

**EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH  
CERN - PS DIVISION**

**CERN/PS 2000-023 (AE)**

**TRANSVERSE ELECTRON COOLING -  
FROM THE TIME EVOLUTION OF THE PROFILES TO DRIFT VELOCITIES OF THE  
OSCILLATION AMPLITUDES**

C. Carli and M. Chanel

**Abstract**

When a beam in a synchrotron is subjected to transverse cooling, the betatron oscillation amplitudes of the individual particles are reduced. A method to establish the velocity of the amplitude reduction as a function of the amplitude itself has been developed. The beam profile can be measured along the cooling process with for example beam ionisation monitors. From successive profiles one computes the time evolution of the amplitude distribution, which in turn allows the determination of the amplitude reduction velocity. This method could be applied to investigate the influence of the transverse electron temperature on electron cooling performance.

7th European Particle Accelerator Conference, 26th-30th June 2000  
Vienna, Austria

Geneva, Switzerland  
10 July 2000

# TRANSVERSE ELECTRON COOLING - FROM THE TIME EVOLUTION OF THE PROFILES TO DRIFT VELOCITIES OF THE OSCILLATION AMPLITUDES

C. Carli and M. Chanel, CERN, Geneva, Switzerland

## Abstract

When a beam in a synchrotron is subjected to transverse cooling, the betatron oscillation amplitudes of the individual particles are reduced. A method to establish the velocity of the amplitude reduction as a function of the amplitude itself has been developed. The beam profile can be measured along the cooling process with for example beam ionisation monitors. From successive profiles one computes the time evolution of the amplitude distribution, which in turn allows the determination of the amplitude reduction velocity. This method could be applied to investigate the influence of the transverse electron temperature on electron cooling performance.

## 1 INTRODUCTION

The scheme to provide ion bunches with sufficient intensity for the future LHC (Large Hadron Collider) requires the accumulate of a low intensity beam from an ECR source in a small synchrotron through the use of electron cooling. A good understanding of electron cooling is essential to optimize this accumulation scheme. In particular, the time to reduce the transverse oscillation amplitudes limits the repetition rate of the multiturn injection and thus the accumulation rate.

The distribution of beam particles in phase space can be described by an oscillation amplitude distribution. In this paper, a method is presented to determine experimentally the drift velocity with which the oscillation amplitudes are reduced, is presented. For this, the time evolution of the profile is observed with, e.g., a BIPM (Beam Ionisation Profile Monitor). From the profiles, the oscillation amplitude densities are computed. The time evolution of the latter allows a determination of the drift velocity of the oscillation amplitudes.

In Sections 2 and 3 the mathematical transformations needed are explained. In Section 4, the method is applied to a beam undergoing electron cooling.

## 2 FROM PROFILES TO AMPLITUDE DISTRIBUTIONS

Particle beams with azimuthal symmetry<sup>1</sup> (for example a beam injected into a synchrotron after filamentation) are uniquely determined by either the amplitude density or the profile. The transformation of the amplitude density  $\rho(A_z)$

<sup>1</sup>Note that any phase space ellipse can be transferred into a circle without affecting the position  $z$

into a profile can be derived with the help of Figure 1. There are  $\rho(A_z) \Delta A_z$  particles with oscillation amplitudes between  $A_z$  and  $A_z + \Delta A_z$  (top picture). These particles occupy the ring-shaped area indicated in the middle. The phase-space density is given by the number of particles divided by the area  $\frac{\rho(A_z) \Delta A_z}{(2\pi A_z \Delta A_z)} = \frac{\rho(\sqrt{z^2 + z'^2})}{(2\pi\sqrt{z^2 + z'^2})}$ . The profile density at a location  $z$  is the number of particles contained in the hatched strip in the middle picture, divided by the width  $\Delta z$ . Thus :

$$g(z) = \frac{1}{\pi} \int_z^R dA_z \frac{\rho_z(A_z)}{\sqrt{A_z^2 - z^2}}$$

where  $R$  denotes the oscillation amplitude of the extreme particles. The inverse transformation from profiles to am-

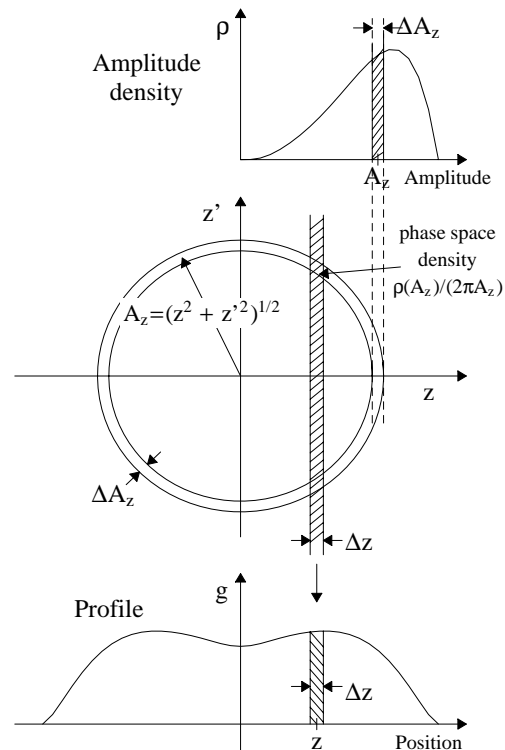


Figure 1: Amplitude distribution (top) and profile (bottom) of a beam with azimuthal symmetry. In this example the beam is hollow, as it may happen after misteering at the injection for cooling investigations.

plitude distributions is more complicated and given by :

$$\rho_z(A_z, t) = -2 \frac{dG_z(A_z, t)}{dA_z}$$

$$\text{with } G_z(A_z, t) = \int_{A_z}^R dz \frac{zg(z, t)}{\sqrt{z^2 - A_z^2}}.$$

The exact derivation[1] involves complicated mathematics, but a "physicist's derivation" of this transformation from profiles to amplitude distributions is given [2].

### 3 FROM AMPLITUDE DISTRIBUTIONS TO AMPLITUDE DRIFT VELOCITIES

When the beam in the synchrotron is subjected to cooling (or to any other process changing the phase space distribution) the profiles vary with time. If the process that changes the particle distribution in phase space (or the amplitude distribution) is dominated by a drift (e.g. friction force due to electron cooling) and not by diffusion, one can derive the drift velocity ( $dA/dt$ ) as a function of the amplitude  $A$  with help of Figure 2. During the time interval  $\Delta t$ , the

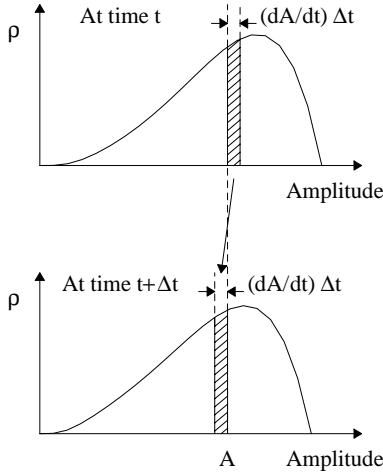


Figure 2: Time evolution of the amplitude distribution.

particles in the hatched strip in the upper part of the figure drift over the amplitude  $A$  and end up in the hatched strip in the bottom part. During this time interval, the surface under the amplitude distribution from  $A$  up to the edge of the beam is reduced by the surface of the hatched strip. This leads to the relation :

$$\frac{dA_z}{dt} \Delta t \rho_z(A_z) = \Delta t \frac{d}{dt} \int_{A_z}^R d\hat{A} \rho_z(\hat{A}) \quad .$$

With the help of the equations from Section refProfToAmp one obtains the amplitude drift velocity :

$$\frac{dA_z}{dt} = \frac{2 (d/dt) G(A_z, t)}{\rho_z(A_z)} = - \frac{(d/dt) G_z(A_z, t)}{(d/dA_z) G_z(A_z, t)}$$

### 4 TRANSVERSE AMPLITUDES AND THEIR DENSITIES

In practice, the ions execute oscillations in both transverse phase spaces and often both horizontal and vertical profile measurements are available. For the following, one assumes that the two dimensional amplitude density is given

by :  $\rho(A_x, A_y) = \rho_x(A_x) \times \rho_y(A_y)$ . This means that one assumes no correlation between the two oscillation amplitudes. The density  $\rho_{\perp}$  of the transverse oscillation amplitudes  $A_{\perp} = \sqrt{A_x^2 + A_y^2}$  is computed by integration over all possible combinations of  $A_x$  and  $A_y$ .

In order to apply the last equation in section 3, one has to compute the function  $G_{\perp}(A_{\perp})$  with the help of the integral :

$$G_{\perp}(A_{\perp}) = \frac{1}{2} \int_{A_{\perp}}^R d\hat{A} \cdot \rho_{\perp}(\hat{A}, t) \quad .$$

### 5 EXAMPLE : SPEED AMPLITUDE DRIFT VELOCITY AT AN ELECTRON COOLER

In order to illustrate the method, it is applied to successive profiles measured with a BIPM during electron cooling. The measurements were done at TSR (Test Storage Ring) in Heidelberg with a BIPM while the carbon beam was subjected to electron cooling. The time interval of 0.1s between two consecutive measurements is given by the integration time of the BIPM.

In this application, one is interested in the reduction of the transverse speed amplitude at the electron cooler. Thus, for both transverse planes, the oscillation amplitudes  $A_z$  at the BIPM are transformed to speed amplitudes  $A_{vz,EC} = (\beta_{rel} c) A_z / \sqrt{\beta_z \beta_{z,EC}}$  at the electron cooler.  $\beta_{rel}$  is the relativistic  $\beta$ -factor of the beam and  $\beta_z$  and  $\beta_{z,EC}$  are the betatron functions at the location of the BIPM and the cooler. From the horizontal and vertical speed amplitude densities, the transverse speed amplitude density  $\rho_v(A_{v,EC})$  is computed with the help of the procedure outlined in Section 4. The quantity  $A_{v,EC} = \sqrt{A_{vx,EC}^2 + A_{vy,EC}^2}$  denotes the transverse speed amplitude.

#### 5.1 A trick to reduce noise

Figure 3 shows the speed amplitude drift velocity as a function of the speed amplitude obtained from two successive profiles. One sees that, for speed amplitudes with a high density, the drift velocities obtained follow a smooth curve, whereas at locations with a small speed amplitude density, the result is very noisy. This is not surprising, because at low amplitude density one has to divide two small quantities. However, during the entire cooling process, the profiles move from large to small speed oscillation amplitudes. Different measurements give precise results at different speed amplitudes. To reduce the noise in the drift velocity  $(d/dt) A_{v,EC}$ , for every amplitude  $A_{v,EC}$ , the mean over all individual curves is computed, with the amplitude density as weighting factor.

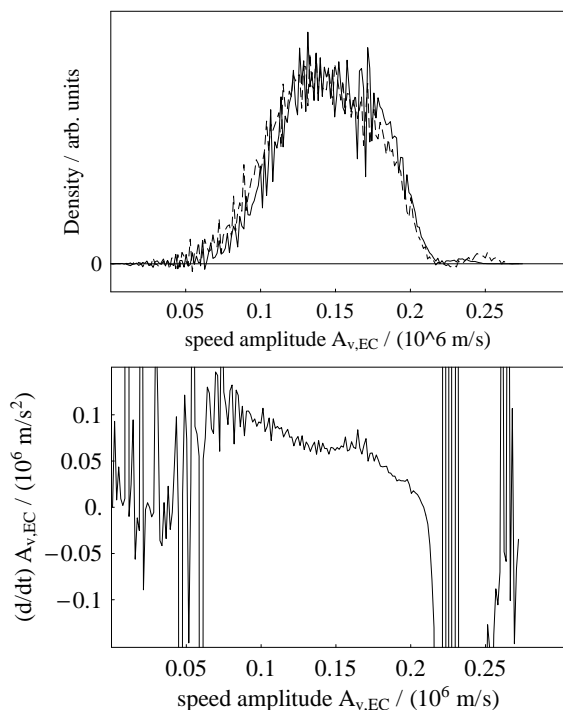


Figure 3: Two consecutive (distant by 0.1 s) amplitude distributions (top) and amplitude drift velocities (bottom) determined from the two profiles underlying these amplitude distributions.

## 5.2 Result

In Figure 4 the quantity  $(m/Z^2) A_{v,EC}$  is plotted as a function of the speed amplitude. Here  $m$  denotes the mass of the beam particle and  $Z$  the charge state. The thin line is the raw result obtained with the procedure explained above and the thick line is the same, after some smoothing. This quantity  $(m/Z^2)dA_{v,EC}/dt$  is not the transverse cooling force (normalized to unit charge state), but closely related to it. In fact<sup>2</sup>, this quantity is obtained from the cooling force by some kind of folding over all possible betatron phases and contains the space charge effect of the electron beam.

From Figure 4 one observes that the drift velocity of the speed amplitude increases to a maximum and then decreases at large amplitudes roughly inversely proportional to the speed amplitude. The fall off of the drift velocity for high speed amplitudes agrees well with theoretical considerations. However, the location of the maximum could not be identified with the RMS spread (neither transverse nor longitudinal) of the electron velocity. One also notes that, for small speed amplitudes, the amplitude reduction does not rise linearly, as one would expect. The reason is that at low speed amplitudes, i.e. if the particle is close to the equilibrium emittance, the result is falsified by diffusion. The cooling is then counteracted by some heating process

<sup>2</sup>At least if the dispersion vanishes at the cooler. If the dispersion is not zero, the longitudinal cooling force may influence the oscillation amplitude.

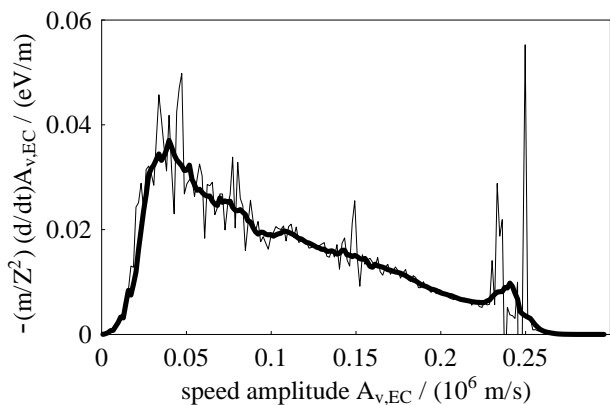


Figure 4: Particle mass times the speed amplitude drift versus speed amplitude at the electron cooler.

(e.g. intra-beam scattering or heating due to the electron beam) and thus, the above analysis no longer holds.

## 6 CONCLUSIONS

A method has been presented, to determine the drift velocities of the transverse oscillation amplitudes, from the time evolution of the profile. This amplitude drift velocity can be translated into a speed amplitude drift at an electron cooling device via the betatron functions and the relativistic  $\beta$  factor. The latter is closely related to the transverse electron cooling force. The oscillation amplitudes and their drift velocities can also be transformed into normalized phase space and are then closely related to the emittance reduction.

To illustrate the usefulness of the method, it has been applied to consecutive profiles measured with electron cooling. From these, it is clearly seen that the speed amplitude drift increases up to a speed corresponding to the transverse electron temperature, and then decreases for higher speed amplitudes.

## 7 ACKNOWLEDGEMENTS

We would like to thank the TSR team for the possibility to carry out measurements with their machine and for their hospitality. We are grateful to Gerard Tranquille for his participation in the measurements.

## 8 REFERENCES

- [1] P.W. Krempf, *The Abel-type Integral Transformation with the Kernel  $(t^2 - x^2)^{-1/2}$  and its Application to Density Distributions of Particle Beams*, CERN internal note MPS/Int. BR/74-1
- [2] C. Carli, M. Lindroos and H. Schönauer, *Comparison of Amplitude Profiles from Beamscope and Wire Scanner Measurements for the vertical Plane in the PSB*, CERN internal note PS/CA/Note 99-11