Anomalous magnetic moment of muon in 3 - 3 - 1 models

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Abstract

A contribution from new gauge bosons in the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ (3 - 3 - 1) models to the anomalous magnetic moments of the muon is calculated and numerically estimated. In the minimal 3 - 3 - 1 model, a lower bound on the bilepton mass at a value of 167 GeV is derived. For an expected precision($\sim 4 \times 10^{-10}$) of the BNL measurements the possible lower bounds on masses of the bileptons in the minimal version and in the version with right-handed neutrinos are around 940 GeV and 250 GeV, respectively.

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1 Introduction

The SuperKamiokande results [1] confirming non-zero neutrino mass call for the standard model (SM) extension. Among the known extensions, the models based on the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ gauge group [2,3] have the following intriguing features: firstly, the models are anomaly free only if the number of families N is a multiple of three. Further, from the condition of QCD asymptotic freedom, which means $N < 5$, it follows that N is equal to 3. The second characteristic is that the Peccei–Quinn [4] symmetry, a solution of the strong CP problem naturally occurs in these models [5]. The third interesting feature is that one of the quark families is treated differently from the other two [6,7]. This could lead to a natural explanation of the unbalancing heavy top quarks in the fermion mass hierarchy [7]. Recent analyses have indicated that signals of new particles in this model, bileptons [8] and exotic quarks [9] may be observed at the Tevatron and the Large Hadron Collider (LHC).

There are two main versions of the 3 - 3 - 1 models: the minimal model in which all lepton components $(\nu, l, (l)^c)_L$ of each family belong to one and same lepton triplet and a variant, in which right–handed neutrinos (r. h. neutrinos) are included, i.e. $(\nu, l, \nu^c)_L$ (hereafter we call it a model with right-handed neutrino [10,11]). New gauge bosons in the minimal model are bileptons $(Y^\pm, X^{\pm\pm})$ carrying lepton number $L = \pm 2$ and Z'. In the

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second model, the bileptons with lepton number $L = \pm 2$ are singly–charged $Y^{\pm}$ and *neutral* gauge bosons X^0, X^{*0} , and both are responsible for lepton–number violating interactions.

With the present group extension there are five new gauge bosons and all these particles are heavy. Getting mass limits for these particles is one of the central tasks of further studies. The anomalous magnetic moments of the muon (AMMM) $a_\mu \equiv (g_\mu - 2)/2$ is one of the most popular values in pursueing this aim. Despite not competitive with the anomalous magnetic moment of the electron (AMME) in precision, the AMMM is much more sensitive to loop effects as well as "New Physics" due to contributions $\sim m_\mu^2$, i.e. $\sim (200)^2$ enhancement in the AMMM relative to the AMME. Therefore the AMMM is a subject of both theoretical and experimental investigations [12]. The $(g_\mu - 2)/2$ was used to get constraints on mass of the bilepton in the minimal version [13]. However in the cited paper a contribution from new neutral gauge boson Z' was not included.

The aim of this work is to calculate the $(g_\mu - 2)/2$ in both 3 - 3 - 1 versions. As a consequence, constraints on the new gauge boson masses are discussed.

Our paper is organized as follows: In Sec. 2 after a brief introduction into the minimal version, we present contributions to the $(g_\mu - 2)/2$ from both bileptons and Z'. Constraints on their masses are also derived. Sec. 3 is devoted to the version with r.h. neutrinos. Finally, our conclusions are summarized in the last section.

2 The $(g_\mu - 2)/2$ in the minimal version

Let us firstly recapitulate the basic elements of the model (for more details see [14]). Three lepton components of each family are in one triplet:

$$f_L^a = (\nu^a, \ l^a, \ (l^c)^a)_L^T \sim (1, 3, 0), \tag{1}$$

where $a = 1, 2, 3$ is the family index. The charged bileptons with lepton number $L = \pm 2$ are identified as follows: $\sqrt{2} \ Y_\mu^- = W_\mu^4 - iW_\mu^5, \sqrt{2} \ X_\mu^{--} = W_\mu^6 - iW_\mu^7$, and their couplings to leptons are given by [15]

$$\mathcal{L}_l^{CC} = -\frac{g}{2\sqrt{2}} \left[\bar{\nu}\gamma^\mu(1 - \gamma_5)C\bar{l}^T Y_\mu^- - \bar{l}\gamma^\mu\gamma_5 C\bar{l}^T X_\mu^{--} + \text{h.c.} \right]. \tag{2}$$

It is to be noted that the vector currents coupled to X^{--}, X^{++} vanish due to Fermi statistics. To get physical neutral gauge bosons one has to diagonalize their mass mixing matrix. That can be done in two steps: At the first, the photon field A_μ and Z, Z' are given by [14]

$$
\begin{aligned}
A_\mu &= s_W W_\mu^3 + c_W \left(\sqrt{3} \ t_W W_\mu^8 + \sqrt{1 - 3 \ t_W^2} \ B_\mu \right), \\
Z_\mu &= c_W W_\mu^3 - s_W \left(\sqrt{3} \ t_W W_\mu^8 + \sqrt{1 - 3 \ t_W^2} \ B_\mu \right), \\
Z'_\mu &= \sqrt{3} \ t_W \ B_\mu - \sqrt{1 - 3 \ t_W^2} \ W_\mu^8,
\end{aligned}
\tag{3}
$$

where, as usual, the notation $s_W \equiv \sin\theta_W$ is used. In the second step, we get the physical neutral gauge bosons Z^1 and Z^2 which are mixtures of Z and Z':

$$
\begin{aligned}
Z^1 &= Z\cos\phi - Z'\sin\phi, \\
Z^2 &= Z\sin\phi + Z'\cos\phi.
\end{aligned}
\tag{4}
$$

The mixing angle ϕ is constrained to be very small, therefore the Z and the Z' can be safely considered as the physical particles.

The gauge interactions for Z' can be written in the form

$$
\mathcal{L}^{NC} = \frac{g}{c_W}\left\{ \bar{f}\gamma^\mu [g'_V(f) + g'_A(f)\gamma_5] f Z'_\mu \right\}.
\tag{5}
$$

The alternative left–right form with coupling coefficients [14]

$$
g'_{L,R}(f) = -\frac{\sqrt{1-4s_W^2}}{2\sqrt{3}} Y(f_{L,R}) + \frac{1-s_W^2}{\sqrt{3(1-4s_W^2)}} N(f_{L,R}),
\tag{6}
$$

has simple relations $g'_V(f) = [g'_R(f) + g'_L(f)]/2$, $g'_A(f) = [g'_R(f) - g'_L(f)]/2$.

Now we calculate contributions from the bileptons and the Z' to the AMMM. It is known that heavy Higgs boson contribution to the AMMM is negligible [16], therefore the relevant diagrams are depicted in Fig.1.

The first three diagrams come from the bileptons and their contributions are found to be

$$
\delta a_\mu^B = \frac{g^2 m_\mu^2}{24\pi^2}\left(\frac{16}{M_X^2} + \frac{5}{4M_Y^2} \right),
\tag{7}
$$

where M_X, M_Y, m_μ stand for masses of the doubly-, singly-charged bileptons and of the muon, respectively. In the limit $m_\mu << M_{Z'}$ where $M_{Z'}$ is the Z' mass, the Z' contribution has the form [17]

$$
\delta a_\mu^{Z'} = \frac{m_\mu^2}{12\pi^2 M_{Z'}^2}\left(g'^2_V - 5g'^2_A \right).
\tag{8}
$$

Applying Eq.(6) we get coupling of the muon to the Z'

$$
g'_V(\mu) = \frac{g}{c_W}\frac{3\sqrt{1-4s_W^2}}{2\sqrt{3}}, \ g'_A(\mu) = \frac{g}{c_W}\frac{\sqrt{1-4s_W^2}}{2\sqrt{3}}.
\tag{9}
$$

Substituting (9) into (8) we obtain the Z' contribution

$$
\delta a_\mu^{Z'} = \frac{g^2}{3c_W^2}\frac{m_\mu^2}{12\pi^2 M_{Z'}^2}(1-4s_W^2).
\tag{10}
$$

Therefore the total contribution from new gauge bosons in the minimal version to the AMMM becomes

$$
\delta a_\mu^{tm} = \frac{G_F m_W^2 m_\mu^2}{3\sqrt{2}\pi^2}\left[\frac{16}{M_X^2} + \frac{5}{4M_Y^2} + \frac{2(1-4s_W^2)}{3c_W^2 M_{Z'}^2} \right],
\tag{11}
$$

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where $G_F/\sqrt{2} = g^2/(8m_W^2)$ is used.

Note that the Z' gives a positive contribution to the AMMM, while the Z gives a negative one as it is well–known in the SM. From Eq. (11) it follows that the bilepton contributions are dominant.

By the spontanous symmetry breaking (SSB) it follows that [18] $|M_X^2 - M_Y^2| \leq \mathcal{O}(m_W^2)$ (more precisely $|M_X^2 - M_Y^2| \leq 3m_W^2$) . Therefore it is acceptable to put $M_X \sim M_Y$ as it was done in [13]. In this approximation, Eq. (7) agrees with the original result in [13], and Eq. (11) becomes

$$\delta a_\mu^{tm} = \frac{G_F m_W^2 m_\mu^2}{\sqrt{2}\pi^2} \left[\frac{23}{4M_Y^2} + \frac{2(1 - 4s_W^2)}{9c_W^2 M_{Z'}^2} \right]. \tag{12}$$

A lower limit $M_Y \sim 230$ GeV at 95% CL can be extracted by the "wrong" muon decay $\mu \to e^- \nu_e \bar{\nu}_\mu$. Combining with the SSB, it follows [14] $M_{Z'} \geq 1.3$ TeV. With the quoted numbers ($M_X = 180$, $M_Y = 230$, $M_{Z'} = 1300$ GeV), the contributions to δa_μ^{tm} from the bileptons and the Z' are 1.04×10^{-8} and 7.76×10^{-13}, respectively. The bilepton contribution is in a range of "New Physics" one [19] $\sim \mathcal{O}(10^{-8})$.

Putting a bound on "New Physics" contribution to the AMMM [20]

$$\delta a_\mu^{New\ Physics} = (7 \pm 8.6) \times 10^{-9}, \tag{13}$$

into the l.h.s of (12) we can obtain a bound on M_Y. In Fig. 2 we plot δa_μ^{tm} as a function of M_Y. For certainty we used $M_{Z'} = 1.3$ TeV quoted above. The horizontal lines are the upper and the lower limit from $\delta a_\mu^{New\ Physics}$.

From the figure we get a lower mass limit on M_Y to be 167 GeV. We recall that this limit is in a range of those obtained from LEP data analysis ($M_Y \geq 120$ GeV) [21].

In the near future, the E-821 Collaboration at Brookhaven would reduce the experimental error on the AMMM to a few $\times 10^{-10}$.

In Fig. 3 we see that δa_μ^{tm} cuts horizontal line I ($\sim 4 \times 10^{-10}$) and line II ($\sim 1 \times 10^{-10}$) at $M_Y \approx 935$ GeV and $M_Y \approx 1870$ GeV, respectively. These lower bounds are much higher than those from the muon experiments.

3 The $(g_\mu - 2)/2$ in the model with r.h. neutrinos

In this version the third member of the lepton triplet is a r. h. neutrino instead of the antilepton l_L^c

$$f_L^a = (\nu^a, l^a, (\nu^c)^a)_L^T \sim (1, 3, -1/3), l_R^a \sim (1, 1, -1). \tag{14}$$

The complex new gauge bosons $\sqrt{2}\, Y_\mu^- = W_\mu^6 - iW_\mu^7, \sqrt{2}\, X_\mu^0 = W_\mu^4 - iW_\mu^5$ are responsible for lepton–number violating interactions. Instead of the doubly–charged bileptons $X^{\pm\pm}$,

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here we have neutral ones X^0, X^{0*}. The SSB gives the bilepton mass splitting [22]

$$|M_Y^2 - M_X^2| \le m_W^2.$$

As before one diagonalizes the mass mixing matrix of the neutral gauge bosons by two steps, and the last one is the same for both versions. At the first step we have

$$A_\mu = s_W W_\mu^3 + c_W \left(-\frac{t_W}{\sqrt{3}} W_\mu^8 + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right),$$

$$Z_\mu = c_W W_\mu^3 - s_W \left(-\frac{t_W}{\sqrt{3}} W_\mu^8 + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right),$$

$$Z_\mu' = \sqrt{1 - \frac{t_W^2}{3}} W_\mu^8 + \frac{t_W}{\sqrt{3}} B_\mu. \tag{15}$$

Due to smallness of mixing angle ϕ we can consider the Z and the Z' as the physical particles. The couplings of fermions with Z' boson are given as follows [23]:

$$g_{L,R}'(f) = c_W^2 \left[\frac{3N(f_{L,R})}{(3 - 4s_W^2)^{1/2}} - \frac{(3 - 4s_W^2)^{1/2}}{2c_W^2} Y(f_{L,R}) \right]. \tag{16}$$

From (16), the couplings of Z' to muon are found to be

$$g_V'(\mu) = \frac{g}{4c_W} \frac{(1 - 4s_W^2)}{\sqrt{3 - 4s_W^2}}, \quad g_A'(\mu) = -\frac{g}{4c_W \sqrt{3 - 4s_W^2}}. \tag{17}$$

Due to its neutrality, the bilepton X^0 does not give a contribution and in this case, the relevant diagrams are only two last (c) and (d). The contribution from the singly–charged bilepton and the Z' in Fig. 1(c) and 1(d) is

$$\delta a_\mu^{tr} = \frac{G_F m_W^2 m_\mu^2}{12\sqrt{2}\pi^2} \left\{ \frac{5}{M_Y^2} - \frac{[5 - (1 - 4s_W^2)^2]}{2c_W^2(3 - 4s_W^2)M_{Z'}^2} \right\}. \tag{18}$$

In the considered version the Z' gives a negative contribution. However, the total value in r.h.s of Eq. (18) is positive (an opposite sign happens when $M_{Z'} \le 0.3 \ M_Y$ which is excluded by the SSB).

Putting the Z' lower mass bound to be 1000 GeV [23] followed from Δm_K and $M_Y = 230$ GeV we get the bilepton and the Z' contributions to δa_μ^{tr}, respectively: 4.75×10^{-10} and -7.87×10^{-12}. This implies that the contribution of the new gauge bosons in the considered version is in two order smaller than an allowed difference between theoretical calculation in the SM and present experimental precision.

However, putting two previous values for δa_μ^{tr} we get lower bounds on the bilepton masses to be about 250 GeV (I) and 500 GeV (II) (see Fig. 4).

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4   Conclusion

In conclusion, we have calculated in detail the second–order contribution from new gauge bosons in the 3 - 3 - 1 models to the AMMM. In the minimal version, contribution from the Z' is positive but suppressed due to a factor $(1 - 4s_W^2)$ while in the version with r.h.neutrinos the contribution Z' is negative. In both cases the bilepton contribution is bigger by an absolute value and the total contribution from the new gauge bosons is positive. Comparing with experimental bounds on the AMMM we get the lower bounds on the bilepton mass: $M_Y > 167$ GeV in the minimal version. For the version with r.h.neutrinos the contribution of the new gauge bosons is in two order smaller than those in the minimal one and it does not allow to get a constraint at the present status on the AMMM.

Although analysis on the AMMM could not give a better limit on the bilepton mass than those from other studies [24], the study on the contribution of new gauge bosons to the AMMM is in its own right very important. However our limit can be made more restrictive by including further experiments.

With the expected experimental error on the AMMM at BNL to be a few $\times 10^{-10}$ the lower bounds on masses of the bileptons in the minimal and in the version with r.h. neutrinos are around 1 TeV and 400 GeV, respectively.

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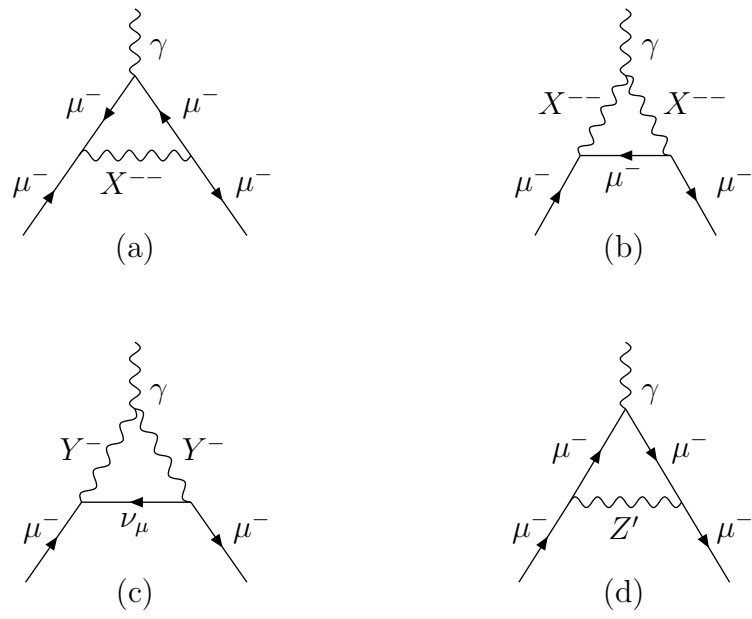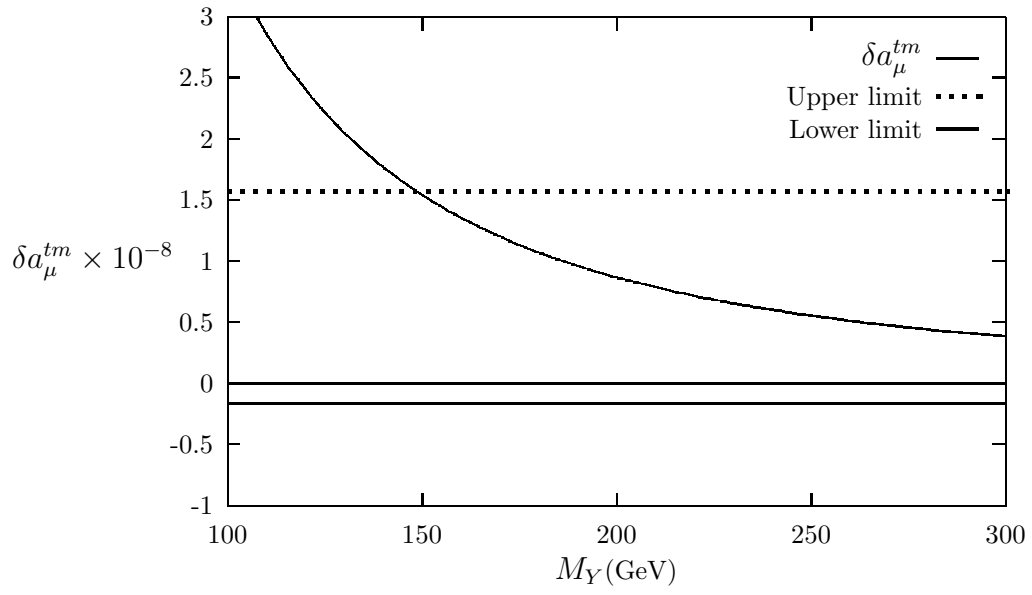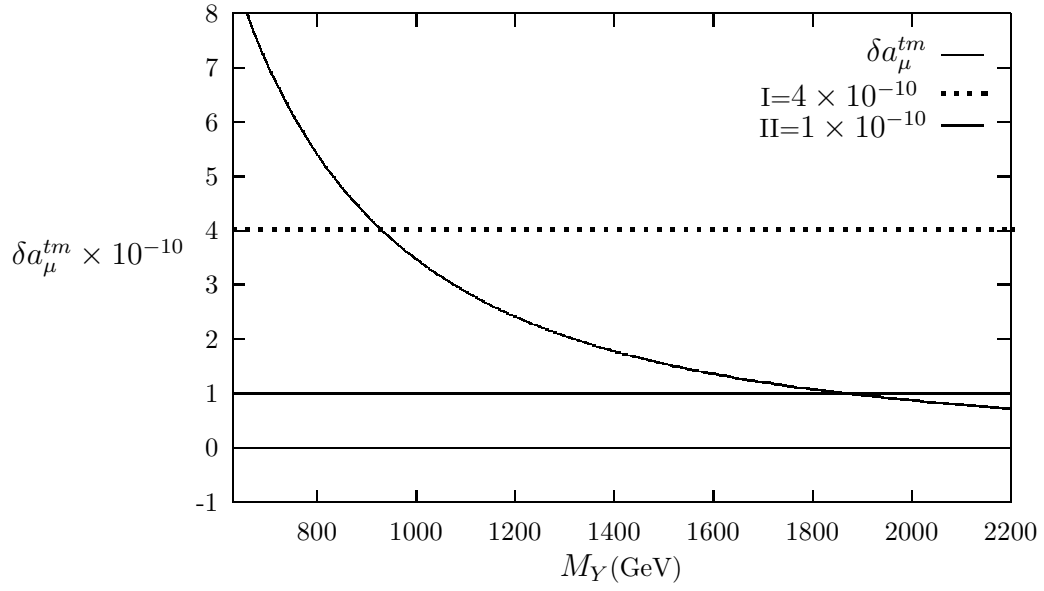
(a)

(b)

(c)

(d)

Figure 1

Figure 2

Figure 3

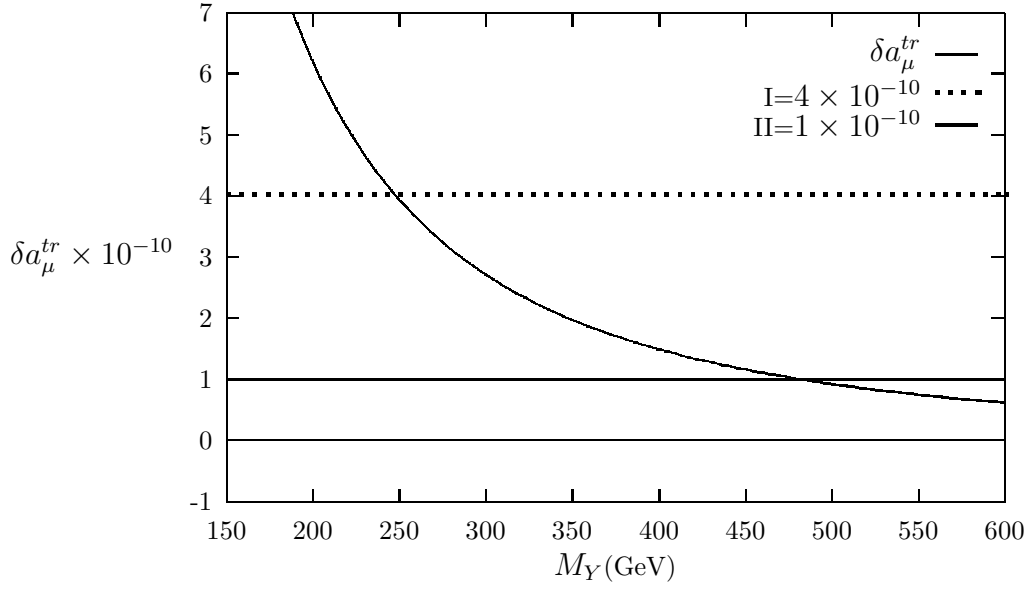Figure 4

FIGURE CAPTIONS

- Fig. 1: Diagrams for the $(g_\mu - 2)/2$
  - (a), (b), (c), (d) in the minimal version
  - (c), (d) in the version with r.h. neutrinos

- Fig. 2: δa_μ^{tm} as a function of M_Y, where $M_{Z'} = 1300$ GeV is used.

- Fig. 3: δa_μ^{tm} as a function of M_Y, where $M_{Z'} = 1300$ GeV is used. Here I and II are two expected BNL experimental values.

- Fig. 4: δa_μ^{tr} as a function of M_Y, where $M_{Z'} = 1000$ GeV is used. Here I and II are two expected BNL experimental values.