RF GYMNASTICS IN SYNCHROTRONS

R. Garoby
CERN, Geneva, Switzerland

Abstract
The RF systems installed in synchrotrons can be used to change longitudinal beam characteristics. 'RF gymnastics' designates manipulations of the RF parameters aimed at providing such non-trivial changes. Some keep the number of bunches constant while changing bunch length, energy spread, emittance or distance between bunches. Others are used to change the number of bunches. After recalling the basics of longitudinal beam dynamics in a hadron synchrotron, this paper deals with the most commonly used gymnastics. Their principles are described as well as performance and limitations.

1. INTRODUCTION

RF systems in synchrotrons are primarily specified for beam acceleration in variable energy machines or for bunching in accumulators. At a later stage of design, and quite often after the machine is built, it frequently becomes necessary to tailor further the longitudinal beam characteristics like bunch length, energy spread, distance between bunches, number of bunches etc. 'RF gymnastics', involving the modulation of the RF parameters, are then considered to help obtain the required performance [1].

As the high-energy frontier gets higher and higher, the cost of an accelerator complex increases, as does the interest in gymnastics, which allows the adaptation of such a facility for purposes not originally foreseen.

2. LONGITUDINAL BEAM DYNAMICS

2.1 Conventions

Synchrotron radiation will not be considered, so the following analysis is only relevant for hadrons.

The longitudinal phase plane has time (or phase) as the *x*-axis and energy (or momentum) as the *y*-axis. The following variables characterize a particle:

- Charge: q
- Rest energy, energy: E_0 , E
- Speed, momentum: v, p
- Relativistic parameters: γ ($\gamma=E/E_0$), β ($\beta=v/c$)
- Revolution period in the synchrotron: T

The synchrotron parameters are the following:

- Momentum compaction factor, transition gamma: α_P, γ_T
- Parameters of the synchronous particle: E_S, v_S, p_S, γ_S, β_S , T_S . The synchronous particle is defined as the particle whose energy E_S and phase ϕ_S (measured with respect to the zero crossing with positive slope of the sinusoidal RF waveform at the lowest harmonic, h_1) are such that the particle sees the same accelerating voltage over successive turns in the accelerator.

The total voltage $V(t)$ results from the contributions of RF systems with voltages $V_1(t)$, $V_2(t)$, etc:

$$V(t) = \sum_{i=1}^{n} V_i(t) \ . \tag{1}$$

If resonant structures are used, the voltage functions are sine waves with h_i periods per revolution and a relative phase θ_i.

$$V_i(t) = \hat{V}_i \sin(h_i \omega_R t + \theta_i) \quad \text{, with} \quad \omega_R = \frac{2\pi}{T_S} \ . \tag{2}$$

2.2 Motion in the longitudinal phase plane

2.2.1 Equations of motion

The motion of particles is analysed in the frame of the synchronous particle. The *x*-coordinate is the phase difference $\Delta\phi = \phi - \phi_S$ measured at the lowest harmonic (h_1), and the energy coordinate is $\Delta E = E - E_S$ (or $\Delta p = p - p_S$). The tracked and the synchronous particles having different revolution periods, the phase difference $\Delta\phi$ changes at every revolution according to Eq. (3):

$$d\Delta\phi = 2\pi h_1 \frac{(T - T_S)}{T_S} = 2\pi h_1 \frac{\Delta T}{T_S} \ . \tag{3}$$

The rate of change of the phase is then:

$$\frac{d\Delta\phi}{dt} = \frac{2\pi h_1}{T_S} \frac{\Delta T}{T_S} \ . \tag{4}$$

In a synchrotron the relative difference in revolution period is proportional to the relative difference in momentum or energy:

$$\frac{\Delta T}{T_S} = \eta \frac{\Delta p}{p_S} = \frac{\eta}{\beta^2} \frac{\Delta E}{E_S} \ , \tag{5}$$

where $\eta = \frac{1}{\gamma_T^2} - \frac{1}{\gamma^2}$.

From Eqs. (4) and (5), the x component of the particle speed is given by

$$\frac{d\Delta\phi}{dt} = 2\pi h_1 \eta \frac{1}{\beta^2 T_S} \frac{\Delta E}{E_S} \ . \tag{6}$$

The y component of the particle speed is the rate of change of its energy with respect to the synchronous particle and is given by

$$\frac{d\Delta E}{dt} = \frac{q}{T_S} \left[V(\Delta\phi + \phi_S) - V(\phi_S) \right] . \tag{7}$$

2.2.2 Case of a single RF harmonic

When a single RF system is used, the voltage can be expressed as

$$V(\phi) = \hat{V} \sin\phi \tag{8}$$

and Eq. (7) simplifies into

$$\frac{d\Delta E}{dt} = \frac{q}{T_S}\hat{V}\left[\sin(\Delta\phi + \phi_S) - \sin\phi_S\right] . \tag{9}$$

The motion described by Eqs. (6) and (9) has the following first integral characterizing closed trajectories of particles oscillating around the synchronous one:

$$\frac{1}{2}\left(\frac{d\Delta\phi}{dt}\right)^2 + \frac{2\pi h_1 \eta q \hat{V}}{\beta^2 T_S^2 E_S}\left[\cos(\Delta\phi + \phi_S) + \Delta\phi \sin\phi_S\right] = \text{constant} . \tag{10}$$

There is a limit to the amplitude of these oscillations. The corresponding trajectory is called the separatrix, and the enclosed region is the bucket, whose area is the acceptance. The separatrix crosses the phase axis at the extreme phase elongation

$$\Delta\phi_{EXT1} = \pi - 2\phi_S . \tag{11}$$

The other extreme phase elongation is the solution of:

$$\cos(\Delta\phi_{EXT2} + \phi_S) + \Delta\phi_{EXT2}\sin\phi_S = -\cos\phi_S + (\pi - 2\phi_S)\sin\phi_S . \tag{12}$$

The extreme excursion in energy is obtained when $\Delta\phi = 0$ rad

$$\Delta E_{MAX} = \sqrt{\frac{E_S \beta^2}{\pi h_1 \eta} q\hat{V}\left[(\pi - 2\phi_S)\sin\phi_S - 2\cos\phi_S\right]} . \tag{13}$$

Figure 1 illustrates the case of a stationary bucket (constant B field in the main dipoles and no acceleration of the synchronous particle) below transition energy (ϕ_S = 0 rad). The separatrix extends from –π to +π radians. The speed of a moving particle inside the bucket is shown. If it is an extreme particle of a stable population, its trajectory is the contour enclosing all others. This set of particles is called a bunch and the area inside the contour is its emittance.

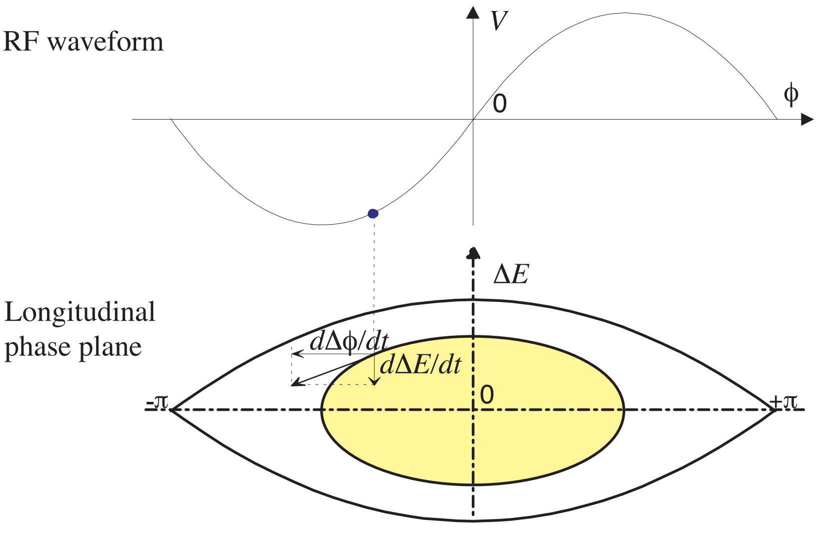

Fig. 1: Trajectories in a stationary bucket

For small amplitude of oscillation, Eqs. (6) and (9) represent a simple harmonic oscillator at the synchrotron frequency ω_S

$$\omega_S = \sqrt{\frac{2\pi h|\eta| q\hat{V}\cos\phi_S}{\beta^2 T_S^2 E_S}} \tag{14}$$

At constant emittance, the peak excursions in phase and energy scale like

$$\Delta\hat{\phi} \propto k \quad \text{and} \quad \Delta\hat{E} \propto \frac{1}{k} \quad \text{with} \quad k = \left[\frac{|\eta|}{E_S q\hat{V}\cos\phi_S}\right]^{\frac{1}{4}} \tag{15}$$

2.3 Effect of changing RF parameters

2.3.1 Adiabaticity

If the RF parameters are changed at a slow rate with respect to the smallest frequency of oscillation of the particles in the bunch, the distribution of particles is continuously at equilibrium and only depends upon the instantaneous value of these parameters. Such an evolution is called 'adiabatic'. The degree of adiabaticity is assessed with the adiabaticity parameter [2] defined as

$$\varepsilon = \frac{1}{\omega_S^2}\left|\frac{d\omega_S}{dt}\right| \tag{16}$$

A process is typically considered adiabatic when $\varepsilon < 0.1$.

2.3.2 Liouville's Theorem

The longitudinal motion that we consider is conservative (i.e. there is no energy dissipation effect like synchrotron radiation). Liouville's Theorem is therefore applicable. This states that the local density of particles in the longitudinal phase plane is always constant [3]. An implicit consequence is that any RF gymnastics is in principle reversible.

When an adiabatic process is used, this helps determine the particle distribution (or bunch shape) in the final state without having to take into account intermediate states (Fig. 2). The area occupied by particles ('emittance') is constant and always limited by a stable trajectory.

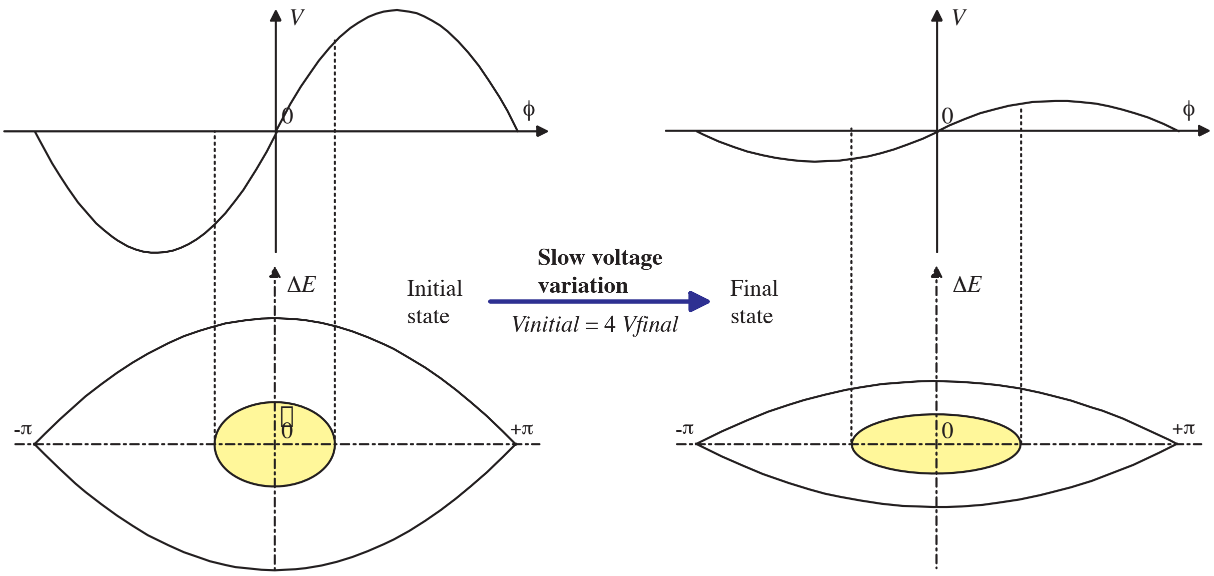

Fig. 2: Adiabatic RF voltage reduction

When a non-adiabatic gymnastic is applied, the consequences are less obvious and detailed tracking is required to evaluate the final particle distribution (Fig. 3). Although the area occupied by particles is also constant, its contour is usually not a stable trajectory in the final state. The final emittance generally has to be considered as increased to the value of the smallest area, limited by a stable trajectory that contains all particles ('macroscopic' emittance).

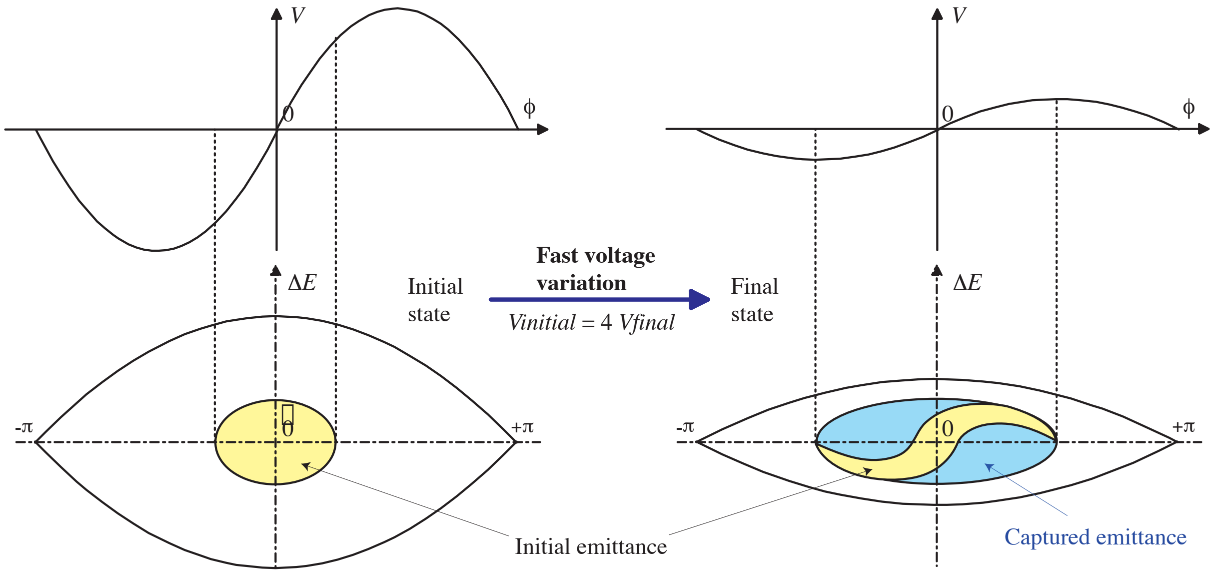

Fig. 3: Non-adiabatic RF voltage reduction

3. SINGLE BUNCH GYMNASTICS

3.1 Bunch compression

To preserve the longitudinal emittance and guarantee reproducible beam performance, the contour of the bunches entering a synchrotron must correspond to stable trajectories in the longitudinal phase plane of that machine. Such a condition is called 'longitudinal matching'. This often requires changing the ratio bunch length/energy spread of the bunches in the previous machine, and generally bunches must be made shorter. When adiabatic variation of the RF voltage cannot be used to provide the proper beam characteristics, non-adiabatic processes are applied. The corresponding gymnastics are called 'bunch compression', 'bunch rotation' or even 'phase rotation' [4, 5].

The principle (Fig. 4) is to let a bunch, initially elongated in phase, rotate in a maximum height bucket, and to eject it when it is at its shortest. Even with a single RF system, various techniques can be used for stretching the bunch:

- Reducing adiabatically the RF voltage V, the bunch length increases in proportion to $V^{-1/4}$ (Eq. (15)). This technique has the drawback of requiring a very large dynamic range in V and of becoming very slow in order to remain adiabatic at low voltages (see Eqs. (14) and (16)).
- Reducing abruptly the RF voltage, a bunch rotation is triggered, which provides, after a quarter of a turn in the phase plane, a bunch length proportional to $V^{-1/2}$. This process is more rapid and efficient than the previous one, but it is more demanding for the transient response of the cavity and the beam servo-loops.
- Switching by π radians the phase of the RF, the bunch becomes centred on the unstable phase and stretches quickly along the separatrix. This technique is also rapid and does not in principle require any voltage change, but it needs a rapid response from the RF system. The fact that the resulting bunch is tilted with respect to the phase axis implies that it will suffer more from non-linearities when rotating in the phase plane for compression.

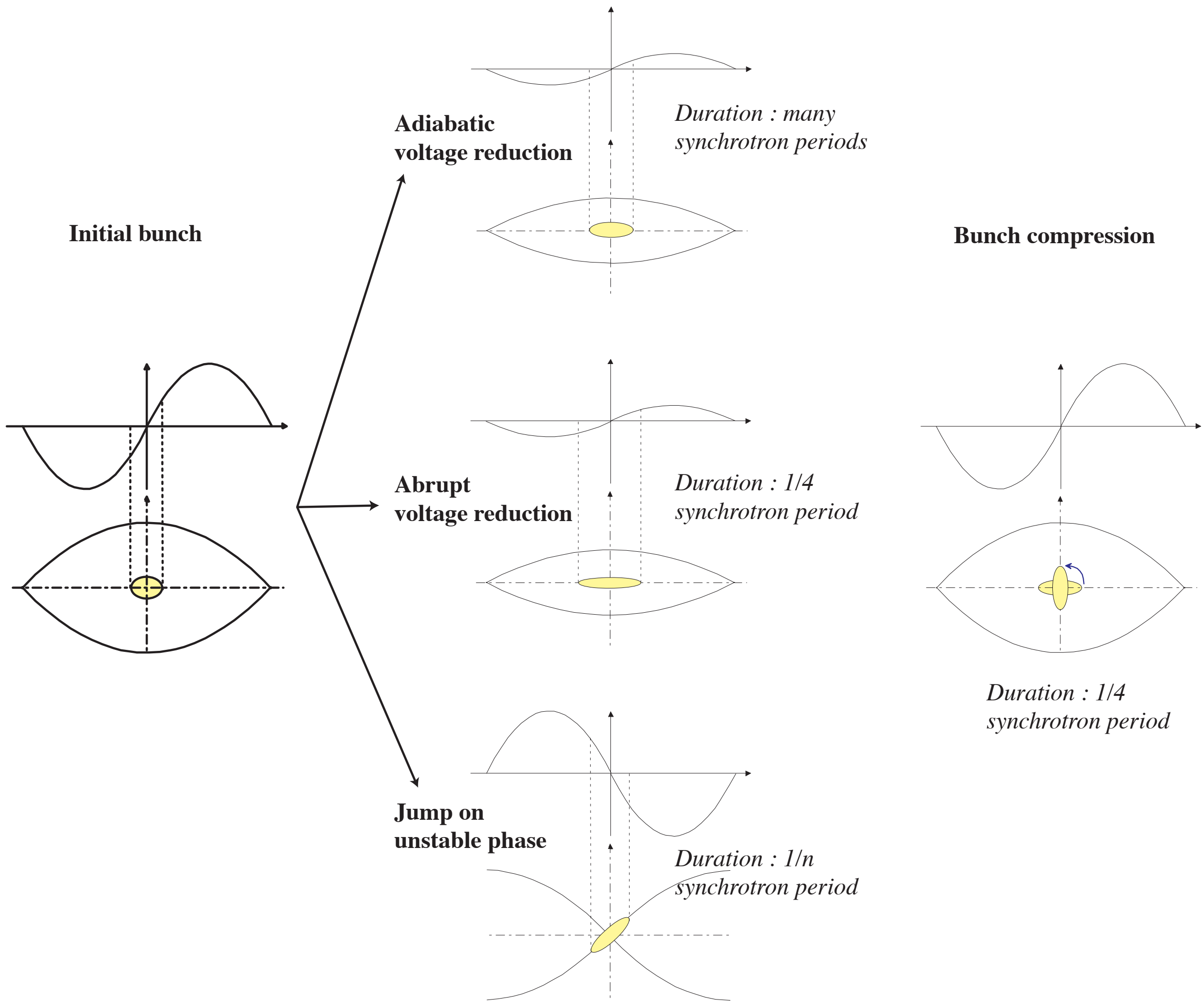

Fig. 4: Bunch compression

The performance of the compression process depends upon the bunch length and the normalized bunch emittance (ratio between emittance and acceptance) during the rotation of the elongated bunch. This is illustrated in Fig. 5, which shows the bunch at the beginning and at the end of rotation. An initially extreme particle along the energy axis (B0) becomes extreme in phase after rotation (B1) under the effect of a quasi-linear focusing voltage, approximated by the tangent at zero phase of the RF sine wave. In contrast, an initially extreme particle along the phase axis (A0) experiences a non-linear focusing voltage during rotation that is always smaller than the previous tangent, and results in a slower motion. In the time it takes for B0 to move to B1, A0 only moves to A1.

For a given normalized emittance, the minimum bunch length is obtained approximately when A1 and B1 are at the same phase. This defines an optimum initial bunch elongation, represented in Fig. 6. This figure also shows the minimum length achieved after rotation and the equilibrium length of a bunch of the same emittance in the rotation bucket. A compression efficiency can be defined as the ratio between that equilibrium bunch length and the length after rotation in optimum conditions. This efficiency is also shown.

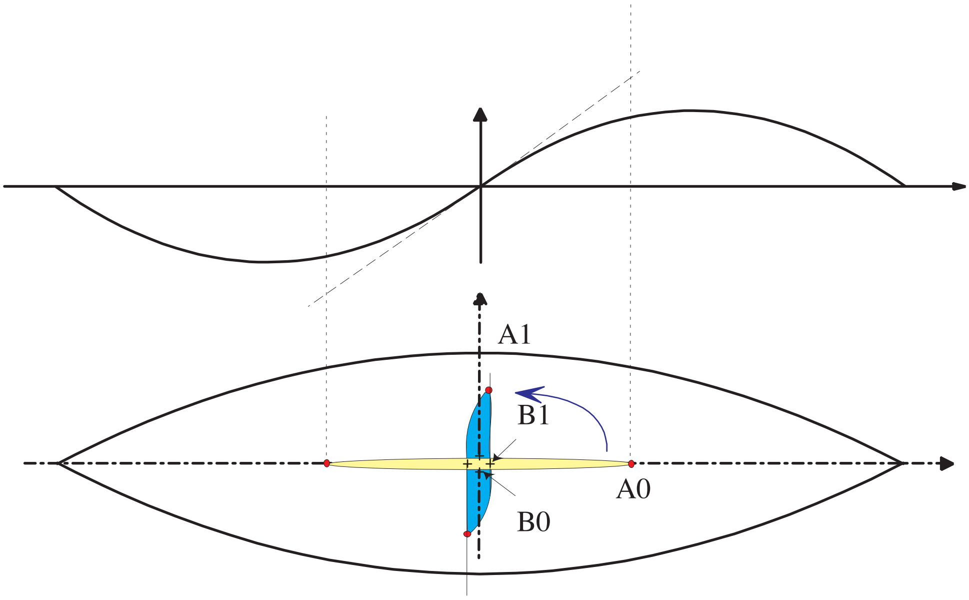

Fig. 5: Optimum bunch rotation

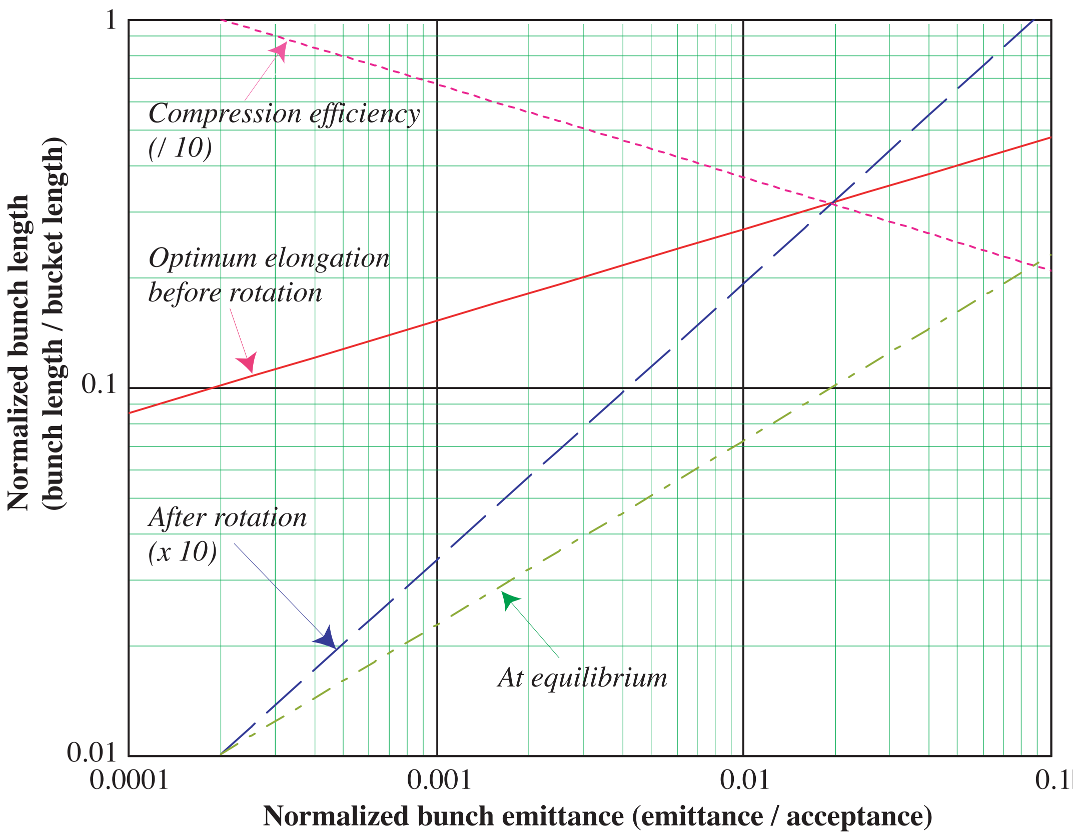

Fig. 6: Bunch rotation parameters

Numerous refinements are possible, using multiple RF harmonics or applying phase and amplitude modulations.

3.2 Longitudinal controlled blow-up

Blow-up techniques have been developed to help stabilize the beam in high-intensity accelerators. They increase the 'macroscopic' emittance in a controlled way while providing an adequate distribution of particles with sharp edges and no tails. A typical and commonly used technique is based on the superposition of a phase-modulated high frequency (V_H, h_H) on the RF normally holding the beam (V_1, $h_1 << h_H$) [6, 7].

The high frequency phase-modulated voltage can be expressed as

$$V_H = \hat{V}_H \sin(h_H \omega_R t + \alpha \sin \omega_M t + \vartheta_H), \tag{17}$$

α being the peak phase modulation, ω_R the modulation frequency, and ϑ_H a phase constant.

This acts as a perturbation to the motion of particles in the bucket of the main RF system. Resonances can be induced that create a redistribution of density in the bunch. Large non-linearities in the motion accelerate filamentation and contribute to the fast disappearance of the density modulations induced by the high-frequency carrier. Of the different distributions that can be obtained, parabolic ones are generally preferred.

The blow-up parameters are in practice optimized either on the real accelerator or using computer simulations. The typical range of values applied in such cases is shown in Table 1.

Smaller harmonic ratio are sometimes used because of hardware constraints, and good quality blow-up can still be obtained after a longer duration.

Table 1: Typical blow-up parameters

	$\frac{\hat{V}_H}{\hat{V}_1}$	$\frac{h_H}{h_1}$	α (rad)	$\frac{\omega_M}{\omega_S}$	Duration
Typical range	0.05 to 0.2	> 10 for fast blow-up	0.8π to 1.2π	2 to 7	$\geq 10\frac{2\pi}{\omega_S}$

4. MULTI-BUNCH GYMNASTICS

4.1 Debunching–rebunching

Debunching–rebunching is the most conventional way to change the number of bunches [5, 8]. It has to take place at constant energy and hence at constant field in the main bending dipoles because of the absence of RF for a significant period of time. At the end of debunching the beam is continuous and ideally has not undergone any azimuthal modulation of the linear density of particles. Rebunching is the reverse process, during which a different RF harmonic number is used, and the beam progressively gets an azimuthal modulation of density and is finally fully bunched on the new harmonic.

Iso-adiabatic debunching is generally used to minimize longitudinal emittance blow-up. The reduction of the RF voltage from V_{I_deb} to V_{F_deb} occurs at constant adiabaticity (see Eq. (16)):

$$V(t) = \frac{V_{I\_deb}}{\left[1 - \left(1 - \sqrt{\frac{V_{I\_deb}}{V_{F\_deb}}}\right)\frac{t}{t_R}\right]^2}, \tag{18}$$

where t_R is the moment of suppression of the RF voltage after reaching the minimum controllable level V_{F_deb}. This is illustrated in Fig. 7. The process takes more time when V_{F_deb} is made smaller:

$$t_R \approx \frac{1}{\omega_S(V_{F\_deb})} \propto \frac{1}{\sqrt{V_{F\_deb}}}. \tag{19}$$

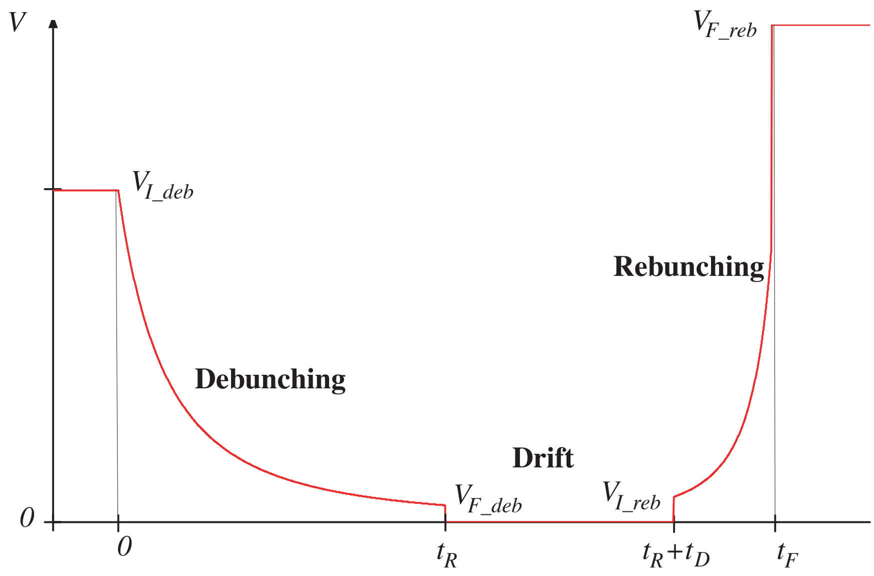

Fig. 7: Voltages for iso-adiabatic debunching–rebunching

During this voltage reduction, the bunch progressively lengthens (proportionally to $V^{-1/4}$ at the beginning according to Eq. (15)). Under the voltage V_{F_deb} the beam is generally still bunched and some time, t_D, is required without voltage for the particles to drift in azimuth and for debunching to be obtained. This results in a blow-up of the macroscopic emittance, which depends upon the normalized bunch emittance in the final bucket (as shown in Fig. 8). In the typical case, where the bunch finally completely fills the bucket, the emittance is multiplied by π/2.

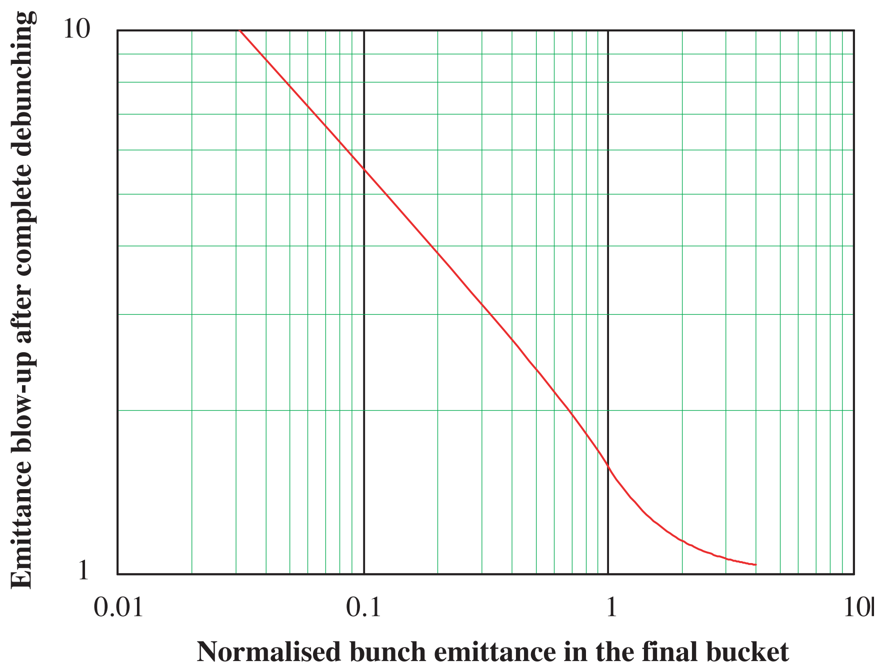

Fig. 8: Emittance blow-up after iso-adiabatic debunching

A reference debunching time can be defined as the time taken for the particles of successive bunches to begin to overlap in azimuth:

$$t_{D\_classic} = \frac{\pi - \Delta\phi}{h\omega_R|\eta|\frac{\Delta p}{p}} \qquad (20)$$

where $\Delta\phi$ and Δp are the full spreads in phase and momentum of the bunch under V_{F_deb}.

A proper debunching requires $t_D >> t_{D\_classic}$.

Iso-adiabatic rebunching is generally used after debunching is completed. It is a time-reversed version of iso-adiabatic debunching, starting abruptly at V_{I_reb} and rising progressively to V_{F_reb}. Similar formulae apply.

4.2 Splitting (Merging)

Splitting is used to multiply the number of bunches by two or three and merging is the reverse process [9, 10]. Although limited in use to circumstances where such ratios are possible, it is an interesting technique compared to iso-adiabatic debunching–rebunching because it can really be made quasi-adiabatic and preserve the emittance.

Splitting bunches in two is obtained using two RF systems with an harmonic ratio of two simultaneously. The bunch is initially held by the first system (V_1, h_1) while the second (V_2, h_2=2 h_1) is stopped. The unstable phase on the second harmonic is centred on the bunch. As the voltage V_2 is slowly increased and V_1 is decreased the bunch lengthens and progressively splits in two as illustrated in Fig. 9.

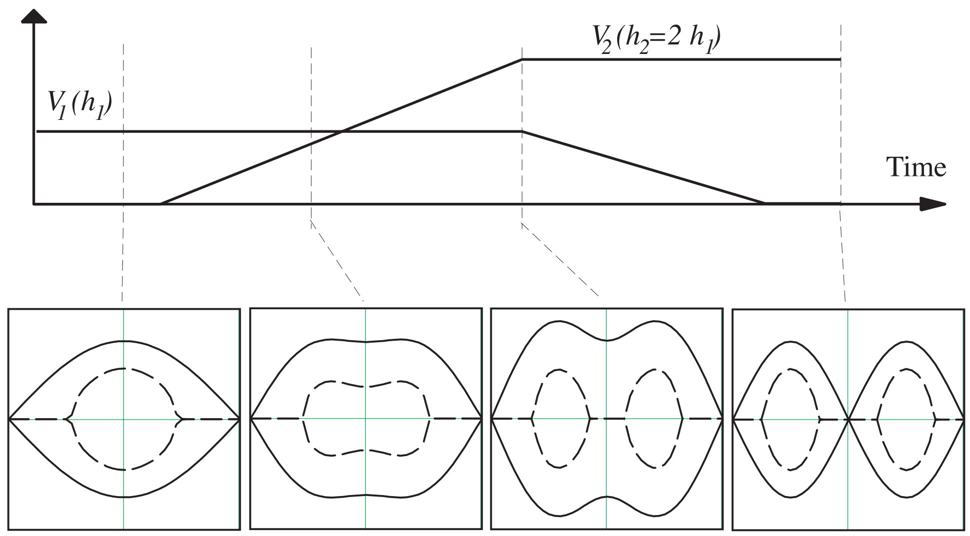

Fig. 9: Bunch splitting in two

When two separate bunches have just begun to form, the voltage on the first harmonic is V_{1_sep}. The normalized emittance at that time is defined as

$$\varepsilon_{sep} = [\text{Total emittance/Acceptance}(V_{1\_sep}, h_1)]\ . \qquad (21)$$

The ratio V_2/ V_{1_sep} at this moment is given in Fig. 10. Good results are consistently obtained for ε_{sep} ~1/3 and using voltage variations thar are linear functions of time with a total duration larger than five synchrotron periods in the bucket (V_{1_sep}, h_1). Each final bunch has half the emittance of the initial bunch, and almost no blow-up is observed.

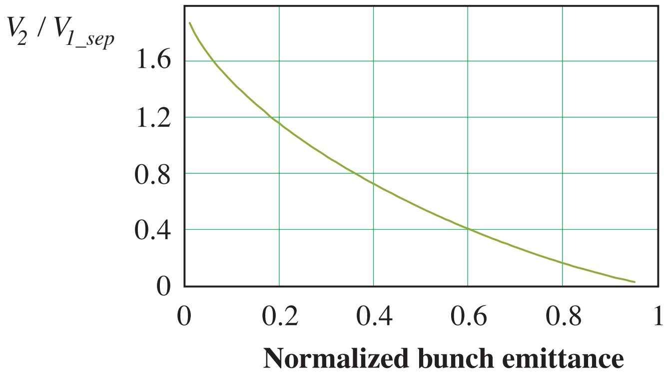

Fig. 10: V_2/V_{1_sep} at the time of bunch separation

An operational implementation of double splitting in the CERN PS is illustrated in Fig. 11. A bunch on h = 8 is split in two on h = 16 within 25 ms and no blow-up can be observed. On the left side of the same figure, the evolution of particle density in the longitudinal phase plane during the process is reconstructed using longitudinal tomography [11].

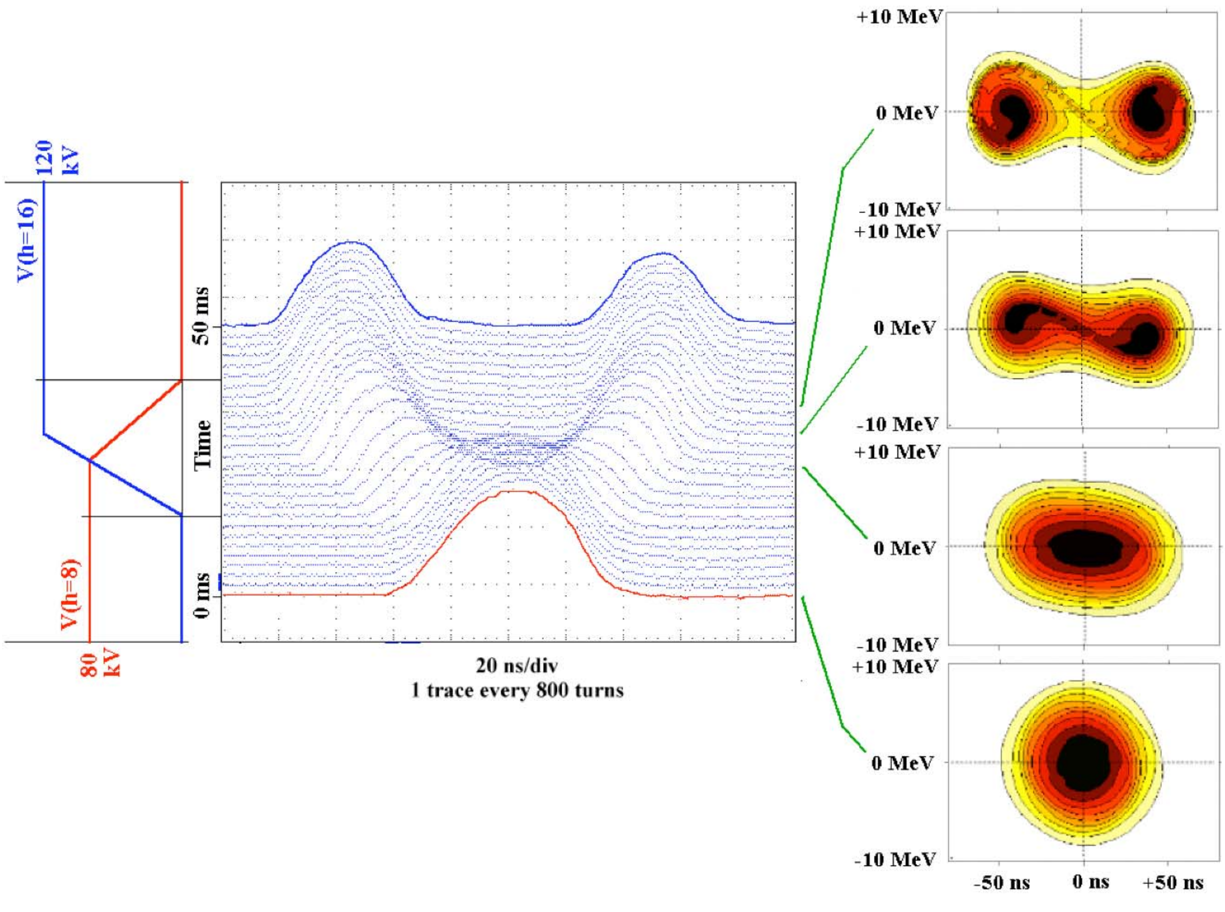

Fig. 11: Example of bunch double splitting from h = 8 to h = 16 in the CERN PS at 3.57 GeV/*c*

Splitting bunches in three has also been demonstrated using three simultaneous RF systems. The relative phases between harmonics as well as the voltage ratios must be precisely controlled for the particles to split evenly into the new bunches and longitudinal emittance to be preserved. As good results as for bunch double splitting have been achieved, and final bunches are one-third the emittance of the original one.

4.3 Batch compression

Batch compression is a process that keeps the number of bunches constant while concentrating them in a reduced fraction of the accelerator circumference [12]. When exercised at a slow enough rate it can be adiabatic and consequently preserve longitudinal emittance.

The principle is slowly to increase the harmonic number of the RF controlling the beam, as shown in Fig. 12. Starting from harmonic h_0, voltage is progressively increased on harmonic $h_1 > h_0$ and decreased to 0 V on h_0, so that harmonic h_1 finally holds the bunches. The phase on h_1 with respect to h_0 must be such that the bunches converge symmetrically towards the centre of the batch.

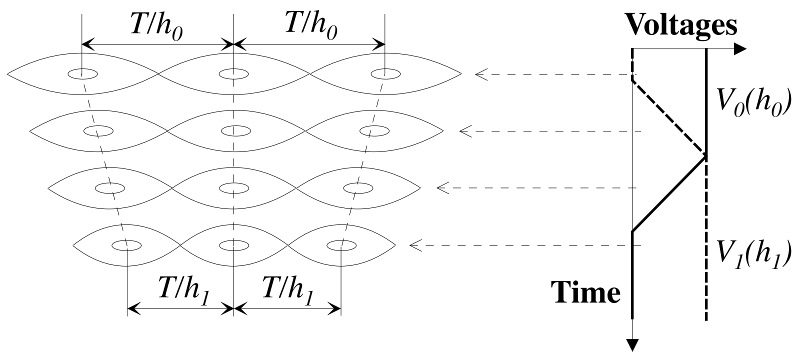

Fig. 12: Batch compression

The amount of compression achievable in a single step is limited by the need to maintain a large enough acceptance for the buckets holding the edge bunches. A consequence is that large compression factors are only obtained after multiple batch compression steps, and complicated manipulations of RF parameters are involved. A typical application is given in Fig. 13, where four bunches on h = 8 are finally brought into four adjacent buckets on h = 20: three groups of RF cavities are used, which help sweep progressively the harmonic seen by the beam from 8 to 20 in steps of two units.

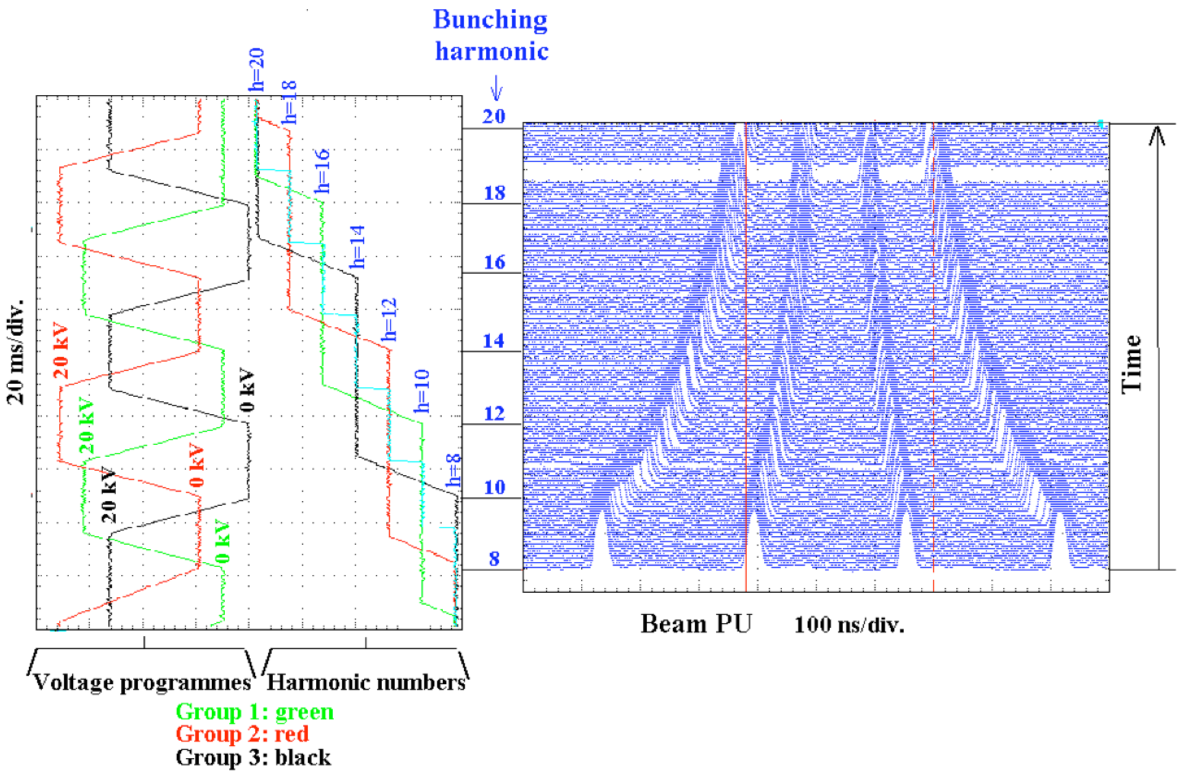

Fig. 13: Example of batch compression from h = 8 to h = 20 in the CERN PS at 26 GeV

4.4 Slip stacking

Slip stacking is used to superimpose two sets of bunches and double the bunch population [13, 14]. It is non-adiabatic and leads to large emittance blow-ups.

The principle is sketched in Fig. 14. Two different RF frequencies are simultaneously applied. If their difference is large enough ($\Delta f > 2f_s$, where f_s is the synchrotron frequency in the centre of an unperturbed bucket of one family), two families of buckets coexist, which drift towards each other because of their frequency difference. Consequently, and provided the acceptance of these buckets ($f = h_0 f_{REV} \pm \Delta f$; V_{drift}) is large enough (acceptance > 2 × emittance), the bunches drift with them and tend to slip past each other. When they are superimposed in azimuth, pairs of bunches can be captured in large buckets centred at the middle frequency ($f = h_0 f_{REV}$; $V_{capture}$).

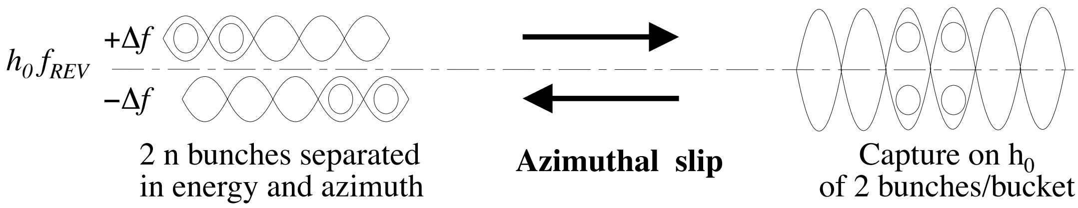

Fig. 14: Slip stacking

Although improvements can be made, like reducing the frequency difference towards the end of the process, the longitudinal contour enclosing a pair of bunches in the final bucket also contains a large area without particles. After filamentation, the macroscopic emittance is much more than doubled and longitudinal density is accordingly reduced.

5. DEBUNCHED BEAM GYMNASTICS

5.1 Barrier / isolated bucket

A single sine wave pulsing at the revolution frequency of the beam generates either an isolated or a barrier bucket, depending upon its polarity and the sign of η (Fig. 15).

In the case of the isolated bucket there is a stable ('synchronous') particle at the central zero crossing of the sine wave. Particles inside the sine wave period can be captured and execute closed trajectories around the particle. Particles outside this bucket move along the full circumference.

In the case of the barrier bucket, the central zero crossing of the sine wave is an unstable position. The stable region is limited by the other zero crossings and extends over all the circumference except the sine wave.

Such a voltage can be obtained from a wide band resonator driven by a high power amplifier or from a limited bandwidth resonator driven by a large current generator.

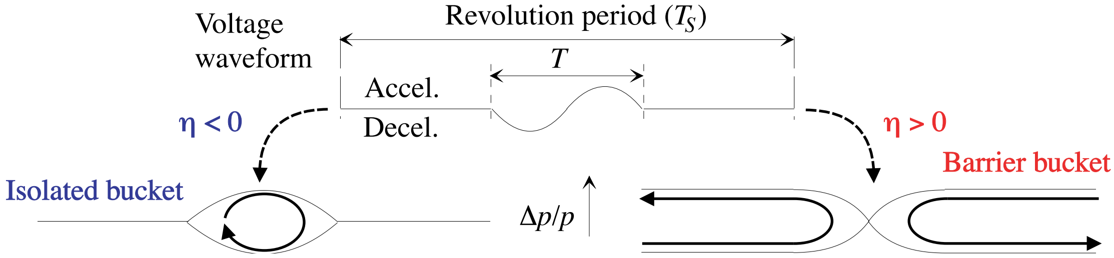

Fig. 15: Isolated / suppressed bucket

Beam dynamics is governed by the equations derived in Section 2.2. An isolated bucket is useful to capture a single bunch of small emittance in the debunched beam stack of an accumulator [8]. Barrier buckets are also typically used for high intensity accumulation, to preserve gaps without beam, and to permit lossless beam transfers [15].

5.2 Phase displacement acceleration

A debunched beam is accelerated (or decelerated) when being traversed by empty RF buckets [16]. This is due to emittance preservation for the empty volume captured by the RF buckets. The resulting change of the stack mean energy is given by

$$\Delta E_{Stack} = A_{bucket} \frac{h}{T} = A_{bucket} f_{RF} \tag{22}$$

A small voltage and a limited frequency range (of a few per cent) are sufficient, while a large beam current and emittance can be handled. However, the acceleration rate is small and the stack tends to degrade progressively as the number of traverses increases.

6. PRACTICAL IMPLEMENTATION

The possible implementation and the effective performance of RF gymnastics in synchrotrons are constrained by a number of practical limitations. In addition to those resulting from the basic hardware capabilities (number of simultaneous frequencies, minimum controllable voltages, etc.) a number of others must also be mentioned:

- Maximum duration at constant field in the dipoles. This may force the use of fast and non-adiabatic techniques or a degraded adiabaticity.
- Beam stability. The quality and reproducibility of performance of the final beam depends on the reproducibility of the initial conditions and the absence of collective beam instabilities during the process.
- Control of the RF parameters. The proper operation of the servo-loops (beam phase, radial, or synchronization loop) all along the gymnastics is critical for performance, and unavoidable transients must be minimized, with their delayed effect quickly damped. Moreover, for good performance at high beam intensity, beam loading in the RF cavities must be minimized, so local RF feedback and 'one-turn delay feedback' are often necessary.
- Variation of the dipole field during the gymnastics. This can be due to drift or ripple of the field in the main dipole, but also to changes of the orbit length induced by orbit bumps.

Solutions exist, but much time can be gained during setting up by a preliminary analysis of the likely disturbances and the direct implementation of adequate corrective measures.

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