

Micrometer Gravitinos and the Cosmological Constant

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Abstract

We compute the 4-dimensional cosmological constant in string compactifications in which the Standard Model fields live on a non-supersymmetric brane inside a supersymmetric bulk. The cosmological constant receives contributions only from the vacuum energy of the bulk supergravity fields, but not from the vacuum energy of the brane fields. The latter is absorbed in a warp factor. Supersymmetry breaking on the brane at the TeV scale implies supersymmetry breaking in the bulk at the micrometer scale. This produces a tiny cosmological constant that agrees with experiment within a few orders of magnitude. Our argument predicts gravitinos and dilatons with masses of order 10^{-2} eV. They should be observable in short-distance tests of Einstein Gravity.

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1. Introduction

The observed smallness of the cosmological constant λ in Einstein's equations poses a fine-tuning problem already in classical field theory coupled to gravity. E.g., when the Higgs field rolls down its potential, the energy density of the vacuum and thus the effective value of λ changes by a large amount. Other expected contributions to λ that are suspiciously absent include those from condensates in QCD.

But perhaps the most mysterious aspect of the problem is that λ does not seem to receive contributions from the quantum mechanical ground state energies

$$\rho_k = \frac{\hbar\omega}{2} \quad , \quad \omega^2 = k^2 + m^2$$

of the oscillators with momentum k of the massless and light fields of the Standard Model. Summing these contributions up to some large-momentum cutoff $\Lambda \gg m$, one finds in the case of a single-component bosonic field [1]:

$$\lambda = 8\pi G \int_0^\Lambda \frac{k^2 dk}{2\pi^2} \rho_k \sim \frac{\Lambda^4}{2\pi} l_P^2 . \quad (1.1)$$

Here, G is the Newton constant and $l_P = \sqrt{\hbar G} \sim 1.7 \cdot 10^{-35} m$ is the Planck length (setting $c = 1$). Experimentally, it presently seems that [2]

$$\lambda \sim (3 \cdot 10^{-33} eV)^2 \sim (1.5 \cdot 10^{26} m)^{-2} \quad (1.2)$$

(setting $\hbar = G = 1$). $10^{26} m$ is the order of magnitude of the curvature radius of the universe, which is roughly the inverse Hubble constant. But even if one considered only the contributions of the two helicity states of the massless photon to the cosmological constant (1.1), then in order to explain such a small value of λ one would need a momentum-cutoff as small as

$$\Lambda \sim \frac{1}{100} eV \sim \frac{1}{20\mu m} .$$

So the minimum wavelength would have to be as large as $20\mu m$. Already the wavelength of visible light is much smaller than $20\mu m$. In this sense the observed smallness of the Hubble expansion parameter seems inconsistent even with what we can see with our bare eyes.

Supersymmetry could explain a zero cosmological constant, because the vacuum energies of the superpartners cancel each other. Supersymmetry looks so much like the missing piece

in the puzzle that it has been questioned whether supersymmetry is really broken [3]. But if it isn't broken, it is of course hard to explain why we do not see superpartners of the Standard Model fields [4].

The recently revived suggestion that we live on a 4-dimensional brane that is embedded in a higher-dimensional bulk theory opens up a new perspective and a way out. It will be proposed below that supersymmetry is indeed unbroken up to micrometer scales – but only in the *bulk* supergravity theory. By a simple argument, supersymmetry breaking in the bulk at micrometer scale is derived from supersymmetry breaking at the *TeV* scale on the brane, which carries the Standard Model fields.

Due to a mechanism first proposed in [5] and re-invented in [6, 7], the vacuum energy of the brane fields is shown not to contribute to the $4d$ cosmological constant. Rather, it is absorbed in the warp factor transverse to the brane. Only the vacuum energy of the bulk supergravity fields is argued to contribute to the $4d$ cosmological constant.

This vacuum energy is estimated. The result is a relation between the Planck mass m_{Planck} , the scale $m_{Brane-Susy}$ of supersymmetry breaking on the brane, the scale $m_{Bulk-Susy}$ of supersymmetry breaking in the bulk sector and the Hubble expansion rate H_0 :

$$\log \frac{m_{Planck}}{m_{Brane-Susy}} \sim \frac{1}{2} \cdot \log \frac{m_{Planck}}{m_{Bulk-Susy}} \sim \frac{1}{4} \cdot \log \left(\frac{m_{Planck}}{H_0} \right) \quad (1.3)$$

(a more detailed relation is given in the text).

Based on the known values of m_{Planck} and H_0 , this relation predicts gravitino and dilaton masses of order $10^{-2}eV$ (which is inside experimental bounds [8]) and a supersymmetry breaking scale on the brane of several *TeV*. Conversely, based on the assumption that supersymmetry is restored in the Standard Model at energies not too much above the weak scale, the relation explains the observed small value of the cosmological constant.

The setup is introduced in section 2. In section 3 it is argued that the $4d$ cosmological constant is zero as long as the bulk supergravity theory is treated classically. In section 4 it is shown that the quantum mechanical ground state energy of the supergravity sector produces a cosmological constant that is within a few orders of magnitude of its observed value. Precise matching yields the predictions for supersymmetry breaking in the Standard Model and the gravitino mass, as explained in section 5. Section 6 contains conclusions.

2. The setup

We consider a 3-brane soliton that is embedded in a $(4 + n)$ -dimensional bulk spacetime (figure 1). We assume that the n extra dimensions are compactified on some manifold \mathcal{M} . The Standard Model fields are assumed to live only on the brane, while gravity lives in the bulk. Let $\text{Vol}(\mathcal{M})$ be the volume of the compactification manifold, and let $\text{Vol}(\mathcal{B})$ be the volume of the ball \mathcal{B} inside \mathcal{M} that intersects with the brane. Since we are going to consider non-supersymmetric branes inside supersymmetric bulks, we will identify the size (i.e. the thickness) of the brane with $l_{\text{Brane-Susy}} \sim m_{\text{Brane-Susy}}^{-1}$, the scale of supersymmetry breaking in the Standard Model. So roughly, $\text{Vol}(\mathcal{B}) \sim (l_{\text{Brane-Susy}})^n$.

Similarly as in [9], because the Einstein action is integrated over $\text{Vol}(\mathcal{M})$ while the Standard Model action is integrated only over $\text{Vol}(\mathcal{B})$, the 4-dimensional Planck length l_{Planck} is related to $l_{\text{Brane-Susy}}$ by

$$\left(\frac{l_{\text{Planck}}}{l_{\text{Brane-Susy}}}\right)^2 = \left(\frac{m_{\text{Brane-Susy}}}{m_{\text{Planck}}}\right)^2 \sim \frac{\text{Vol}(\mathcal{B})}{\text{Vol}(\mathcal{M})} \quad (2.1)$$

(assuming a $(4 + n)$ -dimensional Newton constant of order one).

Let us first consider the supersymmetric version of the story. So we assume that we have a supersymmetric brane inside a supersymmetric compactification manifold. In string theory, this is achieved by considering a compactification on a Calabi-Yau 3-manifold that involves branes parallel to the 4-dimensional space-time, as in [10]. At distances much larger than the size of the compactification manifold, only a four-dimensional supersymmetric effective theory of Standard Model fields plus their superpartners coupled to $4d$ supergravity is seen.

For concreteness, we may assume a metric in the vicinity of the brane of the form

$$ds^2 = dr^2 + f(r) \hat{g}_{\mu\nu} dx^\mu dx^\nu + g(r) d\Omega_5^2$$

where r denotes the distance from the brane, x^μ are the space-time coordinates parallel to the brane, $\hat{g}_{\mu\nu}$ is the $4d$ metric parallel to the brane, and $f(r), g(r)$ are some functions. Supersymmetry of the effective $4d$ theory implies that the $4d$ metric $\hat{g}_{\mu\nu}$ is Ricci-flat (we are assuming that there are no 4-form gauge field strengths or expectation values of other supergravity fields), i.e. the effective $4d$ cosmological constant is zero.

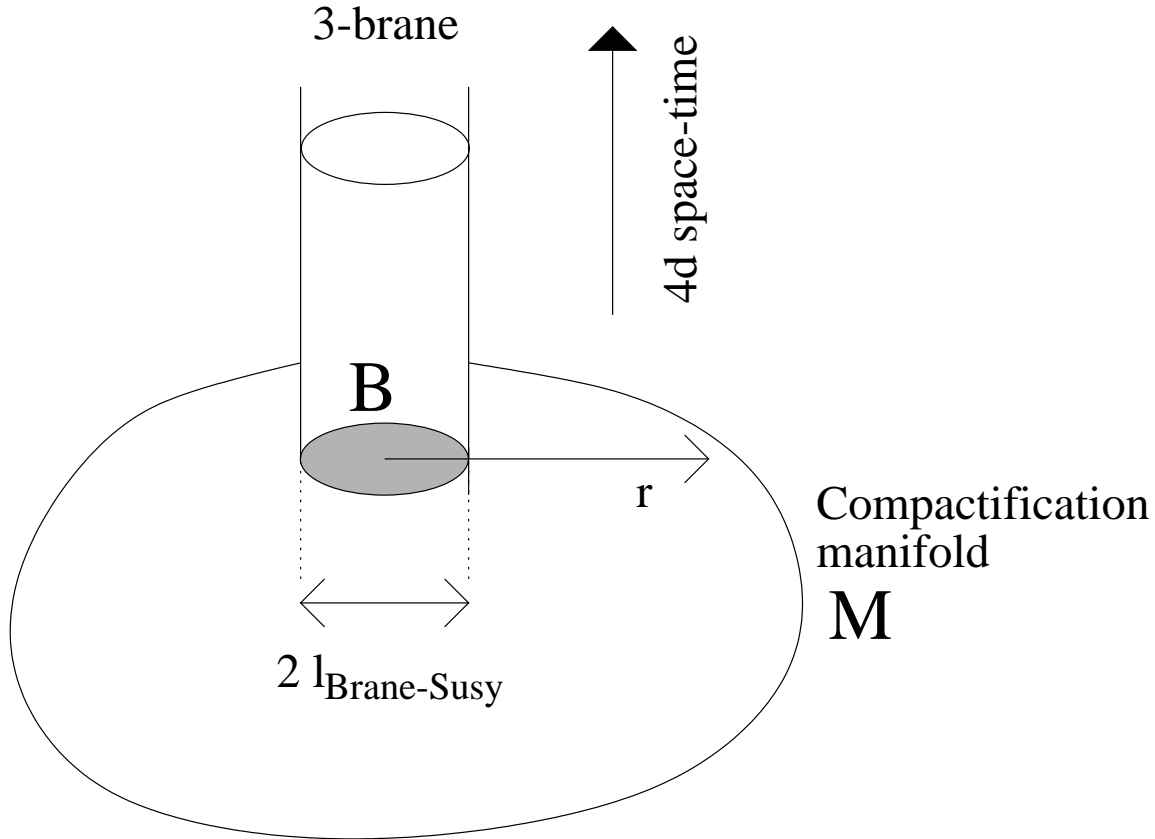


Figure 1: A 3-brane in a $(4 + n)$ -dimensional embedding space.

Now suppose that we cut out a region of radius $l_{Brane-Susy}$ around the brane, and replace the supersymmetric brane soliton solution inside this region by a non-supersymmetric one such that the bulk fields smoothly connect to the supersymmetric solution at $r \geq l_{Brane-Susy}$. In the case of one extra dimension, explicit examples of such supergravity solutions that smoothly interpolate between a supersymmetric and a non-supersymmetric region are the kink solutions of $5d$ gauged supergravity discussed in [11, 12].

Supersymmetry is then broken not only in the bulk theory in the vicinity of the brane. It is also broken in the world-brane theory that contains the Standard Model fields and lives in the non-supersymmetric gravitational background. This will result in a brane vacuum energy of the order of $(m_{Brane-Susy})^4$.

3. Classical Supergravity Approximation

Let us first explain why the vacuum energy on the brane does not curve the $4d$ metric $\hat{g}_{\mu\nu}$ parallel to the brane (i.e., why it does not create a $4d$ cosmological constant) as long as the bulk supergravity theory is treated classically (see [5, 6, 7]). Although the bulk theory is treated classically, the world-brane theory containing the Standard Model fields is assumed to be treated fully quantum mechanically. Corrections from loops of the bulk fields will be discussed in the next section.

The bulk has been separated into two regions: the non-supersymmetric neighborhood of the brane $M^4 \times \mathcal{B}$, where M^4 is the Minkowski space parallel to the brane; and the supersymmetric region, i.e. the rest of the bulk $M^4 \times (\mathcal{M} - \mathcal{B})$. The classical supergravity equations of motion can be solved separately for each region, and can then be matched at their interface at $r = l_{Brane-Susy}$.

In the supersymmetric region, the equations did not change at all by the cut-and-paste procedure that we have performed; they are still the same as for the compactification with a supersymmetric brane. In particular, the $4d$ metric $\hat{g}_{\mu\nu}$ parallel to the brane must still be Ricci-flat.

In the non-supersymmetric region, the brane is a source of vacuum energy $\rho(r)$ of order $(m_{Brane-Susy})^4$ that arises from the world-brane fields. Let us assume that $\rho(r)$ is distributed around $r = 0$, say, in a Gaussian distribution with width of the order $l_{Brane-Susy}$. ρ enters the Einstein equations like an r -dependent cosmological constant:

$$R_{mn} - \frac{1}{2}g_{mn}R = 8\pi G\rho(r) g_{mn} .$$

For simplicity, we focus on the example of a single extra dimension, assume a constant dilaton and neglect the other supergravity fields as in [7]; the generalization is straightforward. We make the metric ansatz

$$ds^2 = dr^2 + e^{2\alpha(r)}\hat{g}_{\mu\nu}dx^\mu dx^\nu .$$

In this ansatz, the $4d$ metric \hat{g} is taken to be r -independent. The 5-dimensional Ricci tensor can be written as

$$R_{\mu\nu}^{(5)} = \hat{R}_{\mu\nu}^{(4)} - \hat{g}_{\mu\nu} e^{2\alpha(r)}(\ddot{\alpha} + 4\dot{\alpha}^2)$$

(a “dot” means $\frac{d}{dr}$). Plugging this into the Einstein equation for the 4-dimensional components (μ, ν) and using the equation for the (r, r) component to eliminate $\ddot{\alpha}$,

$$4(\ddot{\alpha} + \dot{\alpha}^2) = 8\pi G\rho,$$

we obtain:

$$\hat{R}_{\mu\nu}^{(4)} = \hat{g}_{\mu\nu}k^2 \quad \text{where} \quad k^2 = \frac{3}{4}e^{2\alpha}(-6\pi G\rho(r) + 3\dot{\alpha}^2)$$

is an integration constant that is by definition the $4d$ cosmological constant. So the equations for α have a one-parameter family of solutions, labelled by the constant $4d$ curvature k^2 . However, matching at $r = l_{Brane-Susy}$ to the solution in the supersymmetric region requires that we pick the solution that is Ricci-flat in $4d$, i.e. $k = 0$. For this solution, the vacuum energy on the brane is completely absorbed by the warp factor

$$\dot{\alpha}^2 = 2\pi G\rho(r),$$

and therefore does not curve the $4d$ metric parallel to the brane. So the vacuum energy does not lead to a $4d$ cosmological constant.

This is the mechanism of Rubakov and Shaposhnikov [5], recently rediscovered in [6, 7]. We have supplemented it by a matching condition at $r = l_{Brane-Susy}$ that picks out the solution with vanishing $4d$ cosmological constant without fine-tuning. This makes precise the suggestion in [13] of “supersymmetry on the Planck brane”. Higher-order corrections will make the differential equations for α more complicated, and there may be regions in parameter space where no solutions exist [14]; let us assume that conditions are favorable and solutions exist.

4. Supergravity at One Loop

Let us now go beyond the classical supergravity approximation. This is the main new step taken in this paper and it will lead to our main results.

As mentioned in the introduction, the ground state energies

$$\frac{\hbar\omega}{2} \quad \text{with} \quad \omega^2 = k^2 + m^2$$

of modes of light fields with momentum k should give a quantum mechanical contribution to the cosmological constant. We have already demonstrated that the ground state energy of the Standard Model fields does not contribute to the $4d$ cosmological constant, so it only remains to compute the vacuum energy produced by the massless bulk supergravity fields with $m = 0$: the gravitino, the dilaton, antisymmetric tensor fields etc.

As long as supersymmetry is unbroken in the bulk, these vacuum energy contributions cancel. Now, breaking supersymmetry in the region of the bulk near the brane also breaks supersymmetry in the effective $4d$ theory, obtained by integrating over the compactification manifold \mathcal{M} . But because of the small overlap of the wave functions of the supergravity fields with the brane, the mass scale $m_{Bulk-Susy}$ of supersymmetry breaking in the bulk sector of the $4d$ effective theory will be suppressed with respect to the scale of supersymmetry breaking on the brane by the same volume factor that we already found in (2.1),

$$\left(\frac{m_{Bulk-Susy}}{m_{Brane-Susy}}\right)^2 \sim \frac{\text{Vol}(\mathcal{B})}{\text{Vol}(\mathcal{M})} \sim \left(\frac{m_{Brane-Susy}}{m_{Planck}}\right)^2. \quad (4.1)$$

To see this explicitly, let us assume that the gravitino ψ_μ has a large mass of order $m_{Brane-Susy}$ inside the region where supersymmetry is broken: we may take its $(4+n)$ -dimensional Lagrangean to be of the form

$$\gamma^{\mu\nu\sigma\tau} \bar{\psi}_\mu \gamma_\nu \partial_\sigma \psi_\tau + \theta(r_{Brane-Susy} - r) m_{Brane-Susy}^2 \bar{\psi}_\mu \psi^\mu.$$

θ is the step function: $\theta(x) = 0$ for $x < 0$ and $\theta(x) = 1$ for $x \geq 0$. Integrating this Lagrangean over the compactification manifold, the kinetic term acquires a prefactor $\text{Vol}(\mathcal{M})$ while the mass term only acquires a prefactor $\text{Vol}(\mathcal{B})$. After normalizing the gravitino to have a standard kinetic term, its mass is

$$m_{Gravitino}^2 \sim m_{Brane-Susy}^2 \frac{\text{Vol}(\mathcal{B})}{\text{Vol}(\mathcal{M})}.$$

The same argument applies to the dilaton and to the other members of the supergravity multiplet. This yields relation (4.1). So the hierarchy between the scales of supersymmetry breaking in the bulk supergravity sector and supersymmetry breaking in the Standard Model on the brane is the same as the hierarchy between the scale of supersymmetry breaking on the brane and the Planck scale.

Already in the introduction we have discussed the relation (1.1) between the momentum cutoff Λ in the sum over vacuum energies and the value of the cosmological constant λ . In the case of N massless bosonic propagating degrees of freedom, the relation changes to

$$\lambda \sim N \frac{\Lambda^4}{2\pi} l_P^2. \quad (4.2)$$

λ is related to the Hubble expansion rate H_0 of the universe by

$$\lambda = 3\Omega_\Lambda H_0^2 \quad \text{with} \quad \Omega_\Lambda \sim \frac{2}{3}$$

being the value suggested by observation [2]. If we identify the cutoff Λ with the scale $m_{Bulk-Susy}$ of supersymmetry breaking in the bulk, then equation (4.2) implies (converting $l_{Planck} \sim m_{Planck}^{-1}$):

$$\left(\frac{m_{Bulk-Susy}}{m_{Planck}}\right)^2 \sim \frac{6\Omega_\Lambda\pi}{N} \left(\frac{H_0}{m_{Bulk-Susy}}\right)^2. \quad (4.3)$$

Together, with (4.1) this yields the relation claimed in the introduction (where we have set $\Omega_\Lambda = \frac{2}{3}$ and approximated $(\frac{N}{4\pi})^{\frac{1}{2}}$ by 1):

$$\log\left(\frac{m_{Planck}}{m_{BranE-Susy}}\right) = \frac{1}{2} \log\left(\frac{m_{Planck}}{m_{Bulk-Susy}}\right) = \frac{1}{4} \log\left(\sqrt{\frac{N}{6\Omega_\Lambda\pi}} \frac{m_{Planck}}{H_0}\right) \equiv L. \quad (4.4)$$

5. The Numbers

Let us now plug in the numbers. We use

$$m_{Planck} \sim 10^{19} \text{ GeV} \quad (5.1)$$

$$\lambda = \sqrt{3\Omega_\Lambda} H_0 \sim 3 \cdot 10^{-33} \text{ eV} \quad (5.2)$$

What should we insert for N ? Type IIB supergravity multiplets, e.g., have 128 bosonic and 128 fermionic degrees of freedom. Their contributions to λ have opposite sign and depend on their masses after supersymmetry breaking. Without going into details, it seems safe to assume that the effective N is somewhere between 1 and 128, so

$$\sqrt{\frac{N}{2\pi}} \sim 0.4 \quad \text{to} \quad 4$$

Since there may be other hidden factors of $\pi, \frac{1}{2}$, etc. that were missed by our crude analysis, the actual errors in the relation (4.4) may even be somewhat (but not much) larger. Being optimistic about them, we infer that

$$L \sim 15 \quad \text{to} \quad 15.3$$

This yields the predictions

$$m_{Brane-Susy} \sim (5 \text{ to } 10) \text{ TeV} \tag{5.3}$$

$$m_{Bulk-Susy} \sim \left(\frac{1}{4} \text{ to } 1 \right) \cdot 10^{-2} eV . \tag{5.4}$$

The example with the intermediate value of $L = 15.15$ is plotted in figure 2.

So we expect a mass of order $10^{-2} eV$ for the gravitino and for at least some of the other members of the supergravity multiplet. This corresponds to a Compton wavelength of the order of a fraction of a millimeter. Similarly as in the case of millimeter-size extra dimensions [9], the presence in the bulk of gravitinos or dilatons in the micrometer range is not ruled out by experiment: while the brane physics (the Standard Model) has been probed down to the weak scale, the bulk physics (gravity) has only been probed down to centimeter scales. The lower experimental bound on the gravitino mass appears to be only $10^{-5} eV$ [8]. The effects of these new fields might show up in short-distance measurements of gravity in the μm range in the near future [15].

Remarkably, the predicted scale of supersymmetry breaking in the Standard Model is roughly where it is expected to be, in order to insure that the running coupling constants meet in supersymmetric Grand Unification. This is very nontrivial; a priori it could have come out many orders of magnitude off the mark. Reversing the logic, if we assume a probable scale of supersymmetry breaking between 1 and 100 TeV, then we can predict the value of the cosmological constant within a few orders of magnitude of the value that seems to have been measured!

Let us finally note that our derivation and results apply just as well to the case of a single extra dimension as in the Horava-Witten model [16] or in the Randall-Sundrum model [17].

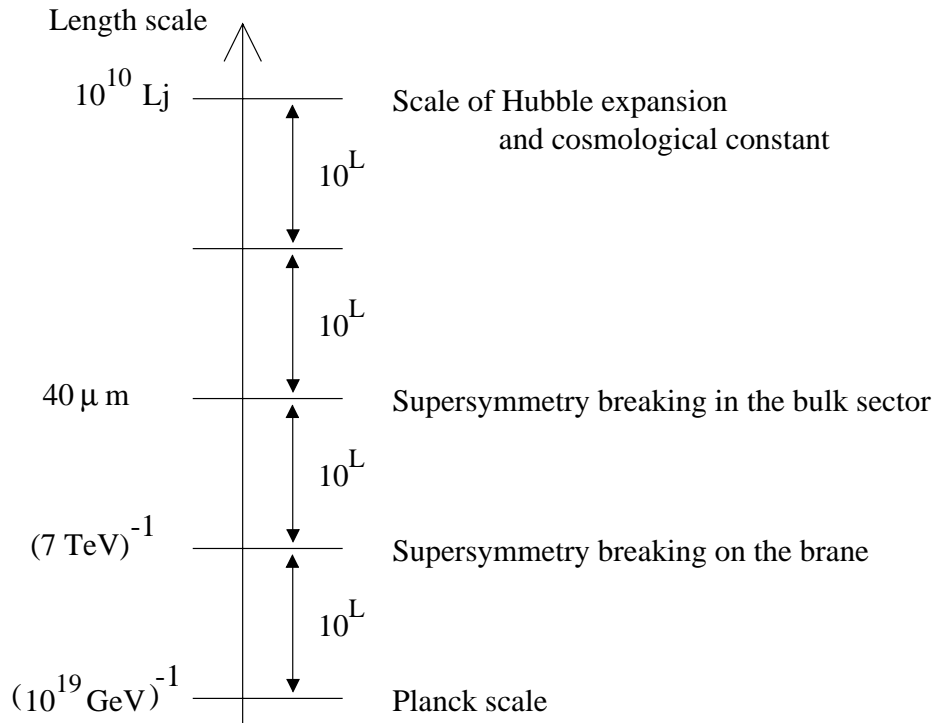


Figure 2: The relation between hierarchies for the intermediate value $L = 15.15$.

Conclusion

It seems that the proposal that we live on a non-supersymmetric brane that is embedded in a supersymmetric higher-dimensional string compactification can explain the observed small value of the cosmological constant under the assumption that supersymmetry in the Standard Model is restored at the scale of several TeV.

This explanation for the small cosmological constant can be tested by searching for signs of a gravitino, a dilaton or other supergravity fields with masses of order $10^{-2} eV$. We can thus look forward to a number of surprises in future tests of Einstein gravity in the micrometer range.

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