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Type-I strings on magnetised orbifolds and brane transmutation

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Abstract

In the presence of internal magnetic fields, a D9 brane can acquire a D5 (or anti-D5) R-R charge, and can therefore contribute to the corresponding tadpole. In the resulting vacua, supersymmetry is generically broken and tachyonic instabilities are present. However, suitable choices for the magnetic fields, corresponding to self-dual configurations in the internal space, can yield new chiral supersymmetric vacua with gauge groups of reduced rank, where the magnetic energy saturates, partly or fully, the negative tension of the $O5_+$ planes. These models contain Green-Schwarz couplings to untwisted R-R forms not present in conventional orientifolds.

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Magnetised tori were considered long ago by Witten [1] in the first attempt to recover four-dimensional chiral spectra from the low-energy field theory of superstrings. More recently, Bachas [2] analysed the effect of Fradkin-Tseytlin deformations [3, 4] on open strings, and showed how their universal magnetic couplings [5] can lead to chiral spectra with broken supersymmetry. However, these models have in general Nielsen-Olesen instabilities [6], that reflect themselves in the emergence of tachyonic modes. This complicates the analysis, and brings about some surprises. For instance, in some cases with extended ($\mathcal{N} = 2, 4$) supersymmetry, where one can analyse the potentials of the tachyonic modes, at the resulting minima supersymmetry is actually restored [7]. The constructions in [1, 2] were both based on the assumption, natural at the time, of a vanishing instanton density for the internal magnetic field. However, we are now accustomed to more general settings, that have naturally emerged from type-I vacua [8], where a non-vanishing instanton density is compensated by the presence of additional branes [9]. This letter is thus devoted to elucidate some peculiar effects of magnetic deformations with non-vanishing instanton number on toroidal and orbifold compactifications of type-I strings. As we shall see, these can result in new vacua with unbroken supersymmetry and Chan-Paton groups of reduced rank, where magnetised D9 branes effectively mimic BPS D5 (anti)branes.

It is by now well appreciated that, in non-trivial gravitational and gauge backgrounds, the Wess-Zumino coupling of [10] endows D branes with R-R charges for forms of different degrees. It is perhaps less appreciated, however, that the Born-Infeld action can turn the non-vanishing vacuum energy of suitable internal magnetic fields into a positive tension capable of recovering the BPS bound for the additional charges. An indirect manifestation of this phenomenon was recently met in [11], where the open descendants of some asymmetric orbifolds with "brane supersymmetry breaking" [12, 13, 14, 15, 16] were built using a magnetised internal space, and where a suitable choice of internal fields played an essential role in saturating all R-R tadpoles with only D9 branes.

Let us begin with some intuitive field theory arguments, well captured by the low-energy effective action for D9 branes in an internal abelian background [5],

$$\mathcal{S}_9 = -T_{(9)} \int_{\mathcal{M}_{10}} \mathrm{d}^{10}x \, e^{-\phi} \sum_{a=1}^{32} \sqrt{-\det\left(g_{10} + q_a F\right)} - \mu_{(9)} \sum_{p,a} \int_{\mathcal{M}_{10}} e^{q_a F} \wedge C_{p+1} + \ldots \quad , \tag{1}$$

where a labels the types of Chan-Paton charges that couple to the magnetic fields with strength q_a,

$$T_{(p)} = \sqrt{\frac{\pi}{2\kappa^2}} \, \left(2\pi\sqrt{\alpha'}\right)^{3-p} = |\mu_{(p)}| \; , \tag{2}$$

with T and μ the tension and the R-R charge for a type-I Dp brane [17], and where κ defines the ten-dimensional Newton constant $G_N^{(10)} = \kappa^2/8\pi$. To illustrate the phe-

[5]The (dimensionless) magnetic fields used in this letter differ from the conventional ones by a $2\pi\alpha'$ rescaling.

nomenon, anticipating the string construction, it suffices to consider the geometry $\mathcal{M}_{10} = \mathcal{M}_6 \times T^2 \times T^2$ with constant abelian magnetic fields H_1 and H_2 lying in the two internal tori. These are effectively monopole fields, and thus satisfy the Dirac quantisation conditions

$$q\, H_i\, v_i = k_i \qquad (i = 1, 2) \ , \tag{3}$$

where, aside from powers of 2π, $v_i = R_i^{(1)} R_i^{(2)}/\alpha'$ are the dimensionless volumes of the two tori of radii $R_i^{(1)}$ and $R_i^{(2)}$, k_i are the degeneracies of the corresponding Landau levels and q is the elementary electric charge for the system. As anticipated, we forego the restriction in [1, 2] and actually pick a pair of abelian fields aligned with the same U(1) subgroup, so that

$$\begin{aligned}
\mathcal{S}_9 \;\; = \;\; & - T_{(9)} \int_{\mathcal{M}_{10}} \mathrm{d}^{10}x\, e^{-\phi} \sqrt{-g_6}\, \sum_{a=1}^{32} \sqrt{(1+q_a^2 H_1^2)(1+q_a^2 H_2^2)} \\
& - 32\, \mu_{(9)} \int_{\mathcal{M}_{10}} C_{10} \;-\; \left(2\pi\sqrt{\alpha'}\right)^4 \mu_{(9)}\, v_1 v_2\, H_1\, H_2 \sum_{a=1}^{32} q_a^2 \int_{\mathcal{M}_6} C_6 \ ,
\end{aligned} \tag{4}$$

where g_6 denotes the six-dimensional space-time metric, and for simplicity we have chosen an identity metric in the internal space. In particular, if the two internal fields have identical magnitudes, for the resulting (anti)self-dual configuration the action becomes

$$\begin{aligned}
\mathcal{S}_9 \;\; = \;\; & - 32 \int_{\mathcal{M}_{10}} \Big(\mathrm{d}^{10}x \sqrt{-g_6}\, T_{(9)}\, e^{-\phi} + \mu_{(9)}\, C_{10}\Big) \\
& - \sum_{a=1}^{32} \left(\frac{q_a}{q}\right)^2 \int_{\mathcal{M}_6} \Big(\mathrm{d}^6 x \sqrt{-g_6}\, |k_1 k_2|\, T_{(5)}\, e^{-\phi} + k_1 k_2\, \mu_{(5)}\, C_6\Big) \ .
\end{aligned} \tag{5}$$

Notice that the Dirac quantisation conditions (3) have compensated the integration over the internal tori, while in the second line the additional powers of α' have nicely converted $T_{(9)}$ and $\mu_{(9)}$ into $T_{(5)}$ and $\mu_{(5)}$. Thus, a D9 brane on a magnetised $T^2 \times T^2$ indeed mimics a D5 brane or a D5 antibrane according to whether the orientations of H_1 and H_2, reflected by the relative sign of k_1 and k_2, are identical or opposite.

We can now turn to the open-string description of this phenomenon. In order to obtain a supersymmetric configuration, we should start from an orbifold that normally requires the introduction of D5 branes. The simplest such instance is the six-dimensional compactification on $(T^2 \times T^2)/Z_2$ with Klein-bottle projection

$$\mathcal{K} = \tfrac{1}{4} \left\{ (Q_o + Q_v)(0;0)\,[P_1 P_2 + W_1 W_2] + 16 \times 2 (Q_s + Q_c)(0;0) \left(\frac{\eta}{\vartheta_4(0)}\right)^2 \right\} \ , \tag{6}$$

that corresponds to the introduction of O9$_+$ and O5$_+$ planes, and thus to a projected $\mathcal{N} = (1,0)$ supersymmetric closed spectrum with one tensor multiplet and 20 hypermultiplets.

In writing this expression, we have endowed the six-dimensional characters of [8] with a pair of arguments, anticipating the effect of the magnetic deformations in the two internal tori. In general

$$
\begin{aligned}
Q_o(\eta;\zeta) &= V_4(0)\,[O_2(\eta)O_2(\zeta)+V_2(\eta)V_2(\zeta)] - C_4(0)\,[S_2(\eta)C_2(\zeta)+C_2(\eta)S_2(\zeta)]\ , \\
Q_v(\eta;\zeta) &= O_4(0)\,[V_2(\eta)O_2(\zeta)+O_2(\eta)V_2(\zeta)] - S_4(0)\,[S_2(\eta)S_2(\zeta)+C_2(\eta)C_2(\zeta)]\ , \\
Q_s(\eta;\zeta) &= O_4(0)\,[S_2(\eta)C_2(\zeta)+C_2(\eta)S_2(\zeta)] - S_4(0)\,[O_2(\eta)O_2(\zeta)+V_2(\eta)V_2(\zeta)]\ , \\
Q_c(\eta;\zeta) &= V_4(0)\,[S_2(\eta)S_2(\zeta)+C_2(\eta)C_2(\zeta)] - C_4(0)\,[V_2(\eta)O_2(\zeta)+O_2(\eta)V_2(\zeta)]\ ,
\end{aligned} \tag{7}
$$

where the four level-one O($2n$) characters are related to the four Jacobi theta functions according to

$$
\begin{aligned}
O_{2n}(\zeta) &= \frac{1}{2\eta^n(\tau)}\left(\vartheta_3^n(\zeta|\tau)+\vartheta_4^n(\zeta|\tau)\right)\ ,\ S_{2n}(\zeta)=\frac{1}{2\eta^n(\tau)}\left(\vartheta_2^n(\zeta|\tau)+i^{-n}\vartheta_1^n(\zeta|\tau)\right)\ , \\
V_{2n}(\zeta) &= \frac{1}{2\eta^n(\tau)}\left(\vartheta_3^n(\zeta|\tau)-\vartheta_4^n(\zeta|\tau)\right)\ ,\ C_{2n}(\zeta)=\frac{1}{2\eta^n(\tau)}\left(\vartheta_2^n(\zeta|\tau)-i^{-n}\vartheta_1^n(\zeta|\tau)\right)\ .
\end{aligned} \tag{8}
$$

Whereas in [1, 2] the internal magnetic two-forms were chosen to satisfy

$$
\mathrm{tr}\ H_i \wedge H_j = 0\ , \tag{9}
$$

here we allow for a non-vanishing instanton density, that in String Theory is naturally compensated by additional unpaired defects (an excess of D5 (anti)branes and/or O5 planes). In particular, as in our field theory considerations, we take the two internal fields aligned with the same U(1) subgroup of SO(32), a choice that in this Z_2 orbifold can preserve at most a U(m) $\times$ U(n) gauge group, with $m+n=16$. In the following, we actually restrict our attention to this maximal case, from which other examples can be obtained via Wilson lines or brane displacements.

In writing the direct-channel annulus amplitude, let us begin by recalling [4] that a uniform magnetic field with components H_1 and H_2 in the two internal tori alters the boundary conditions for open strings, shifting their mode frequencies by

$$
z_i^{\rm L,R} = \frac{1}{\pi}\left[\,\tan^{-1}(q_{\rm L}\,H_i)\ +\ \tan^{-1}(q_{\rm R}\,H_i)\,\right]\ , \tag{10}
$$

where $q_{\rm L}$ ($q_{\rm R}$) denote the charges of the left (right) end of the open string with respect to the U(1) fields H_i. A further novelty [4] is displayed by "dipole" strings, with opposite end charges, whose oscillator modes are unaffected, but whose world-sheet coordinates undergo a complex "boost", so that their Kaluza-Klein momenta m_i are rescaled according to

$$
m_i\ \rightarrow\ \frac{m_i}{\sqrt{1\ +\ q_a^2 H_i^2}}\ . \tag{11}
$$

This rescaling ensures the consistency of the transverse-channel amplitudes, whose lowest-level contributions, aside from a subtlety that we shall discuss later, are to group as usual into perfect squares.

The techniques of [8] determine the direct-channel annulus amplitude

$$
\begin{aligned}
\mathcal{A} &= \tfrac{1}{4}\Big\{(Q_o+Q_v)(0;0)\left[(m+\bar m)^2 P_1P_2 + (d+\bar d)^2 W_1W_2 + 2n\bar n \tilde P_1 \tilde P_2\right] \\
&- 2(m+\bar m)(n+\bar n)(Q_o+Q_v)(z_1\tau;z_2\tau)\frac{k_1\eta}{\vartheta_1(z_1\tau)}\frac{k_2\eta}{\vartheta_1(z_2\tau)} \\
&- (n^2+\bar n^2)(Q_o+Q_v)(2z_1\tau;2z_2\tau)\frac{2k_1\eta}{\vartheta_1(2z_1\tau)}\frac{2k_2\eta}{\vartheta_1(2z_2\tau)} \\
&- \left[(m-\bar m)^2 - 2n\bar n + (d-\bar d)^2\right](Q_o-Q_v)(0;0)\left(\frac{2\eta}{\vartheta_2(0)}\right)^2 \\
&- 2(m-\bar m)(n-\bar n)(Q_o-Q_v)(z_1\tau;z_2\tau)\frac{2\eta}{\vartheta_2(z_1\tau)}\frac{2\eta}{\vartheta_2(z_2\tau)} \\
&- (n^2+\bar n^2)(Q_o-Q_v)(2z_1\tau;2z_2\tau)\frac{2\eta}{\vartheta_2(2z_1\tau)}\frac{2\eta}{\vartheta_2(2z_2\tau)} \\
&+ 2(m+\bar m)(d+\bar d)(Q_s+Q_c)(0;0)\left(\frac{\eta}{\vartheta_4(0)}\right)^2 \\
&+ 2(d+\bar d)(n+\bar n)(Q_s+Q_c)(z_1\tau;z_2\tau)\frac{\eta}{\vartheta_4(z_1\tau)}\frac{\eta}{\vartheta_4(z_2\tau)} \\
&- 2(m-\bar m)(d-\bar d)(Q_s-Q_c)(0;0)\left(\frac{\eta}{\vartheta_3(0)}\right)^2 \\
&- 2(d-\bar d)(n-\bar n)(Q_s-Q_c)(z_1\tau;z_2\tau)\frac{\eta}{\vartheta_3(z_1\tau)}\frac{\eta}{\vartheta_3(z_2\tau)}\Big\}\,,
\end{aligned} \tag{12}
$$

and the corresponding Möbius amplitude

$$
\begin{aligned}
\mathcal{M} &= -\tfrac{1}{4}\Big\{(\hat Q_o+\hat Q_v)(0;0)\left[(m+\bar m)P_1P_2 + (d+\bar d)W_1W_2\right] \\
&- (n+\bar n)(\hat Q_o+\hat Q_v)(2z_1\tau;2z_2\tau)\frac{2k_1\hat\eta}{\hat\vartheta_1(2z_1\tau)}\frac{2k_2\hat\eta}{\hat\vartheta_1(2z_2\tau)} \\
&- \left(m+\bar m+d+\bar d\right)(\hat Q_o-\hat Q_v)(0;0)\left(\frac{2\hat\eta}{\hat\vartheta_2(0)}\right)^2 \\
&- (n+\bar n)(\hat Q_o-\hat Q_v)(2z_1\tau;2z_2\tau)\frac{2\hat\eta}{\hat\vartheta_2(2z_1\tau)}\frac{2\hat\eta}{\hat\vartheta_2(2z_2\tau)}\Big\}\,.
\end{aligned} \tag{13}
$$

Here we have actually resorted to a shorthand notation, where the arguments z_i $(2z_i)$ are associated to strings with one (two) charged ends. Moreover, both the imaginary modulus $\frac{1}{2}it$ of $\mathcal{A}$ and the complex modulus $\frac{1}{2}+\frac{1}{2}it$ of $\mathcal{M}$ are denoted by the same symbol τ, although the proper "hatted" contributions to the Möbius amplitude are explicitly indicated. P_i and W_i are conventional momentum and winding sums for the two-tori, while a "tilde" denotes a sum with momenta "boosted" as in (11). Finally, d (together with its conjugate $\bar d$) is the Chan-Paton multiplicity for the D5 branes, while m and n (together with their conjugates $\bar m$ and $\bar n$) are Chan-Paton multiplicities for the D9 branes. For the sake of brevity, several terms with opposite U(1) charges, and thus with opposite z_i arguments, have been grouped together, using the symmetries of the Jacobi theta-functions.

For generic magnetic fields, the open spectrum is indeed non-supersymmetric and develops Nielsen-Olesen instabilities [6]. As emphasised in [2], the emergence of these tachyonic modes can be ascribed to the magnetic couplings of the internal components of gauge fields. For instance, small magnetic fields affect the mass formula for the untwisted string modes according to

$$\Delta M^2 \;=\; \frac{1}{2\pi\alpha'} \sum_{i=1,2} \left[(2n_i+1)|(q_{\rm L}+q_{\rm R})H_i| + 2(q_{\rm L}+q_{\rm R})\Sigma_i H_i\right] \;, \qquad (14)$$

where the first term originates from the Landau levels and the second from the magnetic moments of the spins Σ_i. For the internal components of the vectors, the magnetic moment coupling generally overrides the zero-point contribution, leading to tachyonic modes, unless $|H_1| = |H_2|$, while for spin-$\frac{1}{2}$ modes it can at most compensate it. On the other hand, for twisted modes the zero-point contribution is absent, since ND strings have no Landau levels. In this case the low-lying space-time fermions, that originate from the fermionic part S_4O_4 of Q_s, are scalars in the internal space and have no magnetic moment couplings. However, their bosonic partners, that originate from O_4C_4, are affected by the magnetic deformations and have mass shifts $\Delta M^2 \sim \pm(H_1 - H_2)$. Therefore, if $H_1 = H_2$ all tachyonic instabilities are indeed absent. Actually, with this choice the supersymmetry charge, that belongs to C_4C_4, is also unaffected[6]. Therefore, a residual supersymmetry is present for the entire string spectrum, and indeed, using the Jacobi identities for non-vanishing arguments [19], one can see that for $z_1 = z_2$ both $\mathcal{A}$ and $\mathcal{M}$ vanish identically. Still, the resulting supersymmetric models are rather peculiar, as can be seen from the deformed tadpole conditions, to which we now turn.

Let us begin by examining the untwisted R-R tadpole conditions. For $C_4S_2C_2$ one finds

$$\Big[m + \bar m + n + \bar n - 32 + q^2 H_1 H_2 (n+\bar n)\Big]\sqrt{v_1 v_2} + \frac{1}{\sqrt{v_1 v_2}}\Big[d + \bar d - 32\Big] = 0 \;, \qquad (15)$$

aside from terms that vanish after identifying the multiplicities of conjugate representations $(m,\bar m)$, $(n,\bar n)$ and $(d,\bar d)$. The additional (untwisted) R-R tadpole conditions from Q_o and Q_v are compatible with (15) and do not add further constraints. This expression reflects the familiar Wess-Zumino coupling of eq. (1), and therefore the various powers of

[6]Type-II branes at angles preserving some supersymmetry were previously considered in [18]. After T-dualities, these can be related to special choices for the internal magnetic fields. Type I toroidal models, however, can not lead to supersymmetric configurations, since the resulting R-R tadpoles require the introduction of antibranes.

H correspond to R-R forms of different degrees. In particular, as we anticipated in our field theory discussion, the term bilinear in the magnetic fields has a very neat effect: it charges the D9 brane with respect to the six-form potential. This can be seen very clearly making use of the quantisation condition (3), that turns the tadpole conditions (15) into

$$\begin{aligned} m + \bar{m} + n + \bar{n} &= 32 \ , \\ k_1 k_2 (n + \bar{n}) + d + \bar{d} &= 32 \ . \end{aligned} \tag{16}$$

Thus, if $k_1 k_2 > 0$ the D9 branes indeed acquire the R-R charge of $|k_1 k_2|$ D5 branes, while if $k_1 k_2 < 0$ they acquire the R-R charge of as many D5 antibranes, in agreement with eq. (5).

The untwisted NS-NS tadpoles exhibit very nicely their relation to the Born-Infeld term in (1). For instance, the dilaton tadpole

$$\left[m + \bar{m} + (n + \bar{n}) \sqrt{(1 + q^2 H_1^2)\,(1 + q^2 H_2^2)} - 32 \right] \sqrt{v_1 v_2} + \frac{1}{\sqrt{v_1 v_2}} \left[d + \bar{d} - 32 \right] \tag{17}$$

originates from $V_4 O_2 O_2$, and can be clearly linked to the derivative of the integrand of $\mathcal{S}_9$, specialised to the form (4), with respect to ϕ. On the other hand, the volume of the first internal torus originates from $O_4 V_2 O_2$, and the corresponding tadpole,

$$\left[m + \bar{m} + (n + \bar{n}) \, \frac{1 - q^2 H_1^2}{\sqrt{1 + q^2 H_1^2}} \, \sqrt{1 + q^2 H_2^2} - 32 \right] \sqrt{v_1 v_2} - \frac{1}{\sqrt{v_1 v_2}} \left[d + \bar{d} - 32 \right] \ , \tag{18}$$

can be linked to the derivative of the Born-Infeld action in (1) with respect to the corresponding breathing mode. A similar result holds for the volume of the second torus, with the proper interchange of H_1 and H_2. For the sake of brevity, we have omitted in these NS-NS tadpoles all terms that vanish using the constraint $n = \bar{n}$. However, the full expression of (18) is rather interesting, since, in contrast with the usual structure of unoriented string amplitudes, it is *not* a perfect square. This unusual feature can be ascribed to the behaviour of the internal magnetic fields under time reversal. Indeed, as stressed long ago in [20], these transverse-channel amplitudes involve a time-reversal operation $\mathcal{T}$, and are thus of the form $\langle \mathcal{T}(B) | q^{L_0} | B \rangle$. Differently from the usual quantum mechanical amplitudes, this type of expression is generally a bilinear, rather than a sesquilinear, form. This, however, is not true in the present examples, where additional signs are introduced by the magnetic fields, that are odd under time reversal. As a result, in deriving from factorisation the Möbius amplitudes for these models, it is crucial to add the two contributions $\langle \mathcal{T}(B) | q^{L_0} | C \rangle$ and $\langle \mathcal{T}(C) | q^{L_0} | B \rangle$, that are different and effectively eliminate the additional terms from the transverse-channel.

Both (18) and the dilaton tadpole (17) simplify drastically in the interesting case $H_1 = H_2$ where, using the Dirac quantisation conditions (3), they become

$$\left[m + \bar{m} + n + \bar{n} - 32 \right] \sqrt{v_1 v_2} \mp \frac{1}{\sqrt{v_1 v_2}} \left[k_1 k_2 (n + \bar{n}) + d + \bar{d} - 32 \right] \ . \tag{19}$$

Thus, they both vanish, as they should, in these supersymmetric configurations, once the corresponding R-R tadpole conditions (16) are enforced.

The twisted R-R tadpoles

$$15\left[\tfrac{1}{4}(m-\bar{m}+n-\bar{n})\right]^2+\left[\tfrac{1}{4}(m-\bar{m}+n-\bar{n})-(d-\bar{d})\right]^2 \tag{20}$$

originate from the sector $S_4O_2O_2$, whose states are scalars in the internal space. They reflect very neatly the distribution of branes among the sixteen fixed points, only one of which accommodates D5 branes in our examples, are not affected by the magnetic fields, and vanish identically for the given unitary gauge groups. In general these breaking terms, that originate from twisted modes flowing in the transverse channel, can be linked to internal curvature contributions to the Wess-Zumino term, here localised at the fixed points: this is actually the reason for the presence of D9 and D5 terms in the same expression in orbifold models. The corresponding NS-NS tadpoles, originating from the $O_4S_2C_2$ and $O_4C_2S_2$ sectors, are somewhat more involved, and after the identification of conjugate multiplicities are proportional to

$$\frac{q\,(H_1-H_2)}{\sqrt{(1+q^2H_1^2)(1+q^2H_2^2)}} \quad . \tag{21}$$

They clearly display new couplings for twisted NS-NS fields that, to the best of our knowledge, were not previously exhibited. Notice that, as expected, for $H_1=H_2$ these twisted tadpoles also vanish.

We can now describe some supersymmetric models corresponding to the special choice $H_1=H_2$. It suffices to confine our attention to the case $k_1=k_2=2$, the minimal Landau-level degeneracies allowed on this Z_2 orbifold. Although the projected closed spectra of all the resulting models are identical, and comprise the $\mathcal{N}=(1,0)$ gravitational multiplet, together with one tensor multiplet and twenty hypermultiplets, the corresponding open spectra are quite different from the standard ones, with a maximal gauge group of rank 32, $\mathrm{U}(16)|_9\times\mathrm{U}(16)|_5$ [21, 22]. Still, they are all free of irreducible gauge and gravitational anomalies, consistently with the vanishing of all R-R tadpoles [23].

A possible solution to the R-R tadpole conditions is $m=13$, $n=3$, $d=4$, that corresponds to a gauge group of rank 20, $\mathrm{U}(13)|_9\times\mathrm{U}(3)|_9\times\mathrm{U}(4)|_5$, with charged hypermultiplets in the representations $(78+\overline{78},1;1)$, in five copies of the $(1,3+\overline{3};1)$, in one copy of the $(1,1;6+\overline{6})$, in four copies of the $(\overline{13},3;1)$, in one copy of the $(13,1;\overline{4})$ and in one copy of the $(1,\overline{3};4)$. Alternatively, one can take $m=14$, $n=2$, $d=8$, obtaining a gauge group of rank 24, $\mathrm{U}(14)|_9\times\mathrm{U}(2)|_9\times\mathrm{U}(8)|_5$. The corresponding matter comprises charged hypermultiplets in the $(91+\overline{91},1;1)$, in one copy of the $(1,1;28+\overline{28})$, in four copies of the $(\overline{14},2;1)$, in one copy of the $(14,1;\overline{8})$, in one copy of the $(1,2;8)$, and in five copies of the $(1,1+\overline{1},1)$. On the other hand, for $m=12$, $n=4$, and thus $d=0$. This is a rather unusual supersymmetric Z_2 model *without* D5 branes, with a gauge group of rank 16, U(12) $\times$ U(4), and charged hypermultiplets in the representations $(66+\overline{66},1)$, in five copies of the $(1,6+\overline{6})$, and in four copies of the $(\overline{12},4)$. A distinctive feature of these spectra is that some of the matter occurs in multiple families. This peculiar phenomenon is a consequence of the multiplicities of Landau levels, that in these Z_2 orbifolds are multiples of two for each magnetised torus. Moreover, it should be appreciated that, in general, the rank reduction for the gauge group is not by powers of two as in the presence of a quantised antisymmetric tensor [24, 13]. Actually, these are not the first concrete examples of brane transmutation in type I vacua but, to the best of our knowledge, they are the first supersymmetric ones. Z_2 orientifolds without D5 branes have recently appeared in [11], where magnetised fractional D9 branes have been used to build six-dimensional asymmetric orientifolds with "brane supersymmetry breaking".

One can also consider similar deformations of the model of [12], that has an $\mathcal{N}=(1,0)$ supersymmetric bulk spectrum with 17 tensor multiplets and four hypermultiplets. This alternative projection, allowed by the constraints in [25], introduces $O9_+$ and $O5_-$ planes and thus requires, for consistency, an open sector resulting from the simultaneous presence of D9 branes and D5 antibranes, with "brane supersymmetry breaking". A magnetised torus can now mimic D5 antibranes provided $H_1=-H_2$, and one can then build several non-tachyonic configurations as in the previous case[7]. A particularly interesting one corresponds to a vacuum configuration without D5 antibranes, where the $O5_-$ charge is fully saturated by magnetised D9 branes. The resulting annulus and Möbius amplitudes can be obtained deforming the corresponding ones in [12], and read

$$
\begin{aligned}
\mathcal{A} \;=\;& \tfrac{1}{4}\Bigg\{(Q_o+Q_v)(0;0)\left[(m_1+m_2)^2 P_1P_2+2n\bar{n}\tilde{P}_1\tilde{P}_2\right]\\
&-\;2(m_1+m_2)(n+\bar{n})(Q_o+Q_v)(z_1\tau;z_2\tau)\frac{k_1\eta}{\vartheta_1(z_1\tau)}\frac{k_2\eta}{\vartheta_1(z_2\tau)}\\
&-\;(n^2+\bar{n}^2)(Q_o+Q_v)(2z_1\tau;2z_2\tau)\frac{2k_1\eta}{\vartheta_1(2z_1\tau)}\frac{2k_2\eta}{\vartheta_1(2z_2\tau)}\\
&+\;\left[(m_1-m_2)^2+2n\bar{n}\right](Q_o-Q_v)(0;0)\left(\frac{2\eta}{\vartheta_2(0)}\right)^2\\
&+\;2(m_1-m_2)(n+\bar{n})(Q_o-Q_v)(z_1\tau;z_2\tau)\frac{2\eta}{\vartheta_2(z_1\tau)}\frac{2\eta}{\vartheta_2(z_2\tau)}\\
&+\;(n^2+\bar{n}^2)(Q_o-Q_v)(2z_1\tau;2z_2\tau)\frac{2\eta}{\vartheta_2(2z_1\tau)}\frac{2\eta}{\vartheta_2(2z_2\tau)}\Bigg\}\quad, \qquad (22)
\end{aligned}
$$

[7] There is a subtlety here. The different GSO projections for strings stretched between a D9 brane and a D5 antibrane would associate the low-lying twisted ND bosons to the characters $O_4S_2(z_1)S_2(z_2)$ and $O_4C_2(z_1)C_2(z_2)$, and thus now the choice $H_1=-H_2$ would eliminate all tachyons even in the presence of D5 antibranes.

and

$$
\begin{aligned}
\mathcal{M} \;=\; & -\tfrac{1}{4}\Big\{(m_1+m_2)(\hat{Q}_o+\hat{Q}_v)(0;0)P_1P_2 \\
& - \;(n+\bar{n})(\hat{Q}_o+\hat{Q}_v)(2z_1\tau;2z_2\tau)\frac{2k_1\hat{\eta}}{\hat{\vartheta}_1(2z_1\tau)}\frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(2z_2\tau)} \\
& + \;(m_1+m_2)\,(\hat{Q}_o-\hat{Q}_v)(0;0)\left(\frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\right)^2 \\
& + \;(n+\bar{n})(\hat{Q}_o-\hat{Q}_v)(2z_1\tau;2z_2\tau)\frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_1\tau)}\frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_2\tau)}\Big\} \quad .
\end{aligned} \tag{23}
$$

In extracting the massless spectra of this class of models, it is important to notice that, at the special point $H_1 = -H_2$, all bosons from Q_o with non-vanishing arguments and all fermions from Q_v with non-vanishing arguments become massive. As a result, all massless fermions arising from strings affected by the internal magnetic fields have a reversed chirality, precisely as demanded by the cancellation of all irreducible anomalies. For $|k_1| = |k_2| = 2$, one can obtain a gauge group $\mathrm{SO}(8)\times\mathrm{SO}(16)\times\mathrm{U}(4)$ and, aside from the corresponding $\mathcal{N}=(1,0)$ vector multiplets, the massless spectrum contains a hypermultiplet in the representation $(8,16,1)$, eight scalars in the $(1,16,4+\overline{4})$, two left-handed spinors in the $(8,1,4+\overline{4})$, and twelve scalars and five left-handed spinors in the $(1,1,6+\overline{6})$. Clearly, supersymmetry is explicitly broken on the magnetised D9 branes. Still, the resulting dilaton potential is effectively localised on the $\mathrm{O5}_-$ plane, since it scales with the internal volume as in the undeformed model of [12].

These configurations present another interesting novelty: they have generalised Green-Schwarz couplings [26, 27] involving gauge fields and untwisted R-R forms, of the type

$$
\mathcal{S}_{\mathrm{GS}} \sim \sum_i \int \epsilon^{\mu_1\ldots\mu_6}\,\epsilon^{I_1\ldots I_4}\,C_{I_1I_2\mu_3\mu_4\mu_5\mu_6}\,\mathrm{tr}\left(F_{\mu_1\mu_2}H^i_{I_3I_4}\right)\,, \tag{24}
$$

while standard orientifolds do not [28]. In six dimensions, these four-forms are actually dual to axions a_{IJ}, and therefore this coupling can be rewritten in the form

$$
\mathcal{S}_{\mathrm{GS}} \sim \sum_i \int \mathrm{tr}(A_\mu Q^i)\;H^i_{IJ}\,\partial^\mu a^{IJ}\,, \tag{25}
$$

where Q^i denote the group generators associated to the internal magnetic fields. Thus, additional U(1) gauge fields can acquire mass by a generalisation of the mechanism in [1, 29], that in type-I strings generally involves several R-R forms. The (non-universal) axions involved in these Higgs mechanisms are the linear combinations $H^i_{IJ}\,a^{IJ}$.

A convenient way to recover these couplings uses, as in [30], a space-time magnetic background $\mathcal{F}$ that, when introduced in the string amplitudes (12) and (13), deforms the space-time theta-functions according to

$$
\frac{1}{\eta^2}\frac{\vartheta_\alpha(0|\tau)}{\eta(\tau)} \;\to\; (q_{\mathrm{L}}+q_{\mathrm{R}})\mathcal{F}\tau\,\frac{\vartheta_\alpha(\epsilon\tau|\tau)}{\vartheta_1(\epsilon\tau|\tau)}\,, \tag{26}
$$

with $\pi\epsilon = \tan^{-1}(q_{\rm L}\mathcal{F}) + \tan^{-1}(q_{\rm R}\mathcal{F})$. As a result, the untwisted R-R tadpoles are modified, and become

$$\Big[m + \bar{m} + n + \bar{n} - 32 + q^2(H_1 H_2 + \mathcal{F} H_1 + \mathcal{F} H_2)(n + \bar{n})\Big]\sqrt{v_1 v_2} \pm \frac{1}{\sqrt{v_1 v_2}}\Big[d + \bar{d} - 32\Big] \ . \tag{27}$$

Using the tadpole conditions (16) and the Dirac quantisation conditions (3), the terms linear in the space-time magnetic field identify the new Green-Schwarz couplings of eq. (24), needed to dispose of the new anomalous U(1) factors.

In conclusion, we have seen how in type I vacua a non-vanishing (anti)instanton density can be used to mimic BPS D5 (anti)branes, and we have exhibited some models with new distinctive features. These include supersymmetric T^4/Z_2 compactifications *without* D5 branes, or with gauge groups of unusual rank, that display new Green-Schwarz couplings of untwisted R-R forms. Several examples of this type can be constructed, both in six and in four dimensions. For instance, in the Z_3 orientifold of [31] magnetic deformations allow the introduction of a *net* number of D5 (anti)branes, a setting to be contrasted with the models of [14, 15, 32], that only involve D5 brane-antibrane pairs. Models with "brane supersymmetry breaking", in particular with additional brane-antibrane pairs, develop NS-NS tadpoles. These tadpoles are not eliminated by the magnetic deformation, and typically result in potentials that, although of run-away type for the dilaton, can in some cases stabilise some geometric moduli [15]. Their presence requires a background redefinition [33], that was recently constructed explicitly in [34] for the model in [35]. In general, these vacua correspond to supergravity models frozen in phases of broken supersymmetry, where the presence of (lower-dimensional) non-supersymmetric couplings renders the field equations naively inconsistent, in complete analogy with ordinary gauge theories frozen in a Higgs phase. Although similar features were previously met in the anomalous Green-Schwarz couplings of [27], the peculiar supergravity models resulting from "brane supersymmetry breaking" clearly deserve further investigation.

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