit $\alpha\to 0$ with

$$e^{\gamma}\tan\alpha=\text{finite}\equiv b\,.$$

In this limit the asymptotic values of the gauge fields simply become

$$B_{02}^{\infty}=E=-b\,, \qquad B_{12}^{\infty}=B=b\,, \qquad A_{03}^{\infty}=\frac{b}{g_s}=-A_{13}^{\infty}\,. \qquad (2.8)$$

We obtain the background

$$
\begin{aligned}
ds^2 &= f^{-1/2}\left[dx_+dx_-+dx_2^2+dx_3^2+\frac{\alpha'^2R^4b^2}{r^4f}dx_-^2\right]+f^{1/2}\left(dr^2+r^2d\Omega_5^2\right), \qquad (2.9)\\
e^{2\phi} &= g_s^2\,, \qquad F_{0123r}=\frac{1}{g_s}\partial_r f^{-1},\\
B &= bf^{-1}\,dx_-\wedge dx_2\,, \qquad A=-\frac{1}{g_s}bf^{-1}\,dx_-\wedge dx_3\,. \qquad (2.10)
\end{aligned}
$$

Note that this is a constant dilaton solution representing a wave travelling on the D3-brane, but with B- and A-fields. The same background is obtained by a similar procedure by starting with the purely electric field configuration $B_{03}\neq 0$ and performing a Lorentz boost. An alternative derivation by dualities is described in subsection 2.3.

Let us consider the decoupling limit $\alpha'\to 0$. Before taking the limit, it is convenient to redefine the coordinate $x_+\to x_+-b^2x_-$. The metric (2.9) takes the form

$$ds^2 = f^{-1/2}\left[dx_+dx_- + dx_2^2 + dx_3^2 - \frac{b^2}{f}dx_-^2\right] + f^{1/2}\left(dr^2 + r^2 d\Omega_5^2\right). \tag{2.11}$$

We set as usual $r = \alpha' u$, with u fixed. In addition, in order to have non-vanishing gauge fields B and A after $\alpha' \to 0$, we must rescale b by introducing $\tilde{b} = \alpha' b$= fixed. So $u, R, \tilde{b}, g_s$ remain fixed. We get the following background:

$$\begin{aligned}
ds^2 &= \alpha'\left(\frac{u^2}{R^2}\left[dx_-dx_+ + dx_2^2 + dx_3^2 - \frac{\tilde{b}^2}{R^4}u^4 dx_-^2\right] + R^2\frac{du^2}{u^2} + R^2 d\Omega_5^2\right), \\
e^{2\phi} &= g_s^2\,, \qquad F_{0123r} = \alpha'^2\frac{4}{g_s R^4}u^3\,, \qquad R^4 = 4\pi g_s N = 2g_{\rm YM}^2 N\,, \\
B &= \alpha'\tilde{b}\frac{u^4}{R^4}\,dx_- \wedge dx_2\,, \qquad A = -\alpha'\frac{\tilde{b}}{g_s}\frac{u^4}{R^4}\,dx_- \wedge dx_3\,, \qquad x_\pm = x_1 \pm x_0\,.
\end{aligned} \tag{2.12}$$

Note that g_s was maintained fixed and $b \to \infty$ in the decoupling limit, in accordance with the field theory analysis of [25].

The above solutions can be easily generalized to the case of the Dp-branes, with $p = 2, \ldots, 5$. A similar procedure gives the following solution representing a wave on the Dp-brane:

$$\begin{aligned}
ds^2 &= f^{-1/2}\left[dx_+dx_- + dx_2^2 + \cdots + dx_p^2 - \frac{b^2}{f}dx_-^2\right] + f^{1/2}\left(dr^2 + r^2 d\Omega_{8-p}^2\right), \\
e^{2\phi} &= g_s^2 f^{\frac{3-p}{2}}\,, \qquad f = 1 + \frac{c_p \alpha'^{5-p} g_{\rm YM}^2 N}{r^{7-p}}\,, \qquad c_p = 2^{7-2p}\pi^{\frac{9-3p}{2}}\Gamma\left(\frac{7-p}{2}\right), \\
B_{-2} &= b f^{-1}\,, \qquad A^{(p-1)}_{-3\cdots} = -\frac{b}{g_s}f^{-1}\,,
\end{aligned} \tag{2.13}$$

where $A^{(p-1)}$ is an RR $(p-1)$-form. Note that for D2-brane we have a one-form A^1_-. There is also the usual RR form which gives the Dp-brane charge. The gauge coupling is

$$g_{\rm YM}^2 = (2\pi)^{p-2} g_s\, \alpha'^{\frac{p-3}{2}}\,. \tag{2.14}$$

The decoupling limit is obtained by rescaling variables as follows:

$$r = \alpha' u\,, \qquad \tilde{b} = \alpha' b\,,$$

and taking the limit $\alpha' \to 0$ with $u, \tilde{b}, g_{\rm YM}^2$ fixed.

We get

$$\begin{aligned}
ds^2 &= \alpha'\frac{u^{\frac{p-3}{2}}}{\sqrt{\lambda}}\left(u^{5-p}\left[dx_-dx_+ + dx_2^2 + \cdots + dx_p^2 - \tilde{b}^2\frac{u^{7-p}}{\lambda}dx_-^2\right] + \lambda\frac{du^2}{u^2} + \lambda d\Omega_{8-p}^2\right), \\
e^{\phi} &= g_{\rm YM}^2\frac{u^{\frac{1}{4}(p-3)(7-p)}}{(2\pi)^{p-2}\lambda^{\frac{1}{4}(p-3)}} = \frac{1}{N}\frac{(\lambda u^{p-3})^{\frac{1}{4}(7-p)}}{c_p(2\pi)^{p-2}}\,, \quad \lambda \equiv c_p g_{\rm YM}^2 N\,, \\
B &= \alpha'\tilde{b}\frac{u^{7-p}}{\lambda}\,dx_- \wedge dx_2\,.
\end{aligned} \tag{2.15}$$

2.2 Light-like non-commutative SYM

The Dp-brane background (2.15) should provide a dual supergravity description of the $(p+1)$-dimensional light-like non-commutative super Yang-Mills theory, where $\theta^{\mu\nu}$ has only non-vanishing $\theta^{+2} = -\theta^{2+}$ components. Since the parameter $\tilde{b}$ can be scaled away from the metric by $x_- \to x_-/\tilde{b}$, $x_+ \to x_+\tilde{b}$, the curvature invariants are independent of $\tilde{b}$. They are functions only of the coordinate u, and therefore they must be the same as in the (commutative) $\tilde{b} = 0$ case. Thus the regime of validity of supergravity approximation is as in the commutative case.

The curvature of the metric in eq. (2.15) has the behavior

$$l_s^2 \mathcal{R} \sim \frac{1}{g_{\rm eff}}\,, \tag{2.16}$$

where we have defined a dimensionless effective gauge coupling $g_{\rm eff}$ [27]

$$g_{\rm eff}^2 = g_{\rm YM}^2\, N u^{p-3}\,. \tag{2.17}$$

As usual, when $g_{\rm eff} \ll 1$ the perturbative field theory description is valid, while when $g_{\rm eff} \gg 1$ the dual supergravity description is valid. The expressions for the dilaton and curvature thus indicate that the phase structure of the non-commutative light-like theory is similar to that of the ordinary Dp-branes.

Let us now consider the case $p = 3$ in detail. The D3-brane background (2.12) provides a dual description of (3+1)-dimensional super Yang-Mills theory with light-like non-commutativity. The solution (2.12) is the first example of a constant dilaton solution describing a non-commutative field theory. Remarkably, this geometry has also constant curvature invariants, e.g. $\mathcal{R}$, $\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}$, and $\mathcal{R}^2_{\mu\nu\rho\sigma}$, which have just the same values as in the $AdS_5 \times S^5$ case. The fact that they are constants can be understood from the invariance of the metric under the scaling $u \to c\, u$, with an appropriate rescaling of x_+, x_-, x_2, x_3. In particular, although the Weyl tensor has some non-vanishing components, the square Weyl tensor $C^2_{\mu\nu\rho\lambda}$ is identically zero. Along with the fact that the dilaton is constant, this suggests a mild energy dependence of the gauge coupling. In the low energy region $u \cong 0$, the metric approaches $AdS_5 \times S^5$, which is consistent with the expectation that the low-energy theory should be described by the usual $\mathcal{N} = 4$ SYM theory.

For $g_s \gg 1$, the dilaton is large and we have to use the S-dual picture. The S-dual background is simply obtained by $g_s \to 1/g_s$ and exchanging the gauge fields B and A, which gives B_{-3} and A_{-2} non-zero components. Therefore the strong coupling limit of SYM theory with light-like non-commutativity is another NCSYM theory with non-commutativity in light-like directions, but with θ^{+3} instead of θ^{+2}. The exchange of 2-3 directions can be undone by an SO(2) rotation in the plane x_2-x_3, since light-like NCSYM theory is invariant under such SO(2) transformations

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(see also [24]). More general $\mathrm{SL}(2,\mathbb{R})$ transformations only mix the B_{-2} and A_{-3} components and introduce a constant RR scalar field χ (which in the low-energy Yang-Mills theory gives rise to a Θ term $\Theta\int\tilde{F}F$).

The supergravity background (2.12) is notably simple, in fact, given by a simple perturbation of the $AdS_5\times S^5$ background:

$$g_{\mu\nu}\longrightarrow g_{\mu\nu}+\delta g_{\mu\nu}\,,\qquad \delta g_{--}=-\tilde{b}^2\frac{u^6}{R^6}\,, \tag{2.18}$$

$$\delta B_{-2}=\tilde{b}\frac{u^4}{R^4}\,,\qquad \delta A_{-3}=-\frac{\tilde{b}}{g_s}\frac{u^4}{R^4}\,. \tag{2.19}$$

The form of the perturbations (2.18), (2.19) implies that as an expansion in the non-commutativity parameter $\tilde{b}$ the first-order perturbation around the $\mathcal{N}=4$ background is exact. Note in comparison that for spatial non-commutativity, say in coordinates x_2, x_3, there are infinite number of terms in the expansion in the non-commutativity parameter around the $\mathcal{N}=4$ background. This suggests that there could exist a simple modification of the $\mathcal{N}=4$ SYM theory lagrangian which turns it into a light-like non-commutative SYM theory. The pure gauge theory part of the dimension six SCFT operator corresponding to δB_{-2} is [28]

$$\mathcal{O}_6=\left[F^{2m}F_{mk}F^{k-}+\frac{1}{4}F_{lm}F^{ml}F^{-2}\right]. \tag{2.20}$$

A question of interest is whether the resulting theory, after adding a perturbation of the form $\mathcal{O}_6\delta B_{-2}$ to the SYM action, is exactly equivalent to the non-commutative field theory action. This is not the case and one can check that with light-like non-commutativity there are still infinite number of terms. In particular, in the abelian case one can compute the NCSYM action using Seiberg-Witten map [3], and show that the lagrangian contains arbitrary powers in θ^{+2}.[2]

Thus, while in the perturbative SYM description there are an infinite number of terms, there seems to be a simplification at strong t' Hooft coupling g_sN. The S-duality symmetry of the theory is not useful in this limit, since the simplification seems to take place at strong 't Hooft coupling g_sN and large N, but $g_s \ll 1$. A possible explanation for the simplification at strong coupling is an existence of a simple resummation of the perturbation series.

The strong coupling expansion is different from that of the ordinary $\mathcal{N}=4$ SYM theory, since the geometries of the corresponding supergravity backgrounds are different. From the field theory point of view, now the lagrangian contains a dimension 6 operator, and the resulting theory is expected to be non-renormalizable. In the spatial NCSYM theory it is important to have an infinite series of terms. If there are only a finite number of terms it is hard to see why renormalization

[2]However, it is interesting to note that in the particular case that $F_{-2}=0$, the series truncates and the full lagrangian contains only the usual SYM action plus a linear term in θ^{+2}.

works. Renormalizability in the light-like non-commutativity seems to work in the same way as in the spatial NCSYM theory, in the sense that the divergences in the planar diagrams are taken care of as in the SYM case. For the non-planar case, as long as $(p_0 - p_1)^2 \neq 0$ there are no divergences, whereas when $(p_0 - p_1)^2 = 0$ we get the divergences that are interpreted as IR divergences.[3] It is worth noting that in a gauge $A_- = 0$ the new interaction (2.20) will always involve multiplicative factors p_-, which may lead to an improvement of the IR behavior of non-planar diagrams. Clearly, a more detailed analysis is needed in order to understand the structure of perturbation theory for the $\mathcal{N} = 4$ SYM theory with light-like non-commutativity.

2.3 M5-brane with light-like C-field

To find a solution representing an M5-brane in the presence of a light-like C-field, we can proceed as above and boost the M5-brane solution with C_{345} component [9], in the direction x_3. This is

$$\begin{aligned} ds^2_{11} &= (kf)^{1/3}\Big[\frac{1}{f}\left(-d\tilde{x}_0^2 + dx_1^2 + dx_2^2\right) + \frac{1}{k}\left(d\tilde{x}_3^2 + dx_4^2 + dx_5^2\right) + \\ &\qquad + dr^2 + r^2 d\Omega_4^2\Big]\,, \end{aligned} \tag{2.21}$$

$$\begin{aligned} f &= 1 + \frac{l_p^3 R^3}{r^3}\,, \qquad k = 1 + \cos^2\alpha\frac{l_p^3 R^3}{r^3}\,, \qquad R^3 = \frac{\pi N}{\cos\alpha}\,, \\ dC_3 &= \sin\alpha\; df^{-1}\wedge d\tilde{x}_0 \wedge dx_1 \wedge dx_2 + \cos\alpha\; 3R^3 l_p^3 \epsilon_4 - \\ &\quad - 6\tan\alpha\; dk^{-1}\wedge d\tilde{x}_3\wedge dx_4 \wedge dx_5\,, \end{aligned} \tag{2.22}$$

where by ϵ_4 we denote the volume form of the 4-sphere, and we have made the Lorentz boost

$$\tilde{x}_0 = \cosh\gamma\, x_0 - \sinh\gamma\, x_3\,, \qquad \tilde{x}_3 = -\sinh\gamma\, x_0 + \cosh\gamma\, x_3\,. \tag{2.23}$$

Now we take the limit $\gamma\to\infty$, $\alpha\to 0$ with $e^\gamma\tan\alpha = b =$ fixed. We get

$$\begin{aligned} ds^2_{11} &= f^{-1/3}\left[dx_+ dx_- + dx_1^2 + dx_2^2 + dx_4^2 + dx_5^2 - \frac{b^2}{f}dx_-^2\right] + \\ &\quad + f^{2/3}\big(dr^2 + r^2 d\Omega_4^2\big)\,, \end{aligned} \tag{2.24}$$

$$\begin{aligned} dC_3 &= bdf^{-1}\wedge dx_-\wedge dx_1\wedge dx_2 + 3R^3 l_p^3\epsilon_4 - \\ &\quad - 6bdf^{-1}\wedge dx_-\wedge dx_4\wedge dx_5\,, \end{aligned} \tag{2.25}$$

where we have redefined $x^+ \to x^+ - b^2 x^-$. This represents a gravitational wave moving parallel to the M5-brane.

[3] We would like to thank J. Gomis for a discussion on this point.

The decoupling limit is taken by rescaling variables as follows:

$$r = l_p^3 u^2 \,, \qquad b = l_p^{-3} \tilde{b} \,, \tag{2.26}$$

and then taking $l_p \to 0$ with fixed $u, R, \tilde{b}$. We obtain

$$\begin{aligned} ds_{11}^2 &= l_p^2 \biggl(\frac{u^2}{(\pi N)^{1/3}} \left[dx_+ dx_- + dx_1^2 + dx_2^2 + dx_4^2 + dx_5^2 - \frac{\tilde{b}^2}{\pi N} u^6 dx_-^2 \right] + \\ &\quad + (\pi N)^{1/3} \left[\frac{4du^2}{u^2} + d\Omega_4^2 \right] \biggr) \,, \end{aligned} \tag{2.27}$$

$$\begin{aligned} dC_3 &= l_p^3 \biggl(\frac{6\tilde{b}u^5}{\pi N} du \wedge dx_- \wedge dx_1 \wedge dx_2 + 3\pi N \epsilon_4 - \\ &\quad - \frac{36\tilde{b}u^5}{\pi N} du \wedge dx_- \wedge dx_4 \wedge dx_5 \biggr) \,. \end{aligned} \tag{2.28}$$

At low energy (small u) the background (2.27), (2.28) reduces to $AdS_7 \times S^4$ as expected. The curvature invariants are the same as in the $AdS_7 \times S^4$ case (again by virtue of the symmetry under rescalings of $\tilde{b}$, combined with a rescaling of coordinates). The background (2.27), (2.28) provides a dual description of the $(0,2)$ theory perturbed by a dimension nine operator. This theory is conjectured to have a matrix-like description as the quantum mechanics on the resolved moduli space of instantons. The light-like C-field is interpreted as the resolution parameters (the Fayet-Iliopoulos parameters of the $0+1$ Yang-Mills theory) [29] (see also [30]).

An alternative derivation of the solution (2.24) is by using as starting point the supergravity solution of M5-branes in the presence of C-field with rank 4 [10]

$$\begin{aligned} ds^2 &= h^{-2/3} \Bigl[f^{-1/3} \left(-dx_0^2 + h dx_{1,2,3,4}^2 + h^2 (dx_5 - C dx_0)^2 \right) + f^{2/3} (dr^2 + r^2 d\Omega_4^2) \Bigr] \,, \\ f &= 1 + \frac{\pi N l_p^3}{\cos^2\theta r^3} \,, \qquad h^{-1} = \sin^2\theta f^{-1} + \cos^2\theta \,, \qquad C = \sin^2\theta f^{-1} \,, \\ C_{012} &= \cos\theta \sin\theta f^{-1} h \,, \qquad C_{345} = \tan\theta f^{-1} h \,, \\ C_{034} &= \cos\theta \sin\theta f^{-1} h \,, \qquad C_{125} = \tan\theta f^{-1} h \,. \end{aligned} \tag{2.29}$$

Dimensional reduction along the direction x_5 gives the D4-brane supergravity background in presence of a B-field with magnetic components. The infinite boost limit can be taken by introducing

$$\tilde{x}_0 = x_0 \cos\theta \,, \qquad \tilde{x}_5 = \frac{x_5}{\cos\theta} \,,$$

and taking the limit $\theta \to \pi/2$ with fixed $\tilde{x}_0, \tilde{x}_5, r$ and *fixed* l_P. In this way one reproduces the background (2.24). The Dp-brane backgrounds can then be obtained by dimensional reduction along either of the coordinates $(1,2,3,4)$ and T-dualities.

2.4 NS5-branes with light-like RR fields

By an S-duality transformation on the D5-brane solution, given by eq. (2.13) with $p = 5$, one finds the solution representing type-IIB NS5-branes in the presence of a light-like RR A-field:

$$
\begin{aligned}
ds^2 &= dx_- dx_+ + \sum_{i=2}^{5} dx_i^2 - \frac{b^2}{f} dx_-^2 + f\left(dr^2 + r^2 d\Omega_3^2\right), \\
f &= 1 + \frac{N l_s^2}{r^2}, \qquad e^{2\phi} = g_s^2 f, \qquad l_s \equiv \sqrt{\alpha'}, \\
A_{-2} &= \frac{b}{g_s} f^{-1}, \qquad A_{-345} = -\frac{b}{g_s} f^{-1}.
\end{aligned}
\tag{2.30}
$$

Using T-duality one can also find the type-IIA NS5-branes with light-like RR fields. T-duality in the direction x_2 gives an NS5-brane in the presence of light-like RR one- and four-forms,

$$
A_- = \frac{b}{g_s} f^{-1}, \qquad A_{-2345} = -\frac{b}{g_s} f^{-1}, \tag{2.31}
$$

with the same metric and dilaton fields. T-duality in the direction x_3 gives an NS5-brane in the presence of light-like RR three-form, with components

$$
A_{-23} = \frac{b}{g_s} f^{-1}, \qquad A_{-45} = -\frac{b}{g_s} f^{-1}. \tag{2.32}
$$

The decoupling limit for these NS5-branes with light-like gauge fields is taken in the same way as that for the usual NS5-branes [25], namely, $g_s \to 0$ and l_s=fixed, but in addition we have to rescale $\tilde{b} = g_s b, r = g_s l_s u$ with fixed $\tilde{b}$ and u. Setting $u = N^{1/2} e^{z/r_0}$, with $r_0 = l_s \sqrt{N}$, we get for the type-IIB NS5-branes (2.30)

$$
\begin{aligned}
ds^2 &= dx_- dx_+ + \sum_{i=2}^{5} dx_i^2 - \tilde{b}^2 e^{2z/r_0} dx_-^2 + dz^2 + r_0^2 d\Omega_3^2, \\
A_{-2} &= \tilde{b}\, e^{2z/r_0}, \qquad A_{-345} = -\tilde{b}\, e^{2z/r_0}, \qquad \phi = -\frac{z}{r_0}.
\end{aligned}
\tag{2.33}
$$

For the type-IIA NS5-branes, the metric and dilaton are the same, and the gauge field components in eqs. (2.31) and (2.32) become $A_- = \tilde{b}\, e^{2z/r_0}$, etc. These backgrounds provide dual descriptions of non-commutative little string theories. The deformation parameters are the light-like RR fields. The phase structure of the theories is the same as the ordinary little string theories. They are characterized by the linear dilaton behavior.

The type-IIB NS5-brane has a DLCQ description as a low energy SCFT of the Coulomb branch of a $(1+1)$-dimensional gauge theory [31, 32, 33]. In this case, the deformation parameters are identified as mass parameters. For the type-IIA NS5-branes, the DLCQ deformation corresponds to turning on a Fayet-Iliopoulos term [33] in the corresponding $(1+1)$-dimensional gauge theory [34, 35].

3. NS5-branes in the presence of RR fields

In this section we will study the theory on the type-II NS5-branes in the presence of different RR field strengths, which can be either electric or magnetic. The supergravity equations of motion require that for NS5-branes in the presence of an RR magnetic (electric) $(p+1)$-form there is also a RR electric (magnetic) $(5-p)$-form with $p = 0, \ldots, 5$. Therefore the theory of NS5-branes in the presence of an electric (magnetic) $(p+1)$-form is the same as the theory on the NS5-branes in the presence of a magnetic (electric) RR $(5-p)$-form. For NS5-branes with a RR 3-form there is no difference between electric and magnetic, so there is only one case to be studied.

The supergravity solution for NS5-branes in the presence of an RR $(p+1)$-form is given by

$$\begin{aligned} ds^2 &= h^{-1/2}\left[-dx_0^2 + \sum_{i=1}^{p} dx_i^2 + h\sum_{j=p+1}^{5} dx_j^2 + f(dr^2 + r^2 d\Omega_3^2)\right], \\ f &= 1 + \frac{Nl_s^2}{\cos\theta\, r^2}, \qquad h^{-1} = \sin^2\theta f^{-1} + \cos^2\theta, \\ A_{0\cdots p} &= \frac{\sin\theta}{g_s} f^{-1}, \qquad A_{(p+1)\cdots 5} = \frac{\tan\theta}{g_s} f^{-1} h, \qquad e^{2\phi} = g_s^2 f h^{(1-p)/2}. \end{aligned} \tag{3.1}$$

For $p = 5$, $A_{(p+1)\cdots 5}$ denotes the RR scalar field. A way to find these solutions is to start with M5-branes in the presence of a C-field (2.21), smeared in some transverse direction. By reducing on this transverse direction, one finds the type-IIA NS5-branes with an electric RR 3-form. Other solutions are generated by T-duality.

The decoupling limit of the above supergravity solution can be defined as the limit $l_s \to 0$, keeping the following quantities fixed:

$$\alpha'_{\text{eff}} = \frac{l_s^2}{\cos\theta}, \qquad u = \frac{r}{l_s^2}, \qquad \tilde{g}\, l_{\text{eff}}^{p-3} = g_s l_s^{p-3}, \tag{3.2}$$

$$\tilde{x}_{0,\ldots,p} = \frac{1}{l_{\text{eff}}} x_{0,\ldots,p}, \qquad \tilde{x}_{(p+1),\ldots,5} = \frac{l_{\text{eff}}}{l_s^2} x_{(p+1),\ldots,5}, \qquad l_{\text{eff}} \equiv \sqrt{\alpha'_{\text{eff}}}. \tag{3.3}$$

In this limit the supergravity solution becomes

$$\begin{aligned} l_s^{-2} ds^2 &= (1 + a^2u^2)^{1/2}\left[-d\tilde{x}_0^2 + \sum_{i=1}^{p} d\tilde{x}_i^2 + \frac{\sum_{j=p+1}^{5} d\tilde{x}_j^2}{1 + a^2u^2} + \frac{N}{u^2}(du^2 + u^2 d\Omega_3^2)\right], \\ A_{0\cdots p} &= \frac{l_s^{(p+1)}}{\tilde{g}} a^2u^2, \qquad A_{(p+1)\cdots 5} = \frac{l_s^{(5-p)}}{\tilde{g}} \frac{a^2u^2}{1 + a^2u^2}, \\ e^{2\phi} &= \tilde{g}^2 \frac{(1 + a^2u^2)^{(p-1)/2}}{a^2u^2}, \quad a^2 = \frac{\alpha'_{\text{eff}}}{N}. \end{aligned} \tag{3.4}$$

These backgrounds provide a supergravity dual description for the ODp theories investigated in [8][4] (with coupling $g_{\text{YM}}^2 = \tilde{g} l_{\text{eff}}^{p-3}$). The scalar curvature of the metric

[4]The supergravity description of ODp with $p = 1, 2$ and $p = 2, 3$ has also been considered in [23, 36], respectively.

is given by

$$l_s^2\mathcal{R} = \frac{1}{N}\frac{c_1 + c_2a^2u^2 + c_3a^4u^4}{(1+a^2u^2)^{5/2}}\,, \tag{3.5}$$

where c_1, c_2, c_3 are numerical constants depending only on p. Therefore for large N the curvature is small.

As an application, let us now consider the absorption cross section of polarized gravitons. This calculation has already been done for the type-IIA NS5-branes in the presence of an RR 3-form and type-IIB NS5-branes in the presence of a magnetic RR 2-form in [36, 37], which correspond to the supergravity dual of OD2 and OD3 theories, respectively. In general one can show that in these backgrounds the scattering potential for a graviton polarized along the brane directions is

$$V(\rho) = -1 + \left(\frac{3}{4} - \omega^2R^2\right)\frac{1}{\rho^2}\,, \quad R^2 = \frac{Nl_s^2}{\cos\theta}\,, \tag{3.6}$$

where $\rho = \omega r$ and ω is the energy of incoming waves. Therefore we see that after the decoupling limit the absorption cross section can be non zero only for waves with energy ω^2 larger than $\sim \frac{1}{N\alpha'_{\text{eff}}}$. Essentially the same effect appears in the little string theories. Following [38], one can see that these theories have a mass gap of order $M^2_{\text{gap}} \sim \frac{1}{N\alpha'_{\text{eff}}}$. To compare the decoupling limit (3.9) with that of ordinary little string theories, it is convenient to describe the decoupling limit in terms of g_s. For $p \leq 2$, one can take the decoupling limit of the NS5-branes in the presence of electric $(p+1)$-form as follows:

$$g_s \longrightarrow 0\,, \qquad \tilde{g}^{\frac{2}{3-p}} = \frac{g_s^{\frac{2}{3-p}}}{\cos\theta}\,, \qquad r = g_s^{\frac{1}{3-p}} l_s u\,. \tag{3.7}$$

with fixed $l_s, u, \tilde{g}$. In this limit the supergravity background (3.1) reduces to the same expression (3.4). This description is equivalent to a rescaling of the coordinates. In this way one has $g_s \to 0$ and l_s fixed, as in the little string theory.

The ODp theories have all the same physics at low energies: for odd p they flow to SYM theory in (5+1)-dimension in the IR; for even p, the theories flow to a fixed point in the IR described by the $(0,2)$ conformal theory.

In the ultraviolet regime, where the effects of non-zero RR fields become important, the different ODp theories exhibit different behaviors, according to the value of p.

For $p \leq 2$ case, the string coupling e^{ϕ} in eq. (3.4) is small in the ultraviolet regime and one can trust the supergravity solution. In this region $u \gg a^{-1}$ the NS5-brane supergravity reduces to a metric describing ordinary Dp-branes smeared in $5-p$ directions. In the particular case of the OD0 theory, the supergravity solution (3.4) provides a supergravity description of a DLCQ compactification of M-theory with N units of DLCQ momentum, in the presence of a transverse M5-brane. The relation between M-theory and type-IIA parameters is as follows:

$$\alpha'_{\text{eff}}\tilde{g}^{2/3} = M_{\text{eff}}^{-2}\,, \qquad \alpha'_{\text{eff}}\tilde{g}^{2} = R_{11}^{2}\,, \tag{3.8}$$

where M_{eff} if the effective eleven-dimensional Planck mass.

For $p = 3$ the dilaton in (3.4) is constant at large u, i.e. $e^{\phi} = \tilde{g}$. For $\tilde{g} \ll 1$ the theory can be described by smeared D3-branes, while for $\tilde{g} \gg 1$ we have to use the S-dual picture describing D5-branes in the presence of a magnetic B-field with rank two. Therefore, in the UV regime, strongly coupled OD3 theory and large N $(5+1)$-dimensional NCSYM theory exhibit a similar behavior. Note that under S-duality the parameters of the theory change as

$$\alpha'_{\text{eff}} \longrightarrow \tilde{g}\alpha'_{\text{eff}}\,, \quad \tilde{g} \longrightarrow \frac{1}{\tilde{g}}\,, \tag{3.9}$$

For $p = 4$ the dilaton is large at $u \gg a^{-1}$. In this case it means that the proper supergravity description is in terms of eleven-dimensional supergravity. From M-theory point of view, the supergravity solution is the bound state of two M5-branes in the directions $(0,1,2,3,4,5)$ and $(0,1,2,3,4,6)$ in the decoupling limit.

For $p = 5$, the dilaton is also large at $u \gg a^{-1}$. The S-dual picture is not useful, since the transformed dilaton field ϕ' is also large in this regime. Indeed, due to the non-zero RR 0-form (of order one), under S-duality we find $e^{\phi'} \sim \tilde{g}au$. This is in agreement with the discussion of [8]. It can be understood from the fact that, under S-duality the system maps to a similar configuration of NS5-branes in the presence of electric RR 6-form. If N is the number of NS5-branes and M the charge induced by RR 6-form one has the relation [14] $\frac{1}{\cos\theta} = g_s\frac{M}{N}$. In the decoupling limit (3.2), one obtains

$$\tilde{g} = \frac{N}{M}\,. \tag{3.10}$$

A similar relation is found for D1-branes in the presence of electric B-field [8, 15].

A T-duality transformation on the background (3.4) implies the following relation between the parameters of ODp theory and OD(p-1) theory:

$$R \longrightarrow \frac{\alpha'_{\text{eff}}}{R}\,, \qquad \tilde{g}^2 \longrightarrow \frac{\alpha'_{\text{eff}}}{R^2}\tilde{g}^2\,. \tag{3.11}$$

Thus, from eqs. (3.8), (3.9) and (3.11), we see that the parameters of OM, NCOS and ODp theories are related in the same way as the corresponding parameters of type IIA, type IIB and M-theory, as expected.

Acknowledgments

We wish to thank O. Aharony, J. Gomis and A. Tseytlin for valuable discussions. Y.O. would like to thank the USC/Caltech center for theoretical physics for hospitality during the course of this work. J.R. would like to thank CERN for hospitality.

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