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Calculation of fermionic two-loop contributions to muon decay*

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Abstract

The computation of the correction Δr in the W-Z mass correlation, derived from muon decay, is described at the two-loop level in the Standard Model. Technical aspects which become relevant at this level are studied, e.g. gauge-parameter independent mass renormalization, ghost-sector renormalization and the treatment of γ_5. Exact results for Δr and the W mass prediction including $\mathcal{O}(\alpha^2)$ corrections with fermion loops are presented and compared with previous results of a next-to-leading order expansion in the top-quark mass.

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1 Introduction

The electroweak Standard Model (SM), together with the theory of strong interactions (Quantum Chromodynamics, QCD), provides a comprehensive description of experimental data with remarkable consistency. In order to further test the validity of the SM predictions and restrict its only missing parameter, the Higgs boson mass, precision measurements play a key role. The analysis of precision observables is sensitive to quantum corrections in the theoretical predictions, which depend on all the parameters of the model. In this way, the top-quark mass, $m_{\rm t}$, had been predicted in the region where it was experimentally found.

The constraints on the Higgs mass, $M_{\rm H}$, are still rather weak since $M_{\rm H}$ appears only logarithmically in the leading order SM predictions. Therefore it is of high interest to further reduce experimental and theoretical uncertainties. An important quantity in this context is the quantum correction Δr in the relation of the gauge boson masses $M_{\rm W}, M_{\rm Z}$ with the Fermi constant $G_{\rm F}$ and the fine structure constant α [1].

This relation is established in terms of the muon decay width, which was first described in the Fermi Model as a four-fermion interaction $\mu^- \to e^- \, \nu_\mu \, \bar{\nu}_e$ with coupling $G_{\rm F}$. This yields for the muon decay width

$$\Gamma_\mu = \frac{G_{\rm F}^2 \, m_\mu^5}{192\pi^3} \; F\left(\frac{m_{\rm e}^2}{m_\mu^2}\right) (1+\Delta q)\,, \tag{1}$$

where $F(m_{\rm e}^2/m_\mu^2)$ subsumes effects of the electron mass on the final-state phase space and Δq denotes the QED corrections in the Fermi Model. Including two-loop QED corrections [2, 3] one obtains from the measurement of the muon decay width [4] $G_{\rm F} = (1.16637 \pm 0.00001)\, 10^{-5}$ GeV^{-2}.

The decay process in the SM, on the other side, involves the exchange of a W boson and additional electroweak higher order corrections. Although not part of the Fermi Model, tree-level W propagator effects are conventionally included in eq. (1) by means of a factor $(1+\frac{3}{5}m_\mu^2/M_{\rm W}^2)$. Yet, this is of no numerical significance. The relation between the Fermi constant $G_{\rm F}$ and the SM parameters is expressed as

$$\frac{G_{\rm F}}{\sqrt{2}} = \frac{e^2}{8 s_{\rm w}^2 M_{\rm W}^2} \; (1+\Delta r)\,. \tag{2}$$

Here the SM radiative corrections are included in Δr (with $s_{\rm w}^2 = 1 - M_{\rm W}^2/M_{\rm Z}^2$).

Since $G_{\rm F}$ is known with high accuracy it can be taken as an input parameter in equation (2) in order to obtain a prediction for the W mass ($M_{\rm W}$ is still afflicted with a considerable experimental error, $M_{\rm W} = 80.419 \pm 0.038$ GeV [5]):

$$M_{\rm W}^2 = M_{\rm Z}^2 \left[\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\alpha\pi}{\sqrt{2} G_{\rm F} M_{\rm Z}^2}\,(1+\Delta r)}\,\right]. \tag{3}$$

Since Δr itself depends on $M_{\rm W}$, eq. (3) is to be understood as an implicit expression.

One expects substantial improvement on the W mass determination from future colliders, which will allow for increasingly stringent constraints on the SM from the comparison of the prediction with the experimental value for $M_{\rm W}$. This requires an accurate theoretical determination of the quantity Δr.

The one-loop result [1, 6] can be split into the following contributions:

$$\Delta r^{(\alpha)} = \Delta\alpha - \frac{c_{\rm w}^2}{s_{\rm w}^2}\Delta\rho + \Delta r_{\rm rem}(M_{\rm H}). \tag{4}$$

Dominant corrections arise from the shift in the fine-structure constant, $\Delta\alpha$, due to large logarithms of light-fermion masses ($\approx 6\%$), and from the leading contribution $\propto m_{\rm t}^2$ to the ρ parameter resulting from the top/bottom doublet, which enters through $\Delta\rho$ ($\approx 3.3\%$). The full $M_{\rm H}$–dependence is contained in the remainder $\Delta r_{\rm rem}$ ($\approx 1\%$).

Furthermore, QCD corrections of $\mathcal{O}(\alpha\alpha_{\rm s})$ [7] and $\mathcal{O}(\alpha\alpha_{\rm s}^2)$ [8] are known. Leading electroweak fermionic $\mathcal{O}(\alpha^2)$ contributions were first taken into account by means of resummation relations [9]. Different approaches for the calculation of electroweak two-loop corrections have been pursued, involving expansions for large values of $M_{\rm H}$ [10] and $m_{\rm t}$ [11, 12, 13].

It turned out that both the leading $\mathcal{O}(\alpha^2 m_{\rm t}^4/M_{\rm W}^4)$ [12] and the next-to-leading $\mathcal{O}(\alpha^2 m_{\rm t}^2/M_{\rm W}^2)$ [13] coefficients in the $m_{\rm t}$ expansion yield important corrections of comparable size. Therefore a complete two-loop calculation of fermionic contributions would be desirable in order to further reduce the theoretical uncertainty, in particular if one considers that these contributions are dominant already in the one-loop result.

A first step into this direction was the determination of the exact Higgs mass dependence of the fermionic $\mathcal{O}(\alpha^2)$ corrections to Δr [15]. Recently the full calculation of these contributions [16] has been accomplished. The results include all electroweak two-loop diagrams with one or two fermion loops without any expansion in $m_{\rm t}$ or $M_{\rm H}$. This talk gives an overview on the techniques and the results of this calculation.

2 Calculational methods

Since in the present calculation all possibly infrared divergent photonic corrections are already contained in the definition (1) of the Fermi constant $G_{\rm F}$ and mass singularities are absorbed in the running of the electromagnetic coupling, $M_{\rm W}$ represents the scale for the electroweak corrections in Δr. Therefore it is possible to neglect all fermion masses except the top quark mass and the momenta of the external leptons so that the muon decay diagrams reduce to vacuum diagrams.

All QED contributions to the Fermi Model have to be excluded in the computation of Δr as they are separated off in the definition of $G_{\rm F}$, see eq. (1). When writing the factor $(1+\Delta q)$ formally as $(1+\Delta\omega)^2$ the decay width up to $\mathcal{O}(\alpha^2)$ can be decomposed as

$$\begin{aligned}
\Gamma_\mu &= \Gamma_{\mu,\text{tree}}(1+\Delta\omega)^2\,(1+\Delta r)^2 \\
&= \Gamma_{\mu,\text{tree}}\Big[1 + 2(\Delta\omega^{(\alpha)} + \Delta r^{(\alpha)}) + (\Delta\omega^{(\alpha)} + \Delta r^{(\alpha)})^2 + 2\Delta\omega^{(\alpha)}\Delta r^{(\alpha)} \\
&\qquad\qquad + 2\Delta\omega^{(\alpha^2)} + 2\Delta r^{(\alpha^2)} + \mathcal{O}(\alpha^3)\Big].
\end{aligned} \tag{5}$$

Apart from the one-loop contributions this includes two-loop QED corrections $\Delta\omega^{(\alpha^2)}$ and mixed contributions of QED and weak corrections $\Delta\omega^{(\alpha)}\Delta r^{(\alpha)}$ which both thus have to be excluded in $\Delta r^{(\alpha^2)}$. For fermionic two-loop diagrams it is possible to find a one-to-one correspondence between QED graphs in Fermi Model and SM contributions.

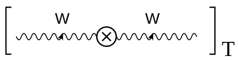
The renormalization is performed in the on-shell scheme. In this context the mass renormalization constants require the computation of two-loop self-energy diagrams with non-vanishing momentum.

All decay amplitudes and counterterm contributions have been generated with the program *FeynArts 2.2* [17]. The amplitudes are algebraically reduced by means of a general tensor integral decomposition for two-loop two-point functions with the program *TwoCalc* [18], leading to a fixed set of standard scalar integrals. Analytical formulae are known for the scalar one-loop [19] and two-loop [20] vacuum integrals whereas the two-loop self-energy diagrams can be evaluated numerically by means of one-dimensional integral representations [21].

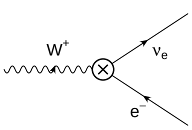
In order to apply an additional check the calculations were performed within a covariant R_ξ gauge, which introduces one gauge parameter $\xi_i, i = \gamma,$ Z,W, for each gauge boson. It has been explicitly checked at the algebraic level that the gauge parameter dependence of the final result drops out.

3 On-shell renormalization

For the determination of the one-loop counterterms (CTs) and renormalization constants the conventions of ref. [22] are adopted. Two-loop renormalization constants enter via the counterterms for the transverse W propagator and the charged current vertex:



$$\Big[\ \cdots\ \Big]_{\mathrm{T}} = \delta Z^{\mathrm{W}}_{(2)}(k^2 - M^2_{\mathrm{W}}) - \delta M^2_{\mathrm{W(2)}} - \delta Z^{\mathrm{W}}_{(1)}\delta M^2_{\mathrm{W(1)}}, \tag{6}$$



$$= i\frac{e}{\sqrt{2}s_{\mathrm{W}}}\gamma_\mu\omega_-\Big[\delta Z_{e(2)} - \frac{\delta s_{\mathrm{W(2)}}}{s_{\mathrm{W}}} + \frac{1}{2}\left(\delta Z^{e\mathrm{L}}_{(2)} + \delta Z^{\mathrm{W}}_{(2)} + \delta Z^{\nu\mathrm{L}}_{(2)}\right) \tag{7}$$
$$+ \text{(1-loop renormalization constants)}\Big].$$

Here, δZ^ϕ denotes the field-renormalization constant of the field ϕ, δM^2_ϕ the corresponding mass CT, and δZ_e the charge-renormalization constant. The numbers in parentheses indicate the loop order. The mixing angle CT, $\delta s_{\mathrm{W(2)}}$, is expressible through the gauge boson mass CTs. Throughout this paper, the two-loop contributions always include the subloop renormalization.

The on-shell masses are defined as the position of the propagator poles. Starting at the two-loop level, it has to be taken into account that there is a difference between the definition of the mass $\widetilde{M}^2$ as the real pole of the propagator D,

$$\Re\left\{D_{\mathrm{T}})^{-1}(\widetilde{M}^2)\right\} = 0, \tag{8}$$

and the real part $\overline{M}^2$ of the complex pole,

$$(D_{\mathrm{T}})^{-1}(\mathcal{M}^2) = 0, \qquad \mathcal{M}^2 = \overline{M}^2 - i\overline{M}\,\overline{\Gamma}. \tag{9}$$

The imaginary part of the complex pole is associated with the width $\overline{\Gamma}$. The definition (9) yields for the W mass CT

$$\delta\overline{M}^2_{\mathrm{W(2)}} = \Re\{\Sigma^{\mathrm{W}}_{\mathrm{T(2)}}(\overline{M}^2_{\mathrm{W}})\} - \delta Z^{\mathrm{W}}_{(1)}\delta\overline{M}^2_{\mathrm{W(1)}} + \Im\{\Sigma^{\mathrm{W}\prime}_{\mathrm{T(1)}}(\overline{M}^2_{\mathrm{W}})\}\,\Im\{\Sigma^{\mathrm{W}}_{\mathrm{T(1)}}(\overline{M}^2_{\mathrm{W}})\}, \tag{10}$$

whereas for the real pole definition the last term of eq. (10) is missing. $\Sigma_{\rm T}^{\rm W}, \Sigma_{\rm T}^{\rm W\prime}$ denote the transverse W self-energy and its momentum derivative. Similar expressions hold for the Z boson.

The W and Z mass CTs determine the two-loop mixing angle CT, $\delta s_{\rm w(2)}$, which has to be gauge invariant since $s_{\rm w}$ is an observable quantity. With the use of a general R_ξ gauge it could be explicitly checked that $\delta s_{\rm w(2)}$ is gauge-parameter independent for the complex-pole mass definition, whereas the real-pole definition leads to a gauge dependent $\delta s_{\rm w(2)}$. This is in accordance with the expectation from S-matrix theory [23], where the complex pole represents a gauge-invariant mass definition.

It should be noted that the mass definition via the complex pole corresponds to a Breit-Wigner parameterization of the resonance shape with a constant width. For the experimental determination of the gauge boson masses, however, a Breit-Wigner ansatz with a running width is used. This has to be accounted for by a shift of the values for the complex pole masses [24],

$$\overline{M} = M - \frac{\Gamma^2}{2M}, \tag{11}$$

which yields the relations

$$\begin{aligned} \overline{M}_{\rm Z} &= M_{\rm Z} - 34.1 \text{ MeV}, \\ \overline{M}_{\rm W} &= M_{\rm W} - 27.4\ (27.0) \text{ MeV} \quad \text{for} \quad M_{\rm W} = 80.4\ (80.2) \text{ GeV}. \end{aligned} \tag{12}$$

For $M_{\rm Z}$ and $\Gamma_{\rm Z}$ the experimental numbers are taken. The W mass is a calculated quantity, and therefore also a theoretical value for the W boson width should be applied here.[1] The results above are obtained from the approximate, but sufficiently accurate expression for the W width,

$$\Gamma_{\rm W} = 3\frac{G_{\rm F} M_{\rm W}^3}{2\sqrt{2}\pi} \left(1 + \frac{2\alpha_{\rm s}}{3\pi}\right). \tag{13}$$

At the subloop level, also the Faddeev-Popov ghost sector has to be renormalized. The gauge-fixing sector for the gauge fields A^μ, Z^μ, $W^{\pm\mu}$ (χ, $\phi^\pm$ denote the unphysical Higgs scalars)

$$\begin{aligned} \mathcal{L}_{\rm gf} &= -\frac{1}{2}\Big((F^\gamma)^2 + (F^{\rm Z})^2 + F^+F^- + F^-F^+\Big), \\ F^\gamma &= (\xi_1^\gamma)^{-\frac{1}{2}}\,\partial_\mu A^\mu + \frac{\xi^{\gamma{\rm Z}}}{2}\partial_\mu Z^\mu, \\ F^{\rm Z} &= \left(\xi_1^{\rm Z}\right)^{-\frac{1}{2}}\,\partial_\mu Z^\mu + \frac{\xi^{{\rm Z}\gamma}}{2}\partial_\mu A^\mu - \left(\xi_2^{\rm Z}\right)^{\frac{1}{2}} M_{\rm Z}\,\chi, \\ F^\pm &= \left(\xi_1^{\rm W}\right)^{-\frac{1}{2}}\,\partial_\mu W^{\pm\mu} \mp i\left(\xi_2^{\rm W}\right)^{\frac{1}{2}} M_{\rm W}\,\phi^\pm \end{aligned} \tag{14}$$

does not need renormalization. Accordingly, one can either introduce the gauge-fixing term after renormalization or renormalize the gauge parameters in such a way that they compensate the renormalization of the fields and masses. Both methods ensure that no counterterms arise from the

[1] In the version of this paper appearing in the proceedings for simplicity a fixed shift of leading order has been used.

gauge-fixing sector but they differ in the treatment of the ghost Lagrangian, which is given by the variation of the functionals F^a under infinitesimal gauge transformations $\delta\theta_b$,

$$\mathcal{L}_{\text{FP}} = \sum_{a,b=\gamma,\text{Z},\pm} \bar{u}^a \, \frac{\delta F^a}{\delta\theta^b} \, u^b. \tag{15}$$

In the latter case, which was applied in this work, additional counterterm contributions for the ghost sector arise from the gauge parameter renormalization. The parameters ξ_i^a in (14) are renormalized such that their CTs $\delta\xi_i^a$ exactly cancel the contributions from the renormalization of the fields and masses and that the renormalized gauge parameters comply with the R_ξ gauge.

4 Treatment of the γ_5–problem

In four dimensions the algebra of the γ_5–matrix is defined by the two relations

$$\{\gamma_5, \gamma_\alpha\} = 0 \qquad \text{for} \qquad \alpha = 1, \ldots, 4 \tag{16}$$

$$\text{Tr}\{\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\} = 4i\epsilon^{\mu\nu\rho\sigma}. \tag{17}$$

It is impossible to translate both relations simultaneously into $D \neq 4$ dimensions without encountering inconsistencies [25].

A certain treatment of γ_5 might break symmetries, i.e. violate Slavnov-Taylor (ST) identities which would have to be restored with extra counterterms. Even after this procedure a residual scheme dependence can persist which is associated with ϵ-tensor expressions originating from the treatment of (17). Such expressions cannot be canceled by counterterms. If they broke ST identities this would give rise to anomalies.

’t Hooft and Veltman [25] suggested a consistent scheme which was formulated by Breitenlohner and Maison [26] as a separation of the first four and the remaining dimensions of the γ-Matrices (HVBM-scheme). It has been shown [27] that the SM with HVBM regularization is anomaly-free and renormalizable. This shows that ϵ-tensor terms do not get merged with divergences.

The naively anti-commuting scheme, which is widely used for one-loop calculations, extends the rule (16) to D dimensions but abandons (17),

$$\{\gamma_5, \gamma_\alpha\} = 0 \qquad \text{for} \qquad \alpha = 1, \ldots, D \tag{18}$$

$$\text{Tr}\{\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\} = 0. \tag{19}$$

This scheme is unambiguous but does not reproduce the four-dimensional case.

In the SM particularly triangle diagrams (Fig. 1) containing chiral couplings are sensitive to the γ_5–problem. For the presented work one-loop triangle diagrams have been explicitly calculated in both schemes. While the naive scheme immediately respects all ST identities the HVBM scheme requires the introduction of additional finite counterterms. Even after this procedure finite differences remain between the results of the two schemes, showing that the naive scheme is inapplicable in this case.

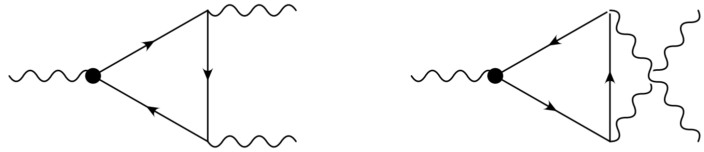


Figure 1: Generic Standard Model triangle diagrams subject to the γ_5–problem.

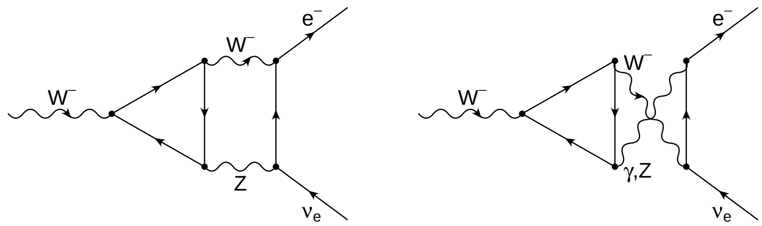


Figure 2: CC vertex diagrams with triangle subgraphs.

In the calculation of Δr triangle diagrams appear as subloops of two-loop charged current (CC) vertex diagrams (Fig. 2). One finds that for the difference terms between both schemes this loop can be evaluated in four dimensions without further difficulties. This can be explained by the fact that renormalizability forbids divergent contributions to ϵ-tensor terms from higher loops in the HVBM scheme. The ϵ-tensor contributions from the triangle subgraph in the HVBM scheme meet a second ϵ-tensor term from the outer fermion lines in Fig. 2, thereby resulting in a non-zero contribution to Δr.

Computations in the HVBM scheme can get very tedious because of the necessity of additional counterterms. Therefore another method shall be examined. One can consider a ”mixed” scheme that uses both relations (16) and (17) in D dimensions despite their mathematical inconsistency. This scheme is plagued by ambiguities of $\mathcal{O}(D-4)$. When applied to the calculation of one-loop triangle diagrams the results immediately respect all ST identities and differ from the HVBM results only by terms of $\mathcal{O}(D-4)$,

$$\Gamma^{\text{HVBM}}_{\Delta(1)} = \Gamma^{\text{mix}}_{\Delta(1)} + \mathcal{O}(D-4). \tag{20}$$

Since for the difference terms the second loop can be evaluated in four dimensions, this also holds for the two-loop CC diagrams,

$$\Gamma^{\text{HVBM}}_{\text{CC(2)}} = \Gamma^{\text{mix}}_{\text{CC(2)}} + \mathcal{O}(D-4). \tag{21}$$

Thus the mixed scheme, despite being mathematically inconsistent, can serve as a technically easy prescription for the correct calculation of the CC two-loop contributions.

5 Results

In the previous sections the characteristics of the calculation of electroweak two-loop contributions to Δr have been pointed out. Combining the fermionic $\mathcal{O}(\alpha^2)$ contributions with the one-loop and

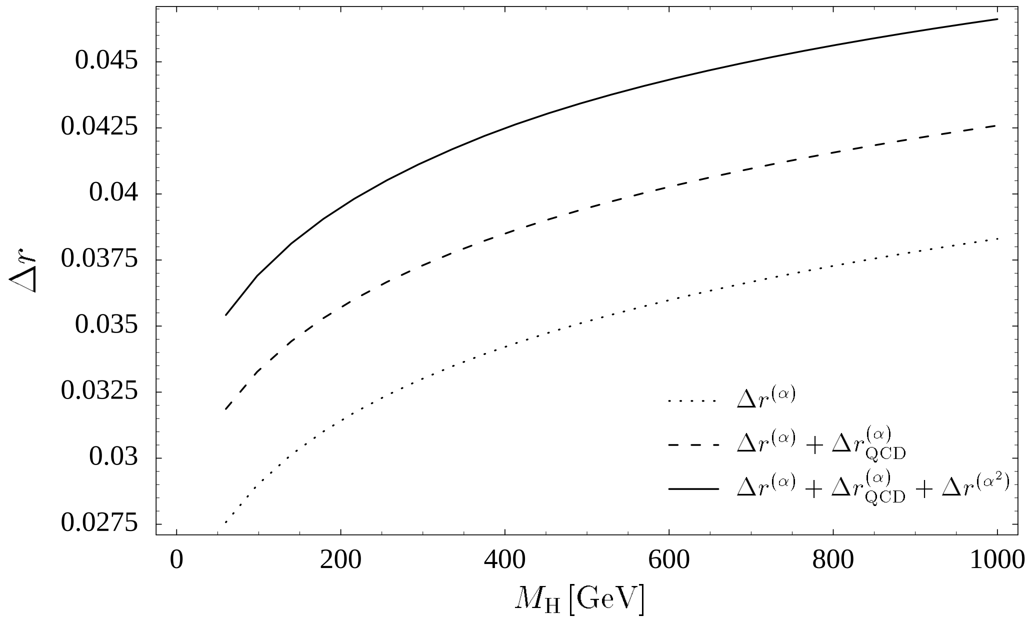


Figure 3: Contribution of one-loop and higher order corrections to Δr.

the QCD corrections yields the total result

$$\Delta r = \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(N_f\alpha^2)} + \Delta r^{(N_f^2\alpha^2)}. \tag{22}$$

Here N_f, N_f^2 symbolize one and two fermionic loops respectively. Fig. 3 shows that both the QCD and electroweak two-loop corrections give sizeable contributions of 10–15% with respect to the one-loop result.

In Fig. 4 the prediction for M_W derived from the result (22) and the relation (3) is compared with the experimental value for M_W. Dotted lines indicate one standard deviation bounds. The main uncertainties of the prediction originate from the experimental errors of $m_\mathrm{t} = (174.3 \pm 5.1)$ GeV [4] and $\Delta\alpha = 0.05954 \pm 0.00065$ [28]. It can be noted that light Higgs masses are favored by this analysis. Further implications of the precision calculation of M_W are discussed in [30].

These results can be compared with the results obtained by expansion of the two-loop contributions up to next-to-leading order in m_t [13, 14]. The predicted values for M_W for several values of M_H are given in Tab. 1. Agreement is found between the results with maximal deviations of about 4 MeV in M_W.

The theoretical uncertainty due to missing higher order contributions can be estimated as follows. The missing $\mathcal{O}(\alpha^2)$ purely bosonic corrections can be judged by means of resummation relations to be very small (< 1 MeV effect on M_W for a light Higgs). An estimate of the $\mathcal{O}(\alpha^3)$ terms can be obtained from the renormalization scheme dependence of the two-loop result yielding about 2–3 MeV, and the missing higher order QCD corrections were estimated to be about 4–5 MeV [31, 32]. Adding this up linearly, one arrives at a total uncertainty of about 7 MeV for the M_W–prediction at low Higgs masses ($M_\mathrm{H} \lesssim 150$ GeV).

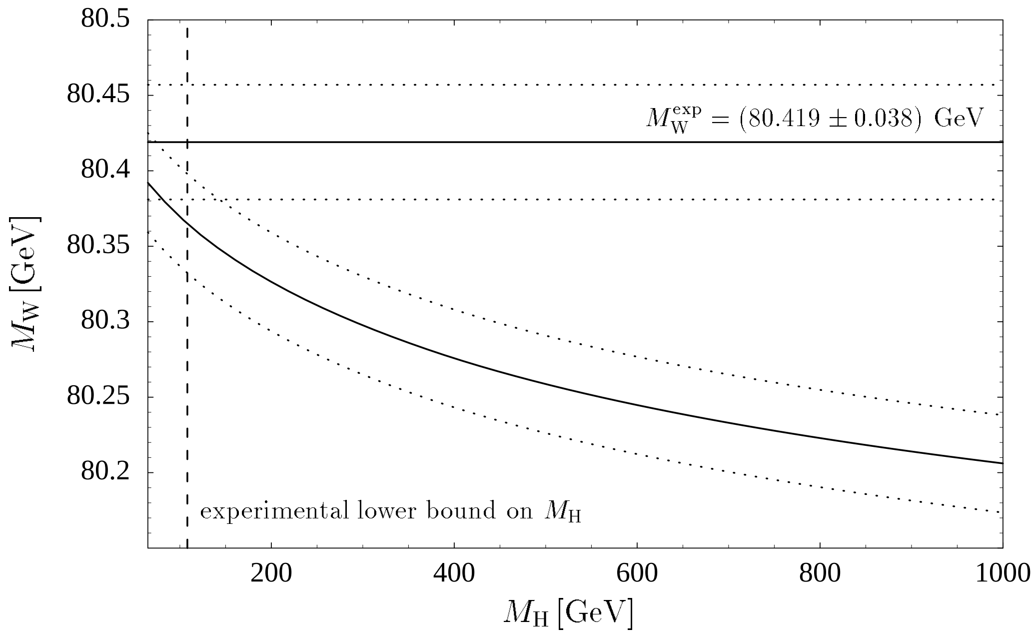


Figure 4: Prediction for M_{W} as function of M_{H} compared with the experimental W mass.

6 Conclusion

In this talk the realization of an exact two-loop calculation of fermionic contributions in the full electroweak SM and its application to the precise computation of Δr were presented. Numerical results and an estimate of the remaining uncertainties were given and might serve as ingredient for future SM fits.

The speaker would like to thank S. Bauberger and D. Stöckinger for valuable discussions.

Table 1: Comparison between M_{W}–predictions from a NLO expansion in m_{t} ($M_{\mathrm{W}}^{\mathrm{expa}}$) and the full calculation ($M_{\mathrm{W}}^{\mathrm{full}}$). δM_{W} denotes the difference. Experimental input values are taken from [14].

M_{H} [GeV]	$M_{\mathrm{W}}^{\mathrm{expa}}$ [GeV]	$M_{\mathrm{W}}^{\mathrm{full}}$ [GeV]	δM_{W} [MeV]
65	80.4039	80.3997	4.2
100	80.3805	80.3771	3.4
300	80.3061	80.3051	1.0
600	80.2521	80.2521	0.0
1000	80.2129	80.2134	−0.5

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