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Supergravity Description of Non-BPS Branes

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Abstract

We construct supergravity solutions that correspond to N Dp-branes coinciding with $\overline{N}$ $\overline{\mathrm{D}p}$-branes. We study the physical properties of the solutions and analyse the supergravity description of tachyon condensation. We construct an interpolation between the brane-antibrane solution and the Schwarzschild solution and discuss its possible application to the study of non-supersymmetric black holes.

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1 Introduction

While a brane breaks half of the space-time supersymmetry, the anti-brane breaks precisely the other half of the supersymmetry. Thus, a system of a brane and anti-brane breaks together all the space-time supersymmetry. The system is not stable, however, since the brane and anti-brane attract each other. This can be understood as the appearance of a tachyon on the world-volume of the branes. It arises from the open string stretched between the brane and the anti-brane and it is charged under the world-volume gauge groups. The decay of the system can be seen by the tachyon rolling down to the minimum of its potential [1]. The phenomenon of tachyon condensation is fairly well studied by now in the open string description [2, 3]. It would be interesting to ask how the phenomenon appears from the closed string viewpoint. One of the aims of this paper is to construct supergravity solutions that correspond to N Dp-branes coinciding with $\overline{N}$ $\overline{\mathrm{D}p}$-branes (anti D-branes) and analyse the supergravity description of tachyon condensation.

While Type IIA (Type IIB) string theory has BPS D-branes of even (odd) dimensions, they also admit non-BPS D-branes of odd (even) dimensions. These branes are not stable. They have been interpreted as the string theoretical analogues of sphalerons in field theory [4]. The families of supergravity solutions that we will discuss contain also backgrounds that correspond to these branes. Stable non-BPS brane configurations are much studied too [5, 6, 7, 8, 9]. However, we will not discuss supergravity backgrounds that correspond to these objects.

Another motivation that we have for studying brane-antibrane solutions is to understand the relation between these solutions and the Schwarzschild black hole solution (see, e.g. [10] for an early indication of such a connection in the context of five-dimensional black holes of Type IIB theory), which may have possible applications in the study of non-supersymmetric black holes.

This paper is organised as follows. In Section 2 we describe the supergravity solution that corresponds to N Dp-branes coinciding with $\overline{N}$ $\overline{\mathrm{D}p}$-branes and its physical properties. In Section 3 we analyse the supergravity description of tachyon condensation. We will also discuss the issue of decoupling and open-closed string duality. In section 4 we describe a general family of supergravity solutions that includes non-Poincare-invariant world-volumes. In particular it contains an interpolation between the brane-antibrane solution and the Schwarzschild solution. We discuss the possible application to the study of non-supersymmetric black holes. Section 5 contains a short discussion of the results.

We note that supergravity descriptions of smeared brane-antibrane configurations have been presented in [11]. We will discuss in this paper the localized ones. Unstable branes on AdS have been analysed in [12].

2 The Supergravity Description

In this section we will describe Type II supergravity solutions that correspond to N Dp-branes coincident with $\overline{N}$ $\overline{\mathrm{D}p}$-branes and their physical properties.

2.1 The Supergravity Solution

The strategy for constructing such solutions will be the following. We know that a brane-antibrane configuration must have the full world-volume Poincare symmetry $ISO(p,1)$[1]. Furthermore, it should have rotational symmetry $SO(9-p)$ in the $9-p$ transverse directions. For $N \neq \overline{N}$, the system will also carry an appropriate RR charge.

We therefore look for the most general solution of Type II A/B supergravity which possess the symmetry

$$\mathcal{S} = ISO(p,1) \times SO(9-p) \ , \tag{1}$$

and carries charge under a RR field[2].

The most general form of the metric, dilaton and RR-field consistent with the symmetry (1) is

$$\begin{aligned} ds^2 &= e^{2A(r)} dx_\mu dx^\mu + e^{2B(r)} \left(dr^2 + r^2 d\Omega_{8-p}^2 \right) \ , \\ \phi &= \phi(r) \ , \\ C^{(p+1)} &= e^{\Lambda(r)}\, dx^0 \wedge dx^1 \wedge \ldots \wedge dx^p \ . \end{aligned} \tag{2}$$

We look for solutions of the form (2), of Type II A/B supergravity Lagrangian, whose

[1] By contrast, a non-extremal Dp-brane breaks $ISO(p,1) \rightarrow ISO(p)$, which is expected of a finite temperature world-volume field theory (see Section 4). Here I stands for "inhomogeneous", referring to the translational symmetries.

[2] Our convention for the RR field and potentials is as follows. For electric p-branes (i.e. for $p = 0,1,2$), the RR field strength is $F_{p+2} \equiv dC^{(p+1)}$. For magnetic p-branes i.e. for $p = 4,5,6$, we interpret $C^{(p+1)}$ as the dual potential, and the RR field-strength will be given by $F_{8-p} \equiv e^{-\frac{(3-p)\phi}{2}} * \left(dC^{(p+1)}\right)$. For 3-branes ($p = 3$) the self-dual field strength is given by $F_5 = \frac{1}{\sqrt{2}} \left(dC^{(4)} + *dC^{(4)}\right)$.

relevant part is given (in the Einstein frame) by

$$S = \frac{1}{16\pi G_N^{10}} \int d^{10}x \sqrt{g} \left(R - \frac{1}{2} \partial_M \phi \partial^M \phi - \frac{1}{2\; n!} e^{a\,\phi} F_n^2 \right) \tag{3}$$

where $a = \frac{5-n}{2}$. The relation between the rank n of the RR field strength F_n and the dimensionality p of the brane has been explained in the footnote 2.

In (2) and in the rest of the paper we represent ten-dimensional coordinates by $x^M, M = 0, \ldots, 9$ and brane world-volume coordinates (including time) by $x^\mu, \mu = 0, 1, \ldots, p$. We will denote the transverse coordinates by $x^i, i = 1, \ldots, 9-p$ or, alternatively, by the polar coordinates $r, \theta_1, \ldots, \theta_{8-p}$ ($r^2 \equiv x^i x^i$).

The equations of motion that follow from (3) for the ansatz (2) are (see, e.g.,[13, 14])

$$\begin{aligned}
&A'' + (p+1)(A')^2 + (7-p)\, A'B' + \frac{8-p}{r}\, A' = \frac{7-p}{16}\, S^2 \ , \\
&B'' + (p+1)A'B' + \frac{p+1}{r}\, A' + (7-p)(B')^2 + \frac{15-2p}{r}\, B' = -\frac{1}{2}\,\frac{p+1}{8}\, S^2 \ , \\
&dA'' + (8-p)B'' + (p+1)(A')^2 + \frac{8-p}{r}\, B' - (p+1)A'B' + \frac{1}{2}\,(\phi')^2 = \frac{1}{2}\,\frac{7-p}{8}\, S^2 \ , \\
&\phi'' + \left((p+1)A' + (7-p)B' + \frac{8-p}{r} \right) \phi' = -\frac{a}{2}\, S^2 \ , \\
&\left(\Lambda' \ e^{\Lambda + a\phi - (p+1)A + (7-p)B}\, r^{8-p} \right)' = 0 \ ,
\end{aligned} \tag{4}$$

where

$$S = \Lambda' \ e^{\frac{1}{2} a\phi + \Lambda - dA} \ . \tag{5}$$

The mathematical solution to this system of differential equations has already been presented in [14]. The solutions depend on three[3] parameters r_0, c_1, c_2 (we have relabelled c_3 of [14] as c_2, and k as $-k$) and are given by

$$\begin{aligned}
A(r) &= \frac{(7-p)(3-p)c_1}{64}\, h(r) \\
&\quad - \frac{7-p}{16}\, \ln \left[\cosh(k\, h(r)) - c_2\, \sinh(k\, h(r)) \right] , \\
B(r) &= \frac{1}{7-p}\, \ln \left[f_-(r) f_+(r) \right] + \frac{(p-3)(p+1)c_1}{64}\, h(r) \\
&\quad + \frac{p+1}{16}\, \ln \left[\cosh(k\, h(r)) - c_2\, \sinh(k\, h(r)) \right] , \\
\phi(r) &= \frac{(7-p)(p+1)c_1}{16} h(r)
\end{aligned}$$

[3] One would naively predict *two* parameters, corresponding to the mass and the charge of the system, based on a suitable generalisation of Birkhoff's theorem. However, the proofs of such uniqueness theorems assume regular manifolds and therefore do not apply here.

$$
\begin{aligned}
&\quad + \frac{3-p}{4} \ln\left[\cosh(k\,h(r)) - c_2 \sinh(k\,h(r))\right], \\
e^{\Lambda(r)} &= -\eta (c_2^2 - 1)^{1/2} \frac{\sinh(k\,h(r))}{\cosh(k\,h(r)) - c_2 \sinh(k\,h(r))}\,, \qquad (6)
\end{aligned}
$$

where

$$
\begin{aligned}
f_\pm(r) &\equiv 1 \pm \left(\frac{r_0}{r}\right)^{7-p}, \\
h(r) &= \ln\left[\frac{f_-(r)}{f_+(r)}\right], \\
k &= \pm\sqrt{\frac{2(8-p)}{7-p} - \frac{(p+1)(7-p)}{16} c_1^2}\,, \\
\eta &= \pm 1\,. \qquad (7)
\end{aligned}
$$

The parameter η describes whether we are measuring the "brane" charge or the "antibrane" charge of the system.

The parameters (r_0, c_1, c_2) appear as integration constants and as such they could be complex, describing a six-dimensional space. However, the reality of the supergravity fields singles out three distinct three-dimensional subspaces I, II and III, as discussed in appendix A. For the rest of our paper, we will concentrate on the physical properties of the solution I where the above three parameters are all real; we will comment on II and III in appendix A. We also note that besides the three continuous parameters r_0, c_1 and c_2, our solution has two additional discrete parameters: $\mathrm{sign}(k), \eta$.

The solution is invariant under three independent Z_2 transformations which act on the space of the parameters

$$
\begin{aligned}
&(\mu, c_1, c_2, \mathrm{sign}(k), \eta) \to (\mu, c_1, -c_2, -\mathrm{sign}(k), -\eta) \\
&(\mu, c_1, c_2, \mathrm{sign}(k), \eta) \to (-\mu, -c_1, c_2, -\mathrm{sign}(k), \eta) \\
&(\mu, c_1, c_2, \mathrm{sign}(k), \eta) \to (-\mu, -c_1, -c_2, \mathrm{sign}(k), -\eta) \\
&\mu \equiv r_0^{7-p}\,.
\end{aligned}
\qquad (8)
$$

For convenience we will fix the above Z_2's by choosing
(a) the positive branch of the square root for k, namely

$$
k = \sqrt{\frac{2(8-p)}{7-p} - \frac{(p+1)(7-p)}{16} c_1^2}\,, \qquad (9)
$$

(b)

$$
c_1 \geq 0\,. \qquad (10)
$$

2.2 Physical Properties

In [14] the physical interpretation of the above three-parameter solution (6),(7) was not presented. We will see that it corresponds to brane-antibrane systems along with condensates.

In a brane-antibrane system, there are two obvious physical parameters N and $\overline{N}$ which are the numbers of branes and antibranes respectively. In the above supergravity solution too, there are two obvious physical parameters: the RR charge Q and the ADM mass M_{ADM}, which clearly depend on N and $\overline{N}$. We will discuss in Section 3 the brane interpretation of the third parameter. Before that, however, it will be useful to discuss Q and M_{ADM} in greater detail.

For convenience, we consider wrapping the spatial world-volume directions on a torus T^p of volume V_p (this is always possible, since the metric and other fields do not depend on these directions). The RR charge Q, defined by an appropriate surface integral over the sphere-at-infinity in the transverse directions (see, e.g. [13]), is given by

$$Q = 2\eta N_p r_0^{7-p} k \sqrt{c_2^2 - 1} \ , \tag{11}$$

where

$$N_p \equiv \frac{(8-p)(7-p)\omega_{8-p} V_p}{128\pi G_N^{10}}, \tag{12}$$

and $\omega_d = \frac{2\pi^{(d+1)/2}}{\Gamma((d+1)/2)}$ is the volume of the unit sphere S^d. We have normalized the charge Q such that the BPS relation becomes $M_{BPS} = Q$.

The ADM mass M is defined, in terms of the Einstein-frame metric, by [15, 16] [4]

$$g_{00} = -1 + \frac{16\pi G_N^{10-p} M}{(8-p)\omega_{8-p} r^{7-p}} + \text{higher order terms} \tag{13}$$

where $G_N^{10-p} = G_N^{10}/V_p$.

This gives us

$$M = N_p r_0^{7-p} \left[\frac{3-p}{2} c_1 + 2 c_2 k \right] . \tag{14}$$

Since the solution is generically non-BPS, M is different from $M_{BPS} \equiv Q$. The mass difference is given by

$$\Delta M \equiv M - M_{BPS} = N_p r_0^{7-p} \left[\frac{3-p}{2} c_1 + 2k(c_2 - \sqrt{c_2^2 - 1}) \right] . \tag{15}$$

[4] This definition differs from the one presented, e.g. in [13], by an overall factor.

In order to have a better understanding of the space of solutions represented by (6),(7), we now consider some special limiting cases.

The BPS case ($\bar{N} = 0$)

Since the BPS Dp-brane clearly respects the symmetry (1), it should be part of our solution space.

We recall [17] that the Dp-brane solution is given by

$$\begin{aligned} ds^2 &= f_p^{\frac{p-7}{8}} dx_\mu dx^\mu + f_p^{\frac{p+1}{8}}(dr^2 + r^2 d\Omega_{8-p}^2), \\ e^\phi &= f_p^{\frac{3-p}{4}} , \\ C^{(p+1)}_{01\ldots p} &= -\eta\frac{1}{2}(f_p^{-1} - 1) , \\ f_p &= 1 + \frac{\mu_0}{r^{7-p}} , \end{aligned} \tag{16}$$

with ADM-mass M_{Dp} and charge Q given by

$$M_{Dp} = Q = \mu_0 N_p , \tag{17}$$

This solution indeed exists in a "scaled neighbourhood" of the point $(r_0, c_1, c_2) = (0, c_m, \infty)$, defined by

$$\begin{aligned} r_0^{7-p} &= \epsilon^{\frac{1}{2}} \bar{r}_0^{7-p} , \\ c_1 &= c_m - \epsilon \frac{8\bar{k}^2}{(p+1)(7-p)c_m}, \\ c_2 &= \frac{\bar{c}_2}{\epsilon} , \end{aligned} \tag{18}$$

where $c_m = (\frac{32(8-p)}{(p+1)(7-p)^2})^{1/2}$ denotes the point where $k = 0$. The second condition is better stated as

$$k = \epsilon^{\frac{1}{2}} \bar{k} . \tag{19}$$

The scaling is defined by the limit $\epsilon \to 0$ such that $\bar{r}_0, \bar{c}_2$ and $\bar{k}$ are fixed.

It is easy to check that the solution (6) reduces to (16) with

$$\mu_0 = 2c_2 k r_0^{7-p} = 2\bar{c}_2 \bar{k} \bar{r}_0^{7-p}. \tag{20}$$

It is useful to consider the three-parameter space of solutions as parameterised by M, Q, c_1. Figure 1 depicts the M, c_1 plane for a given fixed Q. The BPS solution corresponds to the scaled neighbourhood represented by the shaded circle. Other parts of the figure will be explained later.

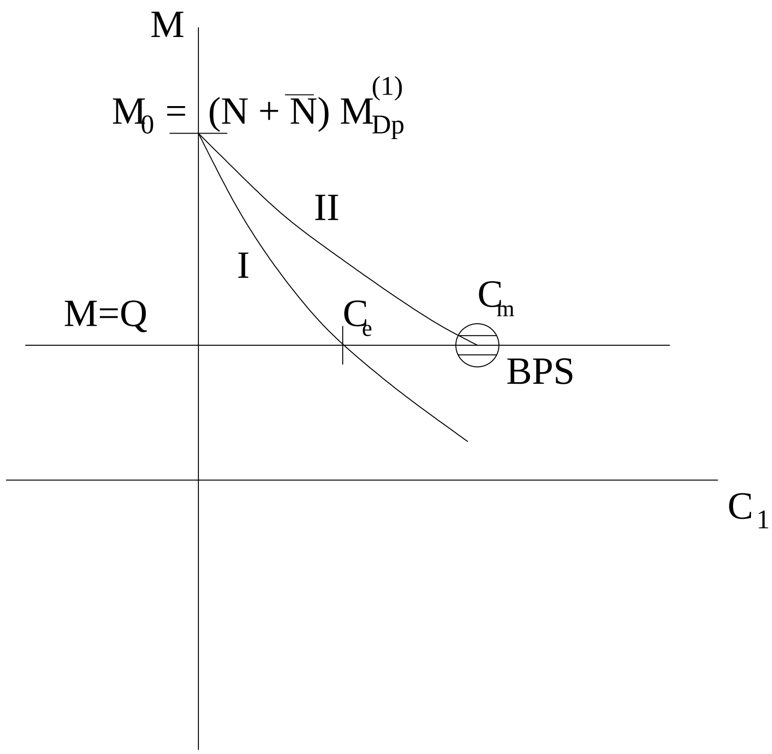

Figure 1: The M, c_1 plane for a given fixed $Q \neq 0$. The BPS solution corresponds to the scaled neighbourhood represented by the shaded circle. Path II represents decay to a BPS D-brane of charge Q.

The Dp-$\overline{\mathrm{D}p}$System ($N = \overline{N}$)

In this case the RR charge $Q \propto (N - \overline{N})$ must vanish. According to (11) this corresponds to the subspace

$$|c_2| = 1 \ . \tag{21}$$

We represent this subspace in Fig 2.

Now (21) implies $c_2 = \pm 1$. As remarked in Section 3 below, the physically relevant choice for $p > 3$ is $c_2 = 1$, while for $p < 3$ it is $c_2 = -1$ (for $p = 3$ the two choices are physically equivalent). To simplify the discussion we will present the formulae in the rest of this section for $p > 3$; it is straightforward to write down the formulae in the other cases.

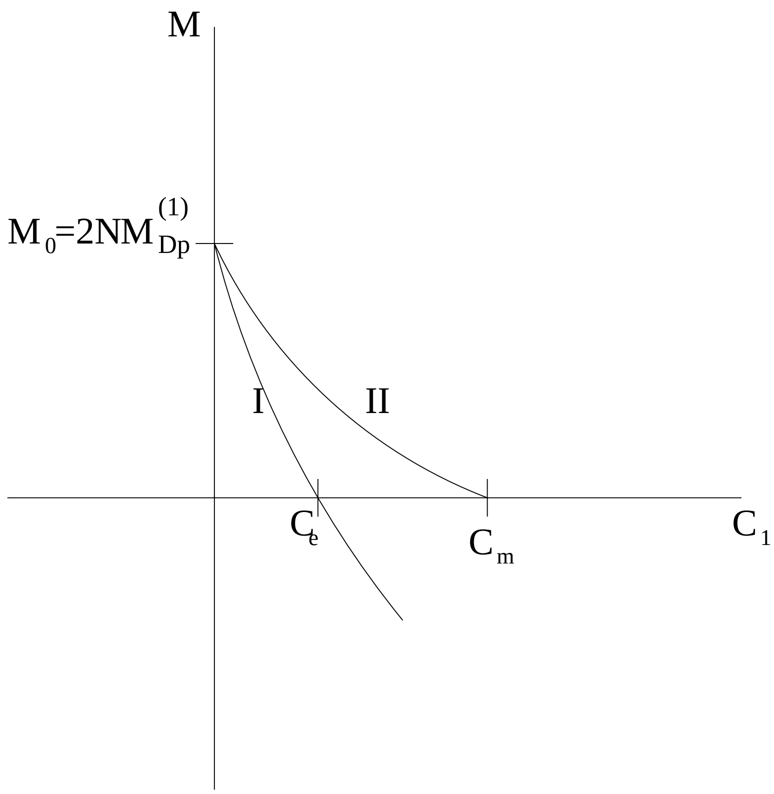

Figure 2: The two-parameter space of solutions for $Q = 0$, as parameterised by M, c_1. Path II represents decay of the brane-antibrane configuration to flat space.

The solution now reads

$$
\begin{aligned}
e^{2A} &= \left(\frac{f_-}{f_+}\right)^{\alpha} \ , \\
e^{2B} &= f_-^{\beta_-} f_+^{\beta_+} \ , \\
e^{\phi} &= (f_-/f_+)^{\gamma} \ , \\
e^{\Lambda} &= 0 \ ,
\end{aligned}
\tag{22}
$$

where

$$
\begin{aligned}
\alpha &= (7-p)\left(\frac{(3-p)c_1 + 4k}{32}\right) \ , \\
\beta_{\pm} &= \frac{2}{7-p} \mp \left(\frac{(p+1)((p-3)c_1 - 4k}{32}\right) \ , \\
\gamma &= \frac{1}{16}\left((7-p)(p+1)c_1 - 4(3-p)k\right) \ .
\end{aligned}
\tag{23}
$$

These represent the most general 2-parameter (r_0, c_1) solution of Type II supergravity with no gauge field and SO(p,1) $\times$ SO(9-p) symmetry.

Consider for instance the case $p = 6$. The solution reads

$$
\begin{aligned}
e^{2A} &= \left(\tfrac{1-r_0/r}{1+r_0/r}\right)^{(4k-3c_1)/32} , \\
e^{2B} &= (1-r_0/r)^{2+7(3c_1-4k)/32} \, (1+r_0/r)^{2-7(3c_1-4k)/32} , \\
e^{\phi} &= \left(\tfrac{1-r_0/r}{1+r_0/r}\right)^{(7c_1+12k)/16}
\end{aligned}
\tag{24}
$$

where $k = \sqrt{4 - 7c_1^2/16}$.

The Einstein metric has a curvature singularity at $r = r_0$. The scalar curvature in (24), e.g., goes as

$$
\mathcal{R} \sim \frac{1}{(r-r_0)^{2+\beta_-}} \ . \tag{25}
$$

The physical regime is $r \geq r_0$. In the case of a single Dp-brane the curvature singularity is resolved by the appropriate inclusion of the brane degrees of freedom. We will discuss this issue in our case later on.

For the specific value

$$
c_1 = 0 \ , \tag{26}
$$

we get

$$
\begin{aligned}
e^{2A} &= \left(\tfrac{1-\frac{r_0}{r}}{1+\frac{r_0}{r}}\right)^{1/4} , \\
e^{2B} &= (1-\tfrac{r_0}{r})^{1/4}(1+\tfrac{r_0}{r})^{15/4} , \\
e^{\phi} &= \left(\tfrac{1-\frac{r_0}{r}}{1+\frac{r_0}{r}}\right)^{3/2} ,
\end{aligned}
\tag{27}
$$

which is the coincident D6-$\overline{\text{D6}}$ solution found by Sen [18] in isotropic coordinates. In Fig 2, this corresponds to the point $(M, c_1) = (M_0, 0)$.

The above observation implies that for $c_1 \neq 0$ we get a generalisation of the coincident D6-$\overline{\text{D6}}$ solution. We will argue in the next section that the parameter c_1 is related to the "vev" [5] of (the zero momentum mode of the) the open string tachyon arising from open strings stretched between the D6 and $\overline{\text{D6}}$ (and more generally between D$p$and $\overline{\text{D}p}$) branes. The Sen solution corresponds to the particular case where the tachyon vev is zero.

Other cases of $\Delta M = 0$

[5] We actually consider generically off-shell values of the tachyon. The issue of why we may have supergravity solutions corresponding to an off-shell tachyon is discussed in Sec 3.1.

Clearly, from (15) we can have

$$M = Q \tag{28}$$

if we have

$$(3-p)/2\ c_1 + 2k(c_2 - \sqrt{c_2^2 - 1}) = 0\ . \tag{29}$$

This solution (represented by $c_1 = c_e$ in Figs 1,2) is nonsupersymmetric. Indeed, there is a range of the parameters (see Figs 1,2) in which

$$M < Q\ , \tag{30}$$

These solutions cannot correspond to physical states of string theory (for $Q = 0$, these correspond to negative ADM mass).

This implies that we expect additional contribution to the ADM mass formula, coming perhaps from a better understanding of the curvature singularity at $r = r_0$. In the case of BPS D-branes or the fundamental string the ADM mass formula as found by the asymptotic behaviour of the Einstein metric does represent the energy-momentum of the source sitting at the curvature singularity. The reason our case is different may have to do with the fact that we have a naked singularity at $r = r_0$; a computation of the Euclidean action similar to that in [19] indeed shows that the action receives contribution not only from $r = \infty$, but also from $r = r_0$.

3 Tachyon Condensation

In the following, we will interpret the 3-parameter family of supergravity solutions as a bound state of N Dp-branes coincident with $\overline{N}$ $\overline{\mathrm{D}p}$-branes, together with a "vev" v of the tachyon condensate. The three parameters r_0, c_1, c_2 will be argued to correspond to various combinations of the three parameters $N, \overline{N}, v$.

3.1 $\langle \mathrm{T} \rangle$ in Supergravity

A system of N Dp-branes on top of $\overline{N}$ $\overline{\mathrm{D}p}$-branes has a tachyon arising from the open string stretched between the Dp-branes and the $\overline{\mathrm{D}p}$-branes. The tachyon T transforms in the $(N, \overline{N})$ (and T^* in $(\overline{N}, N)$) representation of the $U(N) \times U(\overline{N})$ gauge group. Consider first the case $N = \overline{N}$ (the neutral case). The cases that are studied most are $N = \overline{N} = 1$. In this case the tachyon is a complex field (T, T^*) that transforms in the $(1,-1) \oplus (-1,1)$ representation of the $U(1) \times U(1)$ gauge group of the world-volume theory. The brane system is unstable due to the tachyon. The tachyon has a potential $V(T)$ which is a function of $|T|^2$. The Dp-$\overline{\mathrm{D}p}$-branes configuration is expected to decay into the closed string (Type II) vacuum. Such a decay into the vacuum is conjectured to happen through the process of tachyon condensation in which the zero-momentum mode of the tachyon gets a specific vev. In particular, it is conjectured that at the minimum of the tachyon potential, denoted by $|T| = T_0$, the total energy of the system actually vanishes:

$$E = V(T_0) + 2M_{Dp} = 0 \;, \tag{31}$$

where M_{Dp} is the mass of a Dp-brane. Equation (31) has been established numerically to a very high accuracy via open string field theory [3]. When $N > 1$ it was argued in [7] that at the minimum of the potential all the eigenvalues of T_0 are equal. In the following we will denote $\frac{1}{N}\mathrm{Tr}(TT^*)$ by $|T|^2$.

Let us ask ourselves how the above phenomenon appears from the viewpoint of closed string theory. We concentrate on the neutral case first ($Q = 0$) and on the charged case later. There are two ways of looking at the problem:
(a) Real-time: The physical decay process in terms of the brane (open string) variables in which the tachyon rolls down to its minimum is time-dependent. The supergravity background of such a time-dependent brane configuration is naively expected to be time-dependent[6].
(b) Path-in-configuration-space: One can alternatively view the decay as a one-parameter path in the open string configuration space, which for our purposes here is the space of values of $|T|$. Except at the two extremities of the path ($|T| = 0, T_0$), the other values of $|T|$ are not at an extremum of $V(T)$ and is therefore off-shell. Let us ask how such a path would appear in the closed string description. Let us imagine doing an experiment in which gravitons and other massless closed string probes are scattered off the brane-antibrane system for various values of $|T|$ as $|T|$ is varied from 0 to T_0. We will *assume* here that such an experiment makes sense with off-shell values of the tachyon [7]. In principle one can imagine coupling closed string degrees of freedom to the off-shell tachyon through, e.g., the modified DBI action appropriate to brane-antibrane systems. The supergravity

[6]We remark, though, that the exterior geometry of a pulsating spherically symmetric star is given by the static Schwarzschild solution, thanks to Birkhoff's theorem. It is not inconceivable, therefore, to have a time-dependent brane configuration with a static supergravity background for $r > r_0$. In such a case the time-dependence could presumably be discerned at the level of higher mass modes of the closed string (see [20] for a similar analysis where the supergravity background of a BPS state does not see the "polarisation" of the state, although the higher closed string modes see it.)

[7]Coupling on-shell bulk degrees of freedom to off-shell brane degrees of freedom is also familiar from AdS/CFT.

solution away from the brane will have the same symmetry as the brane-antibrane system, namely (1). However, the metric and other fields must reflect the extra parameter $|T|$. We will try to argue that the one-parameter deformation represented by c_1 in our solution corresponds to this $|T|$.

We begin by asking whether we see in the supergravity description an analogue of the tachyon potential. The obvious supergravity counterpart of the total energy E (Eqn. (31)) of the brane-antibrane system is the ADM mass (14). For the suggested identification to be correct we should have

$$M = V(T) + 2NM^{(1)}_{Dp} \; . \tag{32}$$

where by $M^{(1)}$ we mean the ADM mass for a single Dp brane. The supergravity solution in question here is the 2-parameter family (22) of solutions parameterised by (r_0, c_1). Since the left hand side of (32) is the ADM mass (14), viz.

$$M = N_p r_0^{7-p} \left[\frac{3-p}{2} c_1 + \left(\frac{2(8-p)}{7-p} - \frac{(p+1)(7-p)}{16} c_1^2 \right)^{\frac{1}{2}} \right] \; , \tag{33}$$

let us ask whether the the qualitative behaviour of M as a function of c_1 in (33) agrees with the right hand side of (32) for some appropriate identification between c_1 and T.

<u>Comment on branches</u>: As explained in Appendix A, the dependence of the ADM mass on c_1 depends on the specific branch of the solution. In the following we will find that it is for the branch I_{++} for $p > 3$ (and I_{--} for $p < 3$)[8] which lends to a tachyon interpretation. Later on we will briefly comment on the possible interpretation of the other branches.

Once we choose the appropriate branch of the supergravity solution, the qualitative behaviour of M as a function of c_1 (at a fixed r_0) is given by Fig 3.

Consider first the case $p = 6$. When $c_1 = 0$ we have the coincident D6-D$\bar{6}$ solution of Sen [18]. The ADM mass (33) for $p = 6, c_1 = 0$ is $M = 4N_p r_0$. We will argue in Sec. 3.2 that this mass coincides with

$$M = 2NM^{(1)}_{Dp} \; . \tag{34}$$

This implies that $V(T) = 0$ at $c_1 = 0$; since the tachyon potential vanishes only at $T = 0$ [21], we conclude that

$$T = 0 \qquad \text{at} \qquad c_1 = 0 \; . \tag{35}$$

As we will see, the last equation is valid for all p. This will imply that the subspace of our three-parameter solution defined by $c_1 = 0$ represents Dp-$\overline{\mathrm{D}p}$branes with zero value

[8]For $p = 3$ and $Q = 0$ I_{++} and I_{--} are physically indistinguishable.

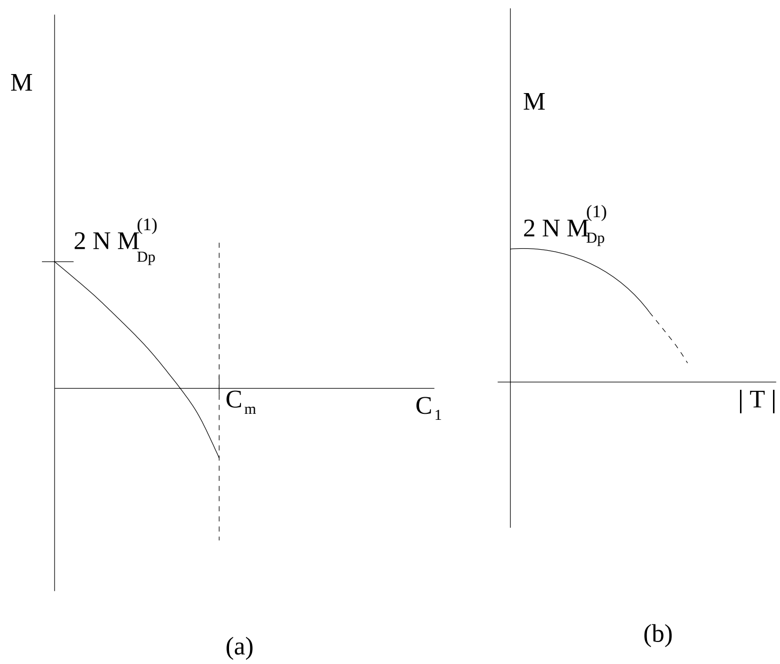

Figure 3: ADM Mass (for a fixed $r_0 > 0$) as a function of (a) c_1 and (b) $|T|$

of the tachyon $|T|$, that is, brane-antibrane configurations which sit at the maximum of the tachyon potential.

Let us now consider small deformations away from $c_1 = 0$. Since $V(T)$ is known to be a function only of $|T|^2$, we expect the ADM mass, and hence c_1, to be a function of $|T|^2$ too. For small deformations, we can write

$$c_1 = a|T|^2 + b|T|^4 + \dots \tag{36}$$

Clearly $a > 0$. It is easy to see that the behaviour of the ADM mass M as a function of $|T|$ (Fig. 3(b)) qualitatively matches the behaviour of $V(|T|)$ near $T = 0$.

Tachyon condensation

In Fig 3(b) we have not plotted the ADM mass in the whole range of $|T|$ because (36) is valid only near $T = 0$. The question then is whether our solution can describe the full double-well potential $V(T)$. In other words, can we describe the process of tachyon condensation all the way to the vacuum?

In Fig 2, vacuum is represented by any point in the line $M = 0$. Any path connecting the point $(M_0, c_1 = 0)$ to this line (e.g. path I or path II) therefore in principle represents a family of supergravity solutions corresponding to a flow of $|T|$ from $|T| = 0$ to $|T| = T_0$.

To know what the actual path is, we need to have a more precise knowledge of mapping (more detailed than (36)) between the open string variables $(N, |T|)$ to the supergravity variables (r_0, c_1). Assuming that such maps exist and are smooth and invertible, the generic form will be

$$\begin{aligned} r_0 &= \tilde{f}_1(N, |T|^2), c_1 = \tilde{f}_2(N, |T|^2) \\ N &= \tilde{g}_1(r_0, c_1), |T| = \tilde{g}_2(r_0, c_1) \end{aligned} \tag{37}$$

These can alternatively be stated as a map $(N, |T|) \to (M, c_1)$:

$$\begin{aligned} M &= f_1(N, |T|^2), c_1 = f_2(N, |T|^2) \\ N &= g_1(M, c_1), |T| = g_2(M, c_1) \end{aligned} \tag{38}$$

Of course (37),(38) should be consistent with (36) near $T = 0$ (we need to consider the coefficients $a, b, \ldots$ to be functions of r_0 or N).

The path I in Fig 2 corresponds, in terms of (37), to $r_0 = \tilde{f}_1(N)$ and $c_1 = \tilde{f}_2(|T|^2)$. This path corresponds to the plot Fig 3(a) of M as a function of c_1 *at fixed* r_0. It has the unphysical feature that it does not stop at $M = 0$ and goes down to the domain of $M < 0$.

Path II in Fig 2 requires the functions $\tilde{f}_{1,2}$ (or the functions $f_{1,2}$) to be necessarily a function of two variables. In other words, the flow of $|T|$ from 0 to T_0 should mean here that both r_0 and c_1 should change appropriately to take the solution to the point $(M, c_1) = (0, c_m)$. The nice feature of this path is that it automatically *ends* at the flat space solution, since c_1 cannot go beyond c_m (actually there is another branch of solution (Branch II, appendix A) for $c_1 > c_m$, but it can be shown that the ADM mass increases for $c_1 > c_m$).

In the absence of a decoupling limit (as we will discuss in Section 3.3) it may not be possible to determine the exact functions mentioned in (37) or (38) and therefore to know any more about the nature of $V(T)$ than what we have already presented here. In any case, if an analysis of brane degrees of freedom is expected to remove the $M < 0$ region, presumably the formulae for the mass will change.

In summary, we see that a path exists (path II in Fig 2) in our space of solutions which describes the flow of $|T|$ from 0 to T_0 and the behaviour of the ADM mass M along this path matches the qualitative features of $V(T)$.

The other branches

In the above we have discussed only the branch I_{++} (see Appendix A for notation) for $p \geq 3$ and I_{--} for $p < 3$. It is easy to see that the behaviour of the branches I_{-+}, I_{+-} are outright unphysical. This leaves I_{--} for $p \geq 3$ and I_{++} for $p < 3$. In this branch (except for $p = 3$) for small deformations of c_1 away from zero, M initially rises beyond the combined rest mass of the brane-antibrane system and then falls again. This seems puzzling since equation (32) does not allow such an increase in the energy of the system. We should recall however that when the vev of the tachyon field is zero the world-volume gauge group is not broken. That means that we are allowed to have other condensates such as a gluon condensate. This can increase the energy of the system. An estimate of such an increase can be obtained from the modified DBI action [22]

$$S = -T_p \int d^{p+1}\sigma e^{-\phi} V(T) \sqrt{\det \left[G_{ij} + 2\pi\alpha' \left(F_{ij} + \partial_i T \partial_j T \right) \right]} \, . \tag{39}$$

The interpretation of the c_1 deformation (for $p \neq 3$) in these branches could therefore be in terms of a gluon condensate. However,it remains a mystery in that case why (a) there is no such phenomenon for $p = 3$ (since the branches I_{++} and I_{--} appear to be identical), and (b) why the ADM mass starts to decrease after a while.

Non-BPS D-branes

Since we are only discussing the tachyon condensate in terms of a real quantity $|T|$ we are left with the possibility that our supergravity solution may represent a real tachyon as well. Recall that a real tachyon characterizes the non-BPS Dp branes, i.e. p odd for IIA and p even for IIB, which are obtained from the Dp-$\overline{\mathrm{D}p}$-brane system by a $(-1)^{F_L}$ projection. So the natural question arises: which brane system does the supergravity solution describe. It is plausible that in the neutral case the solution describes both. In both cases the background has no RR charge, and one expects the full $SO(p,1) \times SO(9-p)$ symmetry. The solution (22) is the most general one that satisfies these conditions. The question is whether the ADM mass of a non-BPS brane (with or without tachyon) occurs in these solutions. We recall that the tension of non-BPS Dp branes (for $N = 1$) is related to the tension of the Dp-$\overline{\mathrm{D}p}$-brane system by $M_{\text{non-BPS}} = \frac{1}{\sqrt{2}} M_{Dp\text{-}\overline{Dp}}$, reflecting a bound system. For $N > 1$ too, the tension of the non-BPS Dp brane system $M^{(N)}_{\text{non-BPS}}$ should be less than that of the combined rest mass $2NM^{(1)}_{Dp}$ of the brane-antibrane system. Since the values of ADM mass discussed in the context of (32) range all the way from $2NM^{(1)}_{Dp}$ to 0, we see that in a suitable range of parameters the solution (22) does have ADM masses that can be fitted to $M = M^{(N)}_{\text{non-BPS}} + \tilde{V}(T)$ where $\tilde{V}(T)$ is the potential for the real tachyon in this case.

The charged case: $Q \neq 0$

In this case we expect the relation

$$M = (N + \overline{N}) M_{Dp}^{(1)} + V(T) \ . \tag{40}$$

where $M_{Dp}^{(1)}$ denotes the ADM mass for a single Dp brane. The analysis of the binding energy in the next section once again suggests that $c_1 = 0$ corresponds to the point where the tachyon potential vanishes, which we expect to be for vanishing tachyon field. The discussion of tachyon condensation is similar to the neutral case. Again path II in Fig 1 is more physical than path I because the former ends at the BPS point and does not go to the region $M < Q$. The qualitative behaviour of M along this path again matches the qualitative features of a tachyon potential which has a local maximum at $|T| = 0$ and a minimum at $|T| = T_0$ where we denote $\frac{1}{N}\mathrm{Tr}(TT^*)$ by $|T|^2$ (we assume that all the eigenvalues of TT^* are the same, namely T_0^2, at the minimum). We expect that at the minimum $V(T) = (|N - \overline{N}| - (N + \overline{N})) M_{Dp}^{(1)}$.

3.2 Dp-brane probes and Binding energy

In the last section we mentioned that $V(T) = 0$ corresponds to $c_1 = 0$. We derive this in the present section.

We will consider the general 3-parameter solution parametrized by (r_0, c_1, c_2). Let us define the binding energy of the Dp-$\overline{\mathrm{D}p}$-branes solution to be

$$E_B = (N + \overline{N}) M_{Dp}^{(1)} - M \ , \tag{41}$$

where M is given by (14) and $M_{Dp}^{(1)}$ represents the rest mass of a single Dp-brane (or $\overline{\mathrm{D}p}$-brane), given by (17) with the scale parameter $\mu_0 = \mu_0^{(1)}$, which depends on g_{str} and p, the dimensionality of the brane.

In view of the equation (32),

$$E_B = -V(T) \ . \tag{42}$$

A straightforward comparison between $(N + \overline{N}) M_{Dp}$ and M of (14) is hampered by the fact that we do not know a priori the relation between the two parameters r_0 and μ_0 that characterise the respective solutions (6) and (16). We will find this relation by the following strategy.

We consider the static force between a Dp-$\overline{\mathrm{D}p}$-branes system and a Dp-brane probe (respectively a $\overline{Dp}$-brane probe) at a distance r. This can be computed in two ways:

(a) From supergravity:

$$S_{\rm probe} = -\frac{1}{g_s l_s^{p+1}} \int d^{p+1}\sigma (e^{-\phi}\sqrt{\hat{G}} \pm C_{p+1}) \tag{43}$$

where $G_{MN} = e^{\phi/2} g_{MN}$ represents the string frame metric corresponding to the solution (6) and $\hat{G}$ is its pull-back to the world-volume. For a Dp(resp. $\overline{\mathrm{D}p}$) probe, we use the upper (resp. lower) sign.

Subtracting the flat space DBI part, and keeping only the leading term in the $1/r$ expansion we get

$$S_{\rm probe} = 2k \frac{V_p}{g_s l_s^{p+1}} (\frac{r_0}{r})^{7-p} (c_2 \mp \sqrt{c_2^2 - 1}) \ . \tag{44}$$

(b) By a string theory computation:

$$\langle Dp D\bar{p} | \exp(-\beta H) | Dp \rangle \tag{45}$$

where the states are regarded as boundary states constructed out of closed-string oscillators. (We consider here the case of the Dp-probe first.) At weak coupling and for $\langle \mathrm{T} \rangle = 0$, the boundary state on the left is given by

$$\langle Dp D\bar{p} | = \langle Dp | \otimes \langle D\bar{p} | \ . \tag{46}$$

We will assume that (46) can be used for computation of the leading term in the $1/r$ expansion for large distances r, when $\langle \mathrm{T} \rangle = 0$ (see [23, 24] for earlier work on connection between boundary states and classical solutions). Since the static force between two Dp-branes vanishes, the computation (b) then reduces, at $\langle \mathrm{T} \rangle = 0$, to

$$\langle D\bar{p} | \exp(-\beta H) | Dp \rangle \ . \tag{47}$$

This latter can be computed at large distances from supergravity, by the DBI action of a Dp-brane probe in the background of a $\overline{\mathrm{D}p}$- brane:

$$S'_{\rm probe} \equiv -\frac{1}{g_s l_s^{p+1}} \int d^{p+1}\sigma [e^{-\phi}\sqrt{\hat{G}} + C^{(p+1)}] \ , \tag{48}$$

where the metric, dilaton and the RR potential are now obtained from (16), with $\mu_0 = \overline{N}\mu_0^{(1)}$. We get, again after subtracting the flat space DBI part, and keeping only the leading term in the $1/r$ expansion,

$$S'_{\rm probe} = \frac{V_p}{g_s l_s^{p+1}} \left(2 \frac{\overline{N}\mu_0^{(1)}}{r^{7-p}} \right) \ . \tag{49}$$

This result holds for the Dp-probe. For the $\overline{\mathrm{D}p}$-probe we need to replace $\overline{N} \rightarrow N$ in the above expression.

Matching (44) and (49) leads to

$$\begin{aligned} N\mu_0^{(1)} &= kr_0^{7-p}(c_2 + \sqrt{c_2^2 - 1}) \ , \\ \overline{N}\mu_0^{(1)} &= kr_0^{7-p}(c_2 - \sqrt{c_2^2 - 1}) \ . \end{aligned} \tag{50}$$

from this we deduce that

$$Q = N_p \mu_0^{(1)}(N - \overline{N}) \tag{51}$$

and

$$\mu_0^{(1)}(N + \overline{N}) = 2kr_0^{7-p}c_2 \ . \tag{52}$$

Using (14), (17) and (50) we can find the zero of the binding energy (41) of the Dp-$\overline{\mathrm{D}p}$bound state. We get

$$E_B = 0 = N_p r_0^{7-p}\left[\frac{3-p}{2}c_1\right] \ . \tag{53}$$

Clearly E_B vanishes at $c_1 = 0$[9]. In view of the identification (42), this implies that

$$c_1 = 0 \Rightarrow V(T) = 0 \ , \tag{54}$$

as promised in the last section. Note that
(a) If we put $c_1 = 0$ in (14) we indeed get $M = M_{Dp} + M_{\overline{Dp}}$, consistent with the vanishing of the binding energy,
(b) Equations (51) and (52) give us essentially $N - \overline{N}$ and $N + \overline{N}$ in terms of the supergravity parameters in the subspace $c_1 = 0$,
(c) The expression for the total mass (52) matches exactly with the BPS mass (20) (recall that at the BPS point $\overline{N} = 0$.

3.3 Open-closed String Duality

In the spirit of the AdS/CFT correspondence (for a review see [25]), it is natural to ask whether we can apply a decoupling limit [26] of the brane modes from the bulk modes to the supergravity description of the Dp-$\overline{\mathrm{D}p}$-branes system. Typically for Dp-branes this

[9]The case $p = 3$ is subtle and we extrapolated the result to this value of p from the other values. An alternative way would presumably be to use some other probe.

is a low energy limit with the resulting background being the near-horizon metric. In the present case, the closest analogue of the near horizon metric is some suitably scaled neighbourhood of $r = r_0$. However, it is easy to see that for the neutral solution (22) there is no such region which by itself is a solution of the supergravity field equations. Also, we cannot find an appropriate rescaling that keeps a metric finite in l_s units as $l_s \to 0$. This means that the interactions between the open and closed strings remain relevant.

Another manifestation of this issue is the form of the potential $V(r)$ for a graviton scattered on the Dp-$\overline{\mathrm{D}p}$-branes. The potential is depicted in figure 4. Near $r = r_0$ it goes like $\frac{-1}{(r-r_0)^2}$ while at infinity it approaches $-\omega^2$ where ω is the frequency of the scattered graviton. The potential poses no barrier for the gravitons sent from infinity to reach the $r = r_0$ and their absorption cross section does not vanish [10]. The absence of a decoupling of the closed strings from the open strings prevents us from making a precise correspondence between the field theory on the Dp-$\overline{\mathrm{D}p}$-branes world-volume and the supergravity background. This suggests that there is also a limitation on the quantitative understanding of the tachyon condensation process by using only the open string description.

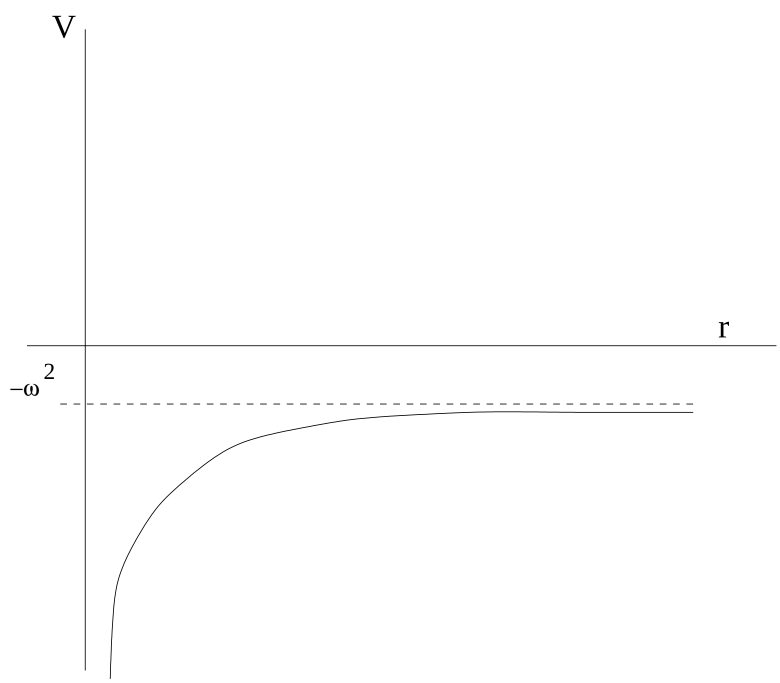

Figure 4: The scattering potential $V(r)$ for gravitons on Dp-$\overline{\mathrm{D}p}$-branes.

[10]For a similar but detailed analysis see [27].

The singularity of the supergravity solution at $r = r_0$ is time-like. Having such a singularity of the classical geometry which we can reach at a finite proper time, there is the natural question whether it is resolved quantum mechanically. One criterion [28] is the existence of a self-adjoint Laplacian. This can still be the case even if the metric is geodesically incomplete. The requirement is the existence of a non-normalizable solution of the wave equation. This criterion is satisfied by our geometry. To see that we consider the Laplace equation in the form

$$\frac{\partial^2 \phi}{\partial t^2} = -A\phi \ . \tag{55}$$

The equation $A\phi = \lambda\phi$ takes the form

$$\rho^\beta \partial_\rho(\rho\partial_\rho\phi) = \lambda\phi \ , \tag{56}$$

where $\beta = 2p - 15 + 2(7-p)(\frac{3-p}{8}c_1 - \frac{k}{2})$ and $\rho = r - r_0$. Defining $z = \sqrt{\lambda}\rho^{(1-\beta)/2}$ we get

$$\phi'' + \frac{\phi'}{z} + \frac{4}{(1-\beta)^2}\phi = 0 \ . \tag{57}$$

This has Bessel function solutions behaving like 1 and $\ln\rho$. The norm of the latter $\int d\rho\rho\rho^{-2}$ diverges.

4 The Four-Parameter Solution

In this section we will briefly describe the most general p-brane solution of Type II string theory in which we relax the requirement of Poincare invariance in the $(p+1)$ dimensional world-volume. In other words, we ask ourselves about the most general solution which respects the symmetry

$$\mathcal{S}' = SO(p) \times SO(9-p) \ , \tag{58}$$

Clearly the previous 3-parameter solution already respects this symmetry and hence should be part of this most general family of solutions. The modified ansatz for the Einstein metric is

$$ds^2 = \exp(2A(r))(-f(r)dt^2 + dx_m dx^m) + \exp(2B(r))\left(dr^2 + r^2 d\Omega^2_{8-p}\right) \ , \tag{59}$$

where we split the world-volume index μ as $0, m = 1, \ldots, p$. The ansatz for the dilaton and the gauge potential remain the same as in (2).

The equations of motion for this ansatz have been written down in appendix B. Once again the mathematical solution of the differential equations has been worked out in [14].

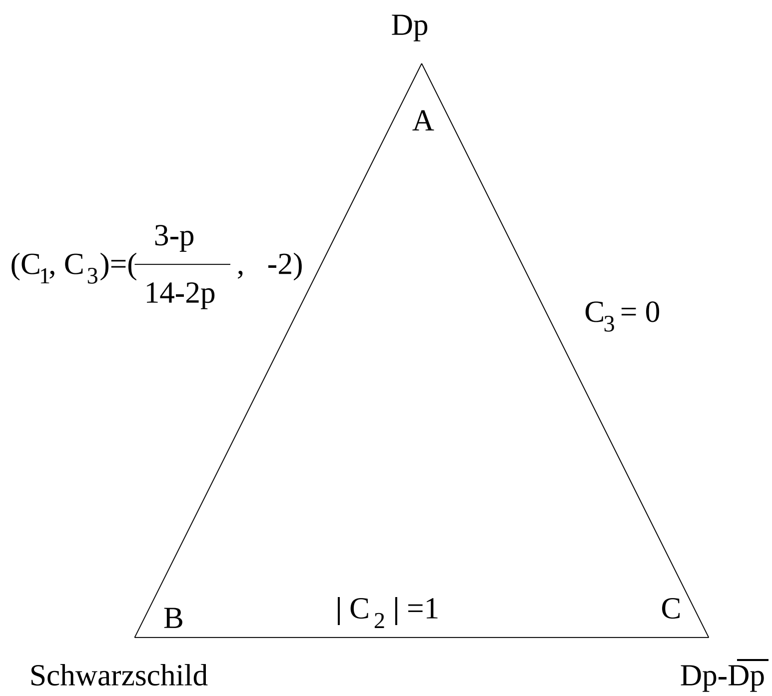

Figure 5: The most general spherically symmetric solution of Type II theories

We write the explicit solution in Appendix B for completeness and discuss here some salient physical features (see Fig 5)

• The general solution has 4 independent parameters (r_0, c_1, c_2, c_3). The Poincare-invariant 3-parameter subspace discussed in the previous sections corresponds to $c_3 = 0$. In Fig 5 this is schematically represented by the arm AC of the triangle ABC. $c_3 \neq 0$ breaks world-volume Poincare invariance.

• The two-parameter subspace $(c_1, c_3) = (\frac{3-p}{2(7-p)}, -2)$ corresponds to the black p-brane solutions of [17]. This has already been identified in [14]. In Fig 5, this is represented by the arm AB of the triangle. Recall that the black p-branes are parameterised by their charge and mass (equivalently r_+, r_-, the outer and inner horizons). Note that the BPS D-brane can be reached as a limit along the arm BA, like it can be reached along CA, although the c_3 values characterising these two arms are different. It is likely that there are continuous families of solutions between BA and CA (corresponding to different c_3 values) which can reach the BPS solution under a limiting procedure.

• The three-parameter subspace defined by $|c_2| = 1$ describes the most general spherically symmetric solution with no gauge fields. This is represented by the arm BC of the triangle. It is well-known that the neutral limit of the black p-brane (point B) corresponds to the Schwarzschild black hole in $10-p$ dimensions ($\times T^p$, assuming a wrapped p-brane). On the other hand, as discussed at great length in this paper, the neutral limit of the arm AC corresponds to the coincident brane-antibrane solutions. The arm BC therefore provides interpolating solutions which connect the brane-antibrane solution to the Schwarzschild solution.

It is clear that there is a rather rich phase structure in Fig 5. Parts of this diagram have obvious decoupling limits and dual field theory descriptions. It would be interesting to chart out these parts completely [29].

Interpolations similar to the arm BC are of paramount importance to the study of the D1/D5 system and the five-dimensional black hole [30]. It has been found that CFT descriptions seem to work in some contexts for non-rotating BTZ black holes which are the analogues of Schwarzschild black holes in AdS_3. An interpolation of such a solution to a brane-antibrane solution of the D1/D5 system would shed light on both brane-antibrane dynamics and nonsupersymmetric black holes.

It has been pointed out by [31] that the equations of motion of the above system are identical to those of a Toda molecule. It is tempting to construct a "mini-superspace" kind of model for this space based on Toda dynamics.

5 Discussion

In this paper we constructed localised supergravity solutions corresponding to bound states of N Dp-branes coinciding with $\overline{N}$ $\overline{\mathrm{D}p}$-branes for $p = 0, 1, \ldots, 6$ (and non-BPS D-branes of odd (even) dimensions of Type IIA (Type IIB) string theory). We constructed these by looking for the most general solution of Type II A/B supergravity which respect world-volume Poincare invariance and rotational invariance in the transverse directions. Contrary to the naive expectation that the solution should have only two parameters corresponding to the charge and the mass, we found that the most general solution has one extra parameter. We found that in the physically relevant branch there are two special values of the extra parameter at which the ADM mass respectively coincides with (a) the combined rest mass of the branes and antibranes, and (b) the mass of the BPS configuration of $N - \overline{N}$ branes [11]. In the case $N = \overline{N}$ (zero RR charge) the point (b) represents flat space. The case $N = \overline{N}$ is extensively studied from the point of view of

[11] for $N > \overline{N}$; for $N < \overline{N}$ these will be $\overline{N} - N$ antibranes.

open strings living on the brane-antibrane system, and we recognised the solutions (a) and (b) as the supergravity background corresponding to the maximum and the minimum of the tachyon potential. This lead us to interpret the extra parameter in our solution as the supergravity manifestation of an expectation value of the tachyon. We matched the qualitative behaviour of the ADM mass as a function of this extra parameter with the behaviour of the tachyon potential $V(T)$.

We noticed the absence of a decoupling of the bulk closed strings from the brane-antibrane open strings. This means that the interactions between the open and closed strings remain relevant and suggets that there is also a limitation on the quantitative understanding of the tachyon condensation process by using only the open string description.

We briefly discussed a more general (four-parameter) space of solutions in which we assume only rotational invariance in the spatial directions on the world-volume. This space includes brane-antibrane pairs, BPS D-branes, the black p-branes of [17] and Schwarzschild black holes. The detailed understanding of this four-parameter space in terms of brane variables is an outstanding problem.

Acknowledgement: We would like to thank A. Kumar and P. Townsend for discussions.

Appendix

A Real Sections of the Supergravity Solution

As remarked in the text, the three parameters (r_0, c_1, c_2) characterising the supergravity solution (6),(7) appear as integration constants in the solution of differential equations and as such could be complex. However, this would generically make the metric, dilaton and gauge field also complex. We find that there are three distinct 3-dimensional domains of (r_0, c_1, c_2), described below as Branches I, II and III, where the supergravity fields remain real.

Branch I:

$$\begin{aligned}
&c_1 \in (0, c_m), \quad c_m = \sqrt{\tfrac{8-p}{8(p+1)(7-p)}} \\
&c_2 \in (-\infty, 1) \cup (1, \infty) \\
&\mu \equiv r_0^{7-p} \in R \\
&\eta = \pm 1
\end{aligned} \tag{60}$$

We will assume in this section that we have already fixed the Z_2 symmetries (8) of the solution by implementing (9),(10). For Branch I, the remaining choices of signs are best discussed by thinking of four sub-branches, depending on whether the signs of $(c_2, \mu \equiv r_0^{7-p})$ are $++, +-, -+$ and $--$ respectively. We denote these as $I_{++}, I_{+-}, I_{-+}, I_{--}$ respectively (each of these will also contain $\eta = \pm$). The formulae for the ADM mass and charge for Branch I is given by (14),(11). Explicitly

$$\begin{aligned}
M &= N_p r_0^{7-p}\left[\tfrac{3-p}{2}c_1 + 2c_2\sqrt{\tfrac{2(8-p)}{7-p} - \tfrac{(p+1)(7-p)}{16}c_1^2}\right] \\
Q &= 2\eta N_p r_0^{7-p}\sqrt{\tfrac{2(8-p)}{7-p} - \tfrac{(p+1)(7-p)}{16}c_1^2}\sqrt{c_2^2 - 1}\ ,
\end{aligned} \tag{61}$$

The behaviour of these functions depends on the signs of c_2 and μ. We find that it is the branch I_{++} for $p = 3, 4, 5, 6$ which lends to a tachyon interpretation (Section 3). For $p = 0, 1, 2, 3$ it is I_{--}.

Branch II

$$\begin{aligned}
&c_1 \in (c_m, \infty) \Rightarrow k = -i\tilde{k}, \tilde{k} = \sqrt{-\tfrac{2(8-p)}{7-p} + \tfrac{(p+1)(7-p)}{16}c_1^2} \\
&c_2 = i\tilde{c}_2, \tilde{c}_2 \in R \\
&\mu \equiv r_0^{7-p} \in R \\
&\eta = \pm 1
\end{aligned} \tag{62}$$

The mass and charge for this branch read

$$
\begin{array}{l}
M = N_p r_0^{7-p}\left[\frac{3-p}{2}c_1 + 2c_2\sqrt{-\frac{2(8-p)}{7-p} + \frac{(p+1)(7-p)}{16}c_1^2}\right] \\
Q = 2\eta N_p r_0^{7-p}\sqrt{\frac{2(8-p)}{7-p} - \frac{(p+1)(7-p)}{16}c_1^2}\sqrt{(\tilde{c}_2)^2+1}\ ,
\end{array}
\tag{63}
$$

Branch III

$$
\begin{array}{l}
c_1 = i\tilde{c}_1, \tilde{c}_1 \in R^+ \\
c_2 = i\tilde{c}_2, \tilde{c}_2 \in R \\
\mu \equiv r_0^{7-p} = -i\tilde{\mu}, \tilde{\mu} \in R \\
\eta = \pm 1
\end{array}
\tag{64}
$$

The mass and charge for this branch read

$$
\begin{array}{l}
M = N_p r_0^{7-p}\left[\frac{3-p}{2}c_1 + 2c_2\sqrt{\frac{2(8-p)}{7-p} + \frac{(p+1)(7-p)}{16}(\tilde{c}_1)^2}\right] \\
Q = 2\eta N_p r_0^{7-p}\sqrt{\frac{2(8-p)}{7-p} + \frac{(p+1)(7-p)}{16}(\tilde{c}_1)^2}\sqrt{(\tilde{c}_2)^2+1}\ ,
\end{array}
\tag{65}
$$

B Details of the 4-parameter solution

The equations of motion that follow from (3) for the ansatz (59) are

$$
\begin{array}{l}
A'' + (p+1)(A')^2 + (7-p)\,A'B' + \dfrac{8-p}{r}\,A' + \dfrac{1}{2}(\ln f)'\,A' = \dfrac{7-p}{16}\,S^2\ , \\
A'' + (p+1)(A')^2 + (7-p)\,A'B' + \dfrac{8-p}{r}\,A' + \dfrac{1}{2}(\ln f)'' + \\
\dfrac{1}{2}(\ln f)'\left((d+1)A' + \dfrac{1}{2}(\ln f)' + (7-p)B' + \dfrac{8-p}{r}\right) = \dfrac{7-p}{16}\,S^2\ , \\
B'' + (p+1)A'B' + \dfrac{p+1}{r}\,A' + (7-p)(B')^2 + \dfrac{1}{2}(\ln f)'\left(B' + \dfrac{1}{r}\right) + \\
\dfrac{15-2p}{r}\,B' = -\dfrac{1}{2}\,\dfrac{p+1}{8}\,S^2\ , \\
dA'' + (8-p)B'' + (p+1)(A')^2 + \dfrac{8-p}{r}\,B' - (p+1)A'B' + \\
\dfrac{1}{2}(\ln f)'' + \dfrac{1}{4}\left((\ln f)'\right)^2 + \dfrac{1}{2}\,(\phi')^2 = \dfrac{1}{2}\,\dfrac{7-p}{8}\,S^2\ , \\
\phi'' + \left((p+1)A' + (7-p)B' + \dfrac{8-p}{r} + \dfrac{1}{2}(\ln f)'\right)\phi' = -\dfrac{a}{2}\,S^2\ , \\
\left(\dfrac{\Lambda'}{f^{\frac{1}{2}}}\,e^{\Lambda + a\phi - (p+1)A + (7-p)B}\,r^{8-p}\right)' = 0\ ,
\end{array}
\tag{66}
$$

where

$$S = \frac{\Lambda'}{f^{\frac{1}{2}}} \;\; e^{\frac{1}{2}a\phi + \Lambda - dA} \; . \tag{67}$$

The solutions [14] depend on four parameters r_0, c_1, c_2, c_3 (we have interchanged the labels c_2, c_3 for convenience, compared to [14]), and are given by

$$\begin{aligned}
f(r) &= e^{-c_3 h(r)}, \\
A(r) &= \frac{(7-p)}{32}\left(\frac{3-p}{2}c_1 + (1 + \frac{(3-p)^2}{8(7-p)})c_3\right) h(r) \\
&\quad -\frac{7-p}{16} \ln\left[\cosh(k\, h(r)) - c_2 \sinh(k\, h(r))\right], \\
B(r) &= \frac{1}{7-p} \ln\left[f_-(r) f_+(r)\right] + \frac{(p-3)}{64}\left((p+1)c_1 - \frac{3-p}{4}c_3\right) h(r) \\
&\quad +\frac{p+1}{16} \ln\left[\cosh(k\, h(r)) - c_2 \sinh(k\, h(r))\right], \\
\phi(r) &= \frac{(7-p)}{16}\left((p+1)c_1 - \frac{3-p}{4}c_3\right) h(r) \\
&\quad +\frac{3-p}{4} \ln\left[\cosh(k\, h(r)) - c_2 \sinh(k\, h(r))\right], \\
e^{\Lambda(r)} &= \eta(c_2^2 - 1)^{1/2} \frac{\sinh(k\, h(r))}{\cosh(k\, h(r)) - c_2 \sinh(k\, h(r))} \; ,
\end{aligned} \tag{68}$$

where

$$\begin{aligned}
f_\pm(r) &\equiv 1 \pm \left(\frac{r_0}{r}\right)^{7-p} \; , \\
h(r) &= \ln\left[\frac{f_-(r)}{f_+(r)}\right] \; , \\
k^2 &= \frac{2(8-p)}{7-p} - c_1^2 + \frac{1}{4}\left(\frac{3-p}{2}\; c_1 + \frac{7-p}{8}\; c_3\right)^2 - \frac{7}{16}\; c_3^2 \; , \\
\eta &= \pm 1 \; .
\end{aligned} \tag{69}$$

The parameter η describes whether we are measuring the "brane" charge or the "antibrane" charge of the system.

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