The Angular Momentum and g_1^p Sum Rules for the Proton* † ‡

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The gauge invariant operator formulation of the angular momentum sum rule $\frac{1}{2} = J_q + J_g$ for the proton is presented and contrasted with the sum rule for the first moment of the polarised structure function g_1^p. The decoupling of the axial charge a^0 from the angular momentum sum rule is highlighted and the possible QCD field-theoretic basis for an angular momentum sum rule of the form $\frac{1}{2} = \frac{1}{2}\Delta q + \Delta g + L_q + L_g$ is critically discussed.

1. Introduction

In this talk, based on work in collaboration with B. White, I review our recent formulation[1] of the angular momentum sum rule for the proton and discuss its relation to the sum rule for the first moment of the polarised structure function g_1^p. In particular, the role of the axial charge a^0, which is measured in the g_1^p sum rule, is highlighted and it is shown how this decouples from the angular momentum sum rule. This emphasises the limitations of attempting to identify a^0 with quark (and gluon) spin and an alternative interpretation in terms of topological charge is briefly reviewed. The angular momentum sum rule is shown to take the simple form $\frac{1}{2} = J_q + J_g$, where J_q and J_g are gauge and Lorentz invariant form factors of the forward matrix elements of local operators, which may reasonably be interpreted as quark and gluon components of the total angular momentum of the proton. Experimentally, they may be measured in, for example, deeply virtual Compton scattering. Their RG evolution properties are derived from a careful analysis of operator mixing. We also discuss critically whether there is indeed a QCD field-theoretic basis for a decomposition of the proton spin into separate quark and gluon spin and orbital angular momentum components, as in the frequently-quoted sum rule $\frac{1}{2} = \frac{1}{2}\Delta q + \Delta g + L_q + L_g$.

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2. The g_1^p sum rule and topological charge

The sum rule for the first moment of g_1^p (see e.g. ref.[2] for reviews of our earlier work and references) is

$$\int_0^1 dx\, g_1^p(x,Q^2) = \frac{1}{12}C_1^{\rm NS}\big(a^3+\frac{1}{3}a^8\big)+\frac{1}{9}C_1^{\rm S}a^0(Q^2) \quad (1)$$

where the C_1 are Wilson coefficients and the flavour singlet axial charge a^0 is defined as the form factor in the forward matrix element of the corresponding axial current,

$$\langle p,s|A_\mu^0|p,s\rangle = a^0 s_\mu \quad (2)$$

where s_μ is the covariant spin vector. Since A_μ^0 is not a conserved current, due to the $U_A(1)$ anomaly

$$\partial^\mu A_\mu^0 - 2n_f Q \sim 0 \quad (3)$$

where $Q = \frac{\alpha_s}{8\pi}\epsilon^{\mu\nu\rho\sigma}{\rm tr}F_{\mu\nu}F_{\rho\sigma}$ is the gluon topological charge density, the form factor a^0 is scale dependent and satisfies the non-trivial RG evolution equation

$$\frac{d}{dt}a^0(Q^2) = \gamma a^0(Q^2) \tag{4}$$

where $t = \ln Q^2/\Lambda^2$ and the anomalous dimension is $\gamma = -n_f \frac{\alpha_s}{2\pi^2}$.

Given a^3 and a^8 from low-energy neutron and hyperon decays, a^0 can be extracted from polarised inclusive DIS processes $e(\mu)p \to e(\mu)X$. The simplest prediction, $a^0 \simeq a^8$, is an immediate consequence of the OZI rule and transcribed into the sum rule for g_1^p gives the Ellis-Jaffe sum rule. However, the flavour singlet pseudovector or pseudoscalar channel is precisely where we would expect to find strong OZI violations related to the $U_A(1)$ anomaly and indeed it is found experimentally that $a^0 \ll a^8$.

Since we can rewrite a^0 using eq.(3) in terms of the matrix element of the topological charge density,

$$a^0 = \langle p, s|Q|p, s\rangle \tag{5}$$

we see immediately that the observed suppression in a^0 is a manifestation of topological charge screening. This interpretation has been developed in a series of papers written in collaboration with Veneziano, Narison and De Florian[2]. Our proposal is that this screening is *universal*, i.e. target-independent, being an intrinsic property of the QCD vacuum itself. Specifically, we showed that (in the chiral limit)

$$a^0 = \frac{1}{2M}2n_f\left[\chi(0)\Gamma_{Qpp} + \sqrt{\chi'(0)}\Gamma_{\Phi_5 pp}\right] \tag{6}$$

where the Γ are suitably defined 1PI vertex functions and $\chi(k^2) = i\int d^4x e^{ik.x}\langle 0|TQ(x)Q(0)|\rangle$ is the *topological susceptibility*, $\chi'(0)$ being its slope at $k = 0$. Since the anomalous chiral Ward identity implies $\chi(0) = 0$ in the chiral limit, the sole contribution to a^0 comes from the second term in eq.(6). Making the motivated assumption that the RG invariant vertex $\Gamma_{\Phi_5 pp}$ obeys the OZI rule to a good approximation, we conjecture that the principal origin of the suppression in a^0 is an anomalously small value of $\chi'(0)$ due to universal topological charge screening by the QCD vacuum.[5] This is anticipated in certain instanton-based models of the QCD vacuum, has been confirmed quantitatively by QCD spectral sum rule calculations, and is currently being investigated in lattice gauge theory.

The QCD parton model gives an alternative interpretation of a^0 through the identification

$$a^0(Q^2) = \Delta q - 2n_f\frac{\alpha_s}{4\pi}\Delta g(Q^2) \tag{7}$$

where $\Delta q = \Delta u + \Delta d + \Delta s$ and $\Delta g(Q^2)$ are the first moments of the polarised flavour singlet quark and gluon distributions and we have used the AB class of renormalisation schemes where Δq is defined to be Q^2 independent. The OZI/Ellis-Jaffe relation $a^0 = a^8$ follows immediately from the assumption that in the proton, $\Delta s = 0$ and $\Delta g = 0$. In this model, $\frac{1}{2}\Delta q$ and Δg are interpreted as the quark and gluon spins, which led to the initial interpretation of the experimental observation $a^0 \ll a^8$ as indicating that the quarks carry only a small fraction of the spin of the proton – the so-called 'proton spin crisis'.

In the rest of this talk, we derive the actual angular momentum sum rule for the proton in terms of gauge invariant operator matrix elements, with particular emphasis on whether and how the axial charge a^0 appears and whether the interpretation of Δq and Δg as spin components can be complemented by corresponding definitions of orbital angular momentum components L_q and L_g.

3. The angular momentum sum rule

The angular momentum sum rule is derived by taking the forward matrix element of the conserved angular momentum current $M^{\mu\nu\lambda}$, defined from the energy-momentum tensor as

$$M^{\mu\nu\lambda} = x^{[\nu}T^{\lambda]\mu} + \partial_\rho X^{\rho\mu\nu\lambda} \tag{8}$$

The inclusion of the arbitrary tensor $X^{\rho\mu\nu\lambda}$ (antisymmetric under $\rho \leftrightarrow \mu$ and $\nu \leftrightarrow \lambda$) just reflects the usual freedom in QFT in defining conserved currents. However, this arbitrariness allows us to

[5] Explanations which favour OZI violations due to a large polarised strange quark component of the proton or the implications of the Skyrme model of proton structure have been recently reviewed in, for example, ref.[3]

write different equivalent expressions for $M^{\mu\nu\lambda}$ as a sum of local operators, suggesting corresponding interpretations of the total angular momentum as a sum of 'components', which we can try to identify as reasonable definitions of quark and gluon spin and orbital angular momentum[4].

The best decomposition is the following:

$$\begin{aligned}M^{\mu\nu\lambda} &= O_1^{\mu\nu\lambda} + O_2^{\mu[\lambda}x^{\nu]} + O_3^{\mu[\lambda}x^{\nu]} + g^{\mu[\lambda}x^{\nu]}\mathcal{L}_{\rm gi} \\ &\quad + \big\{ i\partial^{\{\mu}\bar{c}D^{[\lambda\}}c + \partial^{\{\mu}BA^{[\lambda\}} + g^{\mu[\lambda}\mathcal{L}_{\rm gf}\big\}x^{\nu]} \\ &\quad -\frac{1}{4}\partial_\rho\big[x^{[\nu}\epsilon^{\lambda]\mu\rho\sigma}\bar{\psi}\gamma_\sigma\gamma_5\psi\big\} + {\rm EOM} + \partial_\rho X^{\rho\mu\nu\lambda} \end{aligned} \tag{9}$$

The tensor $X^{\rho\mu\nu\lambda}$ is chosen to cancel the divergence and equation of motion (EOM) terms, while the forward matrix element of the operator $g^{\mu[\lambda}x^{\nu]}\mathcal{L}_{\rm gi}$ vanishes. The term in $\{\ldots\}$ is the contribution from the covariant gauge-fixing and ghost terms in the Lagrangian, but this turns out to be a BRS variation so its matrix element between physical states vanishes. The remaining (bare) operators are gauge invariant:

$$\begin{aligned} O_1^{\mu\nu\lambda} &= \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}\gamma_\sigma\gamma_5\psi \equiv \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}A^0_\sigma \\ O_2^{\mu\lambda} &= i\bar{\psi}\gamma^\mu \overleftrightarrow{D}^\lambda \psi \\ O_3^{\mu\lambda} &= F^{\mu\rho}F_\rho{}^\lambda \end{aligned} \tag{10}$$

We also find it convenient for later use to define $O_4^{\mu\lambda} = \frac{1}{2}O_2^{\{\mu\lambda\}}$. At first sight, $O_1^{\mu\nu\lambda}$, which is just the flavour singlet axial current considered in section 2, looks as if it may be associated with 'quark spin', with $O_2^{\mu[\lambda}x^{\nu]}$ corresponding to a gauge invariant definition of 'quark orbital angular momentum'. This leaves $O_3^{\mu[\lambda}x^{\nu]}$ to be associated with the 'gluon total angular momentum'.

In fact, there is no further decomposition of the gluon contribution as long as we restrict to gauge invariant operators. We can certainly make the alternative decomposition:

$$\begin{aligned} M^{\mu\nu\lambda} &= \\ &\tilde{O}_1^{\mu\nu\lambda} + \tilde{O}_2^{\mu[\lambda}x^{\nu]} + \tilde{O}_3^{\mu\nu\lambda} + \tilde{O}_4^{\mu[\lambda}x^{\nu]} + g^{\mu[\lambda}x^{\nu]}\mathcal{L}_{\rm gi} \\ &+ \big\{A^\mu\partial^{[\lambda}B + i\partial^\mu\bar{c}\partial^{[\lambda}c + i\partial^{[\lambda}\bar{c}D^\mu c + g^{\mu[\lambda}\mathcal{L}_{\rm gf}\big\}x^{\nu]} \\ &+ \partial_\rho\big[x^{[\nu}A^{\lambda]}F^{\mu\rho}\big] + {\rm EOM} + \partial_\rho X^{\rho\mu\nu\lambda} \end{aligned} \tag{11}$$

where

$$\begin{aligned} \tilde{O}_1^{\mu\nu\lambda} &= \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}\gamma_\sigma\gamma_5\psi \\ \tilde{O}_2^{\mu\lambda} &= -i\bar{\psi}\gamma^\mu\partial^\lambda\psi \\ \tilde{O}_3^{\mu\nu\lambda} &= -F^{\mu[\nu}A^{\lambda]} \\ \tilde{O}_4^{\mu\nu} &= F^{\mu\rho}\partial^\lambda A_\rho \end{aligned} \tag{12}$$

and try to identify the first four operators with, respectively, quark spin and orbital and gluon spin and orbital angular momentum. However, even the forward matrix elements of these operators turn out not to be gauge invariant. Moreover, the gauge-fixing and ghost term in $\{\ldots\}$ is no longer a BRS variation so would contribute a non-vanishing 'ghost orbital angular momentum'. Further discussion of the problems with such gauge non-invariant decompositions is given in ref.[1], and from now on we restrict attention to the gauge invariant formulation based on eq.(9).

The next step is to express the matrix elements of the operators $O_1^{\mu\nu\lambda}$, $O_2^{\mu[\lambda}x^{\nu]}$ and $O_3^{\mu[\lambda}x^{\nu]}$ in terms of form factors. There are technical subtleties connected with defining operators of the form Ox and their renormalisation mixing which are explained precisely in ref.[1]. The prescription is essentially to define the forward matrix elements of Ox in terms of the limit of an off-forward matrix element. We therefore write (a little loosely)

$$\langle p|O^{\mu\lambda}x^\nu|p\rangle = -i\frac{\partial}{\partial\Delta_\nu}\langle p|O^{\mu\lambda}|p'\rangle\big|_{p'=p} \tag{13}$$

where $\Delta = p - p'$. We then have[1]

$$\begin{aligned} &\langle p,s|O_1^{\mu\nu\lambda}|p,s\rangle = a^0 M\epsilon^{\mu\nu\lambda\sigma}s_\sigma \\ &\langle p,s|O_2^{\mu[\lambda}x^{\nu]}|p,s\rangle = B_q(0)\frac{1}{2M}p_\rho p^{\{\mu}\epsilon^{[\lambda\}\nu]\rho\sigma}s_\sigma \\ &+\tilde{B}_q(0)\frac{1}{2M}p_\rho p^{[\mu}\epsilon^{[\lambda]\nu]\rho\sigma}s_\sigma - 2D_q(0)M\epsilon^{\mu\nu\lambda\sigma}s_\sigma \\ &\langle p,s|O_3^{\mu[\lambda}x^{\nu]}|p,s\rangle = B_g(0)\frac{1}{2M}p_\rho p^{\{\mu}\epsilon^{[\lambda\}\nu]\rho\sigma}s_\sigma \end{aligned} \tag{14}$$

The crucial observation now follows from the identity

$$O_1^{\mu\nu\lambda} + O_2^{\mu[\lambda}x^{\nu]} = O_4^{\mu[\lambda}x^{\nu]} + {\rm divergence} + {\rm EOM} \tag{15}$$

Since the matrix elements of the divergence and EOM terms vanish, and recalling that the operator $O_4^{\mu\lambda}$ is symmetric in μ, λ, we see that

$$\tilde{B}_q(0) = 0 \qquad\qquad 2D_q(0) = a^0 \tag{16}$$

Thus a^0 appears not only as the unique form factor in the matrix element $\langle O_1 \rangle$ of the axial current, but also as a contribution to $\langle O_2 x \rangle$. It therefore *cancels* from the angular momentum sum rule.

Introducing the notation $J_q = B_q(0)$, $J_g = B_g(0)$, then from eq.(14) we may write the sum rule as

$$\frac{1}{2} = J_q + J_g \tag{17}$$

where J_q and J_g are gauge and Lorentz invariant quantities which may reasonably be identified as total 'quark' and 'gluon' angular momenta respectively. Of course, this is a rather non-rigorous terminology since the corresponding operators $O_2 x$ and $O_3 x$ mix, and indeed $O_2 x$ itself contains an explicit gluon field component in the covariant derivative, but it is convenient and the closest approximation to a quark-gluon decomposition that can be given in an interacting QFT such as QCD. Moreover, as we discuss below, J_q and J_g are still not RG scheme/scale dependent quantities.

The point we wish to stress is that provided we restrict to gauge and Lorentz invariant quantities, the true angular momentum sum rule involves only the two form factors J_q and J_g. The axial charge a^0 is simply not present in the sum rule (17). We return to this point in section 5.

Just as the axial charge form factor a^0 can be measured in polarised inclusive DIS, the angular momentum form factors J_q and J_g can in principle be extracted from measurements of unpolarised off-forward parton distribution functions in processes such as deeply virtual Compton scattering $\gamma^* p \to \gamma p$. The required identifications are

$$-iP^+ \frac{\partial}{\partial \Delta_\mu} \int_{-1}^{1} dx\; x f_{q(g)/p}(x,\xi,\Delta)\big|_{\Delta=0} = J_{q(g)} \frac{1}{M} \epsilon^{+\mu\rho\sigma} P_\rho s_\sigma \tag{18}$$

where $\xi = \frac{q.\Delta}{2q.P}$ and the incoming(outgoing) proton momenta are $P - (+)\Delta$.

These form factors may also be calculated non-perturbatively in lattice gauge theory and some initial results for J_q in the quenched approximation have recently been obtained in ref.[5].

4. Operator mixing and RG evolution

The operators O_1, $O_2 x$ and $O_3 x$ in the angular momentum sum rule renormalise and mix in a non-trivial way. The analysis is made more subtle by the explicit factors of the coordinate x which have to be carefully treated. A detailed discussion is presented in ref.[1] and here we only sketch the main features.

First, note that when inserted into forward matrix elements, operators of the form O_a and $O_i x$ mix with a block triangular structure:

$$\begin{pmatrix} O_a \\ O_i x \end{pmatrix}_R = \begin{pmatrix} Z_{ab}^{-1} & 0 \\ Z_{ib}^{-1} & Z_{ij}^{-1} \end{pmatrix} \begin{pmatrix} O_b \\ O_j x \end{pmatrix}_B \tag{19}$$

since gauge-invariant operators with no factors of x only mix with other similar operators. Then since $O_3^{\mu\lambda}$ is symmetric, it can only mix with the symmetric operators $O_3^{\mu\lambda}$ and $O_4^{\mu\lambda}$, which for forward matrix elements implies that $O_3^{\mu[\lambda}x^{\nu]}$ only mixes with itself and $O_1^{\mu\nu\lambda} + O_2^{\mu[\lambda}x^{\nu]}$. Finally, since the full angular momentum current is conserved and therefore not renormalised, the columns of the mixing matrix must all add to one. This implies the following form for the mixing matrix for forward matrix elements:

$$\begin{pmatrix} O_1^{\mu\nu\lambda} \\ O_2^{\mu[\lambda}x^{\nu]} \\ O_3^{\mu[\lambda}x^{\nu]} \end{pmatrix}_B = \begin{pmatrix} 1+X & 0 & 0 \\ Z-X & 1+Z & -Y \\ -Z & -Z & 1+Y \end{pmatrix} \begin{pmatrix} O_1^{\mu\nu\lambda} \\ O_2^{\mu[\lambda}x^{\nu]} \\ O_3^{\mu[\lambda}x^{\nu]} \end{pmatrix}_R \tag{20}$$

and one-loop calculations show $Y = -\frac{2}{3} n_f \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}$ and $Z = -\frac{8}{3} C_F \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}$. X is due to the anomaly and is $O(\alpha_s^2)$. For the form factors, this gives:

$$\begin{pmatrix} a^0 \\ B_q \\ B_g \end{pmatrix}_B =$$

$$\begin{pmatrix} 1+X & 0 & 0 \\ 0 & 1+Z & -Y \\ 0 & -Z & 1+Y \end{pmatrix} \begin{pmatrix} a^0 \\ B_q \\ B_g \end{pmatrix}_R \qquad (21)$$

We therefore find the evolution equations for the quark and gluon components of the proton angular momentum:

$$\frac{d}{dt}\begin{pmatrix} J_q \\ J_g \end{pmatrix} = \frac{\alpha_s}{4\pi}\begin{pmatrix} -\frac{8}{3}C_F & \frac{2}{3}n_f \\ \frac{8}{3}C_F & -\frac{2}{3}n_f \end{pmatrix}\begin{pmatrix} J_q \\ J_g \end{pmatrix} \qquad (22)$$

together with eq.(4) for a^0. It follows that in the asymptotic limit $Q^2 \to \infty$, the partitioning of quark:gluon angular momenta is $3n_f : 16$ [6]. Interestingly, this is the same result as for the partitioning of momentum obtained from the first moment of the unpolarised pdfs.

5. Partons and orbital angular momentum

We have seen how the g_1^p and angular momentum sum rules involve three Lorentz invariant form factors a^0, J_q and J_g, where $J_{q(g)}$ may be reasonably identified as total quark (gluon) angular momentum components in the sum rule $\frac{1}{2} = J_q + J_g$ while the axial charge a^0, which enters the g_1^p sum rule, decouples and may instead be interpreted in terms of topological charge density. There is no gauge and Lorentz invariant operator identification of 'orbital angular momenta' L_q and L_g.

In the parton model, the axial charge a^0 is interpreted as a sum of polarised quark Δq and gluon Δg distributions as in eq.(7). In the AB scheme, the RG evolution equations are

$$\frac{d\Delta q}{dt} = 0 \qquad \frac{d\Delta g}{dt} = \frac{\alpha_s}{4\pi}\big(3C_F\Delta q + \beta_0 \Delta g\big) \qquad (23)$$

where $\beta_0 = 11 - \frac{2}{3}n_f$, compatible with eq.(4). This evolution can also be directly obtained from the splitting functions.

Now if as the parton model suggests, $\frac{1}{2}\Delta q$ and Δg are to be interpreted as quark and gluon spins, then to complete the angular momentum sum rule we are forced to write

$$\frac{1}{2} = \frac{1}{2}\Delta q + \Delta g + L_q + L_g \qquad (24)$$

where $L_{q(g)}$ are orbital angular momenta. Since there is no intrinsic operator definition of these partonic quantities, the best we can do is to define them to be consistent with the sum rule (17), i.e.

$$L_q = J_q - \frac{1}{2}\Delta q \qquad L_g = J_g - \Delta g \qquad (25)$$

Of course this means the sum rule (24) is not really predictive, since there is no way to independently measure $L_{q(g)}$ – only the form factors $J_{q(g)}$ can be extracted from experiment. Moreover, the identifications (25) are not very natural, since they involve subtracting quantities belonging to form factors for different Lorentz structures. They are therefore frame-dependent, not surprisingly given that spin and orbital angular momentum are associated with different representations of the Lorentz group.

Nevertheless, if we adopt (7),(25),(24) as the best possible gauge-invariant definition of a quark/gluon spin/orbital angular momentum sum rule, we may determine the RG evolution for the components Δq, Δg, L_q and L_g from eqs. (22),(23). We find

$$\frac{d}{dt}\begin{pmatrix} \Delta q \\ \Delta g \\ L_q \\ L_g \end{pmatrix} = \frac{\alpha_s}{4\pi}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 3C_F & \beta_0 & 0 & 0 \\ -\frac{4}{3}C_F & \frac{2}{3}n_f & -\frac{8}{3}C_F & \frac{2}{3}n_f \\ -\frac{5}{3}C_F & -11 & \frac{8}{3}C_F & -\frac{2}{3}n_f \end{pmatrix}\begin{pmatrix} \Delta q \\ \Delta g \\ L_q \\ L_g \end{pmatrix} \qquad (26)$$

which is consistent with the non-renormalisation of the full angular momentum current and may, at least in part, also be derived in a splitting function approach [6].

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