Reflections on Gravity*

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Abstract

A pedagogical description of a simple ungeometrical approach to General Relativity is given, which follows the pattern of well understood field theories, such as electrodynamics. This leads quickly to most of the important weak field predictions, as well as to the radiation damping of binary pulsars. Moreover, certain consistency arguments imply that the theory has to be generally invariant, and therefore one is bound to end up with Einstein's field equations. Although this field theoretic approach, which has been advocated repeatedly by a number of authors, starts with a spin-2 theory on Minkowski spacetime, it turns out in the end that the flat metric is actually unobservable, and that the physical metric is curved and dynamical.

Short sections are devoted to tensor-scalar generalizations, the mystery of the vacuum energy density, and quintessence.

1 Introductory Remarks

I feel very honored by the invitation to give the concluding talk at this exciting workshop. At the same time I feel a bit worried. After the excellent summaries of the various parallel sessions I shall, of course, not give another overview. Before indicating what I will concentrate on, let me begin with a few general remarks.

At the time in 64/65 when I was a Fellow here in the CERN theory group, General Relativity (GR) played virtually no role in high energy physics and

*Concluding talk at the **ESA-CERN WORKSHOP**, CERN, 5-7 April 2000 was even largely unknown to particle physicists. And this remained so for quite some time, essentially until the advent of supergravity theory.

With the unification attempts in the framework of supergravity theory, the revival of (supersymmetric) Kaluza-Klein theories, and finally string (M-) theory, gravitational interactions became an *essential* and *unavoidable* part of fundamental (speculative) physics.

Another reason - more familiar to the participants of this workshop - why gravitational physics has become a central field in present day physics, is the incredible chain of important astronomical discoveries since the early sixties. We are undoubtedly in the middle of a truly Golden Age of astrophysics and cosmology. Soon we shall have a gravitational wave astronomy, allowing us to study highly dynamical strong field processes, like the coalescence of black holes. Here Einstein's equations come into play in their full glory. Surely, gravitational wave searches will also transform GR into a field like other branches of physics, with a healthy interplay of theory and experiment. For the analysis of the expected data, a lot of difficult analytical and numerical work remains to be done.

In this situation every physicist should have some technical understanding of this marvellous relativistic theory of gravity, called GR, about which Dirac once said that it "is probably the greatest scientific discovery that was ever made".

An obstacle for a full understanding of GR has always been the necessity of absorbing first a considerable amount of mathematical machinery. This is, of course, no problem for theoreticians, but experimentalists and astronomers often do not find the time for this. (This is at least true for many people I know.) To some extend, this hurdle can be postponed in an ungeometrical approach to GR, which has been advocated in the course of time by a number of authors, in particular by R.P. Feynman in his Caltech lectures [1]. One may call it the flat spacetime - or the field theoretic approach. I first learned about it in my youth from discussions with M. Fierz. That was shortly after he left CERN as theory director and came to Zurich as Pauli's successor. His ideas were partially worked out in the thesis of W. Wyss [2]. At about the same time W. Thirring was advocating this approach with different emphasis in talks and some publications [3]. S. Weinberg had a related paper [4], in which he made an attempt to develop a quantum theory of a selfinteracting spin-2 field on flat spacetime. (We now know that such theories are unrenormalizable, also for supersymmetric extensions.) The theme was taken up later by S. Deser [5], R.M. Wald [6], and others.

The idea of this alternative approach is to describe gravity - in close analogy to electrodynamics - by a field theory on *flat* Minkowski spacetime. I shall spend much of my time in showing you how this can be done without much effort. One of the interesting lessons of this will be the following: although we shall start with a theory on Minkowski spacetime, it will in the end turn out that the flat metric is actually *unobservable*, and that the physical metric is *curved* and *dynamical*, subject to field equations which agree with Einstein's equations. The physical metric is determined by the spin-2 field of the theory. In other words, the initial flat metric turns out to be a kind of unobservable ether which we will eliminate.

One of the advantages of this field theoretic approach is that it follows the patterns of well understood field theories and may thus be closer to what many of you are used to. Other pros and cons will be discussed later.

To some of you, most of what I am going to say is not new, but I hope that perhaps a majority of the participants of this workshop will afterwards see GR in a somewhat different light. This can also be useful as a starting point for discussing experimental tests. However, since this subject has been thoroughly reviewed by other speakers, I shall confine myself to a few scattered remarks, mostly in connection with tensor-scalar generalizations of GR. I shall end with some comments on the Λ-problem.

2 A field theoretic (pedestrian) approach to GR

A natural question shortly after 1905 was: Why not develop a field theory of gravity in close analogy to electrodynamics? Einstein's first (unpublished) attempts went in this direction.

In trying to do this, it is useful to recall the following avenue to Maxwell's equations.

As starting point we adopt the following three ingredients:

(i) **Electrostatics:**

- field equations: $\mathrm{curl}\boldsymbol{E} = 0, \quad \mathrm{div}\boldsymbol{E} = \rho_e$ (1)
- equation of motion for test particle: $m\ddot{\boldsymbol{x}} = e\boldsymbol{E}.$ (2)

(ii) **Lorentz-invariance** of the theory. Today this is a battle-field-tested general symmetry principle. [Historically, it grew out of electrodynamics, but now we have overwhelming direct experimental evidence for its validity. I do not have to stress this here at CERN.]

(iii) **Charge conservation:**

$$\partial_\mu j^\mu = 0, \quad j^\mu = (\rho_e, \boldsymbol{J}). \tag{3}$$

[This is presumably the most important and most general conservation law of physics.]

On the basis of (i)-(iii), Maxwell's equations are more than compelling. We argue as follows:

The source of the electromagnetic field must be the current j^μ, instead of ρ_e in (1). The Lorentz-invariant generalization of the inhomogeneous equation in (1) will be of the symbolic form $(\partial \cdot F)^\mu = -j^\mu$, where $\partial \cdot F$ denotes the divergence of a tensor field. Clearly, F has to be a tensor field of second rank, and the inhomogeneous field equation is naturally expected to be of the form $\partial_\nu F^{\mu\nu} = -j^\mu$.

Now, following Maxwell, we require that current conservation is an automatic consequence of the field equations. This can only be the case if $F^{\mu\nu}$ is antisymmetric. The field tensor then transforms *irreducibly* with respect to the homogeneous Lorentz group. With similar arguments one is also led to the homogeneous Maxwell equations. Note that Lorentz-invariance implies, in particular, the existence of the magnetic field for nonstatic situations. Similarly, we shall predict in gravity theory the existence of a gravitomagnetic field.

This compelling reasoning of guessing the correct relativistic field equations is now taken as a model for gravity. We start again from three similar ingredients:

(i) **Static limit** (Newtonian theory): The gravitoelectric field $\boldsymbol{g}$ (gravitational acceleration of test bodies) satisfies the field equations:

$$\operatorname{curl} \boldsymbol{g} = 0, \quad \operatorname{div} \boldsymbol{g} = -4\pi G \rho \tag{4}$$

or in terms of the potential φ,

$$\boldsymbol{g} = -\boldsymbol{\nabla}\varphi, \quad \Delta\varphi = 4\pi G\rho. \tag{5}$$

The equation of motion for a test body is universal (weak equivalence principle):

$$\ddot{\boldsymbol{x}} = \boldsymbol{g}(\boldsymbol{x}), \quad (m_i = m_g). \tag{6}$$

(ii) **Lorentz-invariance**;

(iii) **Energy-momentum conservation:**

$$\partial_\nu T^{\mu\nu} = 0. \tag{7}$$

Since this is so similar to ED, one expects at first sight that it should not be difficult to find a satisfactory Lorentz-invariant theory of gravity. Let us try this.

First, we have to find out which type of field has to be chosen. A priori, we have the possibilities 0, 1 or 2 for the spin. Among these the spin-1 option has to be excluded, because it would lead to *repulsion*. That this is unavoidable was already known to Maxwell. Otherwise, the gravitational waves would have negative energy and the world would be unstable. (This will soon become clear.)

2.1 Scalar theory

It is instructive to consider first the simplest case of a scalar theory (Einstein, Nordström, v. Laue [7]). (I do this also because scalar-tensor theories are still under current discussion, see section 3.)

The field equation for the scalar field φ in the *weak* field case (linear field equation) is unique:

$$\Box\varphi = -4\pi G T, \quad T := T^{\mu}_{\mu}. \tag{8}$$

(For a static situation this reduces to the second equation in (5).)

We formulate the equation of motion of a test particle in terms of a Lagrangian. For weak fields this is again unique:

$$L(x^{\mu}, \dot{x}^{\mu}) = -\sqrt{\eta_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}}(1+\varphi), \tag{9}$$

because only for this the Newtonian limit for weak static fields and small velocities of the test bodies comes out right:

$$L(\boldsymbol{x}, \dot{\boldsymbol{x}}) \approx \frac{1}{2}\dot{\boldsymbol{x}}^2 - \varphi + \text{const}.$$

The basic equations (8) and (9) imply a perihelion motion of the planets, but this comes out wrong, even the sign is incorrect. One finds (-1/6) times the value of GR. In spite of this failure I add some further instructive remarks.

First, I want to emphasize that the interaction is necessarily *attractive*, independent of the matter content. To show this, we start from the general form of the Lagrangian density for the scalar theory

$$\mathcal{L} = \frac{1}{2}\partial_{\mu}\phi\,\partial^{\mu}\phi - g\,T\cdot\phi + \mathcal{L}_{mat}. \tag{10}$$

(ϕ is proportional to φ; g is a coupling constant.)

Note first that only g^2 is significant: Setting $\tilde{\phi} = g\phi$, we have

$$\mathcal{L} = \frac{1}{2g^2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} - T\cdot\tilde{\phi} + \mathcal{L}_{mat},$$

involving only g^2. Next, it has to be emphasized that it is not allowed to replace g^2 by $-g^2$, otherwise the field energy of the gravitational field would be negative. (This "solution" of the energy problem does not work.) Finally, we consider the field energy for *static* sources.

The total (canonical) energy-momentum tensor

$$T^\mu_\nu = \frac{\partial\mathcal{L}}{\partial\phi_{,\mu}}\phi_{,\nu} + ... - \delta^\mu_\nu\mathcal{L}$$

gives for the ϕ-contribution:

$$(T_\phi)_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}\partial_\lambda\phi\,\partial^\lambda\phi + \eta_{\mu\nu}g\,T\phi.$$

For the corresponding total energy we find

$$\begin{aligned} E &= \int(T_\phi)_{00}\mathrm{d}^3x = \frac{1}{2}\int\left[(\boldsymbol{\nabla}\phi)^2 + 2g\,T\phi\right]\mathrm{d}^3x \\ &= \frac{1}{2}\int\left[\phi\,(-\Delta\phi) + 2g\,T\phi\right]\mathrm{d}^3x = \frac{1}{2}\,g\int T\phi\,\mathrm{d}^3x. \end{aligned}$$

Since $\Delta\phi = g\,T$, we have

$$\phi(\boldsymbol{x}) = -\frac{g}{4\pi}\int\frac{T(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\,\mathrm{d}^3x'.$$

Inserting this above gives finally

$$E = -\frac{g^2}{4\pi}\frac{1}{2}\int\frac{T(\boldsymbol{x})T(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\mathrm{d}^3x\,\mathrm{d}^3x',$$

showing that indeed the interaction is attractive.

This can also be worked out in quantum field theory by computing the effective potential corresponding to the one-particle exchange diagram with the interaction Lagrangian $\mathcal{L}_{int} = g\,\bar{\psi}\psi\,\phi_{m=0}$. One finds

$$V_{eff} = -\frac{g^2}{4\pi}\frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|},$$

both for fermion-fermion and fermion-antifermion interactions. The same result is found for the exchange of massless spin-2 particles, while for spin-1 we obtain *repulsion* between particles, and attraction between particles and antiparticles.

The scalar theory predicted that there is *no light deflection*, simply because the trace of the electromagnetic energy-momentum tensor vanishes. For this reason Einstein urged in 1913 astronomers (Erwin Freundlich in Potsdam) to measure the light deflection during the solar eclipse the coming year in the Krim. Shortly before the event the first world war broke out. Over night Freundlich and his German colleagues were captured as prisoners of war and it took another five years before the light deflection was observed.

Quite independently of the failures mentioned so far, an even more profound difficulty of the scalar theory is often claimed. The argument is based on the observation that the trace $T(\boldsymbol{x}, t)$ for a moving particle is given by

$$T(\boldsymbol{x}, t) = m\sqrt{1-\boldsymbol{v}^2}\,\delta^3(\boldsymbol{x}-\boldsymbol{z}(t)),$$

where $\boldsymbol{z}(t)$ is the position of the particle, $\boldsymbol{v} = \dot{\boldsymbol{z}}$. The momentum dependence shows that the moving particle generates a *weaker* gravitational field than a particle at rest.

Now, consider two boxes each containing N particles of mass m, initially at rest (gas at zero temperature). We imagine that the two containers 1 and 2 are connected by a rigid rod. The two boxes gravitationally attract each other. We can arrange things such that the forces balance each other and that the system is at rest. Suppose now that the rest mass of a single particle in box 2 is completely transformed into kinetic energy of the remaining particles. This does not change the inertial mass of container 2, but apparently the *active* gravitational mass becomes smaller, and the total system begins to accelerate. Terrible!

There is a subtle error in this argument. The moving particles are bouncing on the wall of box 2. This induces a surface tension, generating an additional gravitational field. It is easy to show that it is because of this that the total system remains at rest.

This nicely illustrates that the equivalence principle is a subtle and profound property of gravity.

Some final remarks on the scalar theory. So far we have only considered weak fields generated by T_{matter}. It is however, more than natural that the energy-momentum tensor of the gravitational field also acts as a source, so that the theory has to be nonlinear. The nonlinear generalization of Nordström's theory was set up by Einstein and Fokker in 1914 [8]. In a non-geometrical (flat-spacetime) formulation the Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi + \mathcal{L}_{mat}\left[\psi; (1+\kappa\varphi)^2\eta_{\mu\nu}\right](1+\kappa\varphi)^4;$$

in particular, the flat metric $\eta_{\mu\nu}$ in $\mathcal{L}_{mat}$ is replaced by $(1+\kappa\varphi)^2\eta_{\mu\nu}$, $\kappa^2 = 8\pi G$.

One can eliminate the Minkowski metric and replace it by a "physical metric":

$$g_{\mu\nu} = (1 + \kappa\varphi)^2\eta_{\mu\nu} \, .$$

For example, only relative to this metric the Compton wave length is constant, i.e., not spacetime dependent.

Einstein and Fokker gave a geometrical formulation of the theory. This can be summarized as follows:

(i) spacetime is conformally flat: Weyl tensor = 0;

(ii) field equation: $R = 24\pi G\, T$;

(iii) test particles follow geodesics.

In adapted coordinates, with $g_{\mu\nu} = \phi^2\eta_{\mu\nu}$, one finds

$$R = -6\eta^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi/\phi^3,$$

and the field equation becomes

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\phi = -4\pi G\phi^3\, T.$$

The Einstein-Fokker theory is generally covariant (as emphasized in the original paper), however, *not* generally *invariant*. I use this opportunity to point out the crucial difference of the two concepts. For a long time people (including Einstein) were not fully aware of this and this caused lots of confusion and strange controversies. (See, e.g., the preface of Fock's book on GR.)

The *invariance* group of a theory is the subgroup of the covariance group that leaves the absolute, non-dynamical elements of the theory invariant. In the Einstein-Fokker theory the conformal structure is an *absolute* element, and therefore the invariance group is the *conformal group*, whence a finite dimensional Lie group. In GR, on the other hand, the metric is entirely dynamical, and therefore the covariance group is at the same time also the invariance group. For this reason, "general relativity" is an appropriate naming, never mind Fock and others.

A very remarkable property of the Einstein-Fokker theory is, that it satisfies even the *strong* equivalence principle. Beside GR this is (to my knowledge) the only theory which does this. The Einstein-Fokker theory shows, and this is a bit puzzling, that the equivalence principle does *not* imply light deflection. (For further discussion of this, see [9].)

This was probably too much on the scalar theory, but I wanted to make some points of general significance which can more easily be explained in this context. Let us now turn to the tensor theory.

2.2 Tensor (spin-2) theory

We are led to study the spin-2 option. (There are no consistent higher spin equations with interaction.) This means that we try to describe the gravitational field by a symmetric tensor field $h_{\mu\nu}$.

Such a field has 10 components. On the other hand, we learned from Wigner that in the massless case there are only *two* degrees of freedom. How do we achieve the truncation from 10 tow 2 ?

Recall first the situation in the *massive* case. There we can require that the trace $h = h^{\mu}_{\ \mu}$ vanishes, and then the field $h_{\mu\nu}$ transforms with respect to the homogeneous Lorentz group irreducibly as $D^{(1,1)}$ (in standard notation). With respect to the subgroup of rotations this reduces to the reducible representation

$$D^1 \otimes D^1 = D^2 \oplus D^1 \oplus D^0.$$

The corresponding unwanted spin-1 and spin-0 components are then eliminated by imposing 4 subsidery conditions:

$$\partial_\mu h^{\mu}_{\ \nu} = 0.$$

The remaining 5 degrees of freedom describe (after quantisation) massive spin-2 particles (Pauli and Fierz [10]; see, e.g., the classical book of G. Wentzel [11]).

In the *massless* case we have to declare certain classes of fields as physically equivalent, by imposing - as in ED - a *gauge invariance*. The gauge transformations are

$$h_{\mu\nu} \longrightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \tag{11}$$

where ξ_μ is an arbitrary vector field.

Let us first consider the *free* spin-2 theory which is unique (Pauli and Fierz):

$$\mathcal{L} = \frac{1}{4} h_{\mu\nu,\sigma} h^{\mu\nu,\sigma} - \frac{1}{2} h_{\mu\nu,\sigma} h^{\sigma\nu,\mu} - \frac{1}{4} h_{,\sigma} h^{,\sigma} - \frac{1}{2} h_{,\sigma} h^{\nu\sigma}{}_{,\nu}\,. \tag{12}$$

Let $G_{\mu\nu}$ denote the Euler-Lagrange derivative of $\mathcal{L}$ (up to a sign),

$$\begin{aligned} G_{\mu\nu} &= \frac{1}{2} \partial^\sigma \partial_\sigma h_{\mu\nu} + \partial_\mu \partial_\nu h - \partial_\nu \partial^\sigma h_{\mu\sigma} - \partial_\mu \partial^\sigma h_{\sigma\nu} \\ &+ \eta_{\mu\nu} \left(\partial^\alpha \partial^\beta h_{\alpha\beta} - \partial^\sigma \partial_\sigma h \right). \end{aligned} \tag{13}$$

The free field equations

$$G_{\mu\nu} = 0 \tag{14}$$

are identical to the linearized Einstein equations and describe, for instance, the propagation of weak gravitational fields.

The gauge invariance of $\mathcal{L}$ (modulo a divergence) implies the identity

$$\partial_\nu G^{\mu\nu} \equiv 0 \tag{15}$$

("linearized Bianchi identity"). [This should be regarded in analogy to the identity $\partial_\mu(\Box A^\mu - \partial^\mu \partial_\nu A^\nu) \equiv 0$ for the left-hand side of Maxwell's equations.]

Let us now introduce couplings to matter. The simplest possibility is the linear coupling

$$\mathcal{L}_{int} = -\frac{1}{2}\kappa\, h_{\mu\nu} T^{\mu\nu}, \tag{16}$$

leading to the field equation

$$G^{\mu\nu} = -\frac{\kappa}{2}\, T^{\mu\nu}. \tag{17}$$

This can, however, not yet be the final equation, but only an approximation for weak fields. Indeed, the identity (15) implies $\partial_\nu T^{\mu\nu} = 0$ which is unacceptable (in contrast to the charge conservation of ED). For instance, the motion of a fluid would then not at all be affected by the gravitational field. Clearly, we must introduce a *back-reaction* on matter. Why not just add to $T^{\mu\nu}$ in (17) the energy-momentum tensor ${}^{(2)}t^{\mu\nu}$ which corresponds to the Pauli-Fierz Lagrangian (12)? But this modified equation cannot be derived from a Lagrangian and is still not consistent, but only the second step of an iteration process:

$$\mathcal{L}^{free} \longrightarrow^{(2)} t^{\mu\nu} \longrightarrow \mathcal{L}^{cubic} \longrightarrow^{(3)} t^{\mu\nu} \longrightarrow \ldots ?$$

The sequence of arrows has the following meaning:

A Lagrangian which gives the quadratic terms ${}^{(2)}t^{\mu\nu}$ in

$$G^{\mu\nu} = -\frac{\kappa}{2}\left(T^{\mu\nu} +^{(2)} t^{\mu\nu} +^{(3)} t^{\mu\nu} + \ldots\right) \tag{18}$$

must be cubic in $h_{\mu\nu}$, and in turn leads to cubic terms ${}^{(3)}t^{\mu\nu}$ of the gravitational energy-momentum tensor. To produce these in the field equation (18), we need quartic terms in $h_{\mu\nu}$, etc. This is an infinite process. By a clever reorganization it stops already after the second step, and one arrives at field equations which are equivalent to Einstein's equations (S. Deser, [5]). The physical metric of GR is given in terms of $\phi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h$ by

$$\sqrt{-g}\, g^{\mu\nu} = \eta^{\mu\nu} - \phi^{\mu\nu}, \quad g := \det(g_{\mu\nu}). \tag{19}$$

At this point one can reinterprete the theory geometrically. Thereby the flat metric disappears completely and one arrives in a pedestrian way at GR.

It should, however, be pointed out that, as a result of gauge invariance, the energy arguments in this reasoning are somewhat ambiguous. In view of this we shall later (section 2.5) discuss another approach.

Let us first pursue the approximate theory, keeping only ${}^{(2)}t^{\mu\nu}$ in (18). The linearized Bianchi identity (15) implies the conservation laws

$$\partial_\nu \left(T^{\mu\nu} +^{(2)} t^{\mu\nu}\right) = 0. \tag{20}$$

This gives

$$\partial_\nu T_\mu^{\ \nu} - \frac{\kappa}{2}\, \partial_\mu h_{\alpha\beta} T^{\alpha\beta} = 0. \tag{21}$$

This is analogous to $\partial_\nu T_\mu^{\ \nu} - F_{\mu\nu} j^\nu = 0$ in ED. For a charged test particle one obtains from this the Lorentz equation of motion: $\frac{\mathrm{d}}{\mathrm{d}\tau}(m u^\mu) = e F^\mu_{\ \nu} u^\nu$. Similarly, from (21) one can derive the following equation of motion for a neutral test particle in a gravitational field $h_{\alpha\beta}$:

$$\frac{\mathrm{d}u_\mu}{\mathrm{d}\tau} + \kappa \left(\partial_\beta h_{\mu\alpha} - \frac{1}{2}\partial_\mu h_{\alpha\beta}\right) u^\alpha u^\beta = 0. \tag{22}$$

Geometrically, this is just the linearization of the geodesic equation for the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \tag{23}$$

(compare this with (19)). We shall soon see that this has not only a formal meaning.

2.3 Further discussion of the linearized theory

The linearization (equations (17) and (22)) is, by the way, already contained in Einstein's Zurich note book from 1912! This is now published in Volume 3 of the *Collected Papers*.

It is convenient to introduce the fields

$$\phi_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h \tag{24}$$

and impose the Lorentz-Hilbert gauge condition

$$\partial_\nu \phi^{\mu\nu} = 0. \tag{25}$$

(This is also contained in the 1912 notes of Einstein.) Then the field equations (17), with expression (13), become simply

$$\Box \phi^{\mu\nu} = -\kappa T^{\mu\nu}. \tag{26}$$

This is again very similar to what we are used to in ED. The retarded integral of the source $T^{\mu\nu}$ describes the emission of gravitational radiation and gives, for instance, the correct damping of binary pulsars.

In the *almost Newtonian limit* we have $T_{00} \approx \rho$, and all other components are much smaller. Then only ϕ_{00} survives:

$$\phi_{00}(\boldsymbol{x}) = -\frac{\kappa}{4\pi}\int \frac{\rho(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\mathrm{d}^3x'.$$

Using also the Newtonian limit of the equation of motion (22), one finds that

$$\kappa\phi_{00} = 4U, \quad U : \text{Newtonian potential}, \tag{27}$$

and

$$\kappa^2 = 16\pi G. \tag{28}$$

Therefore,

$$\kappa h_{\mu\nu} = 2U\delta_{\mu\nu}\,. \tag{29}$$

In the next order (in $1/c$) we encounter for rotating sources the gravitomagnetic field. If the spatial stress T_{ij} can be neglected, the field equations (26) reduce to

$$\Box\phi_{ij} = 0, \quad \Box\phi_{0\mu} = -\kappa T_{0\mu}\,.$$

Thus $A_\mu := -\frac{1}{4}\kappa\phi_{0\mu}$ satisfy Maxwell type equations:

$$\Box A_\mu = J_\mu, \quad \partial^\mu A_\mu = 0,$$

where $J_\mu = 4\pi G T_{0\mu}$ is proportional to the mass-energy current density. (Note that $A_0 = -U$.) It is natural to define "gravitational electric and magnetic fields" $\boldsymbol{E}$ and $\boldsymbol{B}$ by the same formulas in terms of A_μ as in ED.

Let us now assume that the time derivatives of $\phi_{\mu\nu}$ can be neglected (quasi-stationary situations). Then $\Delta\phi_{ij} = 0$ in all space, whence $\phi_{ij} = 0$, and hence A_μ describes the gravitational field. In this approximation, the equation of motion (22) reduces for non-relativistic velocities to

$$\ddot{\boldsymbol{x}} = \boldsymbol{E} + 4\dot{\boldsymbol{x}} \wedge \boldsymbol{B}. \tag{30}$$

The factor 4 in the "magnetic term" reflects the spin-2 character of the gravitational field. The potentials are given by

$$A_0 = -U, \quad A_i(\boldsymbol{x}) = G\int \frac{T_{0i}(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\mathrm{d}^3x'. \tag{31}$$

On the basis of this one obtains immediately the Lense-Thirring precession of a gyroscope, by simply translating the spin precession formula in electrodynamics: Substitute in the well-known formulae

$$\dot{\boldsymbol{S}} = \boldsymbol{\mu} \wedge \boldsymbol{B}, \quad \boldsymbol{\mu} = \frac{e}{2m}\boldsymbol{S}$$

of ED e by m and $\boldsymbol{B}$ by $4\boldsymbol{B}$. This gives the Lense-Thirring precession frequency

$$\boldsymbol{\omega}_{LT} = -2\mathrm{curl}\boldsymbol{A}. \tag{32}$$

The experiment Gravity Probe - B is supposed to measure this effect directly. Its launch is scheduled for early 2001 [12].

Next, we look at the *coupling to the electromagnetic field*:

$$\mathcal{L}_{int} = -\frac{\kappa}{2} h_{\mu\nu} T^{\mu\nu}_{elm}.$$

Maxwell's equations in the presence of the gravitational field $h_{\mu\nu}$ follow from the Lagrangian

$$\begin{aligned} \mathcal{L}_{elm} &= -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu - \frac{\kappa}{2} h_{\mu\nu} T^{\mu\nu}_{elm} \\ &= -\frac{1}{4}\left(1 + \frac{\kappa}{2}h\right)\left(\eta^{\mu\nu} - \kappa h^{\mu\nu}\right)\left(\eta^{\rho\sigma} - \kappa h^{\rho\sigma}\right) F_{\mu\rho}F_{\nu\sigma} - j^\mu A_\mu \\ &= -\frac{1}{4}\sqrt{-g}\, g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} - j^\mu A_\mu. \end{aligned} \tag{33}$$

In the last equality sign we used the metric (23). (All equality signs are meant to hold in lowest order of $h_{\mu\nu}$.) This gives the modified Maxwell equations:

$$\partial_\sigma \left\{ \left(1 + \frac{\kappa}{2}h\right)\left(\eta^{\mu\nu} - \kappa h^{\mu\nu}\right)\left(\eta^{\rho\sigma} - \kappa h^{\rho\sigma}\right) F_{\mu\rho} \right\} = -j^\nu. \tag{34}$$

Expanding this for a diagonal $h_{\mu\nu}$ (as in (29)), one finds the standard form of Maxwell's equations for macroscopic media, with a dielectric constant ϵ and a magnetic permeability μ. For an almost Newtonian situation (equation (29)) these are given by

$$\epsilon = \mu = 1 - 2U. \tag{35}$$

The corresponding refraction index is

$$n = \sqrt{\epsilon\mu} = 1 - 2U. \tag{36}$$

This result implies the correct light deflection, and more generally "all" of gravitational-lensing theory.

P. Schneider discussed in his talk some major applications of gravitational lensing. During the past ~ 15 years this field has rapidly reached an important position in present day astronomy and astrophysics. An exciting new result is the first evidence for gravitational lensing by large-scale structures [13]. This has demonstrated the technical feasibility of using weak lensing surveys to restrict the cosmological parameters. With upcoming wide field CCD cameras much progress can be expected.

Another very interesting recent result is the measurement of galaxy-galaxy weak lensing from Sloan commissioning data which show that galaxy halos are very extended, so much that the assignment to individual galaxies becomes at some distance meaningless [14].

2.4 The renormalized physical metric

Now I come to a conceptually important point: we shall see that the flat Minkowski metric is not observable.

In order to see this we study the behavior of measuring sticks and clocks in a gravitational field. To be specific, we use the hydrogen atom for defining units of length and time. Moreover, we put the H-atom into the gravitational field outside of a spherically symmetric mass distribution of total mass M at the distance R from the center, where

$$\kappa h_{\mu\nu} = -\frac{2GM}{R}\delta_{\mu\nu}\,. \tag{37}$$

In this gravitational field Maxwell's equations (34) imply the following modified Laplace-Poisson equation for the scalar potential φ for the proton:

$$\left(1 + \frac{2GM}{R}\right)\Delta\varphi = -e\,\delta^3(\boldsymbol{x}),$$

whence

$$\varphi = \frac{e}{4\pi\epsilon}\frac{1}{r}, \quad \epsilon = \frac{1}{1 - 2GM/R}. \tag{38}$$

This is the Coulomb potential for the effective charge

$$e_{eff} = \frac{e}{\epsilon}. \tag{39}$$

The equation of motion (22) or (30) reduces to the Newtonian equation

$$m_{eff}\ddot{\boldsymbol{x}} = e_{eff}\frac{\boldsymbol{x}}{4\pi r^3}, \tag{40}$$

with

$$m_{eff} = m\left(1 + 3\frac{GM}{R}\right). \tag{41}$$

These effective quantities determine the *Bohr radius*

$$a_0 = \frac{\hbar^2}{m_{eff}(e_{eff}^2/4\pi)} = \frac{\hbar^2}{me^2/4\pi}\left(1 - \frac{GM}{R}\right) \tag{42}$$

and the *Rydberg frequency*

$$\omega = \frac{1}{2\hbar}m_{eff}\left(\frac{e_{eff}^2}{4\pi\hbar}\right)^2 = \frac{1}{2\hbar}m\left(\frac{e^2}{4\pi\hbar}\right)^2\left(1 - \frac{GM}{R}\right). \tag{43}$$

Thus, using a somewhat unphysical language, we would conclude that the atoms become smaller and frequencies (times) decrease (increase).

It is, however, clearly much more physical, to express this as follows: We maintain that the Bohr radius and the Rydberg frequency define always our units of length and time. This means that we have to rescale the original "unrenormalized" length (r) and time (t) in a spacetime dependent manner:

$$\tilde{r} = r(1 + GM/R), \quad \tilde{t} = t(1 - GM/R). \tag{44}$$

In other words, the *physical metric* is

$$\mathrm{d}\tilde{s}^2 = \eta_{\mu\nu}\mathrm{d}\tilde{x}^\mu \mathrm{d}\tilde{x}^\nu = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu, \tag{45}$$

with

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}\,. \tag{46}$$

Clearly, this is a dynamical field. Let me stress that in the "renormalized" description the speed of light is always 1, while it would be spacetime dependent if we would maintain the fiction of a flat Minkowski metric.

In this sense spacetime is really curved. I illustrate this in Figure 1, which demonstrates the failure of Pythagoras' theorem.

Let me summarize this discussion:

The consequent development of the theory finally made it possible to eliminate the flat Minkowski metric, leading to a description in terms of a curved metric which has a direct physical meaning. The originally postulated Lorentz invariance turned out to be physically meaningless and plays no useful role. The flat Minkowski spacetime becomes a kind of unobservable ether. The conclusion is inevitable that spacetime is a pseudo-Riemannian (Lorentzian) manifold, whereby the metric is a dynamical field, subjected to field equations.

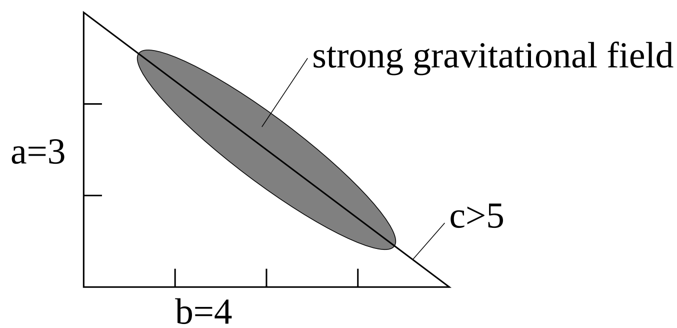

Figure 1: The failure of Pythagoras' theorem in the presence of a gravitational field.

Once this geometrical, Einsteinian point of view is accepted, the field equations are practically unique (up to the cosmological term). For instance, the vacuum field equation of Einstein is the only one for which no derivatives of $g_{\mu\nu}$ higher than second order are allowed (Lovelock theorem). The only freedom is the cosmological term, to which we shall return later.

With this fundamental step we encounter qualitatively new phenomena. Two of the most dramatic ones are:

(i) The appearance of spacetime horizons, in particular for black hole solutions. *Direct* observational evidence for such objects would clearly be most important. The prospects for this look good. In the long run the most important results will come from gravitational wave astronomy, in particular from LISA.

(ii) Since we loose translational invariance of special relativistic field theories, energy-momentum conservation breaks down. This happens in a most dramatic way in inflationary cosmological models. Only for isolated systems we can still define total energy and momentum.

2.5 Uniqueness of nonlinearities and related issues

Before discussing some further important issues, I list the pros and cons of the field theoretic approach.

Advantages:

(i) This follows the pattern of well understood field theories (e.g., electrodynamics).

(ii) It directly predicts that nonrelativistic gravitation is attractive. [How this comes out in Einstein's geometric approach will be explained shortly.]

(iii) It leads unavoidably to a curved spacetime structure.

Note, that the equivalence principle is it not used as a cornerstone.

Disadvantages:

(i) The uniqueness of the nonlinearities is not so clear, while this is one of the most beautiful and convincing aspects of the geometric approach. I shall say more on this below.

(ii) The starting point is somewhat unphysical: Minkowski spacetime is assumed to be a good approximation. While this is the case in most applications, it is certainly not true close to black holes.

Two comments must be added to this.

1. What fixes the *sign* on the right hand side of Einstein's field equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}?$$

Answer: Only for the correct sign do we have a positive energy theorem (PET). The latter says (in an untechnical language):

One cannot construct an object out of "ordinary" matter, i.e., matter with positive local energy density, whose total energy (including gravitational contributions) is negative.

This theorem is far from obvious. (A simplified proof was given by Witten [15], making use of spinors.) To emphasize its significance, I note the following:

(a) If objects with negative total energy (mass) could exist in GR, they would *repel* rather than attract nearby objects

(b) The PET implies, for example, that there are no regular interior solutions for the "anti-Schwarzschild" vacuum solution with negative mass, and hence there is attraction.

This is only true for the correct sign of Einstein's field equation. Changing the sign would lead to repulsion. Since the PET would, however, no more be true, we could presumably extract an *unlimited* amount of energy from a system with negative energy.

With the correct sign this bizarre situation cannot occur in GR. *Physically*, the reason is that as a system is compressed to take advantage of the negative gravitational binding energy, a *black hole* is inevitably formed which has *positive total energy*.

2. **Perturbation consistency and uniqueness**
The uniqueness of Einstein's vacuum equation $G_{\mu\nu}[\boldsymbol{g}] = 0$ (ignoring the Λ-term) can be translated into a perturbative consistency property, which we conversely may impose in the field theoretic approach to guarantee uniqueness. Let me explain this in some detail.

First, we decompose the Einstein tensor as

$$G_{\mu\nu} = G^{(1)}_{\mu\nu} + G^{(2)}_{\mu\nu} + ..., \tag{47}$$

where $G^{(n)}_{\mu\nu}$ contains all terms of power n in the field variable $g_{\mu\nu}$. For the metric we make a perturbative (formal) expansion about the Minkowski metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon g^{(1)}_{\mu\nu} + \epsilon^2 g^{(2)}_{\mu\nu} + ... \tag{48}$$

Inserting this into the vacuum equation leads to an infinite chain of equations:

$$\begin{aligned} &G^{(1)}_{\mu\nu}[\boldsymbol{g}^{(1)}] = 0, \\ &G^{(2)}_{\mu\nu}[\boldsymbol{g}^{(1)}] + G^{(1)}_{\mu\nu}[\boldsymbol{g}^{(2)}] = 0, \\ &.... \end{aligned} \tag{49}$$

Now we take the divergence of the second equation and use the linearized Bianchi identity

$$\partial^\nu G^{(1)}_{\mu\nu} \equiv 0. \tag{50}$$

This gives

$$\partial^\nu G^{(2)}_{\mu\nu}[\boldsymbol{g}^{(1)}] = 0, \tag{51}$$

and looks like an additional requirement for $g^{(1)}_{\alpha\beta}$, besides the linearized Einstein equation (first equation in (49)). That would be dangerous. However, equation (51) is *automatically* satisfied because of the Bianchi identity in second order. The same happens in higher order. Recall in this context, that the Bianchi identity can be regarded as a consequence of gauge invariance (general coordinate invariance).

The idea is now to turn the argument around. The question is: If we impose the correct linearized Einstein equations plus perturbative consistency, do the Einstein equations uniquely follow, if no derivatives higher than second order are allowed?

R. Wald [6] has analyzed this question, and came up with a qualified "yes". There are still some loose ends, but these are presumably not serious. T. Damour informed me at this meeting, that in collaboration with M. Henneaux the remaining gaps have been closed.

So general invariance is unavoidable, at least for reasonable matter couplings[1].

All this reflects once more the rigidity and beauty of GR. This theory must be contained in the low energy limit of any true unification.

Point 2. is also relevant for string-theory: The string excitations contain a massless spin-2 mode, and therefore GR has to be part of the field theoretic limit of string theory.

3 Tensor-scalar generalizations

In the light of the marvellous rigidity of GR and the many tests it has already passed [16], there seem to be no good reasons for studying alternative theories of gravity. There is, however, one class of generalizations which not only has a long tradition, but also new motivation from string theory.

Already in his geometric five-dimensional unification of gravity and electromagnetism, Kaluza [17] automatically got also a scalar component for the 4-dimensional theory. Indeed, the appearance of scalar fields is unavoidable in Kaluza-Klein theories. Later, Jordan [18] tried to make use of the scalar field to obtain a theory in which the gravitational constant is replaced by a dynamical field. This work was criticized by Fierz [19], who noted that Jordan's tensor-scalar theories generically entail unacceptable violations of the equivalence principle. Fierz specialized Jordan's theory such that this was avoided and arrived at a theory which was later called the Brans-Dicke-theory. (More on this, as well as references, can be found in [20].)

[1] There exist nonlinear *vacuum* theories with normal spin-2 gauge invariance: $h_{\mu\nu} \longrightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$. An example is

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(3)}, \quad \mathcal{L}^{(3)} = \left(R^{(1)}\right)^3,$$

where $R^{(1)}$ is the (gauge-invariant) linearized Ricci scalar. Explicitly,

$$\mathcal{L}^{(3)} = \left(\partial^\mu \partial^\nu h_{\mu\nu} - \Box h\right)^3.$$

This is a special case of a class of metrically-coupled tensor-scalar theories which can be characterized by the following two postulates:

(i) **Metric coupling to matter:** Only the "physical" metric (not the scalar field) couples directly to matter, as in GR: $\mathcal{L}_{mat}[\psi; g_{\mu\nu}]$ $(\partial_\mu \to \nabla_\mu)$.

(ii) **Dynamics of $g_{\mu\nu}$, φ:** This is given by the Einstein action for a conformally related metric, $\tilde{g}_{\mu\nu} = f^2(\varphi) g_{\mu\nu}$, plus the kinetic term for φ:

$$S_{grav} = -\frac{1}{16\pi G} \int [R[\tilde{g}] - 2\tilde{g}^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi] \sqrt{-\tilde{g}} \; \mathrm{d}^4 x.$$

In this class of theories the function $f(\varphi)$ is arbitrary. [For the Fierz-Brans-Dicke theory $f(\varphi) = e^{\alpha\varphi}$.]

The observable consequences of these theories have been worked out, in particular, by T. Damour and collaborators (see, e.g., [21]). There is the interesting possibility that the deviations from GR in weak field situations are tiny, but become significant in strong field regions such as the interiors of neutron stars.

I should add that in string theory scalar fields, notably the dilaton field, appear as necessary partners of the metric field. In some scenarios such scalar fields behave in laboratory and solar system measurements as massless fields, and could modify the predictions of GR. In particular, violations of the weak equivalence principle are expected at some level. In the light of this, the STEP experiment is of importance. In string theories, scalar fields typically have couplings not much weaker than gravity. If the test of the equivalence principle is improved by a factor 10^6, this would put a severe restriction on models.

It is, however, more likely that scalar fields are *massive*, in which case the theory is practically equivalent to GR.

Generalized tensor-scalar theories are often used in cosmological model building (inflation, quintessence, etc). Much of this is, however, quite arbitrary and very speculative.

Finally, I should mention that scalar-tensor gravitational waves can have an additional transverse breathing mode. The strength of this mode depends, of course, on the nature of the source.

4 The mystery of the cosmic vacuum energy density

Classically, one may ignore the cosmological term in Einstein's field equation, although there is no good reason for this, since it is allowed by the principles of GR. (Simplicity is not a convincing argument.)

In *quantum theory* the Λ-problem is much worse, because quantum fluctuations are expected to give rise to a non-vanishing vacuum energy density ρ_{vac}, which acts like a cosmological constant. Without gravity, we do, of course, not care about the energy of the vacuum, because only energy *differences* matter. However, even then the quantum fluctuations of the vacuum can be important, as is beautifully demonstrated by the Casimir effect. The radiative corrections to Maxwell's equations, first discussed by Heisenberg and Euler, and later by Weisskopf, can also be interpreted in this manner (see, e.g., [22]).

When we consider the coupling to gravity, the vacuum expectation value of the energy-momentum tensor has the form of the cosmological term (up to higher order curvature contributions):

$$\langle T_{\mu\nu} \rangle_{vac} = g_{\mu\nu}\, \rho_{vac} + \dots .$$

The *effective* cosmological constant, which controls the large scale behavior of the universe, is given by $\Lambda = 8\pi G \rho_{vac} + \Lambda_0$, where Λ_0 is a bare cosmological constant in Einstein's field equations. We know that $\rho_\Lambda \equiv \Lambda/8\pi G$ can not be much larger than the critical density, $\rho_{crit} = 8 \times 10^{-47} h_0^2 GeV^4$ ($h_0 \equiv H_0/100km/s/Mpc$). This is infinitesimal by particle physics standards.

It is recognized since quite some time that this is a profound mystery. Indeed, we expect that quantum fluctuations in the fields of the standard model of particle physics, cut off at about the Fermi scale, contribute to the vacuum energy density, because there is no symmetry principle in this energy range which would require a cancellation of the various contributions (as in strictly supersymmetric theories).

To have some measure, let us compare ρ_{crit} with the condensation energy density of QCD in the broken phase of the chiral symmetry, which is about $\Lambda_{QCD}^4/16\pi^2 \approx 10^{-4} GeV^4$. The discrepancy is at least 40 orders of magnitudes.

So far string theory has not offered convincing clues why the cosmological constant is extremely small (for a recent discussion, see [23]). The main reason is that a low energy mechanism is required, and the low energy physics is described by the standard model.

5 Quintessence

G. Tammann reviewed the recent astronomical evidence for a cosmologically significant vacuum energy density (or some effective equivalent). This arises mainly from the Hubble diagram of type Ia supernovae and from the observed temperature fluctuations of the cosmic microwave background radiation. In particular, the first results from the BOOMERANG experiment have reinforced the evidence.

If the present situation is going to stay we are confronted with the following *cosmic coincidence problem:* Since the vacuum energy is constant in time, while the matter energy density decreases as the universe expands, it is more than surprising that the two are comparable just at the present time, while their ratio has been tiny in the early universe.

Possible ways of avoiding this puzzle have recently been discussed extensively. The general idea is to explain the accelerated expansion of the universe by yet another form of exotic missing energy with negative pressure, called *quintessence*. In concrete models this is described by a scalar field, whose dynamics is such that its energy naturally adjusts itself to be comparable to the matter density today for generic initial conditions.

Let me briefly describe a simple model of this kind [24]. For the dynamics of the scalar field ϕ we adopt an exponential potential

$$V = V_0 \, e^{-\lambda\phi/M_P}.$$

Such potentials often arise in Kaluza-Klein and string theories. Matter is described by a fluid with a baryotropic equation of state: $p_f = (\gamma - 1)\rho_f$.

For a Friedmann model with zero space-curvature, one can cast the basic equations into an autonomous two-dimensional dynamical system for the quantities

$$x(\tau) = \frac{\kappa\dot{\phi}}{\sqrt{6}H}, \quad y(\tau) = \frac{\kappa\sqrt{V}}{\sqrt{3}H},$$

where

$$H = \dot{a}/a, \quad \tau = \log a, \quad \kappa^2 = 8\pi G$$

($a(t)$ is the scalar factor). This system of autonomous differential equations has the form

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = f(x, y; \lambda, \gamma), \quad \frac{\mathrm{d}y}{\mathrm{d}\tau} = g(x, y; \lambda, \gamma),$$

where f and g are polynomials in x and y of third degree, which depend parametrically on λ and γ. The density parameters Ω_ϕ and Ω_f for the field ϕ and the fluid are given by

$$\Omega_\phi = x^2 + y^2, \quad \Omega_\phi + \Omega_f = 1.$$

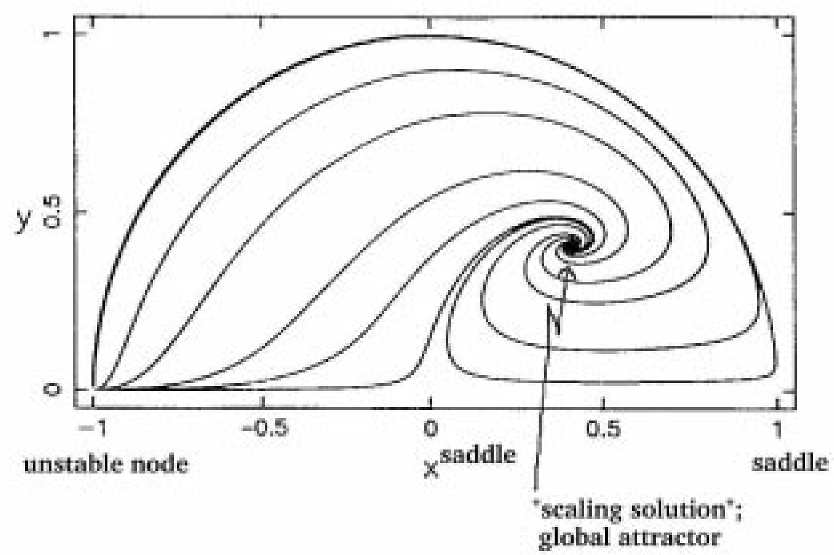

Figure 2: Phase plane for $\gamma = 1$, $\lambda = 3$. The late-time attractor is the scaling solution with $x = y = 1/\sqrt{6}$ (from Ref. [24]).

The interesting fact is that, for a large domain of the parameters λ, γ, the phase portrait has qualitatively the shape of Figure 2. Therefore, under generic initial conditions, there is a global attractor (a node or a spiral) for which $\Omega_\phi = 3\gamma/\lambda^2$. For this "scaling solution" Ω_ϕ/Ω_f remains fixed, and for any other solution this ration is finally approached.

This looks good. However, various complications of the model introduce also unstable directions and the attracting behavior gets lost. Moreover, if a constant of order M^4_{Fermi} (or even m_e^4) would be added to the potential V, the mechanism would not work. In addition, we have to worry about unacceptable changes in the nucleosynthesis results.

* * *

Having expressed once more that we are confronted with a deep mystery, I conclude with the following amusing story:

During the 1920ties most people were convinced that the universe is on the average *static*. The ground-breaking papers of Friedmann and Lemaître were, in fact, largely ignored. In comments to Lemaître during the Solvay Meeting in 1927, Einstein rejected the expanding universe solutions as physically acceptable. It is also not well-known that Hubble interpreted his famous results on the redshift of the radiation emitted by distant nebulae in the framework of the static de Sitter model. However, Lemaître's successful explanation of Hubble's discovery finally changed the viewpoint of the majority of workers in the field. At this point Einstein rejected the cosmological term as superfluous and no longer justified. He published his new view in the "Sitzungsberichte der Preussischen Akademie der Wissenschaften". The correct citation is:

Einstein, A. (1931). Sitzungsber. Preuss. Akad. Wiss. 235-37.

Many people have quoted this paper, but never read it. As a result, the quotations gradually changed in an interesting, quite systematic fashion. Some steps are shown in the following sequence:

- A. Einstein. 1931. Sitzsber. Preuss. Akad. Wiss. ...
- A. Einstein. Sitzber. Preuss. Akad. Wiss. ... (1931)
- A. Einstein (1931). Sber. Preuss. Akad. Wiss. ...
- A. Einstein. .. 1931. Sb. Preuss. Akad. Wis. ...
- A. Einstein. S.-B. Preuss. Akad. Wis. ... 1931
- A. Einstein. S.B. Preuss. Akad. Wiss. (1931) ...
- A. Einstein. and Preuss. S. B. (1931) Akad. Wiss. ...

Presumably, one day some historian of science will try to find out what happened with the young physicist S.B. Preuss, who apparently wrote just one paper and then disappeared from the scene.

With this light note I would like to conclude.

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