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COMPACTON LIKE BREATHERS IN NONLINEAR ANHARMONIC LATTICE WITH NONLOCAL DISPERSIVE INTERACTION

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Abstract

We show the existence and stability of localised breather modes with compact support in an anharmonic lattice with power dependence r^{-s} of the dispersive interactions. We also consider the effect of the harmonic nonlocal dispersive interaction on such solutions.

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1 Introduction

It is now well known that as a result of the interplay between discreteness, dispersion and nonlinear interaction intrinsic localized modes or discrete breathers are induced in translational invariant Hamiltonian lattice. Intense work during the last ten years [1] has addressed and resolved in many cases issues regarding their rigorous existence, numerical constructions, stability, dynamics, thermodynamics, quantum aspects and very recently also the experimental manifestation in specific materials[2]. One aspect that the discrete breathers seem to share in most cases and with other nonlinear localized excitations like solitons in continuum systems is the typical spatial exponential profile which gives them an infinite spatial extent. However, recently Rosenau and Hyman introduced [3] the concept of compactification or strict localization of solitary waves. They discovered that in the continuum systems, the solitary waves can be compactified in the presence of nonlinear dispersion and termed these solitary waves with compact support as compactons. The possibility of existence of such intrinsics localized modes or breathers with compact support in discrete nonlinear anharmonic lattice was predicted in recent studies [4, 5]. More recent work [6] has shown that a class of exact continuous compacton solutions of a low order continuous approximation of the discrete equations of motion, survive in general when substituted in the corresponding discrete equations. What is remarkable is that, it appears, the continuous cosine-shape compacton solutions seems to represent quite well breather solutions in the highly discrete regimes far from the continuous limit. In ref. [7] it is shown that the discrete compact breathers in the nonlinear anharmonic lattice can be generated in the anti-continuous limit through a numerically exact procedure and are generally found to be stable.

In all the above studies of compact breathers in discrete lattice, the non linear dispersive interactions are assumed to be short range and therefore a nearest neighbor approximation is used. However, there exist physical systems such as ionic and molecular crystals where this approximation cannot describe appropriately the physical systems. For example, excitations transfer in molecular crystals and energy transport in biopolymers [8] are due to transition dipole-dipole interaction with $\frac{1}{r^3}$ dependence on the distance r. Also, the DNA molecules have long range Coulomb interaction between them as the molecules contain charged groups. The compact breathers would be ideal for energy storage since due to the lack of exponential tail they would not interact until they are in contact with each other, thus increasing their lifetime and ensuring practical applications such as energy transport, signal processing and communications. However, in these applications, the long range interaction is important, as, for example, in nonlinear fiber optics, the long range interaction imposes a strict limitation on the performance of long haul fiber transmissions [9].

Some studies of solitons and intrinsics localized mode (conventional discrete breathers with exponential tails) in the presence of long range interaction are known. For example, it has been shown [10] that solitary waves like kinks or gap solitons are effected quantitatively by the long range interactions. Gaididei et al [11] also considered the effect of harmonic nonlocal interaction potential in a chain with short range anharmonicity and observed the existence of two types of solitons with characteristically different width and shape for two velocity regions separated by a gap. Similarly, studies on sine-Gordon systems with long range forces was done by Gronbech-Jensen et al [10]. Some results along this direction are also known for discrete breathers (conventional types). For example, a general proof of the existence of breathers in d-dimensional lattice with algebraic decaying interaction are considered in [12]. Similarly, for the discrete nonlinear Schroedinger (DNLS) model with long range interaction varying with the distance as r^{-s}, it has been shown that [13] for sufficiently large s, all features of the model are qualititavely the same as the DNLS model with short range (nearest neighbor) interaction, but for s less than some critical s_{cr} there is an interval of bistability. Similarly, Bonart [13] considered the intrinsic localized modes in the presence of Coulomb interaction. These studies raised an important question: Can a discrete breather with compact support survive the effects of a long range interaction? In this letter we address this problem. We demonstrate the existence of discrete breathers with compact support in the anharmonic nonlinear Klein Gordon lattice with onsite nonlinear substrate potential and with power dependence r^{-s} on the distance r of the nonlinear dispersive interaction. The stationary state of the systems are studied both analytically and numerically. We obtain exact analytic compacton breather solutions of the corresponding equations of motion in the continuum limit in the presence of long range interaction and nonlinear substrate potential. These analytic solutions are then used as an initial condition for the numerical solutions of the discrete equations of motion and numerical simulations show that within some parameter range the compacton breather solutions survive in the presence of lattice discreteness and long range anharmonic interactions . The range of the long range interaction parameter s within which the analytic continuum solutions are shown to exist agrees quite well with that obtained from the numerical calculations.

2 Equations of motion and exact analytic solutions in the continuum limit

The model we study is described by the Hamiltonian

$$\begin{aligned} H &= \sum_n [\frac{p_n^2}{2} + V(\phi_n)] + \sum_{n,m} \frac{k_1}{2} \frac{(\phi_m - \phi_n)^2}{\mid m-n \mid^{s_1}} + \\ & \sum_{n,m} \frac{k_2}{2} \frac{(\phi_m - \phi_n)^2}{\mid m-n \mid^{s_2}} \end{aligned} \tag{1}$$

where the potential $V(\phi_n)$ is a nonlinear onsite potential, $\phi_n(t) \equiv \phi_n$ is the displacement of the n-th unit mass oscillator from its equilibrium position at time t, k_1 and k_2 determine the strength of the harmonic and anharmonic interaction between the oscillators respectively and s_1 and s_2 are the long range interaction parameters, being introduced to consider different physical situations ranging from nearest neighbor ($s = \infty$) , dipole-dipole interaction ($s = 3$), Coulomb interaction ($s = 1$) etc. Tuning the parameters k_1 and k_2 as well as s_1 and s_2, we can study the effect of competition between anharmonicity and dispersion and the interplay of the long range interaction and lattice discreteness and also between harmonic and anharmonic interaction between the particles. The equations of motion are given by

$$\begin{aligned} \ddot{\phi}_n &= -\alpha\phi_n - \beta\phi_n^3 + k_1 \sum_m \frac{(\phi_m - \phi_n)}{\mid m-n \mid^{s_1}} + \\ & k_2 \sum_m \frac{(\phi_m - \phi_n)^3}{\mid m-n \mid^{s_2}} \end{aligned} \tag{2}$$

It is a difficult problem to obtain the analytic solution to these equations of motion. For the case of $k_2 = 0$ (i.e. the case of conventional exponentially decaying discrete breathers in the presence of long range interaction), the analytic solutions have been obtained earlier by approximate methods like the variational method [13] and lattice Greens' function method [14]. The lattice Green's function method describes only the asymptotic properties of the solutions. Implementing such methods for the present problem with $k_2 \neq 0$ (i.e in the presence of long range nonlinear dispersive interaction) is a difficult proposition. However, we realized that if we are only interested in the properties of the localized solutions with analytic behavior, like the exponentially localized solutions, compactons etc., then such solutions in the continuum limit can easily be obtained by a simple Taylor series expansion of the discrete equations of motion. Accordingly, we rewrite eq.(2) as

$$\begin{aligned} \ddot{\phi}_n &= -\alpha\phi_n - \beta\phi_n^3 + k_1 \sum_{p=1}^{\infty} \frac{(\phi_{n+p} + \phi_{n-p} - 2\phi_n)}{\mid p \mid^{s_1}} + \\ & k_2 \sum_{p=1}^{\infty} \frac{[(\phi_{n+p} - \phi_n)^3 + (\phi_{n-p} - \phi_n)^3]}{\mid p \mid^{s_2}} \end{aligned} \tag{3}$$

and expand $\phi_{n\pm p}$ in terms of Taylor series as

$$\begin{aligned} \phi_{n\pm p} &= \phi(x) \pm p\phi'(x) + \frac{p^2}{2}\phi''(x) \\ & \pm \frac{p^3}{6}\phi'''(x) + \frac{p^4}{24}\phi''''(x) + ... \end{aligned} \tag{4}$$

To show that the Taylor series expansion as given above can actually give the analytic localized solutions, we used this expansion to reproduce the exponentially localized solutions of the DNLS with harmonic long range interaction which was earlier obtained by Rasmussen et al using the approximate variational method [13]. For this we substitute equation (4) above in the corresponding discrete equations of motion of DNLS with harmonic long range interaction (eq.(4) in [13]) and then it can easily be checked that the corresponding continuum equations of motion have localized solutions given by $\phi(x) = \sqrt{2}\lambda sech[\frac{\sqrt{\lambda}(x+x_0)}{J\zeta(s-2)}]$, where $\zeta(s)$ is the Riemann zeta function, s is the long range interaction parameter and J is the strength of the long range harmonic interaction. We have also checked that the energy and the excitation number N of the system which corresponds to this localized solution also agrees exactly with the corresponding values obtained with the variational method (eq.(18) in [13]). We now show that the compact breather solutions of the discrete equations of motion (eq.(2)) in the continuum limit can also be obtained using the Taylor series expansion as above. For this we substitute eq.(4) in eq.(3) above to get the corresponding continuum equations for the system as (we first consider the case of $k_1 = 0$, i.e. no harmonic interaction)

$$\frac{\partial^2\phi}{\partial t^2} = -\alpha\phi - \beta\phi^3 + 3k_2\zeta(s_2 - 4)(\frac{\partial\phi}{\partial x})^2\frac{\partial^2\phi}{\partial x^2} \tag{5}$$

To get the compact breather solution we use the ansatz

$$\phi(x,t) = y(x)G(t) \tag{6}$$

Substituting this in eq.(5) we get the equations for $y(x)$ and $G(t)$ respectively as

$$3k_2\zeta(s_2-4)(\frac{\partial y}{\partial x})^2\frac{\partial^2 y}{\partial x^2} - \beta y^3 + Cy = 0$$
$$\ddot{G} + \alpha G + CG^3 = 0 \quad (7)$$

It can easily be checked that the above equation supports compacton breather solutions given by

$$\begin{aligned} \phi(x,t) &= A\ cos(Bx)\ cn[(A^2+\alpha)^{\frac{1}{2}}\sqrt{2}t, k^2], \\ & \quad for \quad \mid Bx \mid \leq \frac{\pi}{2} \\ &= 0, \ otherwise. \end{aligned} \quad (8)$$

where $k = \frac{A}{[2(\alpha+A^2)]^{\frac{1}{2}}}$, cn is Jacobi elliptic function of time and B is the inverse width of the compacton given by

$$B = [\frac{\beta}{3k_2\zeta(s_2-4)}]^{\frac{1}{4}} \quad (9)$$

From eq.(9) we see that for a real width of the compacton, the compacton breather solutions given by eq.(8) above exist only for the long range interaction parameter s_2 within the ranges $s_2 > 5$ and $0 < s_2 < 2$.

Now we consider the case when the harmonic displacive interaction is present (i.e. $k_1 \neq 0$). In the absence of long range interaction, it has been shown [6, 7] that the harmonic displacive interaction destroys the compactons in the discrete anharmonic lattice by progressively turning compact breathers into conventional exponentially decaying ones. However, in the presence of long range interaction, the situation may be different. To examine this, we introduced in the Hamiltonian (eq.(2)) two sets of parameter (k_1, s_1) and (k_2, s_2), so that by suitably tuning these parameters we can compare the effects of different contributions from the harmonic and anharmonic terms in forming the stable compacton breather solutions. Using the Taylor series expansion as above, the continuum equations in this case is given by

$$\begin{aligned} \frac{\partial^2\phi}{\partial t^2} &= -\alpha\phi - \beta\phi^3 + k_1[\zeta(s_1-2)\frac{\partial^2\phi}{\partial x^2} + \\ & \frac{1}{12}\zeta(s_2-4)\frac{\partial^4\phi}{\partial x^4}] + 3k_2\zeta(s_2-4)(\frac{\partial\phi}{\partial x})^2\frac{\partial^2\phi}{\partial x^2} \end{aligned} \quad (10)$$

Using the ansatz $\phi(x,t) = y(x)coswt$ for the compacton breather solutions, the continuoum equations reduces to

$$\begin{aligned} \lambda^2 y - \frac{3}{4}\beta y^3 + k_1[\zeta(s_1-2)\frac{\partial^2 y}{\partial x^2} + \\ \frac{1}{12}\zeta(s_2-4)\frac{\partial^4 y}{\partial x^4}] + 3k_2\zeta(s_2-4)(\frac{\partial y}{\partial x})^2\frac{\partial^2 y}{\partial x^2} = 0 \end{aligned} \quad (11)$$

where $\lambda^2 = w^2 - \alpha$ and we have used the rotating wave approximation to get to this equation. It can again be checked that this equation has compacton solutions of the form

$$\begin{aligned} y(x) &= Acos(Bx), for \mid Bx \mid \leq \frac{\pi}{2} \\ &= 0, \ otherwise \end{aligned} \quad (12)$$

where

$$
\begin{aligned}
B &= [\frac{\beta}{4k_2\zeta(s_2-4)}]^{\frac{1}{4}} \\
A^2 &= \frac{4}{3\beta}[\lambda^2 - k_1\zeta(s_1-2)\sqrt{\frac{\beta}{4k_2\zeta(s_2-4)}} + \\
&\quad \frac{\beta k_1\zeta(s_1-4)}{48k_2\zeta(s_2-4)}]
\end{aligned} \tag{13}
$$

Thus, from the condition of real A and B we see that for the appropriate choice of parameter values, compacton breather solutions of the form $\phi(x,t) = Acos(Bx)coswt$ may exist in the continuum limit for the system containing both the harmonic and anharmonic long range dispersive interactions. We would like to point out that the first derivatives of these solutions is discontinuous at the edge and hence the compacton solutions presented here must be understood in the weak sense. The robustness of these compacton solutions are yet unknown. However, as reported by Rosenau (see ref.12 in [5]), extensive numerical studies of the continuum equations indicate that compactons smoothness at the edge is not indicative of their stability.

3 Numerical analysis

To see whether the above analytic continuum compacton breather solutions survive in the discrete lattice, we have obtained the numerical solutions of the discrete equations of motion (eq.(2)). The initial condition for the numerical solutions is chosen as $\phi_n(t=0) = Acos(Bn)$ where B is the inverse width of the continuum compacton breather as given above. The initial velocity is taken to be zero. To take into account the effect of the long range interaction effectively, the numerical simulations are done over a large lattice with N = 1000 lattice sites. Similarly, to check the stability of the solutions over time, the solutions are evolved over a very long time. First we consider the case when no harmonic terms are present, i.e. $k_1 = 0$. Fig.(1) shows the evolution of the lattice profile of the compacton breather solutions over a time $t = 150T$, where T is the period of the compacton breather. The initial width of the compactons are chosen to be $Lc = \frac{\pi}{B} = 6$ times the lattice spacing, where B is given by eq.(9). The long range parameter s_2 is chosen to be $s_2 = 6$, the amplitude as 0.1 and the results are plotted in Fig.(1a). As can be seen from these figures, the initial analytic continuum compact breather solutions (eq.(8)) remains stable even after a very long time (t=150T) in the discrete lattice. Fig.(1b) is the same as in Fig.(1a) but with $s_2 = 7$. Again we see that the compact breather solutions remain very stable. We have checked that the compacton breather solutions for other values of the long range parameter $s_2 \geq 4$ also remains stable. We have also considered the compact breather solutions with width $Lc = 30$ times the lattice spacing. In this case the results of the numerical simulations show that although the solutions remain stable initially for a time $t = 50T$ (Fig.(2a)), it loses its compact support and develops some structures near its edge after a larger time $t = 150T$ (Fig.(2b)). We also observe that the stability of the compacton breather solutions in the discrete lattice depends on the amplitude of the solutions. For example, the stable compacton solutions with amplitude 0.1 as shown in Fig.(1a) becomes unstable when the amplitude is increased to 1.2. This is shown in Fig.(3). Similarly, from the numerical simulations we find that the compact breather solutions are also stable for parameter s within the parameter range $s < 4$ but with amplitude much smaller than similar solutions for $s > 4$. Fig.(1c) shows the discrete compacton breather solution with amplitude $A = 0.01$ for $s = 1$. This solution becomes unstable if the corresponding amplitude is increased to 0.1 as in Fig.(1a) and Fig.(1b). Finally, the results of the numerical simulations of the discrete equations of motion in the presence of both the harmonic and anharmonic long range dispersive interactions (both $k_1, k_2 \neq 0$) again shows that the discrete compacton breather solutions, even though stable for a time $t = 50T$ initially, loses its compact structure after about $t = 150T$. This is shown in Fig. (4) for the parameter values $s_1 = s_2$ and $k1 < k2$ (Fig.(4a)) and $s_1 > s_2$ and $k1 < k2$ (Fig.(4b)).

4 Conclusions

In conclusion, we have examined here the question of existence and stability of the compacton like breather solutions in the nonlinear anharmonic lattice with nonlocal dispersive interaction with power dependence r^{-s} on the distance. Using a simple Taylor series expansion we derive the equations of motion in the continuum limit and show that the resulting equations support compacton breather solutions in the presence of the nonlocal dispersive interaction. We then numerically solve the discrete equations of motion and observe that the existence and stability of the compacton like breathers in the anharmonic lattice with long range interaction depends crucially on the value of the long range interaction parameter s as well as on the amplitude and width of the solutions. In absence of the harmonic dispersive interaction, the stable compacton like discrete breathers exist for certain ranges of the long range interaction parameter ($s > 4$). For $s < 4$, the simulations on the discrete lattice show that the compacton like breather solutions are stable but with much smaller amplitude than that for $s > 4$. This might indicate that there may be other kinds of solutions (non compact type) for this parameter range. This is also suggested from the results of the earlier studies on harmonic lattice (DNLS) with long range interactions of the type r^{-s} [13, 14], that, there is a transition from the exponential decaying solutions to algebraic decaying solutions around the parameter value $s = 3$. Finally, we considered the effect of the nonlocal harmonic interaction on the stable compactons as obtained above. We find that, even though the solutions are stable initially, it starts developing a tail near the edge of the compacton, thereby destroying the compact nature of the solutions. This is similar to the results obtained from the earlier studies of discrete compacton breathers without the long range interaction [6, 7] which showed that the presence of the harmonic dispersive interaction progressively turns compact breathers to conventional exponentially breathers.

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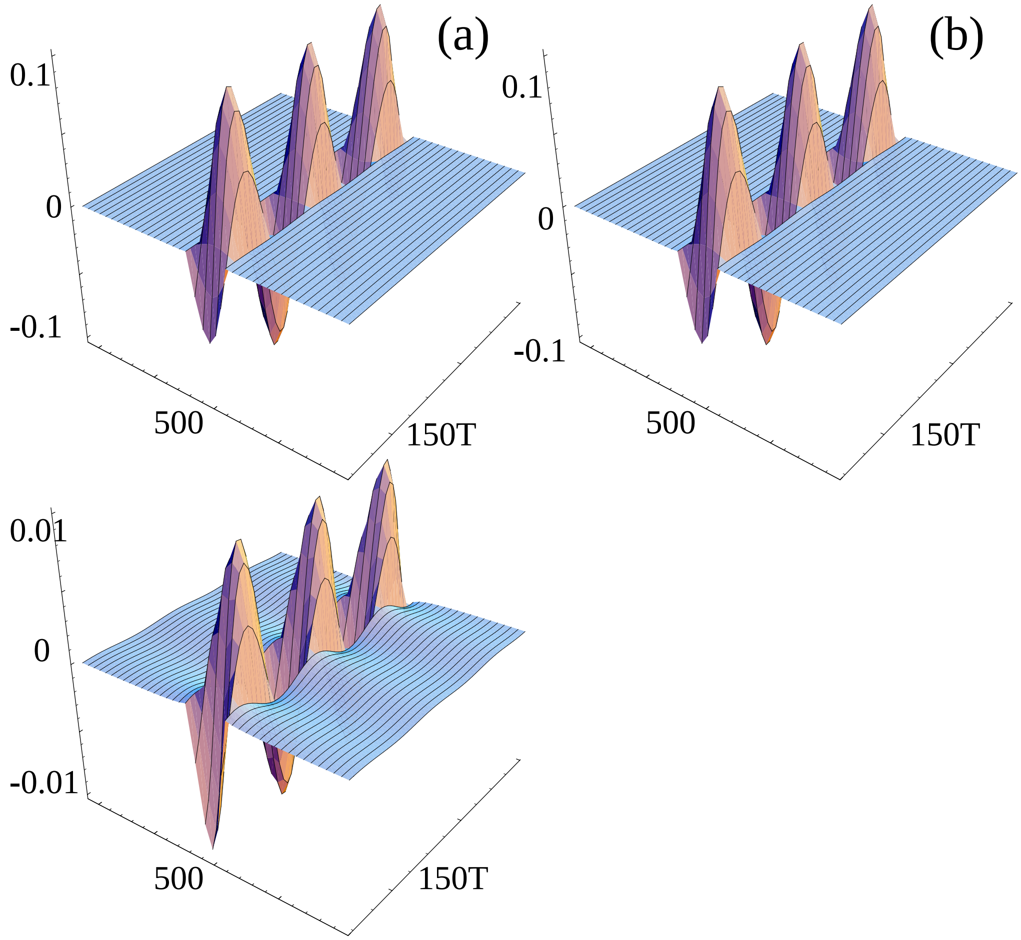

Figure 1: Plot of temporal behaviour of spatial profile of a compacton breather of width Lc = 6 , (a) $s_2 = 6$, A=0.1 (b) $s_2 = 7$, A=0.1 (c) $s_2 = 1$, A=0.01. The central site is at N=500. The solutions are stable.

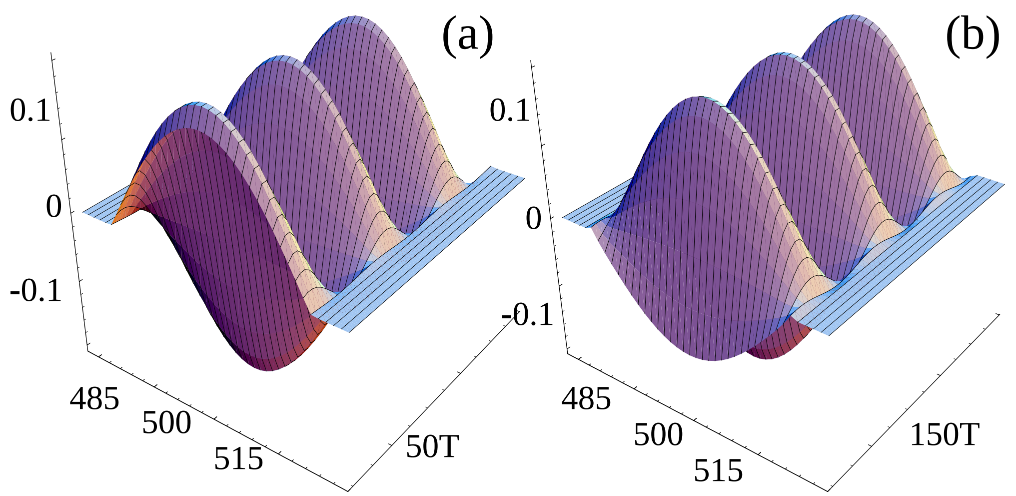

Figure 2: Same as in Fig.(1a) but with Lc = 30; temporal evolutions after (a) t=50T and (b) t=150T. The initial spatial profile of this large width compacton breather loses its shape near its edges.

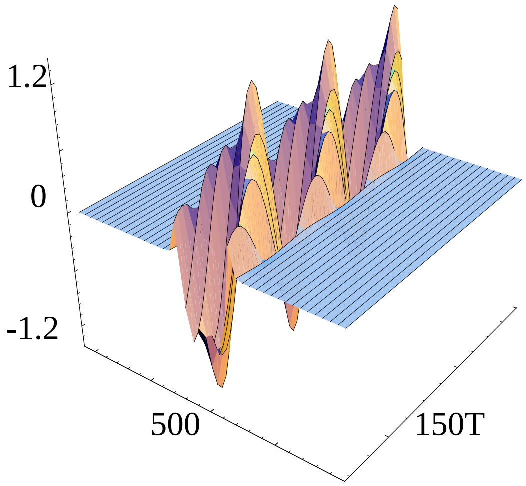

Figure 3: Same as in Fig.(1a) but with amplitude A=1.2. The initial compacton loses its shape and develops a tail at the edge.

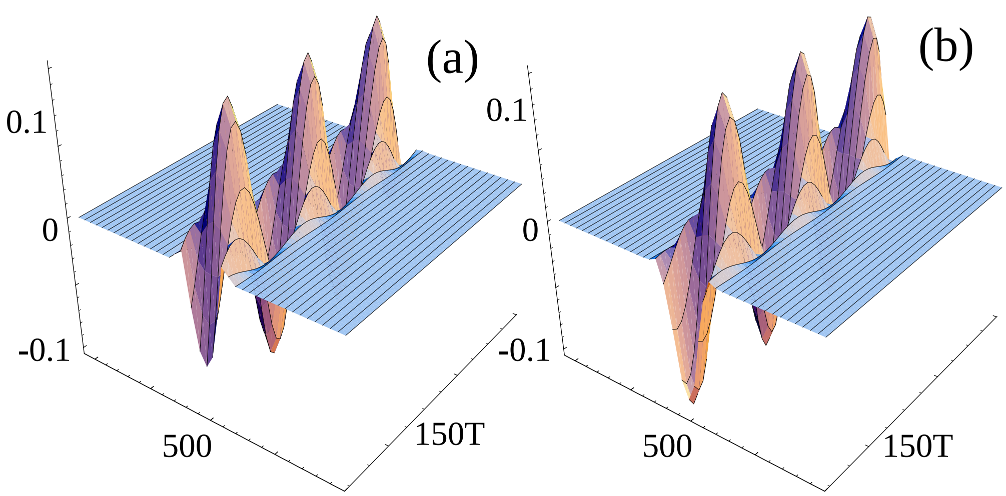

Figure 4: Plot of the temporal evolution of spatial profile of a compacton breather in the presence of both harmonic and anharmonic long range interactions. Here Lc = 6, N=1000 and t=150T, for (a) $s_1 = s_2$ and $k_1 < k_2$ (b) $s_1 > s_2$ and $k_1 < k_2$. The initial compacton solutions loses its shape after a long time.