

Is Nothing Sacred? Vacuum Energy, Supersymmetry and Lorentz Breaking from Recoiling D branes

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Abstract. Classical superstring vacua have zero vacuum energy and are supersymmetric and Lorentz-invariant. We argue that all these properties may be destroyed when quantum aspects of the interactions between particles and non-perturbative vacuum fluctuations are considered. A toy calculation of string/ D -brane interactions using a world-sheet approach indicates that quantum recoil effects - reflecting the gravitational back-reaction on space-time foam due to the propagation of energetic particles - induce non-zero vacuum energy that is linked to supersymmetry breaking and breaks Lorentz invariance. This model of space-time foam also suggests the appearance of microscopic event horizons.

1 Introduction

String theory is our best (only?) hope for a consistent quantum theory of gravity and all the other particle interactions [1]. As such, it should address the non-perturbative nature of the quantum-gravitational vacuum [2]. On basic physical grounds, one would expect the vacuum to exhibit non-perturbative quantum fluctuations, much as the QCD vacuum exhibits non-perturbative topological fluctuations such as instantons. In the case of quantum gravity, the Planck-scale vacuum fluctuations go by the picturesque name of space-time foam [3], which string theory should have the ambition to understand. However, many extant string approaches are limited to the identification of classical string vacua [1,2], and the consideration of the interactions of particles propagating through them.

Certain approaches to quantum gravity, such as the loop description [4], do attempt to understand certain quantum aspects of space-time foam. We have been making [5] parallel attempts in an approach to string theory that introduces on the world sheet a Liouville field [6,7], which is identified with a renormalization

scale and the time coordinate t . This approach accommodates departures from conventional critical string theory, as may be needed to describe transitions between different classical vacua and other non-perturbative phenomena. In this connection, it is encouraging to learn [8] that D branes appear naturally within this world-sheet Liouville approach to non-critical string theory.

Several of the sacred properties of classical string theory may be unsustainable in such an approach. We have argued that it provides a natural description for a system of particles interacting with D branes, which we use to model non-perturbative quantum fluctuations in the space-time background. As a consequence of the induced non-criticality, the vacuum energy becomes non-zero ¹. It is, however, not constant: in our approach [9,10], there is a component in the vacuum energy which relaxes towards zero $\propto 1/t^2$. If this component is dominant today, the vacuum energy is not a ‘cosmological constant’.

In the context of a supersymmetric theory, non-zero vacuum energy carries with it the stigma of supersymmetry breaking. Indeed, as we demonstrate explicitly within our approach [10–12], quantum-gravitational recoil does break supersymmetry. Whether this can be related to the supersymmetry breaking required by the non-observation of supersymmetric partners of the known particles is, however, a weighty subject beyond the scope of this talk.

Non-critical string theory also carries with it the stigma of a breakdown of Lorentz invariance. We are not (sufficiently?) shocked by this corollary. Indeed, our D -brane model is not the only treatment of space-time foam in which the vacuum may acquire non-trivial optical properties such as a refractive index, birefringence and diffusive particle propagation [13–15]. Any and all of these properties violate Lorentz invariance [3,16,17].

The structure of this talk is as follows. In Section 2 we review our description of D -brane interactions and recoil, arguing that they lead in general to departures from criticality that can be accommodated within a world-sheet Liouville approach. We discuss in Section 3 recoil-induced deformations of the background space-time metric. We show in Section 4 how D -brane recoil may induce a non-zero but time-dependent contribution to the vacuum energy, and also a cosmic expansion of our world, viewed as a brane embedded in a higher-dimensional (bulk) space-time. The appearance of a microscopic horizon in this approach to space-time foam is described in section 5, and the breaking of supersymmetry by such a recoil effect is discussed in Section 6. We discuss in Section 7 some observable consequences of our recoil formalism, namely a non-trivial refractive index *in vacuo*, as well as diffusive particle propagation. We provide in Section 8 a brief summary of observational constraints on such possible space-time foam signatures of quantum gravity, and Section 9 discusses the outlook for our approach.

¹ We recall that non-perturbative QCD effects also change the vacuum energy.

2 A Review of the World-Sheet Description of D -brane Interactions

We first review the world-sheet formalism for treating D -brane interactions [18–20,9]. In this approach, the recoil of a D -brane when struck by a closed-string state or by another D -brane is described mathematically by a logarithmic conformal field theory [21]. Such theories lie on the border between finite conformal field theories and general renormalizable two-dimensional quantum field theories. As such, they provide the first stepping-stone away from the conformal field theories used to characterize critical string theories. They are relevant [18–20] to this problem because the recoil process involves a change of state (transition) in the string background, which cannot be described by a conformal field theory. In the language of the world-sheet, this change of state induced by the recoil process can be described as a change in the σ -model background, and as such is a non-equilibrium process. This property is reflected [20,19] in the logarithmic operator algebra itself.

As discussed in references [18,20,19], the recoil of a D -brane string soliton after interaction with a closed-string state, illustrated in Fig. 1, is characterized by a σ model deformed by a pair of logarithmic operators [21]:

$$C_\epsilon^I = \epsilon \Theta_\epsilon(X^I), \quad D_\epsilon^I = X^I \Theta_\epsilon(X^I), \quad I \in \{0, \dots, 3\} \quad (1)$$

defined on the boundary $\partial\Sigma$ of the string world sheet. Here $X^I, I \in \{0, \dots, p\}$ obey Neumann boundary conditions on the string world sheet, and denote the D -brane coordinates, whilst $\epsilon \rightarrow 0^+$ is a regulating parameter and $\Theta_\epsilon(X^I)$ is a regularized Heaviside step function. The remaining $y^i, i \in \{p+1, \dots, 9\}$ in (1) denote the transverse bulk directions.

In the case of D particles [18,20,19], the index I takes the value 0 only, in which case the operators (1) act as deformations of the conformal field theory on the world sheet. The operator

$$u_i \int_{\partial\Sigma} \partial_n X^i D_\epsilon \quad (2)$$

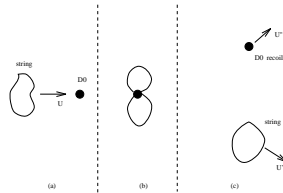


Fig. 1. Schematic representation of the scattering of a closed-string state on a D particle ($D0$ brane) embedded in the target space-time. (a) The closed-string state, moving with velocity U along a given spatial direction, (b) strikes at a given instant in time the $D0$ brane, which then (c) recoils, distorting the space-time around it.

describes the movement of the D brane induced by the scattering, where u_i is its recoil velocity, and $y_i \int_{\partial\Sigma} \partial_n X^i C_\epsilon$ describes quantum fluctuations in the initial position y_i of the D particle. It has been shown rigorously [19] that the logarithmic conformal algebra ensures energy–momentum conservation during the recoil process:

$$u_i = M_D(k_i^1 + k_i^2), \quad (3)$$

where $k^1(k^2)$ is the momentum of the propagating closed string state before (after) the recoil, and $M_D = 1/(\ell_s g_s)$ is the mass of the D brane, where g_s is the string coupling, which is assumed here to be weak enough to ensure that the D brane is very massive, and ℓ_s is the string length.

The second member of the logarithmic pair of σ -model deformations is

$$y_i \int_{\partial\Sigma} \partial_n X^i C_\epsilon, \quad (4)$$

where, in order to realize the logarithmic algebra between the operators C and D , one uses as a regulating parameter [18]

$$\epsilon^{-2} \sim \ln[L/a] \equiv \Lambda, \quad (5)$$

where L (a) is an infrared (ultraviolet) world-sheet cutoff. The recoil operators (4) are relevant, in the sense of the renormalization group for the world-sheet field theory, having small conformal dimensions $\Delta_\epsilon = -\epsilon^2/2$. Thus the σ -model perturbed by these operators is not conformal for $\epsilon \neq 0$, and the theory requires Liouville dressing [6,7,20]. The consistency of this approach is supported by the proof of momentum conservation during the scattering process [19].

The world-sheet renormalization-group β functions of the relevant recoil couplings $g_{Ii}^D \equiv g_{Ii}$, $I \in \{0, \dots, p\}$, $i \in \{p+1, \dots, 9\}$ have the form

$$\beta_{g_{Ii}} = \frac{d}{dt} g_{Ii} = -\frac{1}{2t} g_{Ii}, \quad t \sim \epsilon^{-2}. \quad (6)$$

Thus, one may construct an exactly marginal set of couplings \bar{g}_{Ii} by redefining

$$\bar{g}_{Ii} \equiv \frac{g_{Ii}}{\epsilon}. \quad (7)$$

The renormalized couplings \bar{g}_{0i} were shown [19] to play the rôle of the physical recoil velocity of the D brane, whilst the remaining \bar{g}_{Ii} , $I \neq 0$, describe the folding of the Dp brane for $p \neq 0$ [22]. Here we assume, generalizing [19], that the (bare) recoil couplings for all I are of equal strength and related to the transverse momentum transfer as

$$g_{Ii} = g_s \frac{\Delta P_i}{M_s}, \quad I = 0, \dots, m, \quad i = m+1, \dots, D \quad (8)$$

for a D brane embedded in a D -dimensional space-time.

An important technical remark [18] is now in order: for reasons of convergence of the world-sheet path integral, the Neumann coordinate X^0 must be

Euclideanized. It is only in this case that the identification (5), with $\epsilon^2 > 0$, leads to a mathematically consistent logarithmic algebra of operators. This can be understood simply by the fact that, in the pertinent world-sheet computations of correlation functions of logarithmic operators (1), one encounters [18] the free propagator of the Neumann coordinates X^I :

$$\mathcal{G}_0 = \lim_{\sigma \rightarrow 0} \langle X^I(\sigma) X^J(0) \rangle_* \sim \eta_{IJ} \ln[L/a] \quad (9)$$

where $\langle \dots \rangle_*$ denotes a world-sheet expectation value calculated using the free-string world-sheet action on a flat target space-time manifold $\{X^I\}$, and η^{IJ} is the target-space metric. For Euclidean world sheets, one takes $\eta^{IJ} = \delta^{IJ}$, which is essential for the convergence of world-sheet path-integral expressions entering in the respective correlators.

In our picture, we view the (3+1)-dimensional physical world as a brane. This Euclideanization implies that the (longitudinal) Neumann coordinates define a $D4$ domain wall in the bulk space-time, which, after analytic continuation of the coordinate X^0 , will result in our four-dimensional space-time. However, the analytic continuation takes place only at the very end of the calculations. This is important for our subsequent discussion, and should always be understood in what follows.

3 Deformations of Space-Time Induced by D -Brane Recoil

As discussed in [20,23], the deformations (1) create a local distortion of the space-time surrounding the recoiling folded D -brane, which may be determined using the method of Liouville dressing. In [20,23] we concentrated on describing the resulting space-time in the case when a D -particle defect embedded in a D -dimensional space-time recoils after the scattering of a closed string. To leading order in the recoil velocity u_i of the D particle, the resulting space-time was found, for times $t \gg 0$ long after the scattering event at $t = 0$, to be equivalent to a Rindler wedge, with apparent ‘acceleration’ ϵu_i [23], where ϵ is defined above (5). For times $t < 0$, the space-time is flat Minkowski ².

This situation is easily generalized to Dp branes [22] as seen in Fig. 2. The folding/recoil deformations of the Dp brane (4) are relevant deformations, with anomalous dimension $-\epsilon^2/2$, which disturbs the conformal invariance of the world-sheet σ model, and restoration of conformal invariance requires Liouville dressing [6,7,20], as discussed above. To determine the effect of such dressing on the space-time geometry, it is essential to write [20] the boundary recoil deformations as bulk world-sheet deformations

$$\int_{\partial\Sigma} \bar{g}_{Iz} x \Theta_\epsilon(x) \partial_n z = \int_{\Sigma} \partial_\alpha (\bar{g}_{Iz} x \Theta_\epsilon(x) \partial^\alpha z) \quad (10)$$

² There is hence a discontinuity at $t = 0$, which leads to particle production and decoherence for a low-energy spectator field theory observer who performs local scattering experiments long after the scattering, and far away from the location of the collision of the closed string with the D particle [23].

where the \bar{g}_{Iz} denote the renormalized folding/recoil couplings (7), in the sense discussed in [19]. As we have already mentioned, such couplings are marginal on a flat world sheet. The operators (10) are marginal also on a curved world sheet, provided [7] one dresses the (bulk) integrand by multiplying it by a factor $e^{\alpha_{Ii}\phi}$, where ϕ is the Liouville field and α_{Ii} is the gravitational conformal dimension, which is related to the flat-world-sheet anomalous dimension $-\epsilon^2/2$ of the recoil operator, viewed as a bulk world-sheet deformation, as follows [7]:

$$\alpha_{Ii} = -\frac{Q_b}{2} + \sqrt{\frac{Q_b^2}{4} + \frac{\epsilon^2}{2}} \quad (11)$$

where Q_b is the central-charge deficit of the bulk world-sheet theory. In the recoil problem at hand, as discussed in [23],

$$Q_b^2 \sim \epsilon^4/g_s^2 > 0 \quad (12)$$

for weak folding deformations g_{Ii} , and hence one is confronted with a *supercritical* Liouville theory. This implies a *Minkowskian-signature* Liouville-field kinetic term in the respective σ model [24], which prompts one to interpret the Liouville field as a time-like target-space field. In our context, this will be a *second* time coordinate [5], which is independent of the (Euclideanized) X^0 . The presence of this second ‘time’ does not affect physical observables, which are defined for appropriate slices with fixed Liouville coordinate, e.g., $\phi \rightarrow \infty$, or equivalently $\epsilon \rightarrow 0$. From the expression (12) we conclude (cf. (11)) that $\alpha_{Ii} \sim \epsilon$ to leading order in perturbation theory in ϵ , to which we restrict ourselves here.

We next remark that, as the analysis of [20] indicates, the X^I -dependent field operators $\Theta_\epsilon(X^I)$ scale as follows with ϵ : $\Theta_\epsilon(X^I) \sim e^{-\epsilon X^I} \Theta(X^I)$, where $\Theta(X^I)$ is a Heavyside step function without any field content, evaluated in the limit

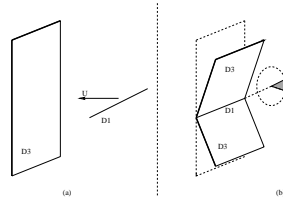


Fig. 2. Schematic representation of the folding effect in D -brane/ D -brane collisions: (a) a $D1$ brane moving with velocity U along a ‘bulk’ direction perpendicular to a Dp brane embedded in a D -dimensional space time strikes the Dp brane (b), which is then folded, and the space-time around it is distorted. The dashed circle around the $D1$ direction in (b) indicates the angular deficit that appears when the bulk direction along which the $D1$ brane was moving is compactified to a circle. A generalization to a higher-dimensional case for the incident brane is straightforward. In that case the deficit (in the compact case) is a higher-dimensional solid hyperangle.

$\epsilon \rightarrow 0^+$. The bulk deformations, therefore, yield the following σ -model terms:

$$\frac{1}{4\pi\ell_s^2} \int_{\Sigma} \sum_{I=0}^3 (\epsilon^2 \bar{g}_{Ii}^C + \epsilon \bar{g}_{Ii} X^I) e^{\epsilon(\phi_{(0)} - X_{(0)}^I)} \Theta(X_{(0)}^I) \partial_{\alpha} \phi \partial^{\alpha} y_i \quad (13)$$

where the subscripts (0) denote world-sheet zero modes, and $\bar{g}_{0i}^C = y_i$.

Upon the interpretation of the Liouville zero mode $\phi_{(0)}$ as a (second) time-like coordinate, the deformations (13) yield metric deformations of the generalized space-time with two times. The metric components for fixed Liouville-time slices can be interpreted in [20] as expressing the distortion of the space-time surrounding the recoiling D -brane soliton.

For clarity, we now drop the subscripts (0) for the rest of this paper, and we work in a region of space-time such that $\epsilon(\phi - X^I)$ is finite in the limit $\epsilon \rightarrow 0^+$. The resulting space-time distortion is therefore described by the metric elements

$$\begin{aligned} G_{\phi\phi} &= -1, & G_{ij} &= \delta_{ij}, & G_{IJ} &= \delta_{IJ}, & G_{iI} &= 0, \\ G_{\phi i} &= (\epsilon^2 \bar{g}_{Ii}^C + \epsilon \bar{g}_{Ii} X^I) \Theta(X^I), & i &= 4, \dots, 9, & I &= 0, \dots, 3 \end{aligned} \quad (14)$$

where the index ϕ denotes Liouville ‘time’, not to be confused with the Euclideanized time which is one of the X^I . To leading order in $\epsilon \bar{g}_{Ii}$, we may ignore the $\epsilon^2 \bar{g}_{Ii}^C$ term. The presence of $\Theta(X^I)$ functions and the fact that we are working in the region $y_i > 0$ indicate that the induced space-time is piecewise continuous³. In the general recoil/folding case considered in this article, the form of the resulting patch of the surrounding space-time can be determined fully if one computes the associated curvature tensors, along the lines of [23].

We next study in more detail some physical aspects of the metric (14), restricting ourselves, for simplicity, to the case of a single Dirichlet dimension z that plays the rôle of a bulk dimension in a set up where there are Neumann coordinates X^I , $I = 0, \dots, 3$ parametrizing a D4 (Euclidean) brane, interpreted as our four-dimensional space-time. Upon performing the time transformation $\phi \rightarrow \phi - \frac{1}{2} \epsilon \bar{g}_{Iz} X^I z$, the line element (14) becomes:

$$\begin{aligned} ds^2 &= -d\phi^2 + \left(\delta_{IJ} - \frac{1}{4} \epsilon^2 \bar{g}_{Iz} \bar{g}_{Jz} z^2 \right) dX^I dX^J + \\ &\left(1 + \frac{1}{4} \epsilon^2 \bar{g}_{Iz} \bar{g}_{Jz} X^I X^J \right) dz^2 - \epsilon \bar{g}_{Iz} z dX^I d\phi, \end{aligned} \quad (15)$$

where ϕ is the Liouville field which, we remind the reader, has Minkowskian signature in the case of supercritical strings that we are dealing with here.

One may now make a general coordinate transformation on the brane X^I that diagonalizes the pertinent induced-metric elements in (15)⁴. For instance,

³ The important implications for non-thermal particle production and decoherence for a spectator low-energy field theory in such space-times were discussed in [23,20], where the D -particle recoil case was considered.

⁴ Note that general coordinate invariance is assumed to be a good symmetry on the brane, away from the ‘boundary’ $X^I = 0$.

to leading order in the deformation couplings $\bar{g}_{Iz}\bar{g}_{Jz}$, one may redefine the X^I coordinates by

$$\begin{aligned} X^I &\rightarrow X^I - \frac{\epsilon^2}{8} z^2 \bar{g}_{Iz} \sum_{J \neq I} \bar{g}_{Jz} X^J, \\ z &\rightarrow z \left(1 + \frac{\epsilon^2}{8} \sum_{I \neq J} \bar{g}_{Iz} \bar{g}_{Jz} X^I X^J \right) \end{aligned} \quad (16)$$

which leaves only diagonal elements of the metric tensor on the (redefined) hyperplane X^I . In this case, the metric becomes, to leading order in g_{Iz}^2 and in the case where $\epsilon \bar{g}_{Iz} z \ll 1$:

$$\begin{aligned} ds^2 &= -d\phi^2 + (1 - \alpha^2 z^2) (dX^I)^2 + (1 + \alpha^2 (X^I)^2) dz^2 - \epsilon \bar{g}_{Iz} z dX^I d\phi, \\ \alpha &= \frac{1}{2} \epsilon \bar{g}_{Iz} \sim g_s |\Delta P_z| / M_s \end{aligned} \quad (17)$$

where the last expression makes it clear that, upon utilizing (7, 8), one can express the parameter α (in the limit $\epsilon \rightarrow 0^+$) in terms of the (recoil) momentum transfer along the bulk direction. As we see later on, this parameter is responsible for the mass hierarchy in the problem, assuming that the string scale M_s is close to Planck mass scale 10^{18} GeV, for ordinary string-theory couplings of order $g_s^2/2\pi = 1/20$.

A last comment, which is important for our purposes here, concerns the case in which the metric (17) is *exact*, i.e., it holds to all orders in $\bar{g}_{Iz} z$. This is the case where there is no world-sheet tree-level momentum transfer. This naively corresponds to the case of static intersecting branes. However, the whole philosophy of recoil [18,19] implies that, even in that case, there are quantum fluctuations induced by the sum over genera of the world sheet. The latter implies the existence of a statistical distribution of logarithmic deformation couplings of Gaussian type about a mean field value $\bar{g}_{Iz}^c = 0$. Physically, the couplings \bar{g}_{Iz} represent recoil velocities of the intersecting branes, hence these Gaussian fluctuations represent the effects of quantum fluctuations about the zero recoil-velocity case, which may be considered as quantum corrections to the static intersecting-brane case. We therefore consider a statistical average $\langle\langle \dots \rangle\rangle$ of the line element (15)

$$\begin{aligned} \langle\langle ds^2 \rangle\rangle &= -d\phi^2 + \left(1 - \frac{1}{4} \epsilon^2 \langle\langle \bar{g}_{Iz} \bar{g}_{Jz} \rangle\rangle z^2 \right) dX^I dX^J + \\ &\left(1 + \frac{1}{4} \epsilon^2 \langle\langle \bar{g}_{Iz} \bar{g}_{Jz} \rangle\rangle X^I X^J \right) dz^2 - \epsilon \langle\langle \bar{g}_{Iz} \rangle\rangle z dX^I d\phi, \end{aligned} \quad (18)$$

where

$$\langle\langle \dots \rangle\rangle = \int_{-\infty}^{+\infty} d\bar{g}_{Iz} (\sqrt{\pi}\Gamma)^{-1} e^{-\bar{g}_{Iz}^2/\Gamma^2} (\dots) \quad (19)$$

and the width Γ has been calculated in [19], is found after summation over world-sheet genera to be proportional to the string coupling g_s . In fact [19], it can be shown that Γ scales as $\epsilon \bar{\Gamma}$, where $\bar{\Gamma}$ is independent of ϵ . This will be important later on, when we consider the identification of ϵ with the target time t .

This is the model of string-inspired space-time foam used in this work. The quantum recoil fluctuations reflect the response of the D brane to the emission of virtual closed strings or other branes.

We see from (19), assuming that $g_{Iz} = |U_i|$ where $U_i = g_s \Delta P_i / M_s$ is the recoil velocity [18,19], that the average line element ds^2 becomes:

$$\begin{aligned} \langle\langle ds^2 \rangle\rangle &= -d\phi^2 + (1 - \alpha^2 z^2) (dX^I)^2 + (1 + \alpha^2 (X^I)^2) dz^2, \\ \alpha &= \frac{1}{2\sqrt{2}} \epsilon^2 \bar{\Gamma} \end{aligned} \quad (20)$$

The definition of α comes from evaluating the quantity $\langle\langle \bar{g}_{Iz}^2 \rangle\rangle$ using the statistical distribution (19). Thus the average over quantum fluctuations leads to a metric of the form (17), but with a parameter α much smaller, being determined by the width (uncertainty) of the pertinent quantum fluctuations [19]. The metric (20) is exact, in contrast to the metric (17) which was derived for $z \ll 1/\alpha$. However, for our purposes below we shall treat both metrics as exact solutions of some string theory associated with the recoil.

An important feature of the line element (20) is the existence of a *horizon* at $z = 1/\alpha$ for *Euclidean* Neumann coordinates X^I . Since the Liouville field ϕ has decoupled after the averaging procedure, one may consider slices of this field, defined by $\phi = \text{const}$, on which the physics of the observable world can be studied. From a world-sheet renormalization-group view point this slicing procedure corresponds to selecting a specific point in the non-critical-string theory space. Usually, the infrared fixed point $\phi \rightarrow \infty$ is selected. In that case, from (5), one considers a slice for which $\epsilon^2 \rightarrow 0$. But any other choice could do, so α may be considered a small but arbitrary parameter of our effective theory. The presence of a horizon raises the issue of how one could analytically continue so as to pass to the space beyond the horizon. The simplest way, compatible, as we shall show later with the low-energy Einstein's equations, is to take the absolute value of $1 - \alpha^2 z^2$ in the metric element (17).

We next note that in [25], where the above metric was initially derived as an extension of the D -particle case of [23], the metric was defined in all space $z \in R$ on a *slice* of the Liouville time $\phi = \text{const}$ by:

$$ds_f^2 = |1 - \alpha^2 z^2| (dX^I)^2 + (1 + \alpha^2 (X^I)^2) dz^2, \quad (21)$$

For small α , which is the case studied here, and for Euclidean Neumann coordinates X^I , the scale factor in front of the dz^2 term does not introduce any singular behaviour, and hence for all qualitative purposes one may study the metric element [25]:

$$ds_f^2 = |1 - \alpha^2 z^2| (dX^I)^2 + dz^2, \quad (22)$$

which shares all the qualitative features of the full metric (21), induced by the recoil process in the case of an uncompactified ‘bulk’ Dirichlet dimension z , as we consider here ⁵.

Such a metric has been shown to satisfy Einstein’s equations in the bulk, provided there is a non-trivial vacuum energy on the brane world at $z = 0$, associated with the excitation due to the recoil [25]. An important point to note is that, formally, our analysis leading to (20) is valid in the region of bulk space-time for which $z > 0$. However, one may consider a *mirror* extension of the space-time to the region $z < 0$, which we assume in this article. From now on, therefore, we treat the metric (22) as being defined over the entire real axis for the bulk coordinate $z \in R$. However, to make contact with the original recoil picture we restrict ourselves to regions of space-time for which $X^I > 0$.

Non-trivial physics is also obtained if, following the spirit of [5], one *identifies*, in the expression for the metric (20), the Liouville mode ϕ with the target time X^0 . In that case, despite the fact that the coordinates X^I are *Euclidean*, one obtains a Minkowskian signature for the (Liouville) time. Notice that the Liouville time now is not fixed, in contrast to the case considered in [25], but ‘runs’ along generalized world-sheet renormalization-group trajectories of the associated Liouville string.

In this case the metric element (20) does not satisfy the classical Einstein’s equations in the bulk, but the ‘off-shell’ Liouville conditions for the σ -model couplings $\{g^i\}$, corresponding to σ -model backgrounds of graviton, dilaton and appropriate matter fields in the low-energy limit of the associated string theory [7,5]:

$$\ddot{g}^i + Q\dot{g}^i = -\tilde{\beta}^i \quad (23)$$

where the dot denotes a derivative with respect to the (world-sheet zero mode of the) Liouville field, $d/d\phi$, and the $\tilde{\beta}^i$ are the appropriate Weyl anomaly coefficients on a flat world-sheet [5], which are related to the ordinary world-sheet renormalization β functions by $\tilde{\beta}^i = \beta^i + \delta g^i$. The quantities δg^i denote terms expressing appropriate target-space diffeomorphism variations of the fields g^i . The presence of $\tilde{\beta}^i$ instead of β^i is necessitated by the target-space diffeomorphism invariance of the σ model ⁶.

In the case of [25], where a slice at an infrared fixed point $\phi \rightarrow \infty$ of the Liouville mode was considered, both sides of (23) *vanish*, corresponding to the independence of the couplings/fields $\{g^i\}$ from the Liouville mode, and the satisfaction of Einstein’s and other field equations derived from a low-energy string effective action.

In the present case, the background (20) is constrained to satisfy (23), with the field ϕ identified as the target-space time X^0 in the solution of (23). This needs a careful consistency check. In general, the problem of solving (23) in an arbitrary number of target space dimensions is very complicated. Even in toy

⁵ For the case of compact dimension z the situation changes drastically, since in that case there are angular deficits in the cycle around z [22].

⁶ Despite the fact that, eventually, it will be broken by the recoiling background, this symmetry may be assumed formally valid for the σ models at hand.

models of Liouville cosmology with two target space-time dimensions [26] the solutions of such equations, with the Liouville mode being identified with the target time X^0 , are but partially known, and only numerically at present.

However, the situation may become simpler in certain regions of the bulk space time. Indeed, far away from the horizon at $|z| = 1/\alpha$, i.e., for $\alpha^2 z^2 \ll 1$, the line element corresponding to the space-time (20) after the identification $\phi = X^0$ becomes:

$$ds^2 \simeq -\alpha^2 z^2 (dX^0)^2 + dz^2 + \sum_{i=1}^3 (dX^i)^2 \quad (24)$$

implying that X^0 plays now the rôle of a *Minkowskian*-signature temporal variable, despite its original Euclidean nature. This is a result of the identification $\phi = X^0$, and the fact that ϕ appeared with Minkowskian signature due to the supercriticality (cf. (12)) of the Liouville string under consideration.

Notice that the space time (24) is flat, and hence it satisfies Einstein's equations, formally. However, the space time (24) has a *conical* singularity when one compactifies the time variable X^0 on a circle of finite radius corresponding to an inverse 'temperature' β . Formally, this requires a Wick rotation $X^0 \rightarrow iX^0$ and then compactification, $iX^0 = \beta e^{i\theta}$, $\theta \in (0, 2\pi]$. The space-time then becomes a *conical* space-time of Rindler type

$$ds_{conical}^2 = \frac{1}{4\pi^2} \alpha^2 \beta^2 z^2 (d\theta)^2 + dz^2 + \sum_{i=1}^3 (dX^i)^2 \quad (25)$$

with deficit angle $\delta \equiv 2\pi - \alpha\beta$. We recall that there is a 'thermalization theorem' for this space-time [27], in the sense that the deficit disappears and the space-time becomes regular, when the temperature is fixed to be

$$T = \alpha/2\pi \quad (26)$$

The result (26) may be understood physically by the fact that α is essentially related to recoil. As discussed in [23], the problem of considering a suddenly fluctuating (or recoiling) brane at $X^0 = 0$, as in our case above, becomes equivalent to that of an observer in a (non-uniformly) accelerated frame. At times long after the collision the acceleration becomes uniform and equals α . This implies the appearance of a non-trivial vacuum [27], characterized by thermal properties of the form (26). At such a temperature the vacuum becomes just the Minkowski vacuum, whilst the Unruh vacuum [27] corresponds to $\beta \rightarrow \infty$. Here we have derived this result in a different way than in [23], but the essential physics is the same. Notice that the presence of such a 'thermal' effect is a manifestation of the breaking of Lorentz invariance induced by D -brane recoil. We study further observational consequences of this effect in Section 7.

4 Vacuum Energy and Expansion of the Universe Induced by D -Brane Recoil

In the picture envisaged above, where our world is viewed as a fluctuating D -brane, one may consider more complicated configurations of intersecting branes. The simplest of all cases is the one depicted in Fig. 3, in which a D particle is embedded in a Euclidean $D4$ brane, which is itself embedded in a higher-dimensional (bulk) space-time.

In this case, any low-energy string state living on the $D3$ brane which scatters off the embedded $D0$ brane will cause recoil of the latter and hence distortion of space-time, according to the above discussion. The distortion is such as to induce a cosmological constant on the $D3$ brane, as discussed in detail in [10]. The simplest case to consider is that in which the embedded defect is very heavy, so that one may ignore its recoil velocity and consider only its quantum fluctuations, which are associated with the uncertainty in its position, i.e., with the C operator in (1). In that case, as discussed in [10], the space-time surrounding the recoiling (fluctuating) D particle becomes at first sight anti-de-Sitter ⁷, with a small negative cosmological constant of the form:

$$V_{cosm\ const}^{bulk} = -4D(D-1)\epsilon^4 \cdot M_s^4 \quad (27)$$

where D is the dimensionality of the target space-time in which the D particle is embedded, and $\epsilon^2 \sim \text{Ln}(L/a)$ is related to the world-sheet size of the non-critical string, and thus to the renormalization-group scale, on account of (5).

However, upon the identification of this scale ϵ^{-2} with the (Euclideanized) target time X^0 , one observes [10] that when one continues analytically the Euclidean time X^0 to a Minkowskian time variable it , the quantity $\epsilon^2 \rightarrow i\epsilon^2$. This implies that the ‘vacuum energy’ due to the recoiling D particle in Fig. 3 becomes *positive*, and relaxes to zero asymptotically as $1/t^2$. As discussed in [10]

⁷ To be precise, the corresponding stringy σ model becomes conformally equivalent (i.e., up to marginal deformations on the world-sheet) to a σ model in an anti-de-Sitter target space-time.

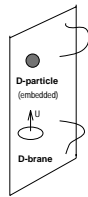


Fig. 3. The world as a $D3$ brane ‘punctured’ by D particles ($D0$ branes). The scattering on the $D0$ brane of string states, either closed (gravitons) or open (matter fields) that live on the $D3$ brane, cause the $D0$ brane to recoil, leading to stochastic effects in the propagation of the low-energy states, as well as to non-zero ‘vacuum’ energy on the $D3$ brane.

such a time dependence may not be inconsistent with recent astrophysical observations [28].

The identification of the scale ϵ^{-2} with the Euclideanized target time X^0 raises the question whether some variants of Einstein's equations are satisfied in this case. We note that, upon this identification, the quantity α in (24) depends on the time t , since by definition it depends on ϵ . Thus, the resulting metric is no longer flat. Even in this case, however, the satisfaction of Einstein's equations is guaranteed in the bulk space-time, as shown in [12], provided there is a non-trivial dilaton field of the form:

$$\varphi \propto \ln t \quad (28)$$

In fact, as emphasized in [12], an important rôle is played by averaging over quantum fluctuations in the position of the $D3$ brane along the bulk direction in the case of Fig. 3. Such fluctuations are properly taken into account by summing up over world-sheet topologies, as explained in [19]. This leads effectively to a dimensional reduction in the associated geometry, given that in this picture the extra (bulk) dimensions are viewed as couplings of a σ model over which a four-dimensional observer averages [12].

If one denotes by σ^2 the corresponding uncertainty in the position of the $D3$ brane along the fifth (bulk) dimension in the geometry of Fig. 3, then the analysis of [12] has demonstrated that the distortion of the (four-dimensional) space-time due to such fluctuations, within the framework of the identification of the scale ϵ^{-2} with the target time t , can be described by the following invariant line element:

$$\langle\langle ds^2 \rangle\rangle^{(4)} = \frac{b^2\sigma^2}{t^2}(dt)^2 - \left| 1 - \frac{b^2\sigma^2}{t^2} \right| \sum_{i,j=1}^3 \delta_{ij} dx^i dx^j, \quad b = \bar{T}/2\sqrt{2} \quad (29)$$

where \bar{T} is related to the uncertainty in the momentum of the $D3$ brane, as defined in (19).

Notice that the product $b^2\sigma^2$ is just the momentum-position uncertainty product of the quantum-fluctuating $D3$ brane, which is saturated by the uncertainty principle in its stringy version, as discussed in [19]. Its precise value depends on the form of the recoiling D -brane state, something which at present cannot be known exactly. It is the lack of such knowledge that prevents us, at present, from determining dynamically the scale α , and thus deriving dynamically (or excluding phenomenologically!) the induced hierarchy of mass scales. In our opinion, this problem is still unsolved, given that even in the superstring/supermembrane scenaria of [29], the induced low-energy scale is introduced in an *ad hoc* manner, constrained only by consistency with phenomenology.

We now remark that, if one ignores the recoil fluctuations of the D particle in Fig. 3, and considers only the fluctuations of the $D3$ brane, then the induced 'vacuum energy' on the brane A has the form:

$$A = \frac{(5b^2 - 8)}{t^2} M_s^4, \quad (30)$$

In addition to this field-independent vacuum energy, one also obtains a positive-definite excitation energy V for the $D3$ brane, due to the fact that the recoiling D brane finds itself in an excited state rather than in its ground state. The result is [12]:

$$V = \frac{(2b^2 + 4)}{t^2} M_s^4, \quad (31)$$

where we have passed to a Minkowskian target time t in (30) and (31).

The analysis of [12] showed that, in the classical limit where the position uncertainty of the $D3$ brane vanishes, $\sigma \rightarrow 0$, the dilaton equation of motion forces a dynamical constraint on the width parameter $b^2 = 8/5$, which, *remarkably*, constrains the cosmological constant Λ to vanish. Notably the above value of b is compatible with the supercriticality of the σ model, which is essential for the self-consistency of the identification of the Liouville mode with the target time X^0 . In the case where $\sigma \neq 0$, one may still demand the cosmological constant to vanish, but then this constrains the uncertainty σ to a particular value that nearly saturates position-momentum uncertainty for the $D3$ brane: $\sigma = 5/(\sqrt{96}M_s) \simeq 1/(\sqrt{2}M_s)$. We remark, comparing the contributions (30, 31) to the vacuum and excitation energies (27), that fluctuating defects on the $D3$ brane in the geometry of Fig. 3 yield contributions to the vacuum energy on the brane that scale similarly with the target time, i.e., as $1/t^2$.

In the case that the recoiling (momentum) fluctuations of the embedded D particle are not ignored, one obtains additional contributions to the four-dimensional ‘vacuum energy’. To see this, we recall that the four-dimensional space-time, on which the defect is embedded, is to be viewed as a bulk space-time from the point of view of the world-sheet approach to the recoil of the D particle. Following the same approach as that leading to (24), involving the identification of the Liouville field with the target time, t , one observes again that there exists an (expanding) horizon, located at $r^2 \equiv x^2 + y^2 + z^2 = t^2/b'^2$ where b' is related to the momentum uncertainty of the fluctuating D particle, and $\{x_i\}$, $i = 1, \dots, 3$ constitute the bulk dimensions, obeying Dirichlet boundary conditions from a world-sheet view point. For the region of space time *inside the horizon* one obtains the following metric on the $D3$ brane, as a result of recoil of the D particle embedded in it:

$$ds^{2(4)} \simeq \frac{b'^2 r^2}{t^2} (dt)^2 - \sum_{i=1}^3 (dx^i)^2, \quad r^2 = \sum_{i=1}^3 x_i^2 < t^2/b'^2 \quad (32)$$

We can show easily that this metric is a solution of Einstein’s equations in a four-dimensional space-time $\{x_i, t\}$, with a vacuum energy $-\Lambda$ that is constant in time, but position-dependent:

$$\Lambda = \frac{2}{r^2}; \quad (33)$$

and a four-dimensional dilaton field of the form:

$$\varphi = \ln r + b' \ln t \quad (34)$$

Incidentally, we remark that the scalar curvature corresponding to the metric (32) has the form $R = -4/r^2$, and as such has a singularity at the initial location $r = 0$ of the D -particle defect, as expected.

We next notice, from the forms of the metrics (29) and (32), that the choice of time $t \sim \epsilon^{-2}$, i.e., directly proportional to the Liouville world-sheet zero mode, is not the appropriate one for a Friedmann-Robertson-Walker (FRW) Universe, which describes to a good approximation the world around us. It is straightforward to see that the metric (29) becomes of FRW type upon the transformation

$$t \rightarrow t_{FRW} = b \sigma \ln t \quad (35)$$

We then observe from (28) that the dilaton is *linear* [24] in this time coordinate: $\varphi \propto t_{FRW}/b\sigma$ and that the scale factor increases as $|1 - b^2\sigma^2 e^{-2t_{FRW}/b\sigma}|$, whilst the various contributions (30, 31) to the vacuum and/or excitation energies become *exponentially suppressed*.

However, we also remark that there is a model in which the $1/t^2$ scaling pertains directly to the Friedmann-Robertson-Walker time coordinate. This is the original case described in [10], where the world was not viewed as a brane, but there were simply D particles embedded in the four-dimensional space-time. In such a case, the recoil of the D -particle defect is also described by a Liouville theory, but in that case the coefficient of the Liouville coordinate is unity, which leads to a direct identification of the Liouville field with Friedmann-Robertson-Walker time.

5 Energy Conditions and Horizons in Recoil-Induced Space-Times

It is interesting to look at the energy conditions of such space times, which would determine whether ordinary matter can exist in such regions. As is well known there are various forms of energy conditions [30], which may be expressed as follows:

$$\begin{aligned} \text{Strong} & : \left(T_{\mu\nu} - \frac{1}{D-2} g_{\mu\nu} T^\alpha_\alpha \right) \xi^\mu \xi^\nu \geq 0, \\ \text{Dominant} & : T_{\mu\nu} \xi^\mu \eta^\nu \geq 0, \\ \text{Weak} & : T_{\mu\nu} \xi^\mu \xi^\nu \geq 0, \\ \text{Weaker} & : T_{\mu\nu} \zeta^\mu \zeta^\nu \geq 0. \end{aligned} \quad (36)$$

where $g_{\mu\nu}$ is the metric, and $T_{\mu\nu}$ is the stress-energy tensor in a D -dimensional space time, including vacuum energy contributions, ξ^μ, η^μ , are arbitrary future-directed time-like or null vectors, and ζ^μ is an arbitrary null vector. The conditions have been listed in decreasing strength, in the sense that each condition is implied by all its preceding ones.

It can be easily seen from Einstein's equations for the metric (32) that inside the horizon $b'^2 r^2 \leq t^2$ the conditions are satisfied, which implies that stable

matter can exist *inside* such regions of the recoil space time. On the other hand, *outside the horizon* the recoil-induced metric assumes the form:

$$ds^{2(4)} \simeq \left(2 - \frac{b'^2 r^2}{t^2}\right) (dt)^2 - \sum_{i=1}^3 (dx^i)^2, \quad r^2 > t^2/b'^2 \quad (37)$$

The induced scalar curvature is easily found to be:

$$R = -4b'^2 (-3t^2 + b'^2 r^2) / (-2t^2 + b' r^2)^2 .$$

Notice that there is a *curvature* singularity at $2t^2 = b'^2 r^2$, which is precisely the point where there is a signature change in the metric (37).

Notice also that, in order to ensure a Minkowskian signature in the space-time (37), one should impose the restriction

$$2 > \frac{b'^2 r^2}{t^2} > 1 ; \quad (38)$$

It can be easily shown that the weaker energy condition (36) can be satisfied for times t such that

$$\frac{b'^2 r^2}{t^2} \simeq 1 + \varepsilon; \quad \varepsilon \rightarrow 0^+ \quad (39)$$

i.e., on the initial horizon. To see this, it suffices to notice that the weaker energy condition reads in this case:

$$(4t^2 - b'^2 r^2) \left(2 - \frac{b'^2 r^2}{t^2}\right) (\zeta^0)^2 \leq b'^2 \left(\sum_{i=1}^3 x^i \zeta^i\right)^2 \quad (40)$$

where we used the fact that ζ^μ is a null vector. The right-hand-side of the above inequality can be bounded from above by

$$3b'^2 r^2 \sum_{i=1}^3 (\zeta^i)^2 = 3b'^2 r^2 \left(2 - \frac{b'^2 r^2}{t^2}\right) (\zeta^0)^2, \quad (41)$$

which, on account of the requirement (40) would imply $(t^2 - b'^2 r^2) \leq 0$. This is in contradiction with the range of validity of (37), unless one lies on the initial horizon (39). Notice that in this region of space-time there is a smooth matching between the interior (32) and the exterior (37) geometries. In such regions of space time, surrounding the recoiling defect, matter can exist in a *stable form*.

The above considerations suggest that matter can be trapped *inside* such horizon regions around a fluctuating D -particle defect. This sort of trapping is interesting for our space-time-foam picture, as it implies that such *microscopic D-brane horizons act in a similar way as black-hole horizons*. The *dynamical* formation of horizons that trap matter around a recoiling heavy D -particle defect is reminiscent of black-hole horizons obeying the cosmic censorship hypothesis. Of course, this situation pertains to a single defect embedded in the four-dimensional space-time. The extension to a multi-defect situation, in which D -particle defects appear as quantum fluctuations rather than real defects, is still

pending, even in a dilute-gas approximation. This would be closer to a realistic space-time foam picture ⁸.

6 Supersymmetry Breaking Induced by D -Brane Recoil

The thermalization of the region of the bulk space time $\alpha^2 z^2 \ll 1$ implied by (26) also implies that any low-energy effective field theory of excitations that are constrained to live on the brane world $\{X^I\}$ at $z = 0$ will be exposed to this (small but) finite temperature. This has important implications for symmetry obstruction, as discussed in [11,31]. The recoiling D -brane world constitutes an excited state, and the finite induced temperature breaks both the Lorentz symmetry and the supersymmetry (if the latter is assumed) of the low-energy effective theory. The symmetry breaking is an obstruction, in the sense of [31], rather than a spontaneous breaking, because the D brane is not in its ground state. Notice that the mathematical origin of the obstruction may be traced back to the impossibility of defining globally covariantly-constant spinors in space-times with deficits [32]. The induced supersymmetry breaking may be studied explicitly using the ‘thermal superspace’ (TS) formalism of [33]. For simplicity, we review here only the basic results of this formalism that are relevant for our purposes. The interested reader is advised to study [33] for further details.

A point in TS is described by, in addition to the usual space-time coordinates, Grassmann parameters that, in contrast to the zero-temperature case, are *time-dependent* and *antiperiodic* in the imaginary time on the interval $(0, \beta]$:

$$\widehat{X} = \left(\widehat{x}^\mu, \widehat{\theta}^\alpha(t), \widehat{\bar{\theta}}^{\dot{\alpha}}(t) \right) \quad (42)$$

Here the \widehat{X} notation denotes TS quantities, and

$$\widehat{\theta}^\alpha(t + i\beta) = -\widehat{\theta}^\alpha(t), \quad \widehat{\bar{\theta}}^{\dot{\alpha}}(t + i\beta) = -\widehat{\bar{\theta}}^{\dot{\alpha}}(t). \quad (43)$$

Due to this time dependence in the parameters $\widehat{\theta}$, the spinor parameters of the infinitesimal transformations $\widehat{\epsilon}$ in thermal supersymmetry are also *time-dependent*, in contrast to the zero-temperature supersymmetric case, where such spinors are space-time constants. For instance, a scalar superfield $\widehat{\phi}(x, \widehat{\theta}, \widehat{\bar{\theta}})$ transforms in a way that is formally analogous to the $T = 0$ case [33], but with the important difference that ϵ is now time-dependent and antiperiodic in temperature:

$$\delta \widehat{\phi} = i \left(\widehat{\epsilon}^\alpha \widehat{Q}_\alpha + \widehat{\bar{\epsilon}}_{\dot{\alpha}} \widehat{\bar{Q}}^{\dot{\alpha}} \right), \quad \widehat{\epsilon}(t + i\beta) = -\widehat{\epsilon}(t) \quad (44)$$

⁸ The above formulae assume that the defects are point-like. However, if one views the defects as having small dimensions, which could come from the compactification of extra dimensions as in the case of string theory, then the analysis of [25] can be appropriately extended to this case in a way that does not qualitatively change the above conclusions.

It should be stressed that the time dependence of the supersymmetry parameters is essential for the breaking of the thermal supersymmetry at finite temperature [33], and, essentially, arises from the difference in boundary conditions between bosons and fermions.

Using such a TS formalism, one may calculate *thermal mass splittings* for the various supermultiplets. For instance, in the case of the four-dimensional Wess-Zumino model, examined in [33], the mass splittings at $T \neq 0$ between the bosonic and fermionic degrees of freedom in the model are of the form:

$$M_{3,n}^B = M_4^2 + \frac{4\pi^2 n^2}{\beta^2}, \quad M_{3,n}^F = M_4^2 + \frac{\pi^2 (2n+1)^2}{\beta^2}. \quad (45)$$

where M_4 is the $T = 0$ mass of the Wess-Zumino model. The notation $M_{3,n}^{B,F}$ denotes the thermal mass of the n 'th mode for bosons (B) or fermions (F) in the effective three-dimensional Euclidean theory at temperature $T = \beta^{-1}$, obtained after the thermal expansion.

It is clear from (45) that the mass degeneracy of the $T = 0$ (rigid) supersymmetry is lifted, signalling *thermal supersymmetry breaking* at the level of the thermal fields. Such a breaking is also manifest in the finite-temperature action of the model, in the sense that the thermal supersymmetric variations of the action are proportional to the time-derivative $\partial_t \hat{\epsilon}$ (in a Wick-rotated time), which is non-trivial. Expressed in terms of Matsubara thermal modes, the total variation of the action is then found to be proportional to $\omega_n^F \equiv \pi^2 \beta^{-2} (2n+1)^2 \sim T$, as a consequence of thermal supersymmetry breaking. Obviously, in the limit $T \rightarrow 0$, the thermal breaking of supersymmetry evaporates.

We now come to our D -brane case, where, as we have seen, there is a ‘thermalization’ of the space-time surrounding a recoiling D brane (26). The lifting of the mass degeneracy discussed above results in a non-trivial contribution to the vacuum energy. It should be noted that the D -brane recoil deformations (1) are essentially higher-genus effects in string theory, as discussed in detail in [5]. Such effects are target-space quantum effects. In ordinary field theories, it is well known how quantum effects contribute to the effective potential, which has crucial consequences for the so-called gauge hierarchy problem in field theories with spontaneous breaking of a gauge symmetry. For (rigid) supersymmetric theories, for instance, one-loop contributions to the effective potential have the generic form [34,35]:

$$V = V_0 + \frac{1}{64\pi^2} \text{Str} \mathcal{M}^0 \cdot \Lambda^4 \cdot \ln \frac{\Lambda^2}{\mu^2} + \frac{1}{32\pi^2} \text{Str} \mathcal{M}^2 \cdot \Lambda^2 + \frac{1}{64\pi^2} \text{Str} \mathcal{M}^4 \cdot \ln \frac{\mathcal{M}^2}{\Lambda^2} + \dots \quad (46)$$

where Λ is an ultraviolet cut-off for the low-energy effective field theory, which, in our context, may be taken to be of order of the string scale M_s , V_0 is the classical potential, the \dots denote Λ -independent contributions, μ is a scale parameter, and the Supertrace $\text{Str} \mathcal{M}^n$ is defined as:

$$\text{Str} \mathcal{M}^n \equiv \sum_i (-1)^{2J_i} (2J_i + 1) m_i^n \quad (47)$$

where the m_i are the mass eigenvalues of the various field species, and the J_i are their spins. In rigid supersymmetric theories, $\text{Str}\mathcal{M}^n = 0$ as a result of the mass degeneracy within the supermultiplets. This is not the case for $n > 0$, if supersymmetry is broken dynamically. In our case of D -brane recoil (45), there is a recoil-induced dynamical split between the modes, due to the breaking of the thermal supersymmetry. In this case the various supertraces involve sums over thermal modes which need regularization. Typically, due to the fermionic contributions, there are $\zeta(0)$ terms appearing, where ζ is the Riemann zeta function. For instance, from (45) one would expect qualitatively a contribution to the effective potential in the Euclidean $d = 3$ -dimensional theory, after thermal expansion, of the form:

$$V \ni \frac{M_{uv}^2}{32\pi^2} \text{Str}\mathcal{M}^2 \sim \sum_{i=1}^{N_s} \frac{b^2 M_{uv}^2}{8\pi^2 t^2} \zeta\left(-1, \frac{1}{4}\right) = -\frac{N_s M_{uv}^2 b^2}{384\pi^2 t^2} \quad (48)$$

where N_s is the number of particle species, and M_{uv} is an ultraviolet cut-off mass scale in the low-energy effective theory. It is natural to identify it with the string (or quantum gravity) scale M_s ⁹. We stress that such contributions should be attributed to loop corrections rather than to the classical potential, as they are associated with the higher-genus nature of the recoiling operators (1), as explained in [5] and mentioned above.

It is an important issue whether, if one selects $T \sim \alpha \sim \mathcal{O}(1)\text{TeV} \ll M_{Planck} \sim 10^{19}$ GeV, the supersymmetric ‘solution’ to the hierarchy problem is stable to higher orders in the loop expansion. For theories with softly-broken rigid supersymmetry this is guaranteed, as the corrections to the Higgs mass coming from the logarithmic terms in (46) are at most of order $\mathcal{O}(M_{SUSY}^2)$ ¹⁰. However, the situation changes drastically in supergravity theories. In general, for local supersymmetry spontaneously broken at a scale m_{SUSY} , a gravitino mass $m_{3/2} \sim m_{SUSY} \sim 1$ TeV $\ll M_{Planck}$ does not necessarily guarantee that the hierarchy is stable under quantum corrections [35].

In our string D -brane-inspired model we naturally have supergravity in the brane world $\{X^I\}, I = 0, \dots, 3$. In such a case a detailed thermal expansion, using and extending TS techniques to include supergravity models [33], should be performed before making any quantitative statement about the phenomenological consequences of the recoil-induced supersymmetry breaking on the brane world. However, some qualitative statements may be made. For instance it is natural to ask (at least qualitatively) whether the hierarchy implied by the presence of the scale $\alpha \ll M_s$, with the string scale $M_s \sim 10^{18}$ GeV, is stable against quantum corrections.

The interesting issue is whether realistic phenomenology can be achieved (at least within the one-loop approximation made in the above treatment of

⁹ In our approach we take M_s to be high, near the Planck scale, and hence we do not distinguish between the gravity and string scales.

¹⁰ We recall that in supersymmetric theories the Λ^4 terms are absent, since there are equal numbers of fermions and bosons in each multiplet, implying that $\text{Str}\mathcal{M}^0 = 0$.

recoiling D -brane world), in the sense that the resulting contributions to the vacuum energy are compatible with astrophysical data and with the scale $\alpha \sim \epsilon \Gamma \sim \mathcal{O}(1)$ TeV expected in the prospective resolution of the gauge hierarchy problem. This depends crucially on our interpretation of ϵ^{-2} as the target time. Indeed, one should note that the width Γ of the Gaussian fluctuations of the recoil momenta ΔP^i entering αM_s , see (20), satisfies an uncertainty relation [19], $\Delta P_i \Delta Y_i \geq \hbar + \mathcal{O}(\ell_s^2 \Delta P_i^2)$, and $\Delta Y_i \geq g_s^{1/3} \ell_s + \dots$, where g_s is the string coupling and $\ell_s \sim M_s^{-1}$ is the string length. Thus ΔP_i can be arbitrarily bigger than M_s , if, for instance, the excited (recoiling) D -brane is localized in the bulk space-time at a distance of order ℓ_s . In principle, of course, just exactly how big ΔP_i would be is determined by the appropriate wave functional of the brane universe, which is beyond any detailed description at present. But phenomenological assumptions may actually shed light on such an important issues in quantum gravity.

In principle, one can arrive in our scenario for space-time foam at a situation where $\alpha \sim \epsilon^2 \overline{T} \sim 1$ TeV, as required by the phenomenological constraints on supersymmetry breaking and the solution to the hierarchy problem, whilst the vacuum energy is compatible with the current observations, simply because there may be cancellations among the various components (30, 31, 27) and (48), which come with different signs, as we have seen. Such deep issues require serious investigations before definite conclusions are reached, but we believe this approach deserves further study.

7 Observable Breakdowns of Lorentz Invariance

In this Section we ignore the possible quantum fluctuations of the $D3$ brane in the geometry of Fig. 3, and concentrate only on the quantum fluctuations of the D particles embedded in the $D3$ brane. This is equivalent to the original case discussed in our earlier work on the subject [9,23], according to which the observed four-dimensional space-time is just punctured with real D -particle defects, without being viewed as a D -brane.

One of the most important consequences of the induced metric (14) is the induced refractive index for massless probes [14] propagating in such space-times, which we now discuss in more detail. It is well known that light propagating through media with non-trivial optical properties may exhibit a frequency-dependent refractive index, namely a variation in the light velocity with photon energy. Another possibility is a difference between the velocities of light with different polarizations, namely birefringence, and a third is a diffusive spread in the apparent velocity of light for light of fixed energy (frequency). Within the framework leading to the induced metric (14), the first [13] and third [5,36] effects have been derived via a formal approach based on a Born-Infeld Lagrangian using D -brane technology¹¹. A different approach to light propagation has been taken in [37], where quantum-gravitational fluctuations in the light-cone have

¹¹ The possibility of birefringence has been raised [15] within a canonical approach to quantum gravity, but we do not pursue such a possibility here.

been calculated. Here we use this formalism to derive a non-trivial refractive index and a diffusive spread in the arrival times of photons of given frequency.

We first review briefly the analysis in [37], which considered gravitons in a squeezed coherent state, the natural result of quantum effects in the presence of black holes. Such gravitons induce quantum fluctuations in the space-time metric, in particular fluctuations in the light-cone [37], i.e., stochastic fluctuations in the velocity of light propagating through this ‘medium’. Following [37], we consider a flat background space-time with a linearized perturbation, corresponding to the invariant metric element $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu = dt^2 - d\vec{x}^2 + h_{\mu\nu}dx^\mu dx^\nu$. Let $2\sigma(x, x')$ be the squared geodesic separation for any pair of space-time points x and x' , and let $2\sigma_0(x, x')$ denote the corresponding quantity in a flat space-time background. In the case of small gravitational perturbations about the flat background, one may expand $\sigma = \sigma_0 + \sigma_1 + \sigma_2 + \dots$, where σ_n denotes the n -th order term in an expansion in the gravitational perturbation $h_{\mu\nu}$. Then, as shown in [37], the root-mean-square (RMS) deviation from the classical propagation time Δt is related gauge-invariantly [37] to $\langle \sigma^2 \rangle$ by

$$\Delta t = \frac{\sqrt{\langle \sigma^2 \rangle - \langle \sigma_0^2 \rangle}}{L} \simeq \frac{\sqrt{\langle \sigma_1^2 \rangle}}{L} + \dots \quad (49)$$

where $L = |x' - x|$ is the distance between the source and the detector.

As commented earlier, one may expect Lorentz invariance to be broken in a generic theory of quantum gravity, and specifically in the recoil context discussed earlier in this paper. In the context of string theory, violations of Lorentz invariance entail the exploration of non-critical string backgrounds, since Lorentz invariance is related to the conformal symmetry that is a property of critical strings. As we have discussed, a general approach to the formulation of non-critical string theory involves introducing a Liouville field [7] as a conformal factor on the string world sheet, which has non-trivial dynamics and compensates the non-conformal behaviour of the string background, and we showed in the specific case of D branes that their recoil after interaction with a closed-string state produces a local distortion of the surrounding space-time (14).

Viewed as a perturbation about a flat target space-time, the metric (14) implies that the only non-zero components of $h_{\mu\nu}$ are:

$$h_{0i} = \epsilon^2 \bar{u}_i t \Theta_\epsilon(t) \quad (50)$$

in the case of D -brane recoil. We now consider [36] light propagation along the x direction in the presence of a D -brane-induced metric fluctuation h_{0x} (50) in flat space, along a null geodesic given by $(dt)^2 = (dx)^2 + 2h_{0x}dt dx$. For large times $t \sim \text{Log}A/a \sim \epsilon^{-2}$ [23], $h_{0x} \sim \bar{u}$, and thus we obtain

$$\frac{cdt}{dx} = \bar{u} + \sqrt{1 + \bar{u}^2} \sim 1 + \bar{u} + \mathcal{O}(\bar{u}^2) \quad (51)$$

where the recoil velocity \bar{u} is in the direction of the incoming light ray. Taking into account energy-momentum conservation in the recoil process, which has been derived in this formalism as mentioned previously, one has a typical order

of magnitude $\bar{u}/c = \mathcal{O}(E/M_D c^2)$, where $M_D = g_s^{-1} M_s$ is the D -brane mass scale, with $M_s \equiv \ell_s^{-1}$. Hence (51) implies a subluminal energy-dependent velocity of light [36,14]:

$$c(E)/c = 1 - \mathcal{O}(E/M_D c^2) \quad (52)$$

which corresponds to a *classical* refractive index. This appears because the metric perturbation (50) is energy-dependent, through its dependence on \bar{u} .

The subluminal velocity (52) induces a delay in the arrival of a photon of energy E propagating over a distance L of order:

$$(\Delta t)_r = \frac{L}{c} \mathcal{O}\left(\frac{E}{M_D c^2}\right) \quad (53)$$

This effect can be understood physically from the fact that the curvature of space-time induced by the recoil is \bar{u} - and hence energy-dependent. This affects the paths of photons in such a way that more energetic photons see more curvature, and thus are delayed with respect to low-energy ones.

The absence of superluminal light propagation was found previously via the formalism of the Born-Infeld lagrangian dynamics of D branes [19,5]. Furthermore, the result (53) is in agreement with the analysis of [13], which was based on a more abstract analysis of Liouville strings. It is encouraging that this result appears also in the more conventional general-relativity approach [37], in which the underlying physics is quite transparent.

In addition to the above mean-field effects, there are *stochastic* fluctuations about this mean value, which arise from the summation over world-sheet topologies. The calculation for the D -brane recoil case yields [5]: $\Delta t = \mathcal{O}\left(g_s \frac{EL}{M_D c^3}\right)$, where the suppression by the extra power of g_s , as compared with the mean-field effect (52), is due to the fact that the phenomenon is associated with higher-order string loops. We note that this second effect is *not* associated with any modification of the dispersion relation of the particle probes, but pertains strictly to fluctuations in the arrival times of photons [5,37]. By construction, the effect is associated with energy-dependent quantum fluctuations [36] around the mean value of the recoil velocity along the incident direction.

8 Observational Limits from Data on Gamma-Ray Bursters

Such speculations may be tested directly using experimental data that are already available. Of particular interest are astrophysical sources of energetic photons that are very distant and have short time structures [14], such as Gamma-Ray Bursters (GRBs), Active Galactic Nuclei (AGNs) [38] and pulsars [39]. A model analysis was presented in presented in [40], where we conducted a study of astrophysical data for a sample of GRBs whose redshifts z are known. Fig. 4 shows the data from a typical burster, GRB 970508. We looked (without success) for a correlation with the redshift, calculating the regression measure shown in Fig. 5 for the effect (52) and also its stochastic counterpart [40]. Specifically,

we looked for linear dependences of a possible delay Δt (and spread $\Delta\sigma$) in the arrival times of photons with higher energies, proportional to E/M (E/Λ) and $\tilde{z} \equiv 2 \cdot [1 - (1/(1+z))^{1/2}] \simeq z - (3/4)z^2 + \dots$, where the mass scales M and Λ might be associated with quantum gravity. In the absence of any such effect, we determined limits on M and Λ by constraining the possible magnitudes of the slopes in linear-regression analyses of the differences between the arrival times and widths of pulses in different energy ranges from five GRBs with measured redshifts, as functions of \tilde{z} . Using the current value for the Hubble expansion parameter, $H_0 = 100 \cdot h_0$ km/s/Mpc, where $0.6 < h_0 < 0.8$, we obtained the following limits [40]

$$M \gtrsim 10^{15} \text{ GeV}, \quad \Lambda \gtrsim 2 \times 10^{15} \text{ GeV} \quad (54)$$

on the possible quantum-gravity effects.

We stress that, as emphasized in [40], it is only after a statistically significant population of GRBs with measured redshifts are studied, and possible systematic effects are excluded, that any safe conclusions about quantum gravity effects could be reached. In this respect, searches for quantum-gravity effects using future satellites such as GLAST and AMS [41,42] would be essential.

Before closing this section, we would like to address some recent comments [16] on the refraction-index effect (52) we have discussed above. As pointed out in [16], this effect may be probed using cosmic rays, leading perhaps to the disappearance of the GZK cutoff on Ultra-High-Energy (UHE) cosmic rays. Since this cutoff is not seen in the data [43], this may be observationally acceptable. Similarly, it has recently been pointed out [44] that there should have been an analogous suppression of 20 TeV photons from the active galaxy Markarian 501, which seem, however, to escape being cut off by the universal infrared background. The persistence of 20 TeV photons could perhaps be explained by Lorentz breaking of the form $\sim E/M_P$.

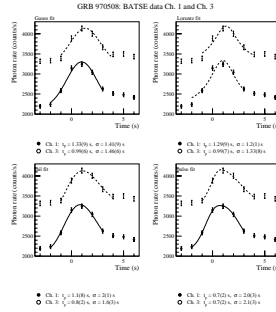


Fig. 4. Time distribution of the number of photons observed by BATSE in Channels 1 and 3 for GRB 970508, compared with the following fitting functions for the observed peaks [40]: (a) Gaussian, (b) Lorentzian, (c) ‘tail’ function, and (d) ‘pulse’ function. We list below each panel the positions t_p and widths σ_p (with statistical errors) found for each peak in each fit. We recall that the BATSE data are binned in periods of 1.024 s.

However, as also pointed out in [16], there is an issue with decays of lower-energy cosmic-ray particles. We observe that there are two additional effects that need to be taken into account in any analysis of such phenomena. One is the possibility/likelihood of stochastic spreading in the velocities of different particles with the same energy, discussed earlier in this paper. Another point is that the possible effects of quantum gravity on decay vertices and interaction processes in ultra-relativistic conditions have not been explored theoretically. We find it quite plausible that more surprises may await us here, and therefore reserve judgement on this issue, limiting ourselves for now to noting that cosmic rays may also provide a valuable window on quantum-gravity effects.

It has also been suggested [45] that neutrino oscillation experiments might also be sensitive to quantum gravity effects. The interpretation of these experiments depends crucially whether there are important flavour differences in any departure from the conventional $E \sim p$ dispersion relation. If not, the neutrino oscillation experiments would be insensitive to such a quantum-gravity effect. On the other hand, any stochastic effect on neutrino propagation could have an important impact on neutrino-oscillation effects, and remains to be analyzed.

9 Outlook

In this talk we have discussed a world-sheet approach to the problem of deformations of space times induced by the recoil of D -brane defects embedded in them during scattering with low-energy string matter (the latter may be real or virtual). As we have stressed here, as well as in our previous works on the subject, the recoil deformations are not simply world-sheet boundary effects, as

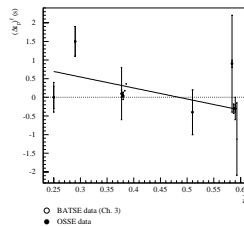


Fig. 5. Values of the shifts $(\Delta t_p)^f$ in the timings of the emission peaks fitted for each GRB studied using BATSE and OSSE data, plotted versus $\tilde{z} = 2(1 - (1 + z)^{-1/2})$, where z is the redshift. The indicated errors are the statistical errors in the ‘pulse’ fits provided by the fitting routine, combined with systematic error estimates obtained by comparing the results obtained using the ‘tail’ fitting function. The values obtained by comparing OSSE with BATSE Channel 3 data have been rescaled by the factor $(E_{min}^{BATSE\ Ch.\ 3} - E_{max}^{BATSE\ Ch.\ 1}) / (E_{min}^{OSSE} - E_{max}^{BATSE\ Ch.\ 3})$, so as to make them directly comparable with the comparisons of BATSE Channels 1 and 3. The solid line is the best linear fit.

one may naively think, but they are associated non-trivially with bulk world-sheet deformations of the corresponding stringy σ models. As a result, conformal invariance in the bulk of the world-sheet is disturbed by their presence, and a standard Liouville theory dressing becomes necessary for its restoration. It is this principle that determines in a unique way the form of the induced deformation in the target-space time surrounding the recoiling D -brane defect.

There are several issues that emerged from our analysis above. The most important of them is the fact that the recoil induces a Rindler structure in the space-time surrounding the D -brane defect. A study of the generic structure of string theories in Rindler spaces is, from a formal view point, a highly-non trivial task. Some critical string theories in Rindler spaces, for given (discrete) values of the acceleration, can be represented as orbifold models [46]. In our case, however, the value of the acceleration α varies continuously with the (world-sheet) renormalization scale, and, as we have seen, a consistent treatment requires the (non-conformal) Liouville formalism. In general, string theories in Rindler-like spaces, or space-times with deficits, are known to have instabilities (tachyons) in their spectrum, even in the supersymmetric case [46], but this in our case is not a drawback, if one takes into account the excited state of the recoiling brane world and its relaxation towards equilibrium.

The presence of such ‘defective’ space-times results in the obstruction of Lorentz symmetry, as well as of rigid (global) supersymmetry at a scale of order of the Rindler ‘acceleration’ α . The symmetry breakdown is an *obstruction* rather than a spontaneous breaking, as the recoiling D -brane is in an excited state rather than its ground state. At present we cannot determine exactly the magnitude of α : this is a dynamical feature of the underlying string theory, and hence can only be determined by knowledge of the pertinent string/ D -brane wavefunctional, which is still lacking.

Another important feature of the recoil-induced space-times is the presence of horizons *inside* which (stable) matter is trapped. Such horizons have great similarities with microscopic space-time boundaries in space-time foam situations.

We have also shown that the recoil-induced space-times possess non-trivial contributions to the vacuum energy, which relax to zero asymptotically in time. In order to carry out realistic phenomenology, it would be necessary to determine exactly the pertinent contributions to the ‘vacuum energy’ on the brane from the various matter fields, which are much more complicated in phenomenologically realistic models than the simplified situation studied above. In general, the presence of matter excitations on the (recoiling or quantum fluctuating) brane may either increase the supersymmetry-breaking scale or reduce the net contribution to the vacuum energy. These issues are left for future work, but we believe that the present considerations motivate further detailed studies along this direction.

Moreover, D -brane recoil appears to induce non-trivial optical properties of space-time, such as a non-zero refractive index and stochastic propagation effects. Such effects are all consequences of the obstructed Lorentz symmetry, and they modify the propagation of matter through such space-times with consequences

that may be observable in the foreseeable future, if the light velocity is modified to $\mathcal{O}(E/M)$ where $M \sim M_P$. We have demonstrated that this is possible in the examples studied in this work. Some of the observable consequences, namely those which are based on observations of light from GRBs and AGNs, have been discussed briefly in this work. Such analyses set at present the limit for the minimum scale of these gravitational effects to $M \gtrsim \mathcal{O}(10^{15})$ GeV.

It should be stressed that, at present, the field is still in its infancy, and may even turn out to be still-born. The mathematical models we have discussed, although consistent, are far from being realistic. However, we believe that the results obtained so far are sufficiently encouraging to motivate further studies in the future. There is even the possibility of experimental verification (or exclusion) of such models, so ‘quantum gravity phenomenology’ may not be an oxymoron.

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