COMBINING QCD MATRIX ELEMENTS AND PARTON SHOWERS

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A new method for combining QCD matrix elements and parton showers in Monte Carlo simulations of hadronic final states is outlined. The aim is to provide at least a leading-order description of all hard multi-jet configurations together with jet fragmentation to next-to-leading logarithmic accuracy, while avoiding the most serious problems of double counting.

1 Introduction

The Monte Carlo simulation of multi-jet final states is a challenging problem in QCD and important for new physics searches. Two extreme approaches to this problem can be formulated as follows. One can use the corresponding matrix elements with bare partons representing jets. Then one must add a model for conversion of the partons into hadrons; any realistic model will include parton showering, and hence extra jet production and potential double counting. Alternatively, one can use the parton model to generate the simplest relevant final state (e.g. $e^+e^- \to q\bar{q}$) and produce addition jets by parton showering. However, this gives a poor simulation of configurations with several widely separated jets.

For earlier work on combining these approaches see refs. [1,2,3,4,5]. Here I outline a method [6] in which the matrix element and parton shower domains are separated at some value y_1 of the k_T (Durham) jet resolution [7]

$$y_{ij} \equiv 2\min\{E_i^2, E_j^2\}(1-\cos\theta_{ij})/Q^2$$

The proposed method has the following features: At $y_{ij} > y_1$ multi-jet cross sections and distributions are given by matrix elements modified by Sudakov form factors. At $y_{ij} < y_1$ they are given by parton showers subjected to a 'veto' procedure, which cancels the y_1 dependence of the modified matrix elements to next-to-leading logarithmic (NLL) accuracy.

Note that the procedure does not aim at a complete description of any configuration to next-to-leading order (NLO) in α_S, although this might be possible after subtracting NLO terms from the Sudakov form factors (see ref. [5]). The main objective is to describe all hard multi-jet configurations to leading order, i.e. $\mathcal{O}(\alpha_S^{n-2})$ for n jets, together with jet fragmentation to NLL accuracy, while avoiding major problems of double counting and/or missed phase-space regions.

2 Modified Matrix Elements

The exclusive e^+e^- n-jet fractions at c.m. energy Q and k_T-resolution $y_1 = Q_1^2/Q^2$ are given to NLL accuracy by [8]

$$
\begin{aligned}
R_2(Q_1,Q) &= [\Delta_q(Q_1,Q)]^2 \\
R_3(Q_1,Q) &= 2\,[\Delta_q(Q_1,Q)]^2 \int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\Delta_g(Q_1,q) \\
R_4(Q_1,Q) &= 2\,[\Delta_q(Q_1,Q)]^2 \left\{ \int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\Delta_g(Q_1,q) \int_{Q_1}^{Q} dq'\,\Gamma_q(q',Q)\Delta_g(Q_1,q') \right. \\
&+ \int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\Delta_g(Q_1,q) \int_{Q_1}^{q} dq'\,\Gamma_g(q',q)\Delta_g(Q_1,q') \\
&+ \left. \int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\Delta_g(Q_1,q) \int_{Q_1}^{q} dq'\,\Gamma_f(q')\Delta_f(Q_1,q') \right\}
\end{aligned}
$$

etc., where $\Gamma_{q,g,f}$ are $q \to qg$, $g \to gg$ and $g \to q\bar{q}$ branching probabilities

$$
\begin{aligned}
\Gamma_q(q,Q) &= \frac{2C_F}{\pi}\frac{\alpha_S(q)}{q}\left(\ln\frac{Q}{q} - \frac{3}{4}\right) \\
\Gamma_g(q,Q) &= \frac{2C_A}{\pi}\frac{\alpha_S(q)}{q}\left(\ln\frac{Q}{q} - \frac{11}{12}\right) \\
\Gamma_f(q) &= \frac{N_f}{3\pi}\frac{\alpha_S(q)}{q}
\end{aligned}
$$

and $\Delta_{q,g}$ are the quark and gluon Sudakov form factors

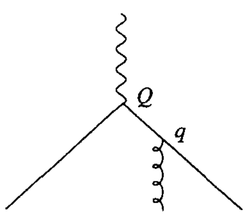
$$
\begin{aligned}
\Delta_q(Q_1,Q) &= \exp\left(-\int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\right) \\
\Delta_g(Q_1,Q) &= \exp\left(-\int_{Q_1}^{Q} dq\,[\Gamma_g(q,Q) + \Gamma_f(q)]\right)
\end{aligned}
$$

with

$$\Delta_f(Q_1,Q) = [\Delta_q(Q_1,Q)]^2/\Delta_g(Q_1,Q)\,.$$

The Sudakov form factor $\Delta_i(Q_1,Q)$ represents the probability for a parton of type i to evolve from scale Q to scale Q_1 without any branching (resolvable at scale Q_1). Thus R_2 is the probability that the produced quark and antiquark both evolve from Q to Q_1 without branching. More generally, the probability to evolve from Q to $q \geq Q_1$ without branching (resolvable at scale Q_1) is $\Delta_i(Q_1,Q)/\Delta_i(Q_1,q)$.

In R_3, the quark q (or antiquark $\bar{q}$) evolves from Q to Q_1 without branching, while the $\bar{q}$ (or q) evolves from Q to q, branches, and the resulting partons evolve from q to Q_1 without branching.



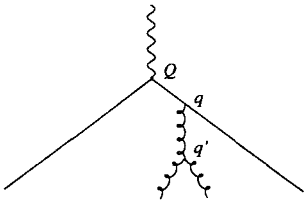
Thus the overall NLL probability is

$$2\Delta_q(Q_1,Q)\frac{\Delta_q(Q_1,Q)}{\Delta_q(Q_1,q)}\Gamma_q(q,Q)\Delta_q(Q_1,q)\Delta_g(Q_1,q)\ ,$$

which gives $R_3(Q_1,Q)$ after integration over $Q_1 < q < Q$.

We can improve the description of 3-jet distributions throughout the region $y_{qg}, y_{\bar{q}g} > y_1$ by using the *full tree-level matrix element squared* $\mathcal{M}_{q\bar{q}g}$ in place of the NLL branching probability $\Gamma_q(q,Q)$. We first generate $q\bar{q}g$ momentum configurations according to $\mathcal{M}_{q\bar{q}g}$, with k_T-resolution cutoff $y_{ij} > y_1 = Q_1^2/Q^2$, then weight each configuration with an extra factor of $[\Delta_q(Q_1,Q)]^2\Delta_g(Q_1,q)$ where $q^2 = \min\{y_{qg}, y_{\bar{q}g}\}Q^2$. For consistency we also use $\alpha_S(q)$ in $\mathcal{M}_{q\bar{q}g}$.

For four or more jets, there are several branching configurations with different colour factors. For example there is a contribution from $q \to qg$ branching at scale q followed by $g \to gg$ at scale q':



The probability of this is

$$\Delta_q(Q_1,Q)\frac{\Delta_q(Q_1,Q)}{\Delta_q(Q_1,q)}\Gamma_q(q,Q)\Delta_q(Q_1,q)\frac{\Delta_g(Q_1,q)}{\Delta_g(Q_1,q')}\Gamma_g(q',q)[\Delta_g(Q_1,q')]^2$$

which contributes to the term with colour factor $C_F C_A$. The product $\Gamma_q(q,Q)\Gamma_g(q',q)$ is an approximation to the full matrix element squared $\mathcal{M}_{q\bar{q}gg}$ in the kinematic region where y_{gg} is smallest interparton separation. Thus it is legitimate in NLLA to replace it by $\mathcal{M}_{q\bar{q}gg}$ in that region. The remaining factor of $[\Delta_q(Q_1,Q)]^2\Delta_g(Q_1,q)\Delta_g(Q_1,q')$ is the extra Sudakov weight to be applied.

In general, the proposed procedure for generating n-parton configurations is thus as follows:

- First distribute the parton momenta according to the relevant n-parton matrix element squared $\mathcal{M}_n$, using a fixed value $\alpha_S(Q_1)$ for the strong coupling.

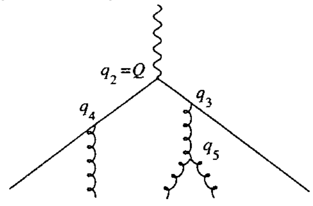
- Use the k_T-clustering algorithm to determine the resolution values $y_2 = 1 > y_3 > \ldots, > y_n > y_1$ at which $2, 3, \ldots, n$ jets are resolved. These give the *nodal values* of $q_j = Q\sqrt{y_j}$ for a tree diagram that specifies the k_T-clustering sequence for that configuration.

- Apply a coupling-constant weight factor of $\alpha_S(q_3)\alpha_S(q_4)\cdots\alpha_S(q_n)/[\alpha_S(Q_1)]^{n-2} < 1$.

- For each internal line of type i from a node at scale q_j to $q_k < q_j$, apply a Sudakov weight factor $\Delta_i(Q_1,q_j)/\Delta_i(Q_1,q_k) < 1$. For an external line from a node at scale q_j, the weight factor is $\Delta_i(Q_1,q_j)$.

Since the weight factors are all less than unity, unweighted events can be generated by rejecting those for which the product of weights is less than a random number.

As an example, the following clustering sequence for $e^+e^- \to q\bar{q}ggg$



has Sudakov weight

$$\frac{\Delta_q(Q_1,Q)}{\Delta_q(Q_1,q_3)}\frac{\Delta_q(Q_1,Q)}{\Delta_q(Q_1,q_4)}\frac{\Delta_g(Q_1,q_3)}{\Delta_g(Q_1,q_5)} \times \Delta_q(Q_1,q_3)\Delta_q(Q_1,q_4)\Delta_g(Q_1,q_4)[\Delta_g(Q_1,q_5)]^2$$
$$= [\Delta_q(Q_1,Q)]^2\Delta_g(Q_1,q_3)\Delta_g(Q_1,q_4)\Delta_g(Q_1,q_5)$$

Note that the weight factor is actually independent of the structure of the clustering tree and is the same as that for the Abelian (QED-like) graph with the same nodal scale values $\{q_j\}$. The clustering of partons will sometimes be 'wrong' but this should not affect LL and NLL terms. Other clustering procedures can be envisaged [4] which should be equivalent in the dominant regions.

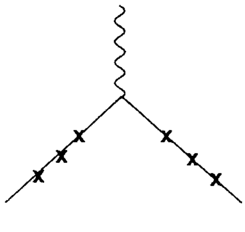
3 Vetoed Parton Showers

Having generated multijet distributions above the resolution value y_1 according to matrix elements modified by form factors, it remains to generate distributions at lower values of y_{ij} by means of parton showers. This should be done in such a way that the dominant (LL and NLL) dependence on the arbitrary parameter y_1 cancels. Any residual dependence on y_1 could be useful for tuning less singular terms to obtain optimal agreement with data.

Note that y_1 must set an upper limit on interparton separations y_{ij} generated in the showers. Otherwise the exclusive jet rates at resolution y_1 could be changed by showering. At first sight, this might suggest that we should evolve the showers from the scale $Q_1 = Q\sqrt{y_1}$ instead of Q. However, this would not lead to cancellation of dependence on $\log y_1$.

Consider, for example, the 2-jet rate at resolution $y_0 = Q_0^2/Q^2 < y_1$. If we start from R_2 at scale Q_1 and then evolve from Q_1 to Q_0, we obtain a 2-jet rate of $[\Delta_q(Q_1,Q)\Delta_q(Q_0,Q_1)]^2$ instead of the correct result $R_2(Q_0,Q) = [\Delta_q(Q_0,Q)]^2$. This is because, although y_{ij} values in the showers are limited by y_1, the *angular regions* in which they evolve should still correspond to scale Q rather than Q_1. Consequently we should allow the showers to evolve from scale Q but *veto* any branching with scale $q > Q_1$ – i.e., the selected parton branching is forbidden but that parton has its scale reset to q for subsequent branching.

The 2-jet rate at any scale $Q_0 < Q_1$ is now given by the sum of probabilities of $0, 1, 2, \ldots$ vetoed branchings (represented by crosses) and no actual resolved branchings:

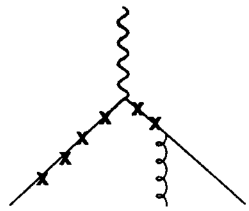


The sum of these probabilities for the quark line is

$$\Delta_q(Q_1,Q)\Delta_q(Q_0,Q)\left\{1+\int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)+\int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\int_{Q_1}^{q} dq'\,\Gamma_q(q',Q)+\cdots\right\}$$

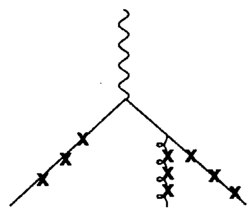
The series sums to $1/\Delta_q(Q_1,Q)$, cancelling the y_1 dependence and giving $\Delta_q(Q_0,Q)$. Similarly for the antiquark line.

For the 3-jet rate at scale $Q_0 < Q_1$ there are two possibilities: either the event is a 2-jet at scale Q_1 and then has one branching resolved at scale Q_0, or it is a 3-jet at scale Q_1 and remains so at scale Q_0. The probability of the first case is



$$2[\Delta_q(Q_1,Q)]^2\left[\frac{\Delta_q(Q_0,Q)}{\Delta_q(Q_1,Q)}\right]^2\int_{Q_0}^{Q_1} dq\,\Gamma_q(q,Q)\Delta_g(Q_0,q)$$

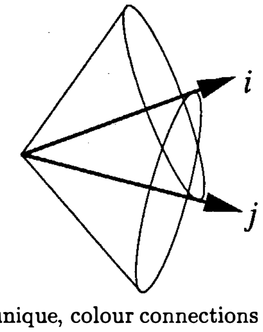
while that of the second case is



$$2[\Delta_q(Q_1,Q)]^2\left[\frac{\Delta_q(Q_0,Q)}{\Delta_q(Q_1,Q)}\right]^2\int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\Delta_g(Q_1,q)\frac{\Delta_g(Q_0,q)}{\Delta_g(Q_1,q)}\,.$$

The sum is indeed y_1-independent and equal to $R_3(Q_0,Q)$. Similarly for higher jet multiplicities.

Notice that evolution after a branching at scale $q > Q_1$ starts at scale q rather than Q or Q_1. In general, vetoed showers should evolve in the *phase space for angular-ordered branching* of each parton[9]. This depends on the colour structure of the matrix element. As illustrated below, the angular region for parton i is a cone bounded by the direction of parton j (and vice-versa), where i and j are colour-connected.



If the colour structure is not unique, colour connections must be selected according to their relative contributions to the matrix element squared, which are well-defined in the limit that the number of colours N_c is large. Corrections to the large-N_c limit are normally of relative order $1/N_c^2$, so this approximation is adequate to $\sim 10\%$. For high parton multiplicity, when the colour structure is not known even at large N_c, it may be possible to use the clustering scheme discussed above as a first approximation in assigning colour connections.

4 Comments/Conclusions

- Modified matrix elements plus vetoed parton showers, interfaced at some value y_1 of the k_T-resolution parameter, should provide a convenient way to describe simultaneously the hard multi-jet and jet fragmentation regions.
- The matrix element modifications are coupling-constant and Sudakov weights computed directly from the k_T-clustering sequence.
- Dependence on y_1 is cancelled to NLLA by vetoing $y_{ij} > y_1$ in the parton showers.
- This prescription avoids double-counting problems and missed phase-space regions.
- In principle one needs the matrix elements $\mathcal{M}_n$ for $y_{ij} > y_1$ at *all values of n*. In practice, if we have $n \leq N$, then y_1 must be chosen large enough for $R_{n>N}(Q_1, Q)$ to be negligible.
- This approach is being implemented (with $N = 5$) in the e^+e^- event generator APACIC++[10].
- It should be possible to extend it to lepton-hadron and hadron-hadron collisions.
- Extension to NLO along the lines of ref.[5] may also be possible.

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