# SOFT PARTON RESUMMATION IN THE CURRENT REGION OF SEMI-INCLUSIVE DEEP INELASTIC SCATTERING ${ }^{\dagger}$ 

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#### Abstract

We discuss resummation of large logarithmic terms that appear in the cross-section of semi-inclusive DIS in the case when the final-state hadron follows the direction of the incoming electroweak vector boson in the c.m. frame of the vector boson and the initial-state proton.


During the past years, the H1 and ZEUS Collaborations at DESY-HERA have put a significant effort ${ }^{1}$ into the experimental study of semi-inclusive deep inelastic scattering (sDIS) $e p \rightarrow e B X$. In this process, in addition to the scattered electron (or positron) $e$, some specific final-state hadron $B$, or the flow of the hadronic energy into a given region of phase space, is observed. The produced data set includes events with a large momentum transfer from the electron to the hadronic system, $Q^{2}=-q^{2} \gg \Lambda_{Q C D}^{2}$. The theoretical description of such events can be attained with the help of methods of perturbative QCD, under the assumption of a single-photon exchange between the scattered electron and the proton. If the renormalization and factorization scales in the calculation are chosen to be of the order of the large photon virtuality $Q$, then the cross-section can be calculated perturbatively as a series in the small QCD coupling $\alpha_{s}$. However, the smallness of $\alpha_{s}$ does not guarantee fast convergence of this series. Even when there is a large momentum scale in the collision,

[^0]
d)

b)

e)

c)

f)

Figure 1: The diagrams contributing to sDIS at the next-to-leading order
it is not hard to imagine a situation in which the sDIS observable receives important contributions from all orders of the perturbative expansion.

To illustrate this point, consider semi-inclusive DIS in the c.m. frame of the photon and the initial proton. The $z$-axis in this frame is defined to follow the direction of the photon's momentum. We assume all particles to be massless.

At the order $\mathcal{O}\left(\alpha_{s}^{0}\right)$ (Fig. 1a), the photon interacts with an initial-state quark $a$ whose momentum is

$$
\begin{equation*}
\vec{p}_{a}=x \vec{P}_{A}=-x \vec{q} ; \quad 0 \leq x \leq 1 \tag{1}
\end{equation*}
$$

The final-state quark $b$, which has momentum equal to $\vec{p}_{a}+\vec{q}=(1-x) \vec{q}$, escapes in the direction of motion of the incoming photon, i.e. along the positive $z$-axis. At the order $\mathcal{O}\left(\alpha_{s}\right)$, one needs to account for QCD loop corrections (Figs. 1b-d), as well as for diagrams with the emission of an additional parton (Figs. 1e,f). The individual diagrams may contain soft singularities, but these cancel in the coherent sum of all contributions. The remaining collinear singularities are absorbed into the parton distribution functions (PDFs), or fragmentation functions (FFs).

While the above computational procedure yields a finite result, it fails to describe accurately the rate of sDIS in the limit when the direction of the resolved final-state parton is close to the one predicted in the leading order subprocess, that is in the direction of the initial electromagnetic current. Mathematically, the higher order corrections in this kinematic region (the current region) of sDIS are dominated by large logarithmic terms $\log ^{m} q_{T} / Q$. Here $q_{T}$ is a variable that is related to the final hadron's pseudorapidity $\eta$ in the $\gamma^{*} p$
c.m. frame:

$$
\begin{equation*}
q_{T}=W e^{-\eta}, \text { with } W^{2}=Q^{2}\left(\frac{1}{x}-1\right) \tag{2}
\end{equation*}
$$

The leading order process has $\eta=+\infty$, i.e. $q_{T}=0$. The singular logarithmic terms in the limit $q_{T} \rightarrow 0$ are therefore given by

$$
\begin{equation*}
\frac{1}{q_{T}^{2}} \sum_{n=1}^{\infty} \alpha_{s}^{n} \sum_{m=1}^{2 n-1} v_{m}^{n} \log ^{m} \frac{q_{T}}{Q} \tag{3}
\end{equation*}
$$

where $v_{m}^{n}$ denote coefficients of order 1 . As can be seen from (3), due to the presence of large logarithms, the higher order contributions are not negligible when $q_{T}$ is small. Thus, they all must be accounted for in order to predict reliably the observables in this kinematical regime.

It is interesting to note that a structure of the singular terms that is similar to the all-order sum (3) also dominates the cross-section for the production of back-to-back hadronic jets at $e^{+} e^{-}$colliders ${ }^{3}$, and the cross-section for small$p_{T}$ vector boson production at hadron colliders ${ }^{4}$. For those two processes, it has been proven that the analog of (3) can be computed in a closed form, by considering a Fourier transform of the cross-section from momentum space to the space of the conjugate variable (the impact parameter) $\vec{b}^{2,3,4}$. The result is called the resummed cross-section. Crossing relations between sDIS, $e^{+} e^{-}$ hadroproduction and vector boson production suggest that the same resummation technique may be successful in improving the theoretical description of the sDIS observables. Indeed, it was shown ${ }^{5}$ that the $b$-space resummation formalism provides a description of the energy flow in the small $-q_{T}$ region of sDIS. This approach was generalized ${ }^{6}$ to describe particle production cross-sections and multiplicities, and a comparison with experimental data was presented.

In the $b$-space formalism ${ }^{4}$, the resummed cross-section in the limit $q_{T} \leq Q$ is written in the form

$$
\begin{equation*}
\sigma(e A \rightarrow e B X) \approx N^{\prime} \int \frac{d^{2} b}{(2 \pi)^{2}} e^{i \vec{q}_{T} \cdot \vec{b}} \tilde{W}_{B A}(b, Q)+\sigma_{B A}^{p e r t}-\sigma_{B A}^{\text {sing }}, \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
\tilde{W}_{B A}(b, Q)=\sum_{a, b, j} e^{-S(b, Q)}\left[D_{B / b} \circ \mathcal{C}_{b / j}^{o u t}\right]\left[\mathcal{C}_{j / a}^{i n} \circ f_{a / A}\right] \tag{5}
\end{equation*}
$$

In this formula, the $\mathcal{C}$-functions, which are convoluted with the $\operatorname{PDFs} f_{a / A}$, or FFs $D_{B / b}$, arise because of collinear radiation along the direction of the initial or final LO quark. The contributions from the sea of soft partons that are not associated with either of the two jet-like structures are absorbed into
the Sudakov factor $S(b, Q)$. Another part of the Sudakov factor is used to parameterize the contribution from non-perturbative physics, which cannot presently be calculated from first principles. The integral over the parameter $\vec{b}$ is combined with the known fixed-order perturbative cross-section $\sigma_{B A}^{p e r t}$. To avoid double counting, one must subtract the singular part of the perturbative cross-section $\sigma_{B A}^{\text {sing }}$, which was already included in the $b$-space integral.

In Fig. 2 we compare experimental results of the H1 Collaboration ${ }^{7}$ to the predictions of the resummation formalism for the $z$-flow (rescaled transverse energy flow in the $\gamma^{*} p$ c.m. frame). The precise definition of the $z$-flow is

$$
\begin{equation*}
\frac{d \Sigma_{z}}{d x d Q^{2} d q_{T}^{2}}=\sum_{B} \int_{z_{\min }}^{1} z \frac{d \sigma(e A \rightarrow e B X)}{d x d z d Q^{2} d q_{T}^{2}} d z \tag{6}
\end{equation*}
$$

where $z=\left(p_{a} \cdot p_{b}\right) /\left(p_{a} \cdot q\right)$ is a variable describing the fragmentation of the final parton into the observed hadrons, and the cross-sections in the limit $q_{T} \rightarrow 0$ are calculated according to (4). Using (2), the experimental values for $\frac{d \Sigma_{z}}{d x d Q^{2} d q_{T}^{2}}$ can be derived from the published pseudorapidity distributions of the transverse energy flow $\left\langle E_{T}\right\rangle$ in the $\gamma^{*} p$ c.m. frame.
As can be seen in Fig. 2, the results of the resummation calculation ${ }^{6}$ are in acceptable agreement with the data. Note that the study of the small $q_{T}$ resummation formalism at HERA has several advantages, including the possibility to test the resummation formula at very small values of Bjorken $x$. It will be interesting to compare the predictions of the resummation formalism for various types of hadronic final states, since this will allow a test of the universality of the non-perturbative Sudakov factor. To summarize, the study of semiinclusive DIS in the current region of





 the $\gamma^{*} p$ c.m. frame may provide important insights on the nature of the soft and collinear parton radiation in the hadronic interactions at high energies.

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[^1]:    Figure 2: Comparison of the resummed $z$-flow with the experimental data from the $\mathrm{H} 1 \mathrm{Col}-$ laboration ${ }^{7}$

