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Macroscopic Interference Effects in Resonant Cavities

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Abstract

We investigate the possibility of interference effects induced by a macroscopic quantum-mechanical superposition of almost orthogonal coherent states - a Schrödinger cat state - in a resonant microcavity. Despite the fact that a single atom, used as a probe of the cat state, on the average only change the mean number of photons by one unit, we show that this single atom can change the system drastically. Interference between the initial and almost orthogonal macroscopic quantum states of the radiation field can now take place. Dissipation under current experimental conditions is taken into account and it is found that this does not necessarily change the interference effects dramatically.

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1 INTRODUCTION

Superpositions of orthogonal states which exhibits macroscopic features appear to be of fundamental importance in recent studies of the foundations of quantum mechanics (see e.g. Ref.[1, 2] and references cited therein). As was noticed by Schrödinger [3], a direct extrapolation of the calculational rules of quantum mechanics to macroscopic systems would lead to the appearance of quantum-mechanical interference effects of classical objects like a living and a dead organism. We have overwhelming empirical evidence that such macroscopic interferences are rare or absent in the real world. It can actually be argued that such quantum mechanical macroscopic superpositions decay with a very short life-time due to environment-induced decoherence [4].

Resonant microcavities can be used to study the behaviour of mesoscopic superpositions coherent states. In order to be specific we will, as an initial state of the cavity radiation field, consider a superposition of coherent states as an example of a Schrödinger "cat" state. The non-dissipative dynamics of the atom-photon interaction, to be described below, is assumed to be described by the Jaynes-Cummings (JC) model [5]. In terms of conventional coherent states $|z\rangle$ parameterised in terms of a complex number z (see e.g. Refs.[6]), this Schrödinger cat state has the following form

$$|z;\phi\rangle = \frac{1}{(2+2\cos\phi\exp(-2|z|^2))^{1/2}}(|z\rangle + e^{i\phi}|-z\rangle) \quad . \tag{1}$$

Such states have been studied in great detail in the literature (see e.g. Refs.[2], [7]-[13]). It can be shown that the results presented below do not crucially depend of the form of the initial cavity state. The essential ingredient is that the overlap probability $|\langle z|-z\rangle|^2 = \exp(-2|z|^2)$ is a sufficiently small number. In our case we will for example consider examples with an average number of photons $\bar{n} = |z|^2 = 49$ and therefore $|\langle z|-z\rangle|^2 \approx 10^{-43}$, indeed a very small number.

The micromaser system [14] is an experimental realisation of the idealised system of a two-level atom interacting with a second quantized single-mode of the electro-magnetic field (for reviews and references see e.g. [15]). The microlaser [16] is the counterpart in the optical regime. Trapping states [17] - [20] have recently been generated in the stationary state of the micromaser system and therefore the generation of Schrödinger cat states in such a system may be feasible [21]. In resonant cavities other possibilities also exist [22] in which case decoherence actually has been studied experimentally. Schrödinger cat states have also been studied experimentally in other systems like in atomic systems [23], and in ion-traps [24]. Related revival phenomena has also been studied for Bose-Einstein condensates [25].

We assume that the atoms which enter the microcavity one at a time, are all prepared in the excited state. Each atom spends a time t in the microcavity interacting with the radiation field. It then leaves the microcavity and the state of the atom is measured. Let $\mathcal{P}_{s_1}(t)$ be the probability that an atom is found in the state $s_1 = \pm$, where $+(-)$ denotes the excited(ground) state, after it leaves the microcavity. Similarly, let $\mathcal{P}_{s_1 s_2}(t)$ be the probability that the next atom is in the state $s_2 = \pm$ if the previous atom has been found in the state s_1. $\mathcal{P}_{s_1}(t)$ exhibits well known revivals (see e.g. Refs.[26]), which has been observed experimentally in microcavity systems [27, 28] and in ion-traps [24]. $\mathcal{P}_{s_1 s_2}(t)$ exhibits in addition so called pre-revivals [29]. In the upper figure of Fig.1 such revivals ($gt_{rev} \approx 2\pi\sqrt{\bar{n}}$) and prerevivals ($gt_{rev} \approx \pi\sqrt{\bar{n}}$) are illustrated for a mesoscopic coherent state with $\bar{n} = 49$, using the physical cavity parameters of Ref.[30]. Damping effects at a non-zero temperature, to be discussed below, are included in Fig.1. In the lower figure of Fig.1 we exhibit the same probabilities but for an even, i.e. $\phi = 0$, Schrödinger cat state with $\bar{n} \approx 49$ and we observe new clear revival phenomena.

It is possible to give a simple physical explanation of these additional revivals, which occur approximately at half of the revival and prerevival times of a coherent state with the same mean-value of photons [31]. It is known that the JC-model has the property that with an initial coherent state of the radiation field, one reaches a pure atomic state at $gt \approx gt_{rev}/2$, independent of the initial atomic state [32]. The coherent states $|z\rangle$ and $|-z\rangle$ used in the construction of the Schrödinger cat state Eq.(1) are approximately orthogonal. These states will then, as independent states, lead to the same atomic state at $gt \approx gt_{rev}/2$ up to a phase and a quantum-mechanical interference pattern will emerge due to the evolution of different "paths" to the same final state. The states $|z\rangle$ and $|-z\rangle$ of the radiation field therefore, in a sense , describe an atomic interferometer. As we will argue below, this interferometer is actually quantum-mechanical since the interference pattern depends on the relative phase ϕ of the Schrödinger cat state Eq.(1).

We therefore have the interesting situation of a superposition of "macroscopic" states, which can be interpreted as a classical interferometer for atoms, and where, in addition, the different "classical slits" of the interferometer interfere quantum-mechanically. With an increasing number of photons in the cavity this interference vanishes rapidly. In fact, if t_{cav} is the decay time of the cavity, decoherence effects will be operative on a time-scale $t_d \approx t_{cav}/\bar{n}(1 + n_b)$, which for $n_b = 0$ agrees with a known result [9]. If $\bar{n}$ is to small revival phenomena will, on the other hand, not be very pronounced. We will argue below that for a realistic experimental situation one can choose a time scale t, such that $t \ll t_d$, and an average number of photons $\bar{n}$ for which the interference effect above still should be realisable in the laboratory.

The present paper is organised as follows. In Section 2 we recapitulate, for the convenience of the reader and in order to define the notation used in the present paper, the basic ingredients in the JC-model. The effect of cavity damping on $\mathcal{P}_{s_1}(t)$ and $\mathcal{P}_{s_1 s_2}(t)$ is discussed in Section 3 extending the analysis of Refs.[7]-[8] to a small and non-zero n_b. In Section 4 we summarise our results together with some final remarks. Some equations used in the main text are summarised in an Appendix.

2 THE DYNAMICAL SYSTEM

The electro-magnetic interaction between a two-level atom, with level separation ω_0, and a single mode of the radiation field in a cavity with frequency ω is described, in the rotating wave approximation, by the JC Hamiltonian [5]

$$H = \omega a^* a + \frac{1}{2}\omega_0 \sigma_z + g(a\sigma_+ + a^*\sigma_-) \quad , \tag{2}$$

where the coupling constant g is proportional to the dipole matrix element of the atomic transition. Here we make use of the Pauli matrices to describe the two-level atom and the notation $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$. The second-quantized single mode electro-magnetic field is described in a conventional manner by means of an annihilation (creation) operator a (a^*), where we have suppressed the cavity mode quantum numbers. For $g = 0$ the atom-field states $|n, s\rangle = |n\rangle \otimes |s\rangle$ are characterised by the quantum number $n = 0, 1, \ldots$ of the oscillator and $s = \pm$ for the atomic levels with energies $E_{n,\pm} = \omega n \pm \omega_0/2$. At resonance $\omega = \omega_0$ the levels $|n-1, +\rangle$ and $|n, -\rangle$ are degenerate for $n \geq 1$ (except for the ground state $n = 0$), but this degeneracy is lifted by the interaction. For an arbitrary coupling g and detuning parameter $\Delta\omega = \omega_0 - \omega$ the system reduces to a 2×2 eigenvalue problem, which may be trivially solved [5]. The result, which describe the entangled system of an atom and the radiation field, is that two new dressed levels, $|n, 1\rangle$ and $|n, 2\rangle$, are formed as superpositions of the previously degenerate ones at resonance according to

$$|\psi_n^+\rangle = \cos\theta_n |n+1, -\rangle + \sin\theta_n |n, +\rangle \quad , \tag{3}$$

$$|\psi_n^-\rangle = -\sin\theta_n |n+1, -\rangle + \cos\theta_n |n, +\rangle \quad , \tag{4}$$

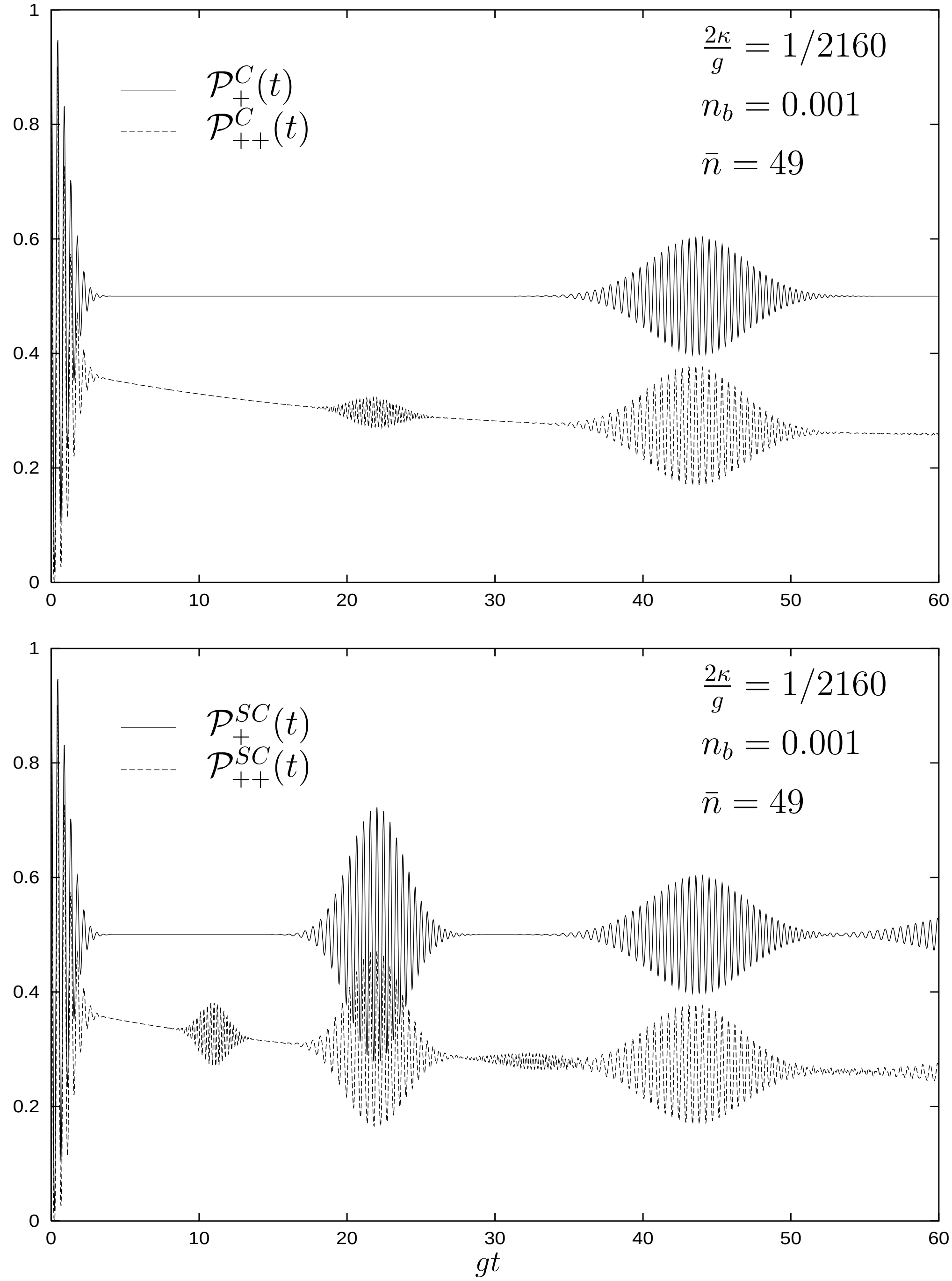

Figure 1: The upper figure shows the revival probabilities $\mathcal{P}_+(t)$ and $\mathcal{P}_{++}(t)$ for (C) a coherent state $|z\rangle$ with a mean number $\bar{n}_C = |z|^2 = 49$ of photons as a function of the atomic passage time gt. The lower figure shows the same revival probabilities for (SC) an even, i.e. $\phi = 0$, Schrödinger cat state $|z;\phi\rangle$ with a mean-value of $\bar{n}_{SC} = |z|^2 \tanh(|z|^2) \approx 49$ of photons. Here we have used physical parameters $\kappa = 8.33\ s^{-1}$ and a Rabi frequency $g = 36\ kHz$ corresponding to the parameters of Ref.[30].

with energies

$$E_n^+ = \omega(n+1/2) + \sqrt{\Delta\omega^2/4 + g^2(n+1)} \quad , \tag{5}$$

$$E_n^- = \omega(n+1/2) - \sqrt{\Delta\omega^2/4 + g^2(n+1)} \quad , \tag{6}$$

respectively. The ground-state of the coupled system is given by $|\psi_0\rangle = |0,-\rangle$ with energy $E_0 = -\omega_0/2$. Here the mixing angle θ_n is given by

$$\tan\theta_n = \frac{2g\sqrt{n+1}}{\Delta\omega + \sqrt{\Delta\omega^2 + 4g^2(n+1)}} \quad . \tag{7}$$

The interaction therefore leads to a separation in energy $\Delta E_n = \sqrt{\Delta\omega^2 + 4g^2(n+1)}$ for the quantum number n. The system performs Rabi oscillations with the corresponding frequency between the original, unperturbed states. The two-level atoms which enter the cavity are assumed to be prepared in the excited state, i.e. the density matrix is of the diagonal form

$$\rho_A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad , \tag{8}$$

The initial density matrix ρ_C of the cavity radiation field is determined by the Schrödinger cat state Eq.(1).

The damping of the cavity is described by a conventional master equation (see e.g. Refs.[33, 34]), i.e. if ρ is the density matrix of the combined atom-field system we have that

$$\frac{d\rho}{dt} = i[\rho, H] - \kappa(n_b+1)(a^*a\rho + \rho a^*a - 2a\rho a^*) - \kappa n_b(aa^*\rho + \rho aa^* - 2a^*\rho a) \quad , \tag{9}$$

where n_b is the average occupation number of thermalized cavity photons at the oscillator frequency and $2\kappa = 1/t_{cav}$ is the decay constant of the cavity.

3 INTERFERENCE EFFECTS

We now define

$$W(t) = e^{iHt}\rho(t)e^{-iHt} \ , \tag{10}$$

which allows us to write the equations for the diagonal elements of Eq. (9) in the form given in the Appendix. By introducing

$$F_n \equiv F_n(t) = \langle \psi_n^+ | W(t) | \psi_n^+ \rangle + \langle \psi_n^- | W(t) | \psi_n^- \rangle \ , \tag{11}$$

for $n \geq 0$ and the ground-state expectation value

$$F_{-1} \equiv F_{-1}(t) = 2 \langle \psi_0 | W(t) | \psi_0 \rangle \ , \tag{12}$$

and by omitting rapidly oscillating terms valid under the assumption $g \ll \kappa$ [7, 8], the equations of motion can be considerably simplified. We obtain

$$\dot{F}_n \equiv \frac{dF_n(t)}{dt} = -\alpha_n F_n + \beta_n F_{n+1} + \gamma_n F_{n-1} \ , \tag{13}$$

where we have defined

$$\begin{aligned} \alpha_n &= 2\kappa \left(2n_b(n+1) + n + \frac{1}{2} \right) \ , \\ \beta_n &= 2\kappa(n_b+1)(n+\frac{3}{2}) \ , \\ \gamma_n &= 2\kappa n_b(n+\frac{1}{2}) \ , \end{aligned} \tag{14}$$

for $n \geq 0$, and where

$$\begin{aligned} \alpha_{-1} &= 2\kappa n_b \ , \\ \beta_{-1} &= 2\kappa(n_b+1) \ , \\ \gamma_{-1} &= 0 \ . \end{aligned} \tag{15}$$

We observe that $\alpha_n - \beta_{n-1} - \gamma_{n+1} = 0$ if $n \geq 1$ and $\alpha_0 - \beta_{-1} - \gamma_1 = -\beta_{-1}/2$. In order to find an approximate solution F_N^* of Eq. (13), valid for sufficiently small n_b, we proceed as follows. For sufficiently large but finite $n = N$, we put $F_{N+1}^* = 0$ as in Ref.[7]. In addition we make use of the approximation $\gamma_n F_{n-1}^* \simeq \gamma_n F_n^*$ for all $n \leq N$. With these approximations the equation of motion for $n = N$ is

$$\dot{F}_N^* = (-\alpha_N + \gamma_N) F_N^* \equiv -\alpha'_N F_N^* \ , \tag{16}$$

with the simple solution

$$F_N^*(t) = F_N^*(0)\, e^{-\alpha'_N t} \ . \tag{17}$$

Next, we solve the equation

$$\dot{F}^*_{N-1} = -\alpha'_{N-1} F^*_{N-1} + \beta_{N-1} F^*_N \ , \tag{18}$$

i.e. by making use of Eq.(17)

$$\ddot{F}^*_{N-1} + \alpha'_{N-1} \dot{F}^*_{N-1} = -\beta_{N-1} \alpha'_N e^{-\alpha'_{N-1} t} F^*_N(0) \ . \tag{19}$$

As one easily can verify, the solution of this equation is

$$F^*_{N-1} = e^{-\alpha'_{N-1} t} \left[\, F^*_{N-1}(0) + \frac{\beta_{N-1}}{\alpha'_{N-1} - \alpha'_N} \left(\, e^{(\alpha'_{N-1} - \alpha'_N) t} - 1 \right) F^*_N(0) \right] \ . \tag{20}$$

Iteration of the previous procedure leads to the approximative solution

$$F^*_n = e^{-2\kappa t \left[(n + \frac{1}{2})(n_b + 1) + n_b \right]} \sum_{j=n}^{N} \frac{(j + \frac{1}{2})! \left(1 - e^{-2\kappa t (n_b + 1)} \right)^{j-n}}{(n + \frac{1}{2})! (j - n)!} p_j \ , \tag{21}$$

valid for $n \geq 0$. Here p_j is the initial photon probability distribution of the cavity radiation field. The approximative solution F^*_n deviates from the exact solution F_n in a manner which can be expressed in terms of the right-hand side in the equation

$$\dot{F}^*_n + \alpha_n F^*_n - \beta_n F^*_{n+1} - \gamma_n F^*_{n-1} = \gamma_n (F^*_n - F^*_{n-1}) \ , \tag{22}$$

by comparing with Eq. (13). By performing a sum on both sides of Eq.(22) we now obtain

$$\beta_{-1} F^*_0 + \sum_{n=0}^{N} \left[\, \dot{F}^*_n + \alpha_n F^*_n - \beta_{n-1} F^*_n - \gamma_{n+1} F^*_n \right] = -\alpha_{-1} \sum_{n=0}^{N} F^*_n \ . \tag{23}$$

By making use of the unitarity relation $\mathrm{Tr}(\rho(t)) = \sum_n p_n = 1$, we can therefore find an explicit expression for F^*_{-1}, i.e.

$$F^*_{-1} = 2 \left(1 - \sum_{n=0}^{N} F^*_n \right) = 2 - e^{-2\kappa n_b t} \sum_{j=0}^{N} \sum_{k=0}^{j} (-1)^k \frac{(j + \frac{1}{2})!}{(j-k)! k! (\frac{1}{2})!} \frac{e^{-\kappa (2k+1)(n_b+1) t}}{k + \frac{1}{2}} \ p_j \ , \tag{24}$$

such that $F^*_{-1}(0) = 0$. Here we observe that F^*_{-1} satisfies the following differential equation

$$\dot{F}^*_{-1} + \alpha_{-1} F^*_{-1} - \beta_{-1} F^*_0 = 4\kappa n_b \ , \tag{25}$$

by making use of Eq.(23). From these considerations we conclude that, when $\gamma_n = n_b = 0$, Eqs. (21) and (24) give the exact solution to the equations of motion. With our method we have therefore found an approximative solution at a non-zero n_b which is consistent with unitarity, as is required in quantum mechanics, and, furthermore, with a small error when n_b is sufficiently small. For the numerical results of the present paper we have indeed verified that the corresponding error can be neglected, at least under the condition as given in the end of the Appendix. In order to proceed further, we also need the result

$$\langle\psi_n^{\pm}|W(t)|\psi_n^{\mp}\rangle = \frac{1}{2}e^{-\alpha_n t}p_n \ , \tag{26}$$

which is easily found from the equations of motion in the Appendix. From the explicit expressions for F_n^*, F_{-1}^* and Eq.(26), which are the major results of the present paper, we see that $t_d \simeq t_{cav}/\bar{n}(n_b+1)$, at least for sufficiently small n_b.

Using the Eqs. (21), (24) and (26) we are now able to evaluate the time evolution of various physical quantities. If for example the first atom is measured to be in the excited state at $t = t_A$, the reduced density of the cavity radiation field is $\rho_\gamma(t_A) = \mathrm{Tr}_A(\rho(t_A)|+\rangle\langle+|) = \langle+|\rho(t_A)|+\rangle$. Within our approximations we then obtain

$$p_n(t_A) = \langle n|\rho_\gamma(t_A)|n\rangle = \frac{1}{2}\left[\ F_n^*(t_A) + e^{-\alpha_n t_A}\cos(2gt_A\sqrt{n+1})\ p_n\right] \ . \tag{27}$$

The probability of finding the atom in an excited state at time t then is

$$\mathcal{P}_+(t) = \frac{1}{2} - \frac{1}{4}F_{-1}^* + \sum_{n=0}^{N}\frac{1}{2}\left[e^{-\alpha_n t}\cos\left(2gt\sqrt{n+1}\right)p_n\right] \ . \tag{28}$$

For the next atom which enters into the cavity, $\rho_\gamma(t_A)$ is then used as the initial density of the radiation field. The probability of finding the next atom in the excited state at time t_B, given that the first atom was in the excited state at time t_A, therefore is

$$\begin{aligned}\mathcal{P}_{++}(t_B) \ &= \ \sum_{n=0}^{N}\frac{1}{2}\Big[\ e^{-2\kappa\left[(n+\frac{1}{2})(n_b+1)+n_b\right](t_B-t_A)} \\ &\times \ \sum_{j=n}^{N}\frac{(j+\frac{1}{2})!\left(1-e^{-2\kappa(n_b+1)(t_B-t_A)}\right)^{j-n}}{(n+\frac{1}{2})!(j-n)!}\ p_j(t_A) \\ &+ \ e^{-\alpha_n(t_B-t_A)}\cos(2g(t_B-t_A)\sqrt{n+1})\ p_n(t_A)\Big] \ . \end{aligned} \tag{29}$$

From now on we choose the atomic passage times t_A and t_B such that $t \equiv t_A = t_B - t_A$.

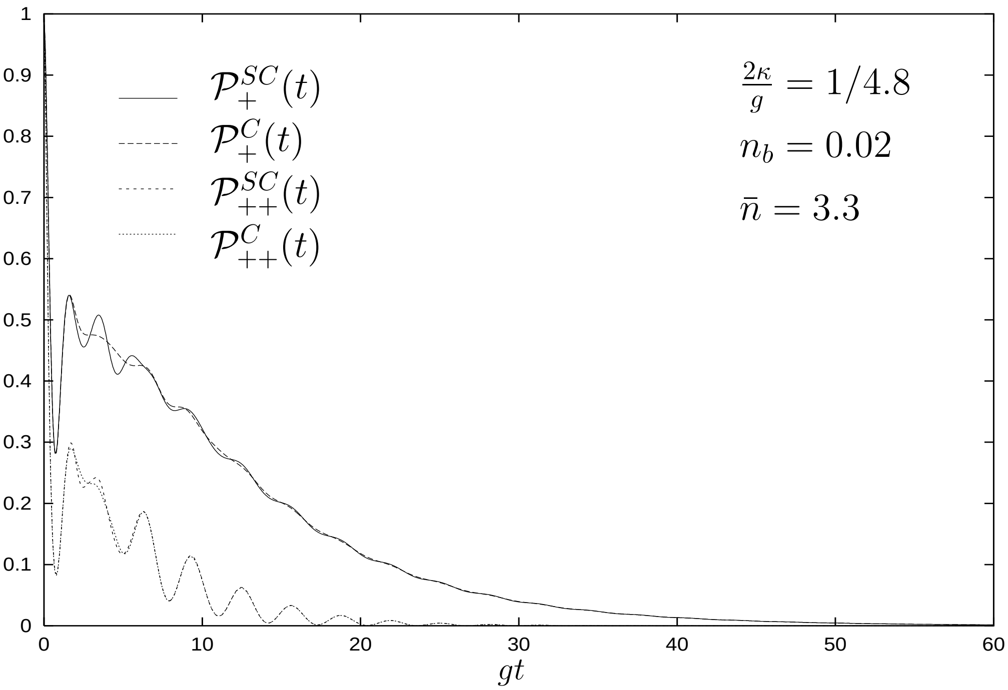

Figure 2: The figure shows the revival probabilities $\mathcal{P}_+(t)$ (upper curves) and $\mathcal{P}_{++}(t)$ (lower curves) for (C) a coherent state $|z\rangle$ and (SC) an even Scrödinger cat state $|z, \phi = 0\rangle$, with a mean number $\bar{n}_C = |z|^2 = 3.3$ and $\bar{n}_{SC} = |z|^2 \tanh(|z|^2) \approx 3.3$, respectively, of photons as a function of the atomic passage time gt. Here we have used physical parameters $\kappa = 2500\ s^{-1}$ and a Rabi frequency $g = 24\ kHz$ corresponding to the parameters of Ref.[22].

4 FINAL REMARKS

In Fig. 1 we show the revival probabilities $\mathcal{P}_+(t)$ and $\mathcal{P}_{++}(t)$ for an initial coherent state or a Schrödinger cat state, where the physical parameters are taken from Ref.[30]. We notice that when the system initially is prepared in a Schrödinger cat state we observe a revival at an earlier time as compared to the case of an initial a coherent state. The first revival time for Schrödinger cat state is $t_{rev}^{SC} = t_{rev}^{C}/2$, where t_{rev}^{C} is the first revival time for the coherent state. Similarly, in Fig. 2, we show the same revival probabilities, but now the parameters are taken from Ref.[22]. Here we see only a minor difference between an initial Scrödinger cat state as compared to an initial coherent state. In Fig. 3 we have, in addition, evaluated the correlation function $\eta(t)$ defined by [22]

$$\eta(t) = \frac{\mathcal{P}_{++}(t)}{\mathcal{P}_{+}(t)} - \frac{\mathcal{P}_{-+}(t)}{\mathcal{P}_{-}(t)}, \tag{30}$$

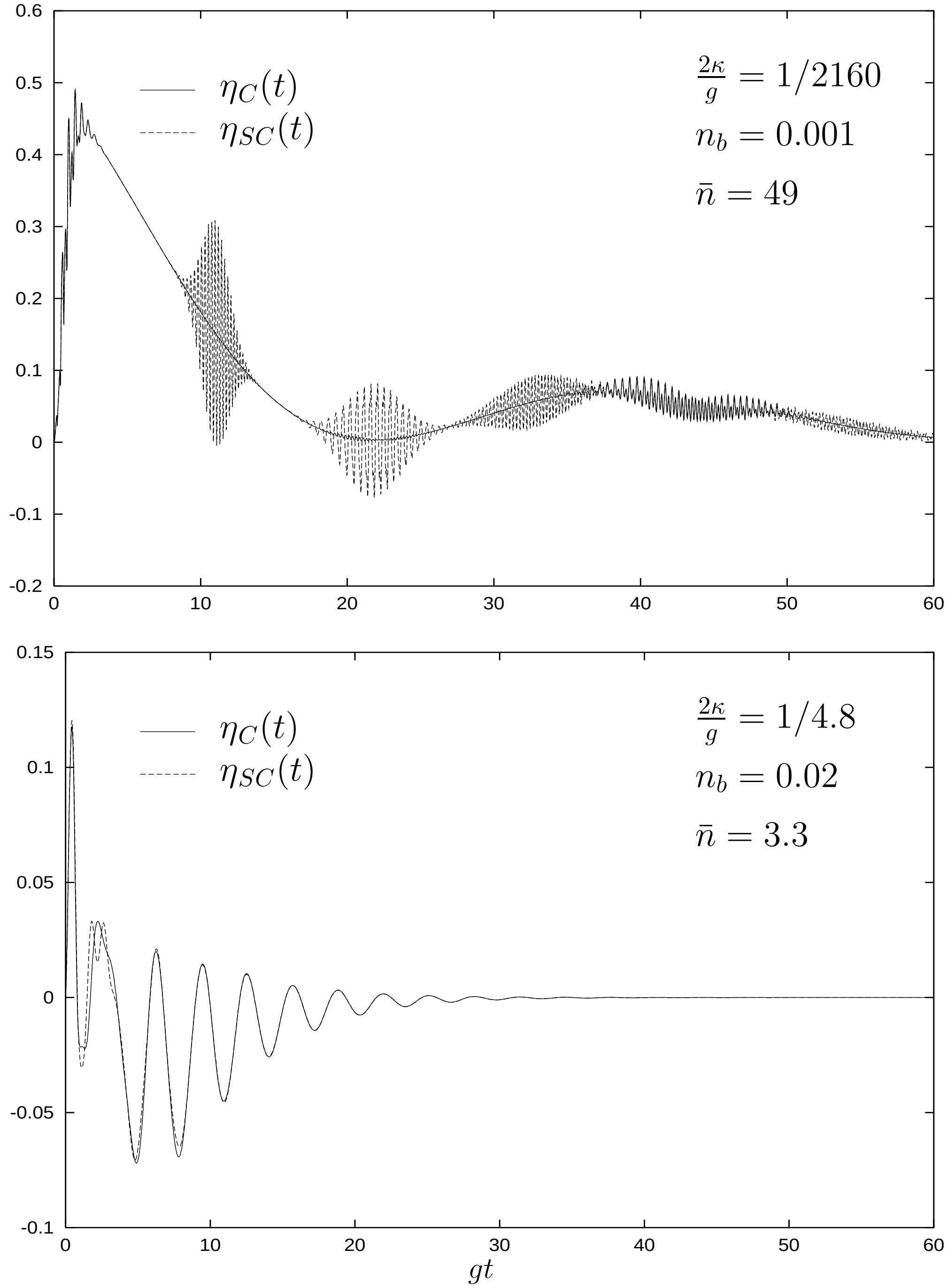

Figure 3: The upper figure shows the correlation function η as a function of the atomic passage time for (C) a coherent state $|z\rangle$ and for (SC) a Schrödinger cat state $|z, \phi = 0\rangle$ with the experimental parameters of Ref.[30] apart from our choice $\bar{n} = 49$. The lower figure shows the same correlation functions, but with the parameters of Ref.[22].

such that $-1 \leq \eta(t) \leq 1$, for an initial coherent and Schrödinger cat state respectively. We notice that $\mathcal{P}_{-+}(t) + \mathcal{P}_{--}(t) = \mathcal{P}_{-}(t) = 1 - \mathcal{P}_{+}(t)$. Again we notice that under the experimental conditions of Ref.[30], with a finite but not to large $\bar{n}$, one is able to very clearly detect the revivals of a Schrödinger cat state at t_{rev}^{SC} by measuring the correlation function $\eta(t)$.

The interference effect we have discussed above depends on the phase ϕ of the Scrödinger cat state. One interesting case occurs when $\phi = \pi/2 \bmod(\pi)$. With regard to revivals, the Scrödinger cat state then behaves like a coherent state, i.e. the revival at t_{SC}^{rev} is now absent. In fact, the phase ϕ essentially only effects the first revival at $gt_{rev} \approx \pi\sqrt{\bar{n}}$. This is easily realised in terms of a Poisson resummation of Eq.(28) [35] for $\bar{n} \gg 1$, i.e.

$$\mathcal{P}_{+}^{SC}(t) \approx \frac{1}{2}e^{-2\kappa n_b t}$$
$$+\ \frac{1}{2}e^{-2\kappa t(2n_b(1+\bar{n}) + \bar{n} + 1/2)}\left[w_0(t) + \sum_{\nu=1}^{N}\Big(\, w_\nu(t) - w_{\nu-1/2}(t)\cos\phi \,\Big)\right] \ . \tag{31}$$

Here

$$w_0(t) = e^{-g^2t^2/2}\cos\left(2gt\sqrt{\bar{n}}\right) \ , \tag{32}$$

describes the initial exponential decay and the various revivals are expressed in terms of

$$w_\nu(t) = p(\frac{g^2t^2}{4\pi^2\nu^2})\frac{gt}{\pi\sqrt{2\nu^3}}\cos\left(\frac{g^2t^2}{2\pi\nu} - \frac{\pi}{4}\right) \ , \tag{33}$$

where, in our case,

$$p(n) = \frac{\bar{n}^n}{n!}e^{-\bar{n}} \ . \tag{34}$$

In Eq.(31) only a leading effect of damping is taken into account in that $F_{-1}^{*} \approx 2(1 - e^{-2\kappa n_b t})$, vaild for $\alpha_{\bar{n}} \ll g$. Numerically Eq.(31) with $N = 3$ is e.g. sufficient to describe $\mathcal{P}_{+}^{SC}(t)$ of Fig.1 very accurately.

We therefore conclude that the two, almost orthogonal, coherent states $|z\rangle$ and $|-z\rangle$ of the Scrödinger cat state Eq.(1), both with a large average number of photons, can act like an "interferometer" for the atoms with regard to the revival at, e.g., $t = t_{SC}^{rev}$. Due to the interaction of the atoms with the radiation field of the cavity, which only changes the average number of photons by a small amount, an interference of the "classical states" corresponding to the quantum coherent states $|z\rangle$ and $|-z\rangle$ is, in addition, induced. In this sense we have therefore obtained a novel quantum-mechanical interferometer which is experimentally feasible. In our numerical examples we have used physical parameters from some recent experiments [22, 30]. but extrapolated the atom transit time t to rather large values of gt. It is, of course, an experimental challenge to obtain a one-atom source and atomic life-times of the atomic states involved such that these large values of gt can be reached.

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APPENDIX

In order to derive the equations of motion in a convenient form, we notice the following useful relations

$$a^*|\psi_n^\pm\rangle = \frac{1}{2}\left(\sqrt{n+1} \pm \sqrt{n+2}\right)|\psi_{n+1}^+\rangle + \frac{1}{2}\left(\sqrt{n+1} \mp \sqrt{n+2}\right)|\psi_{n+1}^-\rangle \ , \qquad \text{(A.1)}$$

and

$$a|\psi_n^\pm\rangle = \frac{1}{2}\left(\sqrt{n} \pm \sqrt{n+1}\right)|\psi_{n-1}^+\rangle + \frac{1}{2}\left(\sqrt{n} \mp \sqrt{n+1}\right)|\psi_{n-1}^-\rangle \ , \qquad \text{(A.2)}$$

from which we derive

$$a^*a|\psi_n^\pm\rangle = \left(n+\frac{1}{2}\right)|\psi_n^\pm\rangle - \frac{1}{2}|\psi_n^\mp\rangle \ . \qquad \text{(A.3)}$$

Using the definition Eq.(10) we then find the following exact differential equations of the diagonal elements of Eq.(9)

$$\begin{aligned}
\langle\psi_n^\pm|\dot{W}(t)|\psi_n^\pm\rangle &= 2\kappa(n_b+1)\Big\{\ \Gamma_{+,n+1}\langle\psi_{n+1}^\pm|W(t)|\psi_{n+1}^\pm\rangle \\
&+\Gamma_{-,n+1}\langle\psi_{n+1}^\mp|W(t)|\psi_{n+1}^\mp\rangle - (n+\frac{1}{2})\langle\psi_n^\pm|W(t)|\psi_n^\pm\rangle \\
&+\frac{1}{4}\Big[\ e^{-2igt\sqrt{n+1}}\langle\psi_n^+|W(t)|\psi_n^-\rangle - e^{-2igt\sqrt{n+2}}\langle\psi_{n+1}^+|W(t)|\psi_{n+1}^-\rangle + c.c.\Big]\Big\} \\
&+2\kappa n_b\Big\{\ \Gamma_{+,n}\langle\psi_{n-1}^\pm|W(t)|\psi_{n-1}^\pm\rangle + \Gamma_{-,n}\langle\psi_{n-1}^\mp|W(t)|\psi_{n-1}^\mp\rangle - (n+\frac{3}{2})\langle\psi_n^\pm|W(t)|\psi_n^\pm\rangle \\
&+\frac{1}{4}\Big[\ e^{-2igt\sqrt{n+1}}\langle\psi_n^+|W(t)|\psi_n^-\rangle - e^{-2igt\sqrt{n}}\langle\psi_{n-1}^+|W(t)|\psi_{n-1}^-\rangle + c.c.\Big]\Big\} \ , \qquad \text{(A.4)}
\end{aligned}$$

for $n \geq 1$ and

$$\begin{aligned}
\langle\psi_0^\pm|\dot{W}(t)|\psi_0^\pm\rangle &= 2\kappa(n_b+1)\Big\{\ \Gamma_{+,1}\langle\psi_1^\pm|W(t)|\psi_1^\pm\rangle + \Gamma_{-,1}\langle\psi_1^\mp|W(t)|\psi_1^\mp\rangle \\
&-\frac{1}{2}\langle\psi_0^\pm|W(t)|\psi_0^\pm\rangle + \frac{1}{4}\Big[\ e^{-2igt}\langle\psi_0^+|W(t)|\psi_0^-\rangle - e^{-2igt\sqrt{2}}\langle\psi_1^+|W(t)|\psi_1^-\rangle + c.c.\Big]\Big\}
\end{aligned}$$

$$+2\kappa n_b \Big\{ \langle\psi_0|W(t)|\psi_0\rangle - 3\langle\psi_0^\pm|W(t)|\psi_0^\pm\rangle + \frac{1}{2}\Big[e^{-2igt}\langle\psi_0^+|W(t)|\psi_0^-\rangle + c.c.\Big] \Big\} . \tag{A.5}$$

A dot denotes differentiation with respect to the time variable t. With the same procedure as above, we also find for $n \geq 1$ the equation

$$\langle\psi_n^\pm|\dot{W}(t)|\psi_n^\mp\rangle = -2\kappa(n_b+1)\Big\{ (n+\frac{1}{2})\langle\psi_n^\pm|W(t)|\psi_n^\mp\rangle$$
$$-\Gamma_{+,n+1}e^{\pm 2igt\left(\sqrt{n+1}-\sqrt{n+2}\right)}\langle\psi_{n+1}^\pm|W(t)|\psi_{n+1}^\mp\rangle - \Gamma_{-,n+1}e^{\pm 2igt\left(\sqrt{n+1}+\sqrt{n+2}\right)}\langle\psi_{n+1}^\mp|W(t)|\psi_{n+1}^\pm\rangle$$
$$-\frac{1}{4}e^{\pm 2igt\sqrt{n+1}}\Big[\langle\psi_n^-|W(t)|\psi_n^-\rangle + \langle\psi_n^+|W(t)|\psi_n^+\rangle - \langle\psi_{n+1}^-|W(t)|\psi_{n+1}^-\rangle - \langle\psi_{n+1}^+|W(t)|\psi_{n+1}^+\rangle\Big]\Big\}$$
$$-2\kappa n_b\Big\{ (n+\frac{3}{2})\langle\psi_n^\pm|W(t)|\psi_n^\mp\rangle - \Gamma_{+,n}e^{\pm 2igt\left(\sqrt{n+1}-\sqrt{n}\right)}\langle\psi_{n-1}^\pm|W(t)|\psi_{n-1}^\mp\rangle$$
$$-\Gamma_{-,n}e^{\pm 2igt\left(\sqrt{n+1}+\sqrt{n}\right)}\langle\psi_{n-1}^\mp|W(t)|\psi_{n-1}^\pm\rangle - \frac{1}{4}e^{\pm 2igt\sqrt{n+1}}\Big[\langle\psi_n^-|W(t)|\psi_n^-\rangle + \langle\psi_n^+|W(t)|\psi_n^+\rangle$$
$$-\langle\psi_{n-1}^-|W(t)|\psi_{n-1}^-\rangle - \langle\psi_{n-1}^+|W(t)|\psi_{n-1}^+\rangle\Big]\Big\} \ , \tag{A.6}$$

as well as

$$\langle\psi_0^\pm|\dot{W}(t)|\psi_0^\mp\rangle = 2\kappa(n_b+1)\Big\{ -\frac{1}{2}\langle\psi_0^\pm|W(t)|\psi_0^\mp\rangle + \Gamma_{+,1}e^{\pm 2igt(1-\sqrt{2})}\langle\psi_1^\pm|W(t)|\psi_1^\mp\rangle$$
$$+\Gamma_{-,1}e^{\pm 2igt(1+\sqrt{2})}\langle\psi_1^\mp|W(t)|\psi_1^\pm\rangle + \frac{1}{4}e^{\pm 2igt}\Big[\langle\psi_0^-|W(t)|\psi_0^-\rangle + \langle\psi_0^+|W(t)|\psi_0^+\rangle$$
$$-\langle\psi_1^-|W(t)|\psi_1^-\rangle - \langle\psi_1^+|W(t)|\psi_1^+\rangle\Big]\Big\} + 2\kappa n_b\Big\{ -\frac{3}{2}\langle\psi_0^\pm|W(t)|\psi_0^\mp\rangle - \frac{1}{2}\langle\psi_0|W(t)|\psi_0\rangle$$
$$+\frac{1}{4}e^{\pm 2igt}\Big[\langle\psi_0^+|W(t)|\psi_0^+\rangle + \langle\psi_0^-|W(t)|\psi_0^-\rangle\Big]\Big\} \ . \tag{A.7}$$

Here we have made use of the notation

$$\Gamma_{\pm,n} = (\sqrt{n+1} \pm \sqrt{n})^2/4 \ . \tag{A.8}$$

With regard to the time-scale $1/\kappa$ of cavity damping, the exponential functions in Eqs.(A.4)-(A.5) and Eqs.(A.6)-(A.7) will vary rapidly and can therefore be neglected provided $\kappa \ll g$, which we assume to be valid.

References

[1] Proceedings of Nobel Symposium **104** "*Modern Studies of Basic Quantum Concepts and Phenomena* ", Eds. E.B. Karlson and E. Brändas, Physica Scripta **T76** (1998) and World Scientific (1998).

[2] V. Bužek and P.L. Knight, "*Quantum Interference, Superpositions States of Light and Nonclassical Effects*", in Progress in Optics **34** (1995) 1, Ed. E. Wolf (Elsevier, Amsterdam, 1995). For an excellent pedagogical account also see C.C. Gerry and P.L. Knight, "*Quantum Superpositions and Schrödinger Cat States in Quantum Optics* ", Am. J. Phys. **65** (1997) 964.

[3] E. Schrödinger, "*Die Gegenwertige Situation in der Quantenmechanik* ", Die Naturwissenshaften **23** (1935) 807, 823 and 844. Also see "*Quantum Theory of Measurement* ", Eds. J.A. Wheeler and W.H. Zurek (Princeton University Press, Princeton, 1983).

[4] A.O. Caldeira and A.J. Leggett, "*Path Integral Approach to Quantum Brownian Motion* ", Physica **A121** (1983)587; E. Joos and H.D. Zeh, "*The Emergence of Classical Properties Through Interaction with the Environment* ", Z. Phys. **B59** (1985) 223; W.H. Zurek, "*Decoherence and the Transition to Classical* ", Phys. Today **44** (Oct. 1991) 36 and "*Preferred States, Predictability, Classicality and the Environment-Induced Decoherence* ", Prog. Theor. Phys. **89** (1993) 281.

[5] E.T. Jaynes and F.W. Cummings, "*Comparison of Quantum and Semiclassical Radiation Theories with Application to the Beam Maser* ", Proc. IEEE **51** (1963) 89.

[6] J.R. Klauder and B.-S. Skagerstam, "*Coherent States-Applications in Physics and Mathematical Physics* " (World Scientific, Singapore, 1985 and Beijing 1988); B.-S. Skagerstam, "*Coherent States - Some Applications in Quantum Field Theory and Particle Physics* " in "*Coherent States: Past, Present, and the Future* ", Eds. D.H. Feng, J.R. Klauder and M.R. Strayer (World Scientific, Singapore, 1994).

[7] R.R. Puri and G.S. Agarwal, "*Collapse and Revival Phenomena in the Jaynes-Cummings Model with Cavity Damping* ", Phys. Rev. **A33** (1986) 3610; "*Finite-Q Cavity Electrodynamics: Dynamical and Statistical Aspects* ", Phys. Rev. **A35** (1987) 3433.

[8] S.M. Barnett and P.L. Knight, "*Dissipation in a Fundamental Model of Quantum Optical Resonance* ", Phys. Rev. **A33** (1986) 2444.

[9] D.F. Walls and G.J. Milburn, "*Effect of Dissipation on Quantum Coherence* ", Phys. Rev. **A31** (1985) 2403; Tran Quang, P.L. Knight and V. Bužek, "*Quantum Collapses and Revivals in an Optical Cavity* ", Phys. Rev. **A44** (1991) 6092; B. Daeubler, H. Risken and L. Schoendorff, "*Analytic Solution for Inversion and Intensity of the Jaynes-Cummings Model With Cavity Damping* ", Phys. Rev. **A46** (1992) 1654; M.S. Kim and V. Bužek, "*Schrödinger-Cat States at Finite Temperature; Influence of a Finite-Temperature Heat Bath on Quantum Interference* ", Phys. Rev. **A46** (1992) 4239 and "*Photon Statistics of Superposition States in Phase-Sensitive Reservoirs* ", Phys. Rev. **A47** (1993) 610; M. Brune, S. Haroche and J.M. Raimond, "*Manipulation of Photons in a Cavity by Dispersive Atom-Field Coupling: Quantum-Nondemolition Measurement and Generation of "Schrödinger Cat" States* ", Phys. Rev. **A45** (1992) 5193; L. Davidovich, M. Brune, J.M. Raimond and S. Haroche, "*Mesoscopic Quantum Decoherence in Cavity QED: Preparation and Decoherence Monotoring Schemes* ", Phys. Rev. **A53** (1996) 1295.

[10] P. Tombesi and D. Vitali, "*All-Optical Model for the Generation and the Detection of Macroscopic Quantum Coherence* ", Phys. Rev. Lett. **77** (1996) 411; G.S. Agarwal, "*Mesoscopic Superpositions of States: Approach to Classicality and Diagonalization in a Coherent State Basis* ", Phys. Rev. **A59** (1999) 3071.

[11] P.J. Bardoff, E. Mayr and W.P. Schleich "*Quantum State Endoscopy: Measurement of the Quantum State in a Cavity* ", Phys. Rev. **A 51** (1995) 4963.

[12] J. Janszky, P. Domokos, S, Szabó and P. Adam "*Quantum-State Engineering via Discrete Coherent-State Superpositions* ", Phys. Rev. **A 51** (1995) 4191.

[13] J.F. Poyatos, J.I. Cirac and P. Zoller, "*Quantum Reservoir Engineering with Laser Cooled Trapped Ions* ", Phys. Rev. Lett. **77** (1996) 4728.

[14] D. Filipowicz, J. Javanainen and P. Meystre, "*The Microscopic Maser* ", Opt. Comm. **58** (1986) 327; "*Theory of a Microscopic Maser* " Phys. Rev. **A34** (1986) 3077.

[15] H. Walther, "*The Single Atom Maser and the Quantum Electrodynamics in a Cavity* ", Physica Scripta **T23** (1988) 165; "*Experiments on Cavity Quantum Electrodynamics* " Phys. Rep. **219** (1992) 263; "*Experiments with Single Atoms in Cavities and*

Traps" in "*Fundamental Problems in Quantum Theory*", Eds. D. M. Greenberger and A. Zeilinger, Ann. N.Y. Acad. Sci. **755** (1995) 133; "*Single Atom Experiments in Cavities and Traps*", Proc. Roy. Soc. **A454** (1998) 431; "*Quantum Optics of a Single Atom*", Laser Physics **8** (1998) 1; Physica Scripta **T76** (1998) 138.

[16] K. An, J.J. Childs, R.R. Dasari and M.S. Feld, "*Microlaser: A Laser with One Atom in an Optical Resonator*", Phys. Rev. Lett. **73** (1994) 3375; K. An "*Atom-Field Interaction: Recent Experiments with the Single-Atom Microlaser*" in "*Applied Field Theory*", Eds. Choonkye Lee, Hyunsoo Min and Q-Han Park (Chungbum Publ. House, Seoul, 1999).

[17] P. Filipowicz, J. Javanainen and P.Meystre, "*Quantum and Semiclassical Steady States of a Kicked Cavity Mode*", J. Opt. Soc. Am. B **3** (1986) 906.

[18] P. Meystre, G. Rempe and H. Walther, "*Very-Low-Temperature Behavior of a Micromaser*", Opt. Lett. **13** (1988) 1078.

[19] P. Meystre and J.J. Slosser, "*Destruction of Quantum Coherence in a Micromaser by Finite Detection Efficiency*", Opt. Comm. **70** (1989) 103.

[20] J.J. Slosser, P. Meystre and S.L. Braunstein "*Harmonic Oscillator Driven by a Quantum Current*", Phys. Rev. Lett. **63** (1989) 934; J.J. Slosser and P. Meystre "*Tangent and Cotangent States of the Electromagnetic Field*", Phys. Rev. A **41** (1990) 3867.

[21] M. Weidinger, B.T.H. Varcoe, R. Heerlein and H. Walther, "*Trapping States in the Micromaser*", Phys. Rev. Lett. **82** (1999) 3795.

[22] M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali, C. Wunderlich, J.M. Raimond and S. Haroche, "*Observing the Progressive Decoherence of the "Meter" in a Quantum Measurement*", Phys. Rev. Lett. **77** (1996) 4887; J.M. Raimond, M. Brune and S. Haroche, "*Reversible Decoherence of a Mesoscopic Superposition of Field States*", Phys. Rev. Lett. **79** (1997) 1964.

[23] M.W. Noel and C.R. Stroud, Jr., "*Excitation of an Atomic Electron to a Coherent Superposition of Macroscopically Distinct States*", Phys. Rev. Lett. **77** (1996) 1913.

[24] D.M. Meekhof, C. Monroe, B.E. King, W.M. Itano and D.J. Wineland, "*Generation of Nonclassical Motional States of a Trapped Atom*", Phys. Rev. Lett. **76** (1996) 1796; "*Demonstration of a Fundamental Quantum Logic gate*", Phys. Rev.

Letters **75** (1995) 4714; C.Monroe, D.M. Meekhof, B.E. King and D.J. Wineland, "*A "Schrödinger Cat" Superposition State of an Atom* ", Science **272** (1996) 1131; C.J. Myatt, B.E. King, Q.A. Turchette, C.A. Sackett, D. Kielpinski, W.M. Itano, C.Monroe and D.J. Wineland, "*Decoherence of Quantum Mechanic Superpositions Through Coupling to Engineered Reservoirs* ", Nature **403** (2000) 269.

[25] L.P. Pitaevskii, "*Bose-Einstein Condensation in a Magnetic Traps - Introduction to the Theory* ", Sov. Phys. Usp. **41** (1998) 569; Yu. Kagan, G.V. Shlyapnikov and J.T.M. Walraven, "*Bose-Einstein Condensation in Trapped Atomic Gases*". Phys., Rev. Lett. **76** (1996) 2670.

[26] I.Sh. Averbukh and N.F. Perelman, "*The Dynamics of Wave Packets of Highly-Excited States Atoms and Molecules* ", Sov. Phys. Usp. **34** (1991) 572 and P.W. Milonni and S. Singh,"*Some Recent Developments in the Fundamental Theory of Light* ", in "*Advances in Atomic. Molecular, and Optical Physics* ", Eds. Sir D. Bates and B. Benderson (Academic Press, 1991).

[27] G. Rempe, H. Walther and N. Klein, "*Observation of Quantum Collapse and Revival in the One-Atom Maser* ", Phys. Rev. Lett. **58** (1987) 353.

[28] M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J.M. Raimond and S. Haroche, "*Quantum Rabi Oscillation: A Direct Test of Field Quantization in a Cavity* ", Phys. Rev. Lett. **76** (1996) 1800.

[29] P. Elmfors, B. Lautrup and B.-S. Skagerstam, "*Correlations as a Handle on the Quantum State of the Micromaser* ", CERN/TH 95-154 (cond-mat/9506058) and Physica Scripta **55** (1997) 724; "*Correlations in the Micromaser*" Phys. Rev. **A54** (1996) 5171.

[30] O. Benson, G. Raithel and H. Walther, "*Quantum Jumps of the Micromaser Field: Dynamical Behavior Close to Phase Transition Points* ", Phys. Rev. Lett. **72** (1994) 3506 and "*Dynamics of the MicroMaser Field* " in "*Electron Theory and Quantum Electrodynamics: 100 Years Later* ", Ed. J.P. Dowling (Plenum Press, New York, 1997).

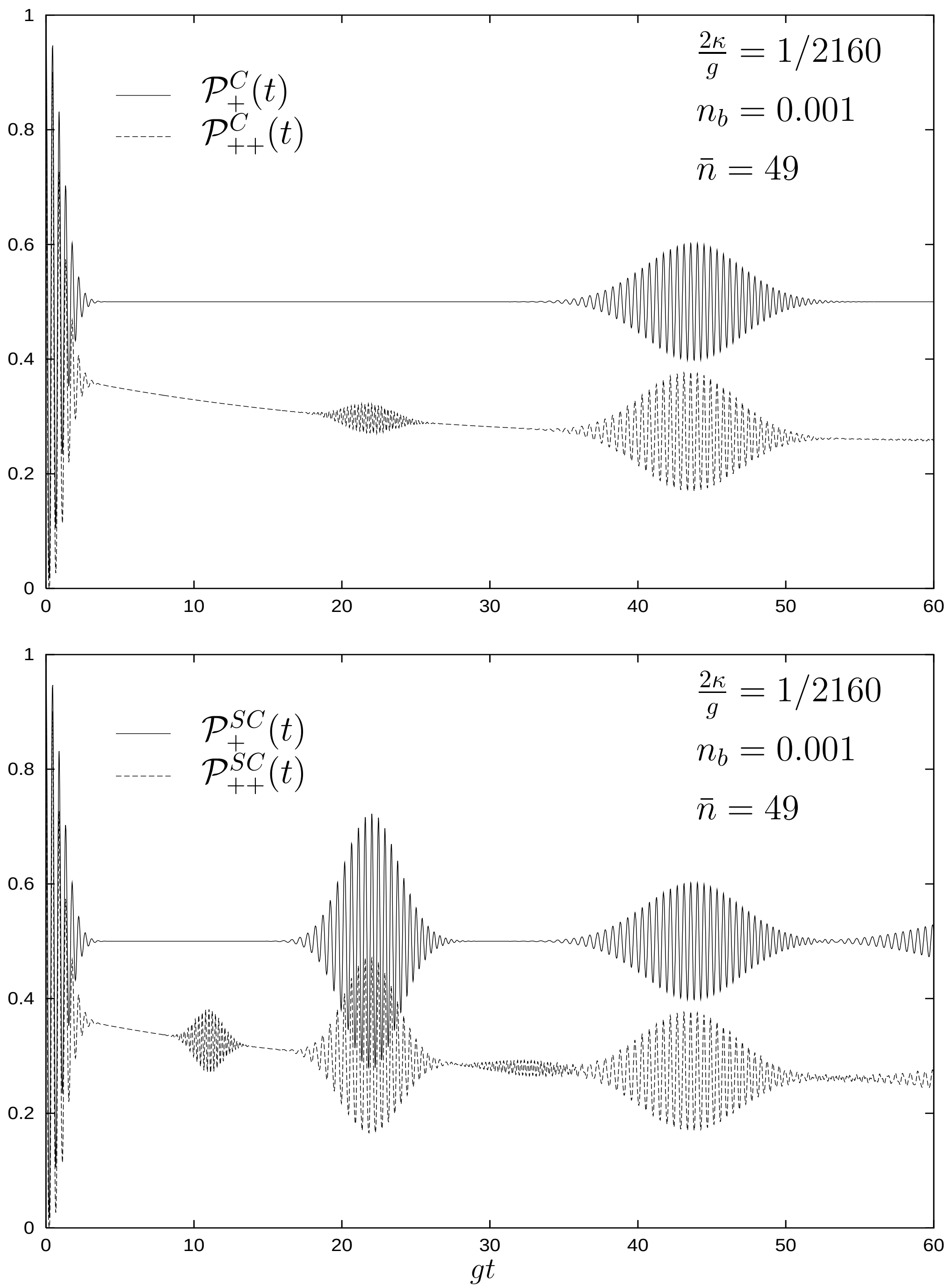
[31] B.-S. Skagerstam in "*Topics in Modern Quantum Optics* " in "*Applied Field Theory* ", Eds. Choonkye Lee, Hyunsoo Min and Q-Han Park (Chungbum Publ. House, Seoul, 1999).

[32] Julio Gea-Banacloche, "*Collapse and Revival of the State Vector in the Jaynes-Cummings Model: An Example of a State Preparation by a Quantum Apparatus* ", Phys. Rev. Lett. **65** (1990) 3385; "*Atom- and Field-State Evolution in the Jaynes-Cummings Model for Large Initial Fields* ", Phys. Rev. **A44** (1991) 5913; Simon J.D. Phoenix and P.L. Knight, "*Establishment of an Entangled Atom-Field State in the Jaynes-Cummings Model* ", Phys. Rev. **A44** (1991) 6023; M. Orszag, J.C. Retamal and C. Saavedra, "*Preparation of a Pure Atomic State* ", Phys. Rev. **A45** (1992) 2118; C.A. Arancibia-Bulnes, H. Moya-Cessa and J.J. Sánchez-Mondragón, "*Purifying a Thermal Field in a Lossless Micromaser* ", Phys. Rev. **A51** (1995) 5032.

[33] G. S. Agarwal, "*Master Equation Methods in Quantum Optics* " in "*Progress in Optics*" **XI**, pp. 1-75, Ed. E. Wolf, (North Holland, 1973).

[34] W. H. Louisell, "*Quantum Statistical Properties of Radiation* " (John Wiley& Sons, 1990); D. F. Walls and G. J. Milburn, "*Quantum Optics*" (Springer, 1995); L. Mandel and E. Wolf, "*Quantum Optics and Coherence*" (Cambridge University Press, 1995); M. O. Scully and M. S. Zubairy, "*Quantum Optics*" (Cambridge University Press, 1996).

[35] See e.g. R. Courant and D. Hilbert, "*Methods of Mathematical Physics* " (Interscience, New York, 1953) and M. Fleischhauer and W.P. Schleich, "*Revivals Made Simple: Poisson Summation Formula as a Key to the Revivals in the Jaynes-Cummings Model* , Phys. Rev. **A47** (1993) 4258.



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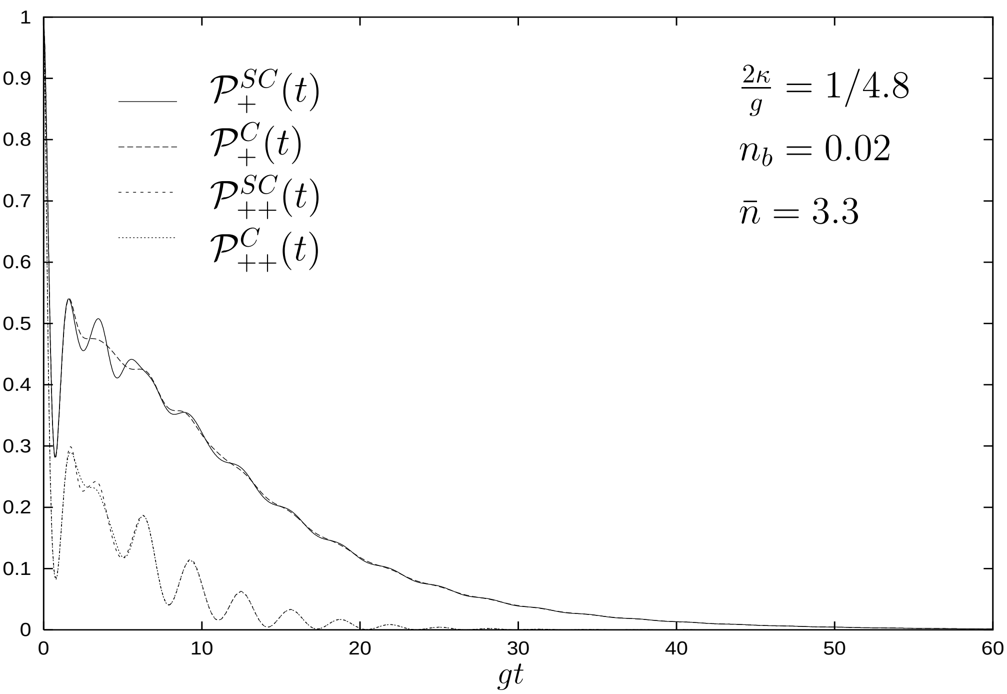

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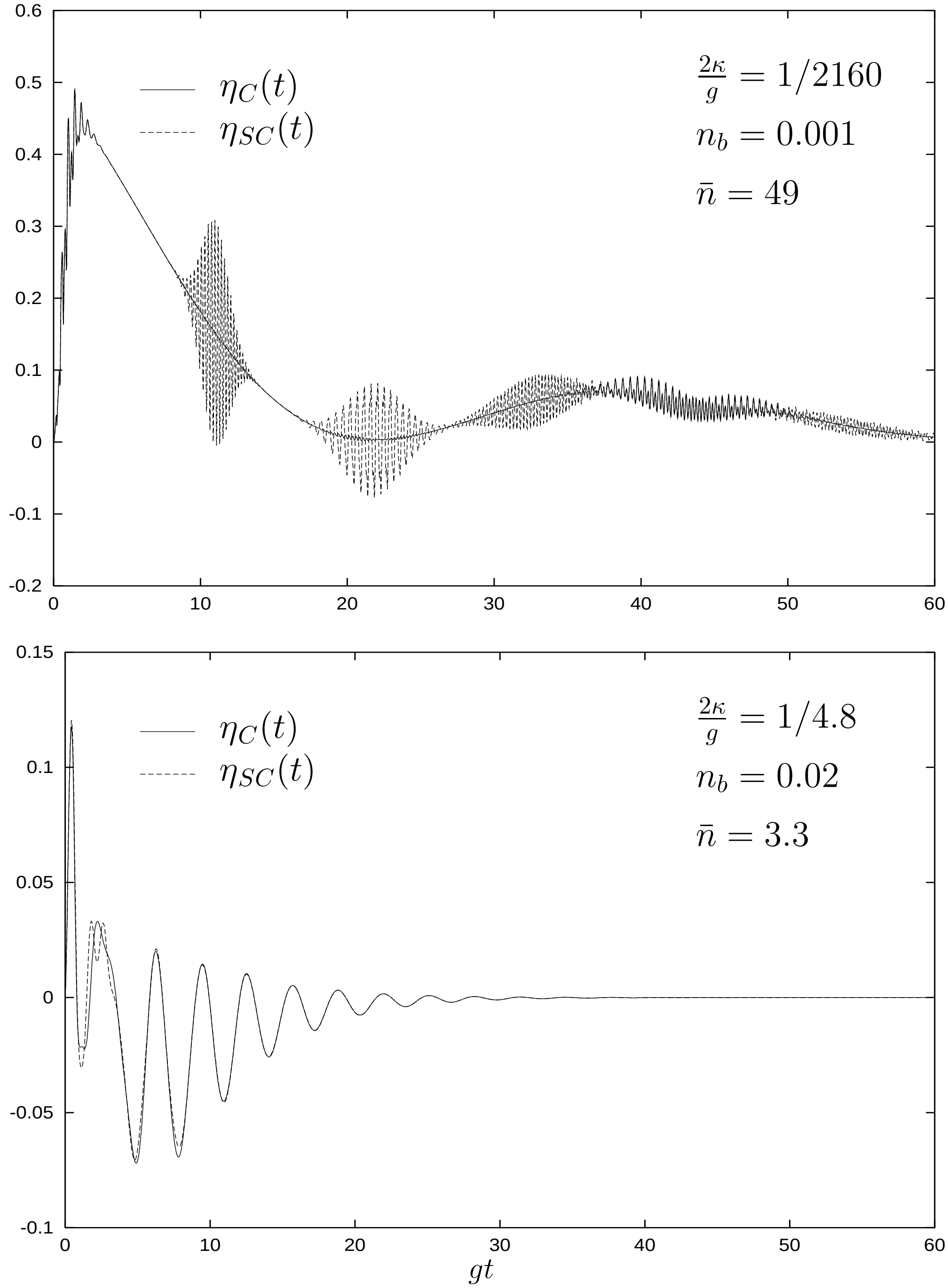

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