View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by CERN Document Server

SO(10) Inspired See-Saw Mechanism

Mario Abud* and Franco Buccella°

* Dipartimento di Scienze Fisiche dell'Università di Napoli
Complesso Universitario di M. St. Angelo
via Cintia, I-80126 Napoli
and INFN, Sezione di Napoli
e-mail :Abud@na.infn.it

° Theory Division, CERN, CH-1211 Geneva 23, Switzerland and
Istituto di Fisica Teorica, Università di Napoli,Italy
e-mail :Buccella@na.infn.it

Abstract

We determine the ν_R Majorana mass matrix from the experimental data on neutrino oscillations in the framework of a see-saw SO(10) model, where we impose the condition $(M^R)_{33} = 0$ to avoid too large fine-tunings in the see-saw formula. We find a class of solutions with the two lowest neutrino masses almost degenerate and the scale of the matrix elements of M^R in the range $10^{11} - 10^{12}$ GeV in agreement with Pati-Salam intermediate symmetry. We find also solutions with smaller neutrino masses, for which the scale of M_R depends on the solution to the "solar neutrino problem" and on the value of the component of ν_e along the highest mass eigenstate, U_{e3}.

1 Introduction

The evidence in favour of oscillations[1] of solar[2] and atmospheric [3] neutrinos with square mass differences orders of magnitude smaller than the square masses of the other fermions strongly supports SO(10) gauge unification[4], where the see-saw mechanism[5] naturally accounts for this large mass gap. In this framework it is important to establish the order of magnitude of the elements of the ν_R's Majorana mass matrix,which is related to the scale of the spontaneous symmetry breaking of the SO(10) generator, B - L [6].
In the following section we shall show the uncertainties for the ν_L effective Majorana mass matrix,which follow not only from the presence of different solutions to the "solar neutrino problem"[7], but also from not knowing the mass of the lightest neutrino mass eigenstate and the relative sign of the three mass eigenvalues. We shall also show that an expansion in the eigenvalues of the Dirac neutrino mass matrix, assumed to be hierarchically ordered according to the family as the other fermions, is unable to reproduce with the first two terms the phenomenological properties of the ν_L effective Majorana mass-matrix, if small mixing angles for the Dirac mass matrices are assumed for leptons in analogy with what we know about the CKM matrix[8] for quarks.
This fact motivates us to take in the third section a vanishing value for M^R_{33}, [9][10][11]. Within the simplifying assumption of a trivial mixing matrix for the Dirac leptons, we are able to implement that condition for all the different solutions of the "solar neutrino problem". In most cases we find solutions with the two lower masses almost degenerate and at the order of magnitude of the highest one and with the highest matrix element of M^R,M^R_{23}, in the range $10^{11} - 10^{12}$ GeV in good agreement with the scale of the spontaneous breaking of B-L, $2.7 \cdot 10^{11}$ GeV [12] in the model with SU(4)xSU(2)xSU(2) [13] intermediate symmetry.We find also solutions with the two lower masses at the order $\sqrt{\Delta m^2_{sun}}$ and in that case the order of magnitude of the largest matrix element of M^R depends on the solution to the "solar neutrino problem" and on the value of U_{e3}.
Finally we give our conclusions.

2 The effective ν_L Majorana mass matrix.

The experimental information from solar and atmospheric neutrinos is such to "almost" allow a model-independent determination of the neutrino mass matrix, within the hypothesis of the existence of only the three active Majorana neutrinos and CP symmetry. The second hypothesis is expected to be at most an approximation, since CP violation in the neutral kaon system most probably comes from a phase in the CKM matrix, as is confirmed by the indication of a large CP-violating asymmetry in the $J/\psi\, K^S$ [14] channel, and a similar phase may be present in the lepton sector, for which, with Majorana neutrinos, two other CP-violating phases are also allowed.

The incompleteness in the determination of the neutrino mass matrix arises from the following facts:

i) Up to now there is only an upper limit for $|U_{e3}|^2 (\leq .05)$ [15][16].

ii) Oscillations depend on Δm^2's and cannot by themselves determine the masses of the neutrino eigenstates.

iii) There are sign ambiguities in the mass-eigenvalues.

Moreover there are five solutions [7] of the "neutrino solar problem" with the oscillation parameters reported in Table I.

Scenario	$\frac{\Delta m_s^2}{eV^2}$	$sin^2(2\theta_s)$	C.L.
LMA	$2.7 \cdot 10^{-5}$	0.79	68 %
SMA	$5.0 \cdot 10^{-6}$	$0.72 \cdot 10^{-2}$	64 %
LOW	$1.0 \cdot 10^{-7}$	0.91	83%
VAC_S	$6.5 \cdot 10^{-11}$	0.72	90 %
VAC_L	$4.4 \cdot 10^{-10}$	0.90	95 %

Table I

For atmospheric neutrino oscillations the relevant parameters are [3]:

$$\Delta m_a^2 \simeq 3.5 \cdot 10^{-3} (eV)^2 \qquad \sin^2(2\theta_a) \simeq 1 \tag{2.1}$$

We shall use this information to determine the effective ν_L Majorana mass matrix.

By assuming CP symmetry, which implies a real symmetric 3×3 Majorana neutrino mass matrix M^L :

$$\begin{aligned} M^L &= (m_{ij})_{i,j=1,2,3} \\ &= \Sigma \, |m_i > m_i < m_i| \end{aligned} \tag{2.2}$$

where the eigenvalues m_i are real, not necessarily positive, numbers.

Our convention is

$$|m_3| > |m_2| > |m_1| \tag{2.3}$$

and we may take $m_3 > 0$, since an overall change of sign of M^L has no physical consequences.

We define

$$\begin{aligned}|m_3> &= \sin\psi|\nu_e> + \cos\psi(\cos\theta|\nu_\mu> + \sin\theta|\nu_\tau>)\\ |m_2> &= -\sin\chi|v_1> + \cos\chi|v_2>\\ |m_1> &= \cos\chi|v_1> + \sin\chi|v_2> \end{aligned} \quad (2.4)$$

where

$$\begin{aligned}|v_1> &= \cos\psi|\nu_e> - \sin\psi(\cos\theta|\nu_\mu> + \sin\theta|\nu_\tau>)\\ |v_2> &= -\sin\theta|\nu_\mu> + \cos\theta|\nu_\tau> \end{aligned} \quad (2.5)$$

The difference $m_3^2 - m_1^2$ has to be identified with Δm_a^2, while for Δm_s^2 one has two options, either $m_3^2 - m_2^2$ or $m_2^2 - m_1^2$. The current interpretation of the atmospheric neutrino anomaly due to a practically maximal mixing between ν_μ and ν_τ, with ν_e playing a marginal role, would imply with the first choice having ν_e components almost completely along the two nearly degenerate higher-mass eigenstates. Since this situation is unnatural in the $SO(10)$ framework, which we will describe in the second part of this paper,we take the option $\Delta m_s^2 = m_2^2 - m_1^2$.

From eqs. 2.2, 2.4 and 2.5, it is easy to get:

$$\begin{aligned}
m_{11} &= m_3\sin^2\psi + m_2\sin^2\chi\cos^2\psi + m_1\cos^2\chi\cos^2\psi\\
m_{22} &= m_3\cos^2\theta\cos^2\psi + \cos^2\theta\sin^2\psi(m_2\sin^2\chi + m_1\cos^2\chi)\\
&+ sin^2\theta(m_2\cos^2\chi + m_1\sin^2\chi) - \sin 2\theta\sin 2\chi\frac{\sin\psi}{2}(m_2 - m_1)\\
m_{33} &= m_3\sin^2\theta\cos^2\psi + sin^2\theta\sin^2\psi(m_2\sin^2\chi + m_1\cos^2\chi) \qquad (2.6)\\
&+ \cos^2\theta(m_2\cos^2\chi + m_1\sin^2\chi) + \sin 2\theta\sin 2\chi\frac{\sin\psi}{2}(m_2 - m_1)\\
m_{12} &= \cos\psi\Big[\sin\psi\cos\theta(m_3 - m_2\sin^2\chi - m_1\cos^2\chi)\\
&\qquad + \sin\theta\frac{\sin 2\chi}{2}(m_2 - m_1)\Big]\\
m_{13} &= \cos\psi\Big[\sin\psi\sin\theta(m_3 - m_2\sin^2\chi - m_1\cos^2\chi)\\
&\qquad - \cos\theta\,\frac{\sin\,2\chi}{2}\,(m_2 - m_1)\Big]\\
m_{23} &= \frac{\sin\,2\theta}{2}\,\Big[m_3\cos^2\psi - m_2\cos^2\chi - m_1\sin^2\chi\\
&\qquad + \sin^2\psi(m_2\sin^2\chi + m_1\cos^2\chi)\Big]\\
&+ \cos 2\theta\,\frac{\sin 2\chi}{2}\,\sin\psi(m_2 - m_1)
\end{aligned}$$

The angles θ and χ should be identified with the mixing angles for atmospheric and solar neutrino oscillations, respectively. The experimental value found for the atmospheric neutrino oscillations, $\sin^2 2\theta_a \simeq 1$, and the upper limit on $|U_{e3}|^2$ imply $\theta = \frac{\pi}{4}$ and a small value for $\sin\psi$.

With hierarchical neutrino masses

$$m_3 >> |m_2| >> |m_1| \tag{2.7}$$

one has

$$\begin{aligned} m_3^2 &\simeq \Delta m_a^2 \\ m_2^2 &\simeq \Delta m_s^2 \end{aligned} \tag{2.8}$$

and by taking $\psi = 0$ for the contributions proportional to m_2 and neglecting $|m_1| << \sqrt{\Delta m_s^2}$, one gets:

$$\begin{aligned} M^L &= \frac{m_3}{2}\cos^2\psi \begin{pmatrix} 2\tan\psi^2 & \sqrt{2}\tan\psi & \sqrt{2}\tan\psi \\ \sqrt{2}\tan\psi & 1 & 1 \\ \sqrt{2}\tan\psi & 1 & 1, \end{pmatrix} \\ &+ \frac{m_2}{2}\cos^2\chi \begin{pmatrix} 2\tan^2\chi & \sqrt{2}\tan\chi & -\sqrt{2}\tan\chi \\ \sqrt{2}\tan\chi & 1 & -1 \\ -\sqrt{2}\tan\chi & -1 & 1, \end{pmatrix}. \end{aligned} \tag{2.9}$$

We now consider the possibility, often advocated, that the two lower masses are almost degenerate and larger than $\sqrt{\Delta m_s^2}$, but smaller than $\sqrt{\Delta m_a^2}$ [9] [10] [11]. In that case, one has still $\Delta m_a^2 = m_3^2$, but we can neglect Δm_{sun}^2 with respect to m_1^2 and m_2^2. The first term in eq. 2.9 remains the same, while the other one takes a different form, depending on the relative sign of m_2 and m_1.We will consider $m_1 m_2 < 0$, since this assumption will be necessary in the following and get

$$\frac{m_2\cos 2\chi}{2} \begin{pmatrix} 1 & \sqrt{2}\tan 2\chi & -\sqrt{2}\tan 2\chi \\ \sqrt{2}\tan 2\chi & 1 & -1 \\ -\sqrt{2}\tan\ 2\chi & -1 & 1, \end{pmatrix} \tag{2.10}$$

Again for the term proportional to m_2 in eq. 2.10 we made the approximation to take $\psi = 0$.

We now consider the possibility advocated by Georgi and Glashow [17], with the motivation of having a sizeable neutrino contribution to the hot dark matter, of almost degenerate square masses for the neutrinos, larger than their differences. We have four options, according to the relative signs of the m_i's. By taking again $m_1 < 0 < m_2$ and $\psi = 0$ for the part proportional to Δm_a^2, we get :

$$M^L = m_3\cos^2\chi \begin{pmatrix} \tan^2\chi - 1 & \sqrt{2}\cos\psi\tan\chi & -\sqrt{2}\cos\psi\tan\chi \\ \sqrt{2}\cos\psi\tan\chi & 1 & \tan^2\chi \\ -\sqrt{2}\cos\psi\tan\chi & \tan^2\chi & 1, \end{pmatrix}$$

$$
\begin{aligned}
&+ \quad m_3 \cos^2\chi \sin\psi \begin{pmatrix} 0 & \sqrt{2}\cos\psi & \sqrt{2}\cos\psi \\ \sqrt{2}\cos\psi & -2\tan\chi & 0 \\ \sqrt{2}\cos\psi & 0 & 2\tan\chi, \end{pmatrix} \\
&+ \quad 2m_3 \cos^2\chi sin^2\psi \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{1}{2}, \end{pmatrix} \\
&+ \quad \frac{\Delta m_a^2 \cos 2\chi}{4m_3} \begin{pmatrix} 2 & -\sqrt{2}\tan 2\chi & -\sqrt{2}\tan 2\chi \\ -\sqrt{2}\tan 2\chi & 1 & 1 \\ -\sqrt{2}\tan 2\chi & 1 & 1, \end{pmatrix}. \qquad (2.11)
\end{aligned}
$$

Let us consider

$$
\begin{aligned}
m_{\nu_e\nu_e} &= m_3(-\cos 2\chi + 2\cos^2\chi\sin^2\psi \\
&+ \sin^2\chi \frac{\Delta m_s^2}{2m_3^2} + \frac{\Delta m_a^2}{2m_3^2}\cos^2\chi) \qquad (2.12)
\end{aligned}
$$

for which there is an experimental upper limit coming from the study of neutrinoless double beta-decay $|m_{\nu_e\nu_e}| < .2\ eV$ [19]. To get the cancellation between $-\cos 2\chi$ and $2\cos^2\chi\sin 2\psi$, χ should be near to $\frac{\pi}{4}$, more precisely :

$$
\sin^2\psi = \frac{\tan^2\chi - 1}{2} \qquad (2.13)
$$

But the r.h.s. of eq. 2.13 takes at least the central value .22 (for $\sin^2 2\theta_s =$ .91), larger than the upper limit for $\sin^2\psi$, which implies [18]

$$
|m_3| \simeq |\frac{m_{\nu_e\nu_e}}{2\cos^2\chi\sin^2\psi - \cos 2\chi}| \leq .85 eV \qquad (2.14)
$$

The situation is similar for the case with $m_1 > 0 > m_2$. In conclusion many options are open for the neutrino mass matrix and even an exact determination of the mixing matrix and of the square mass differences is unable to resolve the ambiguity associated to the value of $|m_1|$ and to the relative signs of the masses [20].Also the same Majorana mass matrix for the ν_L's may be obtained with different choices of the Dirac lepton matrices and of M^R.

It has been suggested since a long time [21] that the peculiar properties of neutrino mixing, almost maximal for atmospheric, rather large for all the solutions to the "solar neutrino problem", except the MSW small angle solution, should be related to the fact that, differently from the case for the charged fermions, their masses arise from the see-saw mechanism [5].The later has the indisputable merit of providing a reason for the small value of the neutrino masses, especially in the framework of $SO(10)$ unified theories, which predict the existence of left-handed antineutrinos with Majorana masses related to the spontaneous breaking of $B-L$ symmetry [6].

Unified $SO(10)$ theories are suitable for the study of fermion masses as all the fermions of one family belong to a single representation, the spinorial

16 [4]. Indeed, by classifying the Higgs doublet responsible for the breaking of the electroweak symmetry in the vector representation (10), one obtains,together with the celebrated(but actually not so successfull) relationship [22],

$$\frac{m_\tau}{m_b}(unification) = 1 \tag{2.15}$$

while the analogous relationship, $\frac{m_{\nu_\tau}}{m_t}(unification) = 1$, is turned by the seesaw mechanism into the intriguing prediction of very small neutrino masses. To correct $\frac{m_\mu}{m_s} = \frac{m_e}{m_d} = 1$, some component of the electroweak Higgs, at least along the 126, should be introduced [23]

$$\begin{aligned}(16 \times 16)_S &= 126 + 10 \\ (16 \times 16)_A &= 120\end{aligned} \tag{2.16}$$

Without adopting a particular scheme, we shall limit ourselves to assume that the Dirac neutrino mass matrix, once diagonalized by the biunitary transformation, gives rise to a hierarchical relationship for the elements of the diagonal matrix similar to that existing for the other fermions, and the matrix corresponding to the CKM, which gives the misalignment with the corresponding leptons, has small non diagonal matrix elements.

$$\begin{aligned}M^L &= -m^D (M^R)^{-1} (m^D)^T \\ m^D &= U_L m_{diag} U_R^+\end{aligned} \tag{2.17}$$

with

$$m_{diag} = diag(\mu_1, \mu_2, \mu_3) \tag{2.18}$$

and we assume $\mu_1 << \mu_2 << \mu_3$. From eqs. 2.17 it is easy to derive:

$$M^L = -U_L\, m_{diag}\, U_R^+\, (M^R)^{-1}\, U_R^\star\, m_{diag}\, U_L^T \tag{2.19}$$

which is simplified by defining

$$\tilde{N} = U_R^+\, M^{R^{-1}}\, U_R^\star \tag{2.20}$$

into

$$M^L = -U_L\, m_{diag}\, \tilde{N}\, m_{diag}\, U_L^T \tag{2.21}$$

With our hypothesis of CP conservation, U_R and U_L are, in fact, real orthogonal matrices and $\tilde{N}$ is symmetric real. We may develope M^L as a quadratic form in the μ_i's:

$$\begin{aligned}-M^L_{\ell\ell'} &= (U_L)_{\ell 3}(U_L)_{\ell' 3}\tilde{N}_{33}\mu_3^2 \\ &+ ((U_L)_{\ell 2}(U_L)_{\ell' 3} + (U_L)_{\ell 3}(U_L)_{\ell' 2})\tilde{N}_{23}\mu_2\mu_3 \\ &+ (U_L)_{\ell 2}(U_L)_{\ell' 2}\tilde{N}_{22}\mu_2^2 + \cdots\end{aligned} \tag{2.22}$$

Should the matrix elements of $\tilde{N}$ be of the same order, it would be a reasonable approximation to take only the terms proportional to μ_3^2, which would give for $-M^L$ the expression:

$$\tilde{N}_{33}\mu_3^2 \begin{pmatrix} (U_L)_{13}^2 & (U_L)_{13}(U_L)_{23} & (U_L)_{13}(U_L)_{33} \\ (U_L)_{13}(U_L)_{23} & (U_L)_{23}^2 & (U_L)_{23}(U_L)_{33} \\ (U_L)_{13}(U_L)_{33} & (U_L)_{23}(U_L)_{33} & (U_L)_{33}^2 \end{pmatrix} \tag{2.23}$$

The matrix defined by eq. 2.23 should have eigenvalues $\mu_3^2\ \tilde{N}_{33}$, corresponding to the eigenvector

$$\begin{pmatrix} U_{13} \\ U_{23} \\ U_{33} \end{pmatrix}, \tag{2.24}$$

and twice the eigenvalue 0. The eigenvector in eq. 2.24 should be identified with:

$$\begin{pmatrix} \sin\psi \\ \cos\psi\cos\theta \\ \cos\psi\sin\theta \end{pmatrix}, \tag{2.25}$$

which, for $\theta = \frac{\pi}{4}$, has almost equal second and third components, implying $U_{23} \simeq U_{33}$, in disagreement with our assumption that the mixing angles of Dirac neutrinos with the corresponding leptons are small.

Let us consider also the term proportional to $\mu_2\ \mu_3$, and, for brevity, define:

$$\begin{aligned} M &= \tilde{N}_{33}\,\mu_3^2 \\ \tilde{M} &= \tilde{N}_{23}\,\mu_2\mu_3. \end{aligned} \tag{2.26}$$

In such a case, the neutrino mass matrix would be:

$$\begin{aligned} & M \begin{pmatrix} (U_L)_{13}^2 & (U_L)_{13}(U_L)_{23} & (U_L)_{13}(U_L)_{33} \\ (U_L)_{13}(U_L)_{23} & (U_L)_{23}^2 & (U_L)_{23}(U_L)_{33} \\ (U_L)_{13}(U_L)_{33} & (U_L)_{23}(U_L)_{33} & (U_L)_{33}^2 \end{pmatrix} \\ + & \tilde{M} \begin{pmatrix} 2(U_L)_{13}(U_L)_{12} & (U_L)_{13}(U_L)_{22} + (U_L)_{12}(U_L)_{23} & (U_L)_{12}(U_L)_{33} + (U_L)_{13}(U_L)_{32} \\ (U_L)_{13}(U_L)_{22} + (U_L)_{12}(U_L)_{23} & 2(U_L)_{22}(U_L)_{23} & (U_L)_{22}(U_L)_{33} + (U_L)_{23}(U_L)_{32} \\ (U_L)_{12}(U_L)_{33} + (U_L)_{13}(U_L)_{32} & (U_L)_{22}(U_L)_{33} + (U_L)_{23}(U_L)_{32} & 2(U_L)_{32}(U_L)_{33} \end{pmatrix} \end{aligned} \tag{2.27}$$

which has a vanishing eigenvalue and the other two given by:

$$\frac{M \pm \sqrt{M^2 + 4\tilde{M}^2}}{2} \tag{2.28}$$

By taking M positive, one should identify

$$\Delta m_a^2 = \frac{1}{4}\ [M + \sqrt{M^2 + 4\tilde{M}^2}]^2, \tag{2.29}$$

while for Δm_s^2 ($<< \Delta m_a^2$) one should have

$$\Delta m_s^2 = \frac{1}{4}[\sqrt{M^2 + 4\tilde{M}^2} - M]^2 \tag{2.30}$$

$$\frac{\Delta m_s^2}{\Delta m_a^2} \simeq \frac{\Delta m_s^2 \Delta m_a^2}{(\Delta m_a^2)^2 + (\Delta m_s^2)^2} = \\ \frac{2\tilde{M}^4}{2M^4 + 8\tilde{M}^2 M^2 + 4\tilde{M}^4} \simeq \left(\frac{\tilde{M}}{M}\right)^4 \tag{2.31}$$

In the most favourable case, large angle MSW, it would give:

$$\left(\frac{\tilde{M}}{M}\right) = \left(\frac{1.6 \cdot 10^{-5}}{3.5 \cdot 10^{-3}}\right)^{\frac{1}{4}} \sim \frac{1}{3.8} \tag{2.32}$$

Let us see how large the non-diagonal matrix elements of U_L should be in order to give rise to a neutrino mass matrix, which has the property of having almost equal matrix elements in 22 and 33 positions.We should have

$$M\,(U_L)_{23}^2 + 2\tilde{M}\,(U_L)_{23}(U_L)_{22} = M(U_L)_{33}^2 + 2\tilde{M}(U_L)_{33}(U_L)_{32} \tag{2.33}$$

which is impossible to obtain with small values of the non-diagonal matrix elements, since

$$(U_L)_{23}(M(U_L)_{23} + 2\tilde{M}(U_L)_{22}) << (U_L)_{33}(M(U_L)_{33} + 2\tilde{M}(U_L)_{32}) \tag{2.34}$$

So the expansion with only the two terms is not able to reproduce a neutrino mass matrix consistent with the experimental information. Therefore, we must now consider further terms, i.e., those proportional to μ_2^2 and to $\mu_1\mu_3$. If we define:

$$\begin{aligned} \sigma &= \tilde{N}_{22}\,\mu_2^2 \\ \tau &= \tilde{N}_{13}\,\mu_1\,\mu_3 \end{aligned} \tag{2.35}$$

in the limit $(U_L)_{ij} = \delta_{ij}$, we would just find the matrix proposed by Stech[10]

$$\begin{pmatrix} 0 & 0 & \tau \\ 0 & \sigma & \mu \\ \tau & \mu & M \end{pmatrix} \tag{2.36}$$

which, to reproduce the maximal mixing for ν_μ and ν_τ, requires the equality of the terms proportional to μ_2^2 and μ_3^2, respectively.
From this discussion it follows that the term proportional to μ_3^2 does not play a dominant role in the effective ν_L Majorana mass matrix not only with respect to the one proportional to $\mu_2\,\mu_3$, but also with respect to the one

proportional to μ_2^2.For a diagonal lepton Dirac matrix the term μ_3^2 appears in the expression of M^R_{33}, multiplied by$(M^{L^{-1}})_{33}$. So to reduce the contribution of μ_3^2 we take a small value for $(M^{L^{-1}})_{33}$.
In the following section we shall impose the vanishing of $(M^{L^{-1}})_{33}$ and consequently of $(M^R)_{33}$ with a diagonal neutrino Dirac matrix given by:

$$m_\nu^D = \frac{m_\tau}{m_b} diag(m_u, m_c, m_t) \tag{2.37}$$

3 See-Saw model in SO(10) with diagonal neutrino Dirac matrix and $M^R_{33} = 0$

We have seen in the previous section that M^L, derived from a Dirac neutrino mass matrix with the properties of the quark mass matrix, hierarchical relationship for its eigenvalues and small mixing angles, is not dominated by the term proportional to $(m^D_{\nu_\tau})^2$,which is as important as the term proportional to $(m^D_{\nu_\mu})^2$.A way to implement this property, the non-dominance of the term proportional to $(m^D_{\nu_\tau})^2$, is found by considering the inverse of the first of eqs. 2.17:

$$M^R = -m_\nu^D (M^L)^{-1} m_\nu^D. \tag{3.1}$$

with m_ν^D given by eq. 2.37:

$$\begin{aligned}
(\frac{m_b}{m_\tau m_u})^2 (M^R)_{11} &= -\frac{1}{m_1} \cos^2\psi \cos^2\chi - \frac{1}{m_2} \cos^2\psi \sin^2\chi - \frac{1}{m_3} \sin^2\psi \\
\frac{(\frac{m_b}{m_\tau})^2}{m_u m_c} (M^R)_{12} &= \frac{1}{\sqrt{2} m_1} \cos\psi \cos\chi (\cos\chi \sin\psi + \sin\chi) \\
&\quad - \frac{1}{\sqrt{2} m_2} \cos\psi \sin\chi (\cos\chi - \sin\psi \sin\chi) - \frac{1}{2\sqrt{2} m_3} \sin(2\psi) \\
\frac{(\frac{m_b}{m_\tau})^2}{m_u m_t} (M^R)_{13} &= \frac{1}{\sqrt{2} m_1} \cos\psi \cos\chi (\cos\chi \sin\psi - \sin\chi) \\
&\quad + \frac{1}{\sqrt{2} m_2} \cos\psi \sin\chi (\cos\chi + \sin\psi \sin\chi) - \frac{1}{2\sqrt{2} m_3} \sin(2\psi) \\
(\frac{m_b}{m_\tau m_c})^2 (M^R)_{22} &= -\frac{1}{2m_1} (\cos\chi \sin\psi + \sin\chi)^2 \qquad (3.2) \\
&\quad - \frac{1}{2m_2} (\cos\chi - \sin\psi \sin\chi)^2 - \frac{1}{2m_3} \cos^2\psi \\
\frac{(\frac{m_b}{m_\tau})^2}{m_c m_t} (M^R)_{23} &= \frac{1}{2m_1} (\sin^2\chi - \cos^2\chi \sin^2\psi) \\
&\quad + \frac{1}{2m_2} (\cos^2\chi - \sin^2\psi \sin^2\chi) - \frac{1}{2m_3} \cos^2\psi
\end{aligned}$$

$$(\frac{m_b}{m_\tau m_t})^2 (M^R)_{33} = -\frac{1}{2m_1}(\sin\chi - \cos\chi\sin\psi)^2 - \frac{1}{2m_2}(\cos\chi + \sin\chi\,\sin\psi)^2 - \frac{1}{2m_3}\cos^2\psi$$

From eqs. 3.3, it is easy to see that

$$|(M^R)_{11}| < \frac{(\frac{m_\tau m_u}{m_b})^2}{|m_1|} \tag{3.3}$$

which implies [24], in the absence of cancellations for $(M^R)_{33}$ [1]:

$$|\frac{(M^R)_{11}}{(M^R)_{33}}| \simeq 2\frac{m_u^2}{m_t^2} \simeq 2\left(\frac{\frac{8}{3}\text{MeV}}{180\text{GeV}}\right)^2 \simeq 4\cdot 10^{-10} \tag{3.4}$$

To prevent the appearence of such unnatural small factor we impose the vanishing of the r.h.s of last equation in 3.3:

$$\frac{(\sin\chi - \cos\chi\sin\psi)^2}{m_1} + \frac{(\cos\chi + \sin\psi\sin\chi)^2}{m_2} + \frac{\cos^2\psi}{m_3} = 0 \tag{3.5}$$

which requires that not all the m_i's have the same sign and

$$\frac{m_b^2}{m_\tau^2 m_c m_t}(M^R)_{23} = \frac{\sin 2\chi\sin\psi}{2}(\frac{1}{m_1} - \frac{1}{m_2}) - (\sin\psi)^2(\frac{(\cos\chi)^2}{m_1} + \frac{(\sin\chi)^2}{m_2}) - \frac{(\cos\psi)^2}{m_3} \tag{3.6}$$

By neglecting the terms proportional to powers of $\sin\psi$, we get

$$|(M^R)_{23}| \leq 7\cdot 10^{11}\text{GeV} \tag{3.7}$$

as in [11].

With $\sin\psi = 0$, eqs. 3.3 read:

$$\begin{aligned}
(\frac{m_b}{m_\tau m_u})^2 (M^R)_{11} &= -\frac{1}{m_1}\cos^2\chi \\
\frac{(\frac{m_b}{m_\tau})^2}{m_u m_c}(M^R)_{12} &= \frac{\cos\chi\sin\chi}{\sqrt{2}}(\frac{1}{m_1} - \frac{1}{m_2}) \\
\frac{(\frac{m_b}{m_\tau})^2}{m_u m_t}(M^R)_{13} &= \frac{\cos\chi\sin\chi}{\sqrt{2}}(\frac{1}{m_1} + \frac{1}{m_2}) \\
(\frac{m_b}{m_\tau m_c})^2 (M^R)_{22} &= -\frac{1}{2m_1}\sin^2\chi - \frac{1}{2m_2}\cos^2\chi - \frac{1}{2m_3} \\
\frac{(\frac{m_b}{m_\tau})^2}{m_c m_t}(M^R)_{23} &= \frac{1}{2m_1}\sin^2\chi + \frac{1}{2m_2}\cos^2\chi - \frac{1}{2m_3} \\
(\frac{m_b}{m_\tau m_t})^2 (M^R)_{33} &= -\frac{1}{2m_1}\sin^2\chi - \frac{1}{2m_2}\cos^2\chi - \frac{1}{2m_3}
\end{aligned} \tag{3.8}$$

[1] Since the ratios of the quark masses have a negligeable dependence on the scale, we can take the values given in [24]

In order to get $(M^R)_{33} = 0$, one should have

$$\frac{\sin^2\chi}{m_1} + \frac{\cos^2\chi}{m_2} + \frac{1}{m_3} = 0 \tag{3.9}$$

We look for its solutions with m_i 's constrained by the relations:

$$\begin{aligned} m_2^2 &= m_1^2 + \Delta m_s^2 \\ m_3^2 &= m_1^2 + \Delta m_a^2 \end{aligned} \tag{3.10}$$

With this we do not want to impose that $(M^R)_{33}$ vanishes exactly. But, once a solution with no large differences between the matrix elements of M^R has been found, we can allow for a value of $(M^R)_{33}$ of the same order of the other matrix elements, with a small change for the left-handed neutrino Majorana mass matrix.
It is obvious that to obey eq. 3.9 the m_i 's cannot have the same sign and, as $|m_3| > |m_2| > |m_1|$, m_1 and m_2 should have opposite signs.This fact has been pointed out in [25]. From eq. 3.9 we get:

$$\tan^2\chi = -\frac{m_1(m_3 + m_2)}{m_2(m_3 + m_1)} \tag{3.11}$$

which,when $|m_2| << |m_3|$ simplifies to

$$\tan^2\chi = -\frac{m_1}{m_2} \tag{3.12}$$

in agreement with the expectation that large mixing angles correspond to almost degenerate masses of the mixed states [26]. One has another approximate solution with almost degenerate lightest neutrino mass eigenstates $(|m_2| \simeq -m_1 = m)$

$$m = \cos(2\chi)m_3 \tag{3.13}$$

These approximate solutions correspond to exact solutions of eq. 3.9. In fact, with equal signs for m_2 and m_3 we may write eq. 3.9 in the form

$$\sin\chi^2 + \cos\chi^2\frac{m_1}{m_2} + \frac{m_1}{m_3} = 0 \tag{3.14}$$

which has one and only one solution for negative m_1 since its l.h.s. is an increasing function of m_1 in the range $(-\infty, 0)$ varying from $-2\cos^2\chi$ to $\sin^2\chi$ [2]. In the case of the same sign for m_1 and m_3 it is convenient to rewrite eq. 3.9 in the form

[2]Notice that eqs. 3.10 imply that when $|m_1|$ goes to ∞ the ratios $\left|\frac{m_1}{m_2}\right|$ and $\left|\frac{m_1}{m_3}\right|$ go to 1.

$$\sin^2\chi\frac{m_2}{m_1}+\cos^2\chi+\frac{m_2}{m_3}=0 \qquad (3.15)$$

With its l.h.s., in the range $(-\infty,\ -\sqrt{\Delta m_s^2})$ for m_2, going from $-2(\sin^2\chi)$ to $-\infty$. But, if $\cos^2\chi$ is sufficiently larger than $\sin^2\chi$, can reach positive values, implying two solutions for eq. 3.9, approximately given by eqs. 3.12 and 3.13, respectively.
For the MSW solutions one has to take $\cos^2\chi > \sin^2\chi$ and therefore one has three solutions. For the vacuum solutions, which allows both signs for $\cos(2\chi)$, one has also a solution with $\sin^2\chi > \cos^2\chi$. We wish to extend the previous analysis to consider non vanishing values of $|\sin\psi|$. In that case, when

$$(\sin\chi-\sin\psi\cos\chi)^2 > (\cos\chi+\sin\psi\sin\chi)^2 \qquad (3.16)$$

one has only one solution of eq. 3.9; instead, when the r.h.s. of the inequality 3.16 is sufficiently larger than the l.h.s. + 1, there are two solutions. We have performed a numerical study of the solutions of eq. 3.6 with $|\sin\psi| = k(.075)$, k=0,...,3. To get a general view of our solutions, we give in Table II, for each solar neutrino oscillation scenario, the matrix where the smallest value for the ratios of the moduli of non-vanishing matrix elements of M^R (which is, with the only exeption of VACL solution, $\frac{M^R_{11}}{M^R_{23}}$) takes the largest value and the ones where the max (ie. highest) matrix element of M^R takes the smallest or largest value. From Table II we see that the matrices with smallest excursion for their matrix element correspond to small values for ${m_1}^2$ and to $\psi = 0$, with the only exception of the LMW solution, where $|\sin\psi| = 0.075$. The matrices with the smallest value of M^R_{23} are found for the large ${m_1}^2$ solutions. Finally the largest values of M^R_{23} are found at the boundary of the allowed values for ψ and in correspondence of the small ${m_1}^2$ solutions. Since our motivation for setting $M^R_{33} = 0$ is also based on the demand of dealing with M^R matrices with not so disparate orders of magnitude for its matrix elements, we favour small ψ and ${m_1}^2$ choices (in particular, the small MSW solution with the ratio between the smallest and the largest matrix elements of M^R given by $4.3\cdot 10^{-4}$). We never find neutrino masses large enough to be of cosmological relevance. The maximum value found for $\sum_i |m_i|$ being .6 eV, corresponding to the boundary of allowed values of the small angle MSW solution.

4 Conclusions

The hypothesis of a vanishing M^R_{33}, assumed in[9][10][11] and here motivated by the requirement of not having too large fine tunings in the see-saw formula may be implemented in all five solutions to the "solar neutrino problem".In general one has solutions with large $\sim (\Delta m_a^2)$ or small $\sim (\Delta m_s^2)$ for the square mass of the lightest neutrino mass eigenstate. For large values of m_1^2 the max matrix element of M^R (M_{23}^R) has a value $\sim 7 \cdot 10^{11}$ GeV, a scale found in a recent work [11] and in agreement with the spontaneous scale of B-L symmetry breaking found in a SO(10) model [12] with SU(4) x SU(2) x SU(2) intermediate symmetry [13]. Moderate values for the matrix elements of M^R have been also found in [9] [10] and recently by Akhmedov, Branco and Rebelo [27]. For small values for m_1^2 one still finds the same order of magnitude for the large angle MSW solution in the entire range of the allowed values for ψ; the same happens for the other MSW solutions for $\psi = 0$. The other MSW solutions correspond to larger values for the scales, especially for the largest allowed values of $|\sin\psi|$. The solutions corresponding to small m_1^2 at ψ=0 have the appealing feature of having not too different orders of magnitude for the matrix elements of M^R.

Acknowledgements: One of us (F.B.) acknowledges G. Fiorentini for interesting discussions.

Scenario $\Delta m_s^2(\mathrm{eV}^2)$ $\sin^2(2\theta_s)$	smallest ratio (matrix elements in GeV)	smallest max $\lvert (M_R)_{ij} \rvert$ (matrix elements in GeV)	largest max $\lvert (M_R)_{ij} \rvert$ (matrix elements in GeV)
LMA $\Delta m_s^2 = 2.7.10^{-5}$ $\sin^2(2\theta_s) = .79$	$\begin{bmatrix} 4.1.10^5 & -8.4.10^7 & -2.6.10^{10} \\ \# & -5.6.10^9 & 1.4.10^{11} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{.075};\ \lvert m_1\rvert = \mathbf{2.6.10^{-3}eV}$	$\begin{bmatrix} 4.1.10^5 & -8.4.10^7 & -2.6.10^{10} \\ \# & -5.6.10^9 & 1.4.10^{11} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{.075};\ \lvert m_1\rvert = \mathbf{2.6.10^{-3}eV}$	$\begin{bmatrix} 2.2.10^6 & -4.3.10^8 & 5.8.10^{10} \\ \# & 6.1.10^{10} & 6.2.10^{12} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{.225};\ \lvert m_1\rvert = \mathbf{5.5.10^{-4}eV}$
SMA $\Delta m_s^2 = 5.10^{-6}$ $\sin^2(2\theta_s) = .0072$	$\begin{bmatrix} -4.3.10^8 & 3.7.10^9 & -10^{12} \\ \# & 0 & -6.9.10^{11} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{0};\ \lvert m_1\rvert = \mathbf{4.2.10^{-6}eV}$	$\begin{bmatrix} -9.10^3 & 1.4.10^5 & -4.7.10^7 \\ \# & 3.10^7 & -2.10^{11} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{.225};\ \lvert m_1\rvert = \mathbf{.19eV}$	$\begin{bmatrix} 2.4.10^7 & -1.4.10^9 & -2.5.10^{11} \\ \# & 4.2.10^{10} & 2.3.10^{13} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{.225};\ \lvert m_1\rvert = \mathbf{7.10^{-5}eV}$
LOW $\Delta m_s^2 = 10^{-7}$ $\sin^2(2\theta_s) = .91$	$\begin{bmatrix} -4.10^6 & 1.3.10^9 & 3.6.10^{11} \\ \# & 0 & -6.7.10^{11} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{0};\ \lvert m_1\rvert = \mathbf{2.10^{-4}eV}$	$\begin{bmatrix} -3.10^4 & 2.10^7 & -5.10^9 \\ \# & 0 & -6.6.10^{11} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{0};\ \lvert m_1\rvert = \mathbf{1.86.10^{-2}eV}$	$\begin{bmatrix} -1.6.10^7 & 4.10^9 & -6.9.10^{11} \\ \# & -6.3.10^{11} & 6.5.10^{13} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{.225};\ \lvert m_1\rvert = \mathbf{6.2.10^{-5}eV}$
VAC_S $\Delta m_s^2 = 6.5.10^{-11}$ $\sin^2(2\theta_s) = .72$	$\begin{bmatrix} -4.8.10^8 & 7.9.10^{10} & -2.10^{13} \\ \# & 0 & -6.8.10^{11} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{0};\ \lvert m_1\rvert = \mathbf{2.6.10^{-6}eV}$	$\begin{bmatrix} -2.6.10^4 & 8.5.10^6 & -2.3.10^9 \\ \# & 0 & -5.9.10^{11} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{0};\ \lvert m_1\rvert = \mathbf{3.7.10^{-2}eV}$	$\begin{bmatrix} -1.8.10^9 & 3.3.10^{11} & -4.1.10^{13} \\ \# & -4.5.10^3 & 3.9.10^{15} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{.225};\ \lvert m_1\rvert = \mathbf{7.10^{-7}eV}$
VAC_L $\Delta m_s^2 = 4.4.10^{-10}$ $\sin^2(2\theta_s) = .9$	$\begin{bmatrix} -6.8.10^7 & 2.10^{10} & -5.6.10^{12} \\ \# & 0 & -6.7.10^{11} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{0};\ \lvert m_1\rvert = \mathbf{1.3.10^{-5}eV}$	$\begin{bmatrix} -9.10^3 & 4.3.10^6 & -1.4.10^9 \\ \# & -9.2.10^8 & -3.1.10^{11} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{.225};\ \lvert m_1\rvert = \mathbf{7.10^{-2}eV}$	$\begin{bmatrix} -2.6.10^8 & 6.5.10^{10} & -10^{13} \\ \# & -10^{13} & 10^{15} \\ \# & \# & 0 \end{bmatrix}$ $\lvert\sin\psi\rvert = \mathbf{.225};\ \lvert m_1\rvert = \mathbf{3.9.10^{-6}eV}$

Table II

The matrices M_R obeying eq. 3.6 for the five scenarios. For each solution the corresponding values of $|\sin\psi|$ and $|m_1|$ are given. As M_R is symmetric, repeated matrix elements are denoted by #.

References

[1] B. Pontecorvo, J. Exptl. Theoret. Phys. **33**, 549 (1957) and **34**, 247 (1958) [Sov Phys. JETP. **6**, 429 (1958) and **7**, 172 (1958)] ZETF **53**, 1717 (1967) [Sov. Phys. JETP **26**, 984 (1968); Z.Maki, M.Nakagawa and S.Sakata Progr. Th. Phys. **28**, 870 (1962); V. Gribov and B. Pontecorvo, Phys. Lett. **B28**, 493 (1969); S.M.Bilenky and B.Pontecorvo Phys. Lett. **B61**, 248 (1976).

[2] B.Pontecorvo, Report PD-205 National Research Council of Canada, Division of Atomic Energy, Chalk River (1946); R. Davis Jr *et al.*, Phys. Rev. Lett. **20**, 1205 (1968); Homestake Coll.: R. Davis Jr *et al.* Prog. Part. Nucl. Phys. **32**,13 (1994); Gallex Coll.: P. Anselmann *et al.* Phys. Lett. **B342**, 440 (1995); W. Hampel *et al.*, Phys. Lett. **B447**, 127 (1999); Sage Coll.: J. N. Abdurashitov *et al.* Phys. Lett. **B328**, 234 (1994); Kamiokande Coll.: Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996); SuperKamiokande Coll.: Y. Fukuda *et al.* Phys. Rev. Lett. **81**, 1158 (1998).

[3] Kamiokande Coll.: K.S.Hirata *et al.*, Phys. Lett. **B205**, 416 (1998) and **B280**, 145 (1998); M.B.Cooper *et al.*, Phys. Rev. Lett. **66**, 2561 (1991); R. Becker-Szendy *et al.*, Phys. Rev. **D46**, 3720 (1992); SuperKamiokande Coll.: Y. Fukuda *et al.* Phys. Rev. Lett. **81**, 397 (1998); Macro Coll.: M. Ambrosio *et al*, Phys. Lett. **B343**, 451 (1998); See also *Neutrino* 98, Nucl. Phys. B (Proc. Suppl.) **77** (1999).

[4] H. Georgi, in *Particles and Fields* (AIP, New York, 1975); H. Fritzsch and P. Minkowski, Ann. Phys. **93**, 193 (1975).

[5] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979); T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe, eds. O. Sawada and A. Sugamoto (KEK, 1979).

[6] D. Chang, R. N. Mohapatra and M. K. Parida Phys.Rev.Lett. 52 (1984) 1072 and Phys. Rev. **D30**. 1052 (1984) F. Buccella, L. Cocco, A. Sciarrino, and T. Tuzi Nucl.Phys. B27 (1986) 559 R. N. Mohapatra and M. K. Parida, Phys. Rev. **D47**, 264 (1993) F. Acampora, G. Amelino-Camelia, F. Buccella, O.Pisanti,L.Rosa and T.Tuzi Nuovo Cimento **A108**, 375 (1995).

[7] J. N. Bahcall, P. I. Krastev and A. Y. Smirnov, Phys. Lett. **B477**, 401 (2000).

[8] N. Cabibbo Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa Progr. Th. Phys. **49**, 652 (1973).

[9] M. Jezabek and Y. Sumino, Phys. Lett. **B440**, 327 (1998).

[10] B. Stech, Phys. Lett. **B465**, 219 (1999).

[11] M. Abud, F. Buccella, D. Falcone, G. Ricciardi and F.Tramontano, Mod. Phys. Lett. **A15** , 15 (2000).

[12] F. Buccella and O. Pisanti, hep-ph/9910447, Symposium in honour of S. Bilenky, Turin, Italy (1999).

[13] J. C. Pati and A. Salam, Phys. Rev. **D10**, 275 (1974).

[14] T. Affolder *et al.*, CDF Collaboration, Phys. Rev. **D61**, 072005 (2000).

[15] M. Apollonio *et al.*, CHOOZ Collaboration, Phys. Lett. **B420**, 397 (1998).

[16] S. M. Bilenky and C. Giunti, Phys. Lett. **B444**, 379 (1998).

[17] H. Georgi and S.L. Glashow, Phys. Rev. **D61**, 097301 (2000).

[18] F. Vissani, JHEP **06**, 022 (1999).

[19] L. Baudis *et al.*, Phys. Rev. Lett. **83**, 41 (1999).

[20] G. Altarelli and F. Feruglio, Phys. Lett. **B439**, 112 (1998); R. Barbieri, G. Ross and A. Strumia, JHEP 9910, 020 (1999).

[21] J. Harvey, D.B. Reiss and P. Ramond, Nucl. Phys. B **199**, 223 (1982). A.Yu.Smirnov, Phys. Rev. **D48**, 3264 (1993) and Nucl. Phys. **B466**, 25 (1996).

[22] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** , 438 (1974); M. S. Chanowitz, J. Ellis, M. K. Gaillard, Nucl. Phys, **B128**, 506 (1977); A. J. Buras, J. Ellis, M. K. Gaillard, D. V. Nanopoulos, Nucl. Phys, **B135**,66 (1978).

[23] H. Georgi and C. Jarslkog, Phys. Lett. **86**, 297 (1979).

[24] H. Fusaoka and Y. Koide, Phys. Rev. **D57**, 3986 (1998).

[25] D. Falcone Phys. Lett. B **475**, 92 (2000) and hep-ph/0002242 .

[26] J. Ellis, G.K. Leontaris, S. Lola and D.V. Nanopoulos, Eur. Phys. J.**C9**, 389 (1999).

[27] E. Kh. Akhmedov, G. C. Branco and M. N. Rebelo Phys. Lett. **B478**, 215 (2000).