Why odd-space and odd-time dimensions in even-dimensional spaces?*

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Abstract

We are answering the question why 4-dimensional space has the metric $1+3$ by making a general argument from a certain type of equations of motion linear in momentum for any spin (except spin zero) in any even dimension d. All known free equations for non-zero spin for massless fields belong to this type of equations. Requiring Hermiticity[1] of the equations of motion operator as well as irreducibility with respect to the Lorentz group representation, we prove that only metrics with the signature corresponding to q time + $(d-q)$ space dimensions with q *being odd* exist. Correspondingly, in four dimensional space, Nature could only make the realization of $1+3$-dimensional space.

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[1]This is a generalization of an earlier work which shows that without assuming the Lorentz invariance -which in the present work *is* assumed- the Weyl equation follows using Hermiticity.

Introduction:

One of the most exciting open questions in physics and cosmology [1] is why the metric of our World is the Minkowski one. With the metric of only the Euclidean signature, for example, the World would have almost no dynamics (classically) for $p_1^2+p_2^2+p_3^2+p_4^2=m^2$, and massless particles like photons, gluons and gravitons would not even exist (onshell) (provided that $p^a,\ a\in\{1,2,3,4\}$ are real quantities, which of course is understood, since otherwise the signature loses sense). Concerning the problem of dynamics, two space and two time signatures would work but other problems would occur [2], like the problem of causality.

The argument presented here comes entirely from considering the *internal space*,i.e. spin, degrees of freedom. Also with internal degrees of freedom arguments, S. E. Rugh and one of us [3] argue that, if one requires the equation operator to be Hermitian, then although the Lorentz invariance is assumed to be broken at the outset, an equation which is Lorentz invariant and can be interpreted as the Weyl equation follows as an especially stable possibility, while the space-time is (1+3)-dimensional [4], [5], [6]. J. Greensite [7] [8] has argued for the signature of (1+3) by assuming that the Lorentz symmetry is a dynamical quantity. Hawking [1], on the other hand has pointed to the Wheeler-De-Witt equation, in which the signature cannot even be seen, and so, he says, it should not matter whether the metric is either Euclidean or Minkowskian. Tegmark [2] has given antropical principle arguments in a spirit of random dynamics, very much like that by one of us [4] [6]. Weinberg has an argument for one-time signature in string theory [9]. Penrose and Rindler have some remark on the special properties of the experimental signature and dimension for the Weyl tensor [10].

The present work is based on considering free equations of motion for an arbitrary spin and dimension and signature. We shall use a special form of such equations put forward by one of us, who [11–15] has proposed the approach in which all the internal degrees of freedom are described as the dynamics in the space of anticommuting coordinates, that is, the Lorentz symmetry in the internal space of anticommuting coordinates manifests in the four-dimensional space as spins and charges for either fermions or bosons. This approach offers an easy and elegant way to define the equations of motion for fermions or bosons to any dimension d (either even or odd). We have made use in this paper of some of the results presented in the references [12,16–19].

In this paper, we are presenting a general proof that under certain assumptions, for any even-dimensional space, the number of, say, time dimensions must be odd. We take the starting point of writing a rather general form of equations of motion

$$\mathbf{B}\mathcal{P}^{a_0}\ \psi=0,$$

where

$$\mathcal{P}^{a_0}=p^{a_0}\ +\ i\ \alpha\ \mathbf{S}^{a_0 i}\ p^i\eta^{ii},\quad i\neq a_{a_0},\tag{1}$$

which is obeyed by all known free fields (the Weyl fields, the Yang-Mills fields), except a scalar one, like the Higgs one (if it exists). Here p^a is the d-momentum, $\mathbf{S}^{ab}$ are the Lorentz generators in internal space; it is the Lorentz generators acting on the spin states. The operator $\mathbf{B}$ is a spin-space matrix, which because of our assumption about "irreducibility" of the representation is a function of the $\mathbf{S}^{ab}$'s, but not (necessarily) Lorentz invariant. The constant α is equal to the inverse of the (maximal) helicity of the field and is for fermions equal to two, for vectors, equal to one, and so on. In fact equation (1) is obtainable by rewriting equations of motion like the Weyl equations and the Maxwell equations. We also take gravity as a free field and only care for it as gravitons. The general proof follows if the following basic assumptions are fulfilled: 1. *Equations of motion are of the form* (1). (This assumption implies that the equations of motion are linear in the d-momentum and thus that we consider massless particles. It also implies that the equation content is Lorentz invariant, but that the form of it is not manifestly Lorentz invariant.) 2. *The equations of motion operator is Hermitian.* 3. *The equations of motion operator operates only inside an irreducible representation with respect to the Lorentz group.* We find the support for the above assumptions in the Standard Model of the electroweak interaction, which supposes four-dimensional space-time. We shall argue for them in a longer article [20].

The internal Lorentz symmetry: The generators of the Lorentz transformations in the internal space $\mathbf{S}^{ab}$ fulfil the commutation relations

$$[\mathbf{S}^{ab}, \mathbf{S}^{cd}] = i(\eta^{ad}\mathbf{S}^{bc} + \eta^{bc}\mathbf{S}^{ad} - \eta^{ac}\mathbf{S}^{bd} - \eta^{bd}\mathbf{S}^{ac}). \tag{2}$$

We recognize the generators $\mathbf{S}^{ab}$ to be of the spinorial character S^{ab}, if they fulfil the following relations

$$\{S^{ab}, S^{ac}\} = \frac{1}{2}\,\eta^{aa}\,\eta^{bc}, \quad \text{no summation over a.} \tag{3}$$

With the appropriate choice of the inner product [11,12,15,17,16] we can make a definition of the Hermiticity of the spin-space Lorentz generators $\mathbf{S}^{ab}$ as follows

$$(\mathbf{S}^{ab})^{+} = \eta^{aa}\,\eta^{bb}\,\mathbf{S}^{ab}, \tag{4}$$

where $^{+}$ stays for Hermitian conjugation. (This definition agrees with the Hermeticity properties of $S^{ab} = -\frac{i}{4}[\gamma^a, \gamma^b]$, if expressed in terms of the γ^a matrices: $(S^{ab})^{+} = \eta^{aa}\eta^{bb}S^{ab}$.) We shall further comment on the inner product and the Hermiticity conditions later. For our proof the expression for the Casimir for the Lorentz group, operating only in internal space

$$\mathbf{\Gamma}^{(int)} = (-i)^{n+1}\frac{(2i)^n}{(2n)!}\,\epsilon_{a_1 a_2 a_3 a_5,\ldots,a_{2n-1}a_{2n}}\mathbf{S}^{a_1 a_2}\mathbf{S}^{a_3 a_4}\cdots\mathbf{S}^{a_{2n-1}a_{2n}} \tag{5}$$

will be needed. Taking into account the Hermiticity properties of the generators $\mathbf{S}^{ab}$ from Eq.(4), one finds that

$$(\mathbf{\Gamma}^{(int)})^{+} = (-)\,(\prod_b\,\eta^{bb})\,\mathbf{\Gamma}^{(int)};\;\; b \in \{1, 2, \cdots, 2n\}. \tag{6}$$

$\mathbf{\Gamma}^{(int)}$ defines left ($\mathbf{\Gamma} = -1$) and right ($\mathbf{\Gamma} = 1$) handed representations.

Higher spins:

All the definitions presented above (except Eq.(3), which is only valid for spinors) are valid for any spin in any dimensional space-time. To treat higher spins we use the Bargmann and Wigner construction [21,22], making higher spin fields out of spinorial fields by constructing the fields with many spinorial indices of left - $\alpha, \beta, \gamma, \ldots$ - and right - $\dot{\alpha}, \dot{\beta}, \dot{\gamma}, \ldots$ handedness and generalize it in higher dimensions (we allow besides the totally symmetrized case, which is the only case needed in four-dimensions) to the antisymmetrized case and to all possible mixed symmetrized cases belonging to various Young tableaux Y and $\hat{Y}$ for left- and right-handed representations, respectively

$$\psi^{(Y\hat{Y})}_{\alpha_1\alpha_2\cdots\alpha_n\dot{\alpha}_1\dot{\alpha}_2\cdots\dot{\alpha}_m} = \mathcal{N} \sum_{P,\hat{P}} sign^{(Y)}(P)\; sign^{(\hat{Y})}(\hat{P})\; \psi_{\alpha_{P(1)}\alpha_{P(2)}\cdots\alpha_{P(n)}\alpha_{\dot{\hat{P}}(1)}\alpha_{\dot{\hat{P}}(2)}\cdots\alpha_{\dot{\hat{P}}(m)}}, \tag{7}$$

where P and $\hat{P}$ run over the permutations, determined by the Young tableaux Y and $\hat{Y}$, respectively and the two $sign^{(\;)}$ are determined by the two Young tableaux Y and $\hat{Y}$, respectively, so that we are projecting out a particular irreducible representation of the Lorentz group. $\mathcal{N}$ takes care of the normalization of the spin field. The generators of the Lorentz transformation for any representation $\mathbf{S}^{ab}$ are constructed using the spinorial generators S^{ab} (Eq.(3))

$$\mathbf{S}^{ab} = \sum^{n+m}{}_{k=1}\, I_{(1)} \otimes I_{(2)} \otimes \cdots \otimes S^{ab}{}_{(k)} \otimes I_{(k+1)} \cdots \otimes I_{(m+n)}, \tag{8}$$

where $\otimes$ means the direct product and $I_{(k)}$ stays for the unit matrix acting on the index α_k or $\dot{\alpha}_k$ and so do $S^{ab}{}_{(k)}$. It is easily shown that $\mathbf{S^{ab}}$ obey the algebra of the Lorentz group (Eq.(2)).

Our theorem giving an odd number of time-dimensions in even-dimensional space-time:

Theorem: Assuming for any *irreducible representation* (for any spin) the equations of motion (Eq.(1))

$$\mathbf{B}\mathcal{P}^{a_0}\psi = \mathbf{B}(p^{a_0} + i\alpha\mathbf{S}^{a_0 i}p^i\, \eta^{ii})\psi = 0 \tag{9}$$

and letting the signature of space-time be defined so that the bilinear form in p^{a_0}

$$\psi^{+}\, (p^{a_0} - i\alpha\mathbf{S}^{a_0 i}\, p^i\, \eta^{ii})\mathcal{P}^{a_0}\, \psi, \tag{10}$$

has the signature of the Klein-Gordon equation for the space-time signature in question, it follows that the time-dimension q is odd, provided that the equations of motion operator is Hermitian

$$\mathbf{B}\mathcal{P}^{a_0} \;=\; (\mathbf{B}\mathcal{P}^{a_0})^{+}. \tag{11}$$

The statement above the bilinear form should even include the fact that the positive and the negative definite subspaces of dimensions say q and $d-q$ can be chosen independently of the internal space state $|\psi>$.

We shall first give a proof of the theorem for the spinorial case, since for this case it is simple and transparent.

Proof for the Weyl equation case:

We recognize that the operator in Eq.(10) in the Weyl case equals the equations of motion operator of the Klein-Gordon equation $p^a p^b \eta_{ab}$, provided that $\alpha = 2$, which is a real number and that Eq.(3), which defines the anticommutation relations for spinors, is taken into account. When taking into account also the Hermiticity condition (Eq.(11)) we have

$$B = B^{+}, \qquad -(BS^{a_0 i})^{+} = BS^{a_0 i}. \tag{12}$$

It follows that

$$B\ S^{a_0 i}\ + S^{a_0 i}\ \eta^{a_0 a_0}\ \eta^{ii}\ B\ =\ 0, \tag{13}$$

which means that B either commutes or anticommutes with $S^{a_0 i}$, depending on whether a_0 and i have the same or the opposite signature.

Using Jacobi identities $[A,[B,C]] + [C,[A,B]] + [B,[C,A]] = 0$ and $[A,\{B,C\}] + [C,\{A,B\}] + [B,\{C,A\}] = 0$ we derive the following generalization of Eq.(13)

$$\begin{aligned} [B, S^{ab}] = 0, &\quad \text{for } (-1)^{\delta_{a_0 a}+\delta_{a_0 b}}\eta^{aa}\eta^{bb} = 1 \\ \{B, S^{ab}\} = 0, &\quad \text{for } \eta^{aa}\eta^{bb}\ (-1)^{\delta_{a_0 a}+\delta_{a_0 b}} = -1. \end{aligned} \tag{14}$$

Let us now deduce the commutation versus anticommutation relations of B and $\Gamma^{(int)}$, the Casimir of the Lorentz group, (Eq.(5)). Taking into account Eq.(14) we find

$$\begin{aligned} [B, \Gamma^{(int)}] = 0, &\quad \text{for} \quad (-1)\ \prod_b \eta^{bb} = 1 \\ \{B, \Gamma^{(int)}\} = 0, &\quad \text{for} \quad (-1)\ \prod_b \eta^{bb} = -1. \end{aligned} \tag{15}$$

Since B was assumed to depend only on S^{ab}, which means that the equations of motion operator $B\mathcal{P}^{a_0}$ operates within only the irreducible representation, then B should commute with $\Gamma^{(int)}$. Unless $\Gamma^{(int)} = 0$, which can cause problems, we can now conclude from Eq.(15) that

$$(-1)\ \prod_b \eta^{bb} = 1, \tag{16}$$

which can only be true if the number of time coordinates and the number of space coordinates are odd.

This finishes the proof for fermions, that is for the Weyl case. Due to the generalized Bargmann-Wigner proposal for the description of any spin field in d dimensions out of the Weyl spinors (Eq.(7)), we would intuitively conclude that since α is real for the Weyl equations of motion operator and since $\mathbf{S}^{ab}$ is a linear composition of $S^{ab}{}_{(k)}$'s, which each act on different spinor indices, α should be real for any spin equations of motion operator and accordingly Eqs.(12, 13,14,15) as well as the equation (16) should be valid for any spin in any dimension.

The general proof, which we present below, ensures that α of Eq.(1) is real for any spin.

Proof for the general case:

In order to perform the general proof of our signature theorem we need a lemma, that is, an extension of a well-known theorem about representations of compact groups to the non-compact groups, which tells us thatany representation of a compact group can be considered unitary with respect to an appropriate inner product. The latter is constructed by averaging or integrating over an arbitrary measure.

We present a slightly extended theorem as a lemma formulated for the Lorentz group SO(q,d-q) with q time and $(d-q)$ space dimensions:

Lemma:

Let $\mathbf{S}^{ab}$ make up a (finite dimensional) representation of the Lorentz group Lie algebra (Eq.(2)). Then there exists an operator $\mathbf{V}$ such that

$$(\mathbf{S}^{ab})^+ = \eta^{aa}\,\eta^{bb}\,\mathbf{V}\mathbf{S}^{ab}\mathbf{V}^{-1}. \tag{17}$$

Proof of lemma: For real ω_{ab} the Lie algebra consisiting of elements of the form $\omega_{ab}\mathbf{S}^{ab}$ is the Lie algebra for the group $SO(q, d-q)$, but if we instead let

$$\text{for } \eta^{aa}\,\eta^{bb} = +1 \;: \quad \omega_{ab} \text{ real} \tag{18}$$

$$\text{for } \eta^{aa}\,\eta^{bb} = -1 \;: \quad \omega_{ab} \text{ purely imaginary,} \tag{19}$$

then the group generated will be the compact group $SO(d)$ instead. In other words, for a choice of the square roots, the Lie algebra consisting of the elements

$$\hat{\omega}_{ab}\,\sqrt{\eta^{aa}\,\eta^{bb}}\,\mathbf{S}^{ab} \tag{20}$$

forms a Lie algebra representation for the compact group $SO(d)$ when the $\hat{\omega}_{ab}$ run through real values. On the compact group $SO(d)$ we can apply the idea of averaging over the Haar measure an arbitrary inner product which, for instance, may be the one represented by the unit matrix $\mathbf{1}$; that is to say, we construct the "averaged" inner product as expressed by a matrix:

$$\mathbf{K} := \int d^{Haar}g\;(e^{i\hat{\omega}_{ab}(g)\sqrt{\eta^{aa}\eta^{bb}}\mathbf{S}^{ab}})^+\;\mathbf{1}\;e^{i\hat{\omega}_{ab}(g)\sqrt{\eta^{aa}\eta^{bb}}\mathbf{S}^{ab}}. \tag{21}$$

We have now ensured that the generators of the compact group are Hermitian with respect to the inner product defined by $\mathbf{K}$, becuase $\mathbf{K}$ constructed as an average over the whole compact group must be left invariant under similarity transformations with representatives $e^{i\hat{\omega}_{ab}\mathbf{S}^{ab}}$ of elements of this compact group. This in turn implies Hermiticity of the generators for this group with respect to $\mathbf{K}$ being used as the inner product, i.e.

$$(e^{i\hat{\omega}_{ab}\sqrt{\eta^{aa}\eta^{bb}}\mathbf{S}^{ab}})^+\;\mathbf{K}\;e^{i\hat{\omega}_{ab}\sqrt{\eta^{aa}\eta^{bb}}\mathbf{S}^{ab}} \;=\; \mathbf{K} \tag{22}$$

or equivalently

$$\mathbf{K}\,\sqrt{\eta^{aa}\eta^{bb}}\mathbf{S}^{ab} - (\sqrt{\eta^{aa}\eta^{bb}}\mathbf{S}^{ab})^+\,\mathbf{K} = 0. \tag{23}$$

Since we had chosen definite values for the square roots, we see that we get the statement of the lemma by dividing equation (23) by $\mathbf{K}$ from the right and putting $\mathbf{V} = \mathbf{K}$. This ends the proof of the lemma.

Corollary to lemma:

$\mathbf{V}$ came out Hermitian and positive definite as an operator, and the constructions $\sqrt{\mathbf{V}}\;\mathbf{S}^{ab}\;\sqrt{\mathbf{V}^{-1}}$ will obey Hermiticity relations of the form of Eq.(4).

The corollary is easily seen by dividing by $\sqrt{\mathbf{V}}$ to the left and $\sqrt{\mathbf{V}^{-1}}$ to the right equation (17).

Another useful corollary is the following

Corollary:

For the anticommutators of the generators of the Lorentz group we have the operator-inequalities

$$\sqrt{\mathbf{V}}(-\eta^{aa}\eta^{bb}(\mathbf{S}^{ab})^2 - \eta^{cc}\eta^{dd}(\mathbf{S}^{cd})^2)\sqrt{\mathbf{V}^{-1}} \leq \sqrt{\mathbf{V}}\sqrt{\eta^{aa}\eta^{bb}\eta^{cc}\eta^{dd}}\{\mathbf{S}^{ab},\mathbf{S}^{cd}\}\sqrt{\mathbf{V}^{-1}} \leq$$
$$\leq \sqrt{\mathbf{V}}(\eta^{aa}\eta^{bb}(\mathbf{S}^{ab})^2 + \eta^{cc}\eta^{dd}(\mathbf{S}^{cd})^2)\sqrt{\mathbf{V}^{-1}} \qquad (24)$$

and thus the signs of the expression in Eq.(10) for the positive and negative coordinate subspaces are not changed by ignoring the anticommutator terms in an expansion of the product of operators.

This corollary is rather easily shown by remarking that the squares of the Hermitian operators $\sqrt{\mathbf{V}}\ (\sqrt{\eta^{aa}\,\eta^{bb}}\ \mathbf{S}^{ab}\ \pm\ \sqrt{\eta^{cc}\,\eta^{dd}}\ \mathbf{S}^{cd})\ \sqrt{\mathbf{V}^{-1}}$, Hermitian according to the first corollary, must be positive (semi) definite as operators (matrices).

From this last corollary we easily see that in the general case of arbitrary representation we can ignore the commutator terms in equation (10) when investigating the condition for the signature, at least if we consider $|\psi>$ as being an eigenstate of the Hermitian operator $\sqrt{V}$. Then namely the extra $\sqrt{\mathbf{V}}$ and $\sqrt{\mathbf{V}^{-1}}$ would in fact effectively be replaced by even positive numbers and it would not matter for conclusions on the sign of the expression in Eq.(10).

Then the requirement that the sign of Eq.(10) for, say, such $|\psi>$'s that are eigenstates of $\sqrt{\mathbf{V}}$ must be the same for p^{a_0} alone different from zero and for p^i alone different from zero in the case of $\eta^{a_0a_0}\eta^{ii} = +1$, while they should lead to opposite signs in the case $\eta^{a_0a_0}\eta^{ii} = -1$, gives us that α is *real*.

A problem may be if it is really possible that the expression of Eq.(10) for all $|\psi>$ can come to show just the signature of the group $SO(q, d-q)$. However, the logic here is that we just have assumed that it is so, since the motivation for our requirements about the bilinear form is a replacement for assuming the Klein-Gordon equation to be valid as a consequence of our equation of motion.

Having now achieved the real α and the Hermiticity properties (17) the condition for positivity of Eq.(1) is easily seen to be that

$$\mathbf{B}^{+} \ = \ \mathbf{B} \qquad (25)$$

and

$$(i\,\mathbf{B}\,\alpha\,\mathbf{S}^{a_0i})^{+} = -i\eta^{a_0a_0}\,\eta^{ii}\,\mathbf{V}\mathbf{S}^{a_0i}\mathbf{V}^{-1}\,\alpha\,\mathbf{B} \ = \ i\,\mathbf{B}\,\alpha\,\mathbf{S}^{a_0i}, \qquad (26)$$

the latter equation of which is equivalent to that

$$[\mathbf{V}^{-1}\,\mathbf{B},\ \mathbf{S}^{a_0i}] = 0 \ \text{ for } \eta^{a_0a_0}\,\eta^{ii} = -1 \qquad (27)$$

while

$$\{\mathbf{V}^{-1}\,B,\ \mathbf{S}^{a_0i}\} = 0 \ \text{ for } \eta^{a_0a_0}\,\eta^{ii} = +1. \qquad (28)$$

But that is to say that we reached the same commutation or anticommutation conditions (13) as for the Weyl equation case, except that now we have the conditions for $\mathbf{V}^{-1}\,\mathbf{B}$ rather than simply for $\mathbf{B}$ itself. That does not matter, however, since with an irreducible representation even $\mathbf{V}$ should be expressed by the generators and must commute - not anticommute (unless the Casimir is zero) - with the Casimir operators, in particular with $\mathbf{\Gamma}^{(int)}$.

Hereby the proof ends also for the general case.

Concluding remarks:

We have presented in this letter the general proof, valid for all spins (except for spin zero) and all even dimensions, that an equations of motion operator can be linear in the p-momentum, Hermitian and operate within only the irreducible representations of the Lorentz group, if space has an odd number of time dimensions and accordingly also an odd number of space dimensions. It is the spin degrees of freedom which determine the signature of space-time.

The assumptions of the linearity in the p-momentum, Hermiticity and irreducibility properties of the equations of motion operator are rather mild assumptions. In fact we have been inspired to make them by the Standard Electroweak Model assumptions, and we shall discuss the arguments favouring them in a longer article [20]. Our general proof answers one of the most exciting open questions of science, namely why our space-time signature is $3+1$, pointing out that there are internal degrees of freedom that are responsible for the signature. Before concluding, we would only point out that all the known elementary particles of the Standard Model (assuming $d = 4$) belong (before switching on the interactions) - not counting the Higgs as known - belong to one of the following groups:

1) Either to the spin-$\frac{1}{2}$ Weyl particles, described by the Weyl equations of left- and right-handed irreducible representations (Eq.(9)), with $B = 1$ or $B = \Gamma^{(int)}$ (which is known as γ^5) and $\alpha = 2$, leading to left-handed ($\Gamma = -1$) fermions of left helicity ($\frac{\overrightarrow{p}.\overrightarrow{S}}{|\overrightarrow{p}.\overrightarrow{S}|} = -1$) or to right-handed ($\Gamma = 1$) fermions of right helicity ($\frac{\overrightarrow{p}.\overrightarrow{S}}{|\overrightarrow{p}.\overrightarrow{S}|} = 1$) while anti-fermions if left-handed ($\Gamma = -1$) have right-helicity ($\frac{\overrightarrow{p}.\overrightarrow{S}}{|\overrightarrow{p}.\overrightarrow{S}|} = 1$) and if right-handed ($\Gamma = 1$) have left helicity ($\frac{\overrightarrow{p}.\overrightarrow{S}}{|\overrightarrow{p}.\overrightarrow{S}|} = -1$), (The Standard Model postulates that no right-handed fermions or anti-fermions exist, which would carry the weak charge. This feature of the Standard Model has appreciable experimental support.)

2) Or to the spin-one linear Yang-Mills equations of the Maxwell type with $\alpha = 1$ in Eq.(1) and again with $\mathbf{B} = 1$ or $\mathbf{B} = \mathbf{\Gamma}^{(int)}$, which are less known equations [16,19] and which lead to equations

$$\begin{pmatrix} p^0 + i\overrightarrow{p}\times & 0 \\ 0 & p^0 - i\overrightarrow{p}\times \end{pmatrix} \begin{pmatrix} \overrightarrow{\mathcal{E}}_L \\ \overrightarrow{\mathcal{E}}_R \end{pmatrix} = 0.$$

Assuming $\overrightarrow{\mathcal{E}}_L = \overrightarrow{\mathcal{E}} + i\overrightarrow{\mathcal{B}}$ and $\overrightarrow{\mathcal{E}}_R = \overrightarrow{\mathcal{E}} - i\overrightarrow{\mathcal{B}}$ the last equation leads to equations of motion for electric and magnetic fields, which both obey Maxwell equations. We shall comment on these equations in a longer article [20].

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