Gauging the Full R-Symmetry Group in Five-dimensional, $\mathcal{N} = 2$ Yang-Mills/Einstein/tensor Supergravity[1]

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Abstract

We show that certain $5d$, $\mathcal{N} = 2$ Yang-Mills/Einstein supergravity theories admit the gauging of the *full* R-symmetry group, $SU(2)_R$, of the underlying $\mathcal{N} = 2$ Poincaré superalgebra. This generalizes the previously studied Abelian gaugings of $U(1)_R \subset SU(2)_R$, and completes the construction of the most general vector and tensor field coupled $5d$, $\mathcal{N} = 2$ supergravity theories with gauge interactions. The gauging of $SU(2)_R$ turns out to be possible only in special cases, and leads to a new type of scalar potential. For a large class of these theories the potential does not have any critical points.

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1 Introduction

Five-dimensional gauged supergravity theories have been subject to a renewed intense interest during the last three years. They offer an important tool in the study of the AdS/CFT-correspondence [1, 2, 3, 4] and have, more recently, been discussed as a potential framework for a string/M-theoretic embedding of the Randall/Sundrum (RS-) scenario [5, 6].

Whereas the original discontinuous RS-model [5] could recently be embedded into $5d$, $\mathcal{N} = 2$ gauged pure supergravity on $\mathbb{R}^4 \times S^1/Z_2$ [7], a realization in terms of a *smooth* ("thick") domain wall solution seemed to be incompatible with a variety of scalar potentials of known matter coupled $\mathcal{N} = 2$ supergravity theories [8, 9, 10].

Since the most general $5d$, $\mathcal{N} = 2$ gauged supergravity theory has not yet been constructed[4], it is, however, still unclear how general these no-go theorems really are. A construction of the most general type of these theories should therefore help to settle this question, and might also be interesting for (bulk-)matter coupled generalizations of the discontinuous model of [5, 7]. At the same time, a complete knowledge of $\mathcal{N} = 2$ gauged supergravities might also contribute to a better understanding of various aspects of the $\mathcal{N} = 8$ theory (like e.g. the structure of its vacua) with possible implications for the AdS/CFT-correspondence.

Motivated by these and other applications, we have recently studied the possible gaugings of vector and tensor field coupled $5d$, $\mathcal{N} = 2$ supergravity theories. All these theories (including those involving tensor multiplets) can be derived from the *ungauged* $\mathcal{N} = 2$ Maxwell/Einstein supergravity theories of ref. [11]. These theories describe the coupling of Abelian vector multiplets to $\mathcal{N} = 2$ supergravity and have a global symmetry group of the form $SU(2)_R \times G$. Here, G is the subgroup of the isometry group of the scalar field target space that extends to a symmetry group of the full Lagrangian, and $SU(2)_R$ denotes the automorphism group ("R-symmetry group") of the $5d$, $\mathcal{N} = 2$ Poincaré superalgebra.

In [12] we generalized the earlier work [13] and constructed all possible gaugings of subgroups of $U(1)_R \times G$, where $U(1)_R \subset SU(2)_R$ denotes the Abelian subgroup of $SU(2)_R$. In particular, we also covered the case when the gauging of a subgroup of G involves the dualization of some of the vector fields to "self-dual" [14] antisymmetric tensor fields, a mechanism that is well-known from the maximally extended gauged supergravities in $d = 7$ [15] and $d = 5$ [16, 17, 18] dimensions.

Thus, the only gaugings that have not yet been covered in this framework, are those involving gaugings of the *full* R-symmetry group $SU(2)_R$. It is the purpose of this paper to close this gap. This will complete the construction of the possible gaugings of the entire vector/tensor sector of $\mathcal{N} = 2$ matter coupled supergravity theories in five dimensions.

[4]See note added.

2 Gauging the full R-symmetry group $SU(2)_R$

The gauging of $SU(2)_R$ is a little less straightforward than gaugings of subgroups of $U(1)_R \times G$, as we shall now explain.

The supermultiplets we are dealing with are ($\mu, \nu, \dots$ and $m, n, \dots$ denote curved and flat spacetime indices, respectively):

(i) The $\mathcal{N} = 2$ supergravity multiplet, containing the graviton (fünfbein) e_μ^m, two gravitini Ψ_μ^i ($i, j, \dots = 1, 2$) and one vector field A_μ

(ii) The $\mathcal{N} = 2$ vector multiplet, comprising one vector field A_μ, two spin-1/2 fermions λ^i ($i, j, \dots 1, 2$) and one real scalar field φ

(iii) The $\mathcal{N} = 2$ "selfdual" tensor multiplet consisting of two real two-form fields $B_{\mu\nu}^{(1)}$, $B_{\mu\nu}^{(2)}$; four spin-1/2 fermions $\lambda^{(1)i}$, $\lambda^{(2)i}$ ($i, j, \dots = 1, 2$) and two real scalar fields $\varphi^{(1)}$, $\varphi^{(2)}$.

Of all the above fields, only the gravitini and the spin-1/2 fermions transform non-trivially under $SU(2)_R$ (they are forming doublets labelled by the index $i = 1, 2$). In particular, all the vector fields are *singlets* under $SU(2)_R$. In order to gauge a non-Abelian symmetry group like $SU(2)_R$, however, one needs vector fields that transform in the *adjoint* representation of the gauge group.

The only way to solve this problem is to identify $SU(2)_R$ with an $SU(2)$ subgroup of the scalar manifold isometry group G and to gauge both $SU(2)$'s simultaneously. In other words, $SU(2)_R$ cannot be gauged by itself, rather one has to gauge a diagonal subgroup of $SU(2)_R \times SU(2)_G \subset SU(2)_R \times G$. The most natural starting point for a gauging of $SU(2)_R$ is therefore a "Yang-Mills/Einstein supergravity theory" (see [13, 12, 19] for details on this terminology) in which a subgroup $K \supset SU(2)_G$ of G is gauged. In order to be as general as possible, we consider the case when the supersymmetric gauging of $K \subset G$ requires the introduction of tensor fields (the case without tensor fields can easily be recovered as a special case). At this point we require the gauge group K only to have an $SU(2)$ subgroup $SU(2)_G \subset K$, but leave it otherwise undetermined.

We start by recalling some relevant properties of Yang-Mills/Einstein supergravity theories with tensor fields (see [12, 19] for details). Yang-Mills/Einstein supergravity theories with tensor fields describe the coupling of n vector multiplets and m self-dual tensor multiplets to supergravity. Consequently, the field content of these theories is

$$\{e_\mu^m, \Psi_\mu^i, A_\mu^I, B_{\mu\nu}^M, \lambda^{i\tilde{a}}, \varphi^{\tilde{x}}\} \tag{2.1}$$

where

$$I, J, K \dots \quad = \quad 0, 1, \dots n$$

$$\begin{aligned} M, N, P \ldots &= 1, 2, \ldots 2m \\ \tilde{a}, \tilde{b}, \tilde{c}, \ldots &= 1, \ldots, \tilde{n} \\ \tilde{x}, \tilde{y}, \tilde{z}, \ldots &= 1, \ldots, \tilde{n}, \end{aligned}$$

with $\tilde{n} = n + 2m$.

Note that we have combined the 'graviphoton' with the n vector fields of the n vector multiplets into a single $(n+1)$-plet of vector fields A_μ^I labelled by the index I. Also, the spinor and scalar fields of the vector and tensor multiplets are combined into $\tilde{n}$-tupels of spinor and scalar fields. The indices $\tilde{a}, \tilde{b}, \ldots$ and $\tilde{x}, \tilde{y}, \ldots$ are the flat and curved indices, respectively, of the $\tilde{n}$-dimensional target manifold $\mathcal{M}$ of the scalar fields. The metric, vielbein and spin connection on $\mathcal{M}$ will be denoted by $g_{\tilde{x}\tilde{y}}$, $f_{\tilde{x}}^{\tilde{a}}$ and $\Omega_{\tilde{x}}^{\tilde{a}\tilde{b}}$, respectively.

The fields that transform non-trivially under K are the gauge fields themselves as well as the tensor fields $B_{\mu\nu}^M$, the spin-1/2 fields $\lambda^{i\tilde{a}}$ and the scalar fields $\varphi^{\tilde{x}}$. The K-gauge covariant derivatives of these fields are as follows (∇ denotes the ordinary spacetime covariant derivative, and g is the coupling constant of the gauge group K)

$$\begin{aligned} \mathcal{D}_\mu \lambda^{i\tilde{a}} &\equiv \nabla_\mu \lambda^{i\tilde{a}} + g A_\mu^I L_I^{\tilde{a}\tilde{b}} \lambda^{i\tilde{b}} \\ \mathcal{D}_\mu \varphi^{\tilde{x}} &\equiv \partial_\mu \varphi^{\tilde{x}} + g A_\mu^I K_I^{\tilde{x}} \\ \mathcal{D}_\mu B_{\nu\rho}^M &\equiv \nabla_\mu B_{\nu\rho}^M + g A_\mu^I \Lambda_{IN}^M B_{\nu\rho}^N. \end{aligned} \tag{2.2}$$

Here, $K_I^{\tilde{x}}$ are the Killing vector fields on $\mathcal{M}$ that generate the subgroup K of its isometry group. The φ-dependent matrices $L_I^{\tilde{a}\tilde{b}}$ and the *constant* matrices Λ_{IN}^M are the K-transformation matrices of $\lambda^{i\tilde{a}}$ and $B_{\mu\nu}^M$, respectively.

We denote the curls of A_μ^I by $F_{\mu\nu}^I$ and combine the non-Abelian field strengths $\mathcal{F}_{\mu\nu}^I = F_{\mu\nu}^I + g f_{JK}^I A_\mu^J A_\nu^K$ with the antisymmetric tensor fields $B_{\mu\nu}^M$ to form the tensorial quantity

$$\mathcal{H}_{\mu\nu}^{\tilde{I}} := (\mathcal{F}_{\mu\nu}^I, B_{\mu\nu}^M), \qquad (\tilde{I}, \tilde{J}, \tilde{K}, \ldots = 0, \ldots, n+2m).$$

The general Lagrangian of a Yang-Mills/Einstein supergravity theory with tensor fields is then given by [12]

$$\begin{aligned} e^{-1}\mathcal{L} = &-\frac{1}{2}R(\omega) - \frac{1}{2}\bar{\Psi}_\mu^i \Gamma^{\mu\nu\rho} \nabla_\nu \Psi_{\rho i} - \frac{1}{4} \overset{\circ}{a}_{\tilde{I}\tilde{J}} \mathcal{H}_{\mu\nu}^{\tilde{I}} \mathcal{H}^{\tilde{J}\mu\nu} \\ &-\frac{1}{2}\bar{\lambda}^{i\tilde{a}} \left(\Gamma^\mu \mathcal{D}_\mu \delta^{\tilde{a}\tilde{b}} + \Omega_{\tilde{x}}^{\tilde{a}\tilde{b}} \Gamma^\mu \mathcal{D}_\mu \varphi^{\tilde{x}} \right) \lambda_i^{\tilde{b}} - \frac{1}{2} g_{\tilde{x}\tilde{y}} (\mathcal{D}_\mu \varphi^{\tilde{x}})(\mathcal{D}^\mu \varphi^{\tilde{y}}) \\ &-\frac{i}{2}\bar{\lambda}^{i\tilde{a}} \Gamma^\mu \Gamma^\nu \Psi_{\mu i} f_{\tilde{x}}^{\tilde{a}} \mathcal{D}_\nu \varphi^{\tilde{x}} + \frac{1}{4} h_{\tilde{I}}^{\tilde{a}} \bar{\lambda}^{i\tilde{a}} \Gamma^\mu \Gamma^{\lambda\rho} \Psi_{\mu i} \mathcal{H}_{\lambda\rho}^{\tilde{I}} \\ &+\frac{i}{2\sqrt{6}} \left(\frac{1}{4} \delta_{\tilde{a}\tilde{b}} h_{\tilde{I}} + T_{\tilde{a}\tilde{b}\tilde{c}} h_{\tilde{I}}^{\tilde{c}} \right) \bar{\lambda}^{i\tilde{a}} \Gamma^{\mu\nu} \lambda_i^{\tilde{b}} \mathcal{H}_{\mu\nu}^{\tilde{I}} \\ &-\frac{3i}{8\sqrt{6}} h_{\tilde{I}} \left[\bar{\Psi}_\mu^i \Gamma^{\mu\nu\rho\sigma} \Psi_{\nu i} \mathcal{H}_{\rho\sigma}^{\tilde{I}} + 2 \bar{\Psi}^{\mu i} \Psi_i^\nu \mathcal{H}_{\mu\nu}^{\tilde{I}} \right] \end{aligned}$$

$$
\begin{aligned}
&+\frac{e^{-1}}{6\sqrt{6}}C_{IJK}\varepsilon^{\mu\nu\rho\sigma\lambda}\left\{F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda+\frac{3}{2}gF^I_{\mu\nu}A^J_\rho(f^K_{LF}A^L_\sigma A^F_\lambda)\right.\\
&\qquad\qquad\qquad\qquad\left.+\frac{3}{5}g^2(f^J_{GH}A^G_\nu A^H_\rho)(f^K_{LF}A^L_\sigma A^F_\lambda)A^I_\mu\right\}\\
&+\frac{e^{-1}}{4g}\varepsilon^{\mu\nu\rho\sigma\lambda}\Omega_{MN}B^M_{\mu\nu}\mathcal{D}_\rho B^N_{\sigma\lambda}\\
&+g\bar{\lambda}^{i\tilde{a}}\Gamma^\mu\Psi_{\mu i}W_{\tilde{a}}+g\bar{\lambda}^{i\tilde{a}}\lambda^{\tilde{b}}_i W_{\tilde{a}\tilde{b}}-g^2P
\end{aligned}
\tag{2.3}
$$

with $e\equiv\det(e^m_\mu)$. The transformation laws are (to leading order in fermion fields)

$$
\begin{aligned}
\delta e^m_\mu &= \frac{1}{2}\bar{\varepsilon}^i\Gamma^m\Psi_{\mu i}\\
\delta\Psi^i_\mu &= \nabla_\mu\varepsilon^i+\frac{i}{4\sqrt{6}}h_{\tilde{I}}(\Gamma_\mu{}^{\nu\rho}-4\delta^\nu_\mu\Gamma^\rho)\mathcal{H}^{\tilde{I}}_{\nu\rho}\varepsilon^i\\
\delta A^I_\mu &= \vartheta^I_\mu\\
\delta B^M_{\mu\nu} &= 2\mathcal{D}_{[\mu}\vartheta^M_{\nu]}+\frac{\sqrt{6}g}{4}\Omega^{MN}h_N\bar{\Psi}^i_{[\mu}\Gamma_{\nu]}\varepsilon_i+\frac{ig}{4}\Omega^{MN}h_{N\tilde{a}}\bar{\lambda}^{i\tilde{a}}\Gamma_{\mu\nu}\varepsilon_i\\
\delta\lambda^{i\tilde{a}} &= -\frac{i}{2}f^{\tilde{a}}_{\tilde{x}}\Gamma^\mu(\mathcal{D}_\mu\varphi^{\tilde{x}})\varepsilon^i+\frac{1}{4}h^{\tilde{a}}_{\tilde{I}}\Gamma^{\mu\nu}\varepsilon^i\mathcal{H}^{\tilde{I}}_{\mu\nu}+gW^{\tilde{a}}\varepsilon^i\\
\delta\varphi^{\tilde{x}} &= \frac{i}{2}f^{\tilde{x}}_{\tilde{a}}\bar{\varepsilon}^i\lambda^{\tilde{a}}_i
\end{aligned}
\tag{2.4}
$$

with

$$
\vartheta^{\tilde{I}}_\mu\equiv-\frac{1}{2}h^{\tilde{I}}_{\tilde{a}}\bar{\varepsilon}^i\Gamma_\mu\lambda^{\tilde{a}}_i+\frac{i\sqrt{6}}{4}h^{\tilde{I}}\bar{\Psi}^i_\mu\varepsilon_i. \tag{2.5}
$$

The various scalar field dependent quantities $\overset{\circ}{a}_{\tilde{I}\tilde{J}}$, $h_{\tilde{I}}$, $h^{\tilde{I}}$, $h^{\tilde{a}}_{\tilde{I}}$, $h^{\tilde{I}\tilde{a}}$ and $T_{\tilde{a}\tilde{b}\tilde{c}}$ that contract the different types of indices are already present in the corresponding *ungauged* MESGT's and describe the "very special"geometry of the scalar manifold $\mathcal{M}$ (see [11] for details). These ungauged MESGT's also contain a constant symmetric tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$. If the gauging of K involves the introduction of tensor fields, the coefficients of the type C_{MNP} and C_{IJM} have to vanish [12]. The only components that survive such a gauging are thus C_{IJK}, which appear in the Chern-Simons-like term of (2.3), and C_{IMN}, which are related to the transformation matrices of the tensor fields by

$$
\Lambda^M_{IN}=\frac{2}{\sqrt{6}}\Omega^{MP}C_{IPN}.
$$

Here Ω^{MN} is the inverse of Ω_{MN}, which is a (constant) invariant antisymmetric tensor of the gauge group K:

$$
\Omega_{MN}=-\Omega_{NM},\qquad \Omega_{MN}\Omega^{NP}=\delta^P_M. \tag{2.6}
$$

The quantities $W^{\tilde{a}}(\varphi)$ and $W^{\tilde{a}\tilde{b}}(\varphi)$ and the scalar potential $P(\varphi)$ are due to the gauging of K in the presence of the tensor fields, and are given by

$$\begin{aligned} W^{\tilde{a}} &= -\frac{\sqrt{6}}{8} h^{\tilde{a}}_{M} \Omega^{MN} h_N \\ W^{\tilde{a}\tilde{b}} &= -W^{\tilde{b}\tilde{a}} = i h^{J[\tilde{a}} K_J^{\tilde{b}]} + \frac{i\sqrt{6}}{4} h^J K_J^{\tilde{a};\tilde{b}} \\ P &= 2 W^{\tilde{a}} W^{\tilde{a}}, \end{aligned} \tag{2.7}$$

where the semicolon denotes covariant differentiation on the target space $\mathcal{M}$.

We will now use the above theory as our starting point for the additional gauging of $SU(2)_R$. To this end, we first split the index I of the $(n+1)$ vector fields A_μ^I according to

$$I = (A, I'),$$

where $A, B, C, \ldots \in \{1, 2, 3\}$ are the indices corresponding the three gauge fields of $SU(2)_G \subset K$, and $I', J', K', \ldots$ label the remaining $(n-2)$ vector fields.

In order to gauge $SU(2)_R$, we use the gauge fields A_μ^A to covariantize the K-covariant derivatives of the fermions also with respect to $SU(2)_R$, i.e., we make the replacements

$$\nabla_\mu \Psi_\nu^i \longrightarrow \mathfrak{D}_\mu \Psi_\nu^i := \nabla_\mu \Psi_\nu^i + g_R A_\mu^A \Sigma_{Aj}^{\ i} \Psi_\nu^j \tag{2.8}$$

$$\nabla_\mu \varepsilon^i \longrightarrow \mathfrak{D}_\mu \varepsilon^i := \nabla_\mu \varepsilon^i + g_R A_\mu^A \Sigma_{Aj}^{\ i} \varepsilon^j \tag{2.9}$$

$$\begin{aligned} \mathcal{D}_\mu \lambda^{i\tilde{a}} \longrightarrow \mathfrak{D}_\mu \lambda^{i\tilde{a}} &:= \mathcal{D}_\mu \lambda^{i\tilde{a}} + g_R A_\mu^A \Sigma_{Aj}^{\ i} \lambda^{j\tilde{a}} \\ &\equiv \nabla_\mu \lambda^{i\tilde{a}} + g A_\mu^I L_I^{\tilde{a}\tilde{b}} \lambda^{i\tilde{b}} + g_R A_\mu^A \Sigma_{Aj}^{\ i} \lambda^{j\tilde{a}} \end{aligned} \tag{2.10}$$

in the Lagrangian (2.3) and the transformation laws (2.4). Here, g_R denotes the $SU(2)_R$ coupling constant, and the $\Sigma_{Aj}^{\ i}$ $(i, j, \ldots = 1, 2)$ are the $SU(2)_R$ transformation matrices of the fermions, which can be chosen as

$$\Sigma_{Aj}^{\ i} = \frac{i}{2} \sigma_{Aj}^{\ i} \tag{2.11}$$

with $\sigma_{Aj}^{\ i}$ being the Pauli matrices. The indices $i, j, \ldots$ are raised and lowered according to

$$X^i = \varepsilon^{ij} X_j\,, \qquad X_i = X^j \varepsilon_{ji}$$

(which implies that the Σ_{Aij} are symmetric in i and j).

The above replacements break supersymmetry, but the latter can be restored by adding

$$\begin{aligned} e^{-1}\mathcal{L}' &= g_R \bar{\Psi}_\mu^i \Gamma^{\mu\nu} \Psi_\nu^j R_{0ij}(\varphi) + g_R \bar{\lambda}^{i\tilde{a}} \Gamma^\mu \Psi_\mu^j R_{\tilde{a}ij}(\varphi) \\ &+ g_R \bar{\lambda}^{i\tilde{a}} \lambda^{j\tilde{b}} R_{\tilde{a}\tilde{b}ij}(\varphi) - g_R^2 P^{(R)}(\varphi), \end{aligned} \tag{2.12}$$

to the Lagrangian and by adding

$$\begin{aligned} \delta' \Psi_{\mu i} &= \frac{2}{3} g_R R_{0ij}(\varphi) \Gamma_\mu \varepsilon^j \\ \delta' \lambda_i^{\tilde{a}} &= g_R R_{ij}^{\tilde{a}}(\varphi) \varepsilon^j \end{aligned} \tag{2.13}$$

to the transformation laws.

The quantities R_{0ij}, $R_{ij}^{\tilde{a}}$, $R_{\tilde{a}\tilde{b}ij}$ and the additional potential term $P^{(R)}$ are fixed by supersymmetry:

$$R_{0ij} = i\sqrt{\frac{3}{8}} h^A \Sigma_{Aij} \tag{2.14}$$

$$R_{ij}^{\tilde{a}} = h^{A\tilde{a}} \Sigma_{Aij} \tag{2.15}$$

$$R_{\tilde{a}\tilde{b}ij} = -\frac{1}{3} \delta_{\tilde{a}\tilde{b}} R_{0ij} - i\sqrt{\frac{2}{3}} T_{\tilde{a}\tilde{b}\tilde{c}} R_{ij}^{\tilde{c}} \tag{2.16}$$

$$P^{(R)} = -\frac{16}{3} R_{0j}^{\ i} R_{0\ i}^{\ j} - R^{\tilde{a}i}{}_j R^{\tilde{a}j}{}_i. \tag{2.17}$$

Supersymmetry also requires

$$g_R = g \tag{2.18}$$

$$f_{I'B}^A = f_{I'J'}^A = 0 \tag{2.19}$$

$$[\Sigma_A, \Sigma_B]_{ij} = f_{AB}^C \Sigma_{Cij} \tag{2.20}$$

$$\Sigma_{Aij,\tilde{x}} = 0. \tag{2.21}$$

(The structure constants of the type $f_{I'A}^{J'}$ do not necessarily have to vanish for supersymmetry. If they do vanish, however, K is a direct product of $SU(2)_G$ and some other group K'. Otherwise K is a semi-direct product of the form $(SU(2)_G \times S) \odot T$, where $\odot$ denotes semi-direct product.)

The following constraints are consequences of the above and are needed in the proof of supersymmetry:

$$R_{ij}^{\tilde{a}} K_J^{\tilde{a}} = -\sqrt{\frac{3}{2}} \Sigma_{Aij} f_{JK}^A h^K \tag{2.22}$$

$$-i\sqrt{\frac{3}{2}} h^{B\tilde{a}} f_{BC}^A h^C \Sigma_{Aij} = \frac{10}{3} W^{\tilde{a}} R_{0ij} + 2R_{ij}^{\tilde{b}} W_{\tilde{a}\tilde{b}} + 2R_{\tilde{a}\tilde{b}ij} W^{\tilde{b}} \tag{2.23}$$

$$R_{ij;\tilde{x}}^{\tilde{a}} = i f_{\tilde{x}}^{\tilde{a}} R_{0ij} - i R_{\tilde{a}\tilde{b}ij} f_{\tilde{x}}^{\tilde{b}} \tag{2.24}$$

$$R_{0ij,\tilde{x}} = -\frac{i}{2} R_{\tilde{x}ij}. \tag{2.25}$$

Furthermore the $\delta\varphi^{\tilde{x}}$ variation of $P^{(R)}$ and similar terms requires that

$$\mathrm{tr}(\Sigma_A \Sigma_B \Sigma_C) = \frac{1}{2} f_{AB}^{D} \mathrm{tr}(\Sigma_C \Sigma_D) \tag{2.26}$$

(as well as $\mathrm{tr}(\Sigma_A) = 0$), which can be directly verified in the basis (2.11).)

Even though the constraints from supersymmetry allows K to be a semi-direct product group, we shall restrict ourselves to gauge groups that are not of the semi-direct type. In this case K is a *direct* product of $SU(2)_G$ with another group. We can thus confine ourselves to the case $K = SU(2)_G$, since additional factors don't change the structure of the above theory very much.

Now to be able to gauge $K = SU(2)_G$, the isometry group of the scalar manifold that extends to a symmetry of the MESGT must have a $SU(2)$ subgroup such that three of the vector fields of the theory transform in the adjoint representation of $K = SU(2)_G$. For the generic Jordan family of MESGT's with the scalar manifold $SO(n-1,1) \times SO(1,1)/SO(n-1)$ such a subgroup exists for all theories with $n > 3$. Similarly for the generic non-Jordan family with the scalar manifold $SO(n,1)/SO(n)$ one can gauge $SU(2)_G$ whenever $n > 3$.

Of the magical N=2 MESGT's, all but the the one defined by the Jordan algebra of real symmetric (3×3) matrices, J_3^R, admit such a gauging.

Finally, all the members of the infinite family with $SU(N)$ isometries ($N > 3$) [12] also admit a gauging of $SU(2)_R$.

For the generic Jordan and the generic non-Jordan families one can choose the $SU(2)_G$ subgroup of the the isometry group such that all the other fields are inert under it, i.e. one does not have to dualize any vector field to tensor fields. On the other hand, the gauging of $SU(2)_G$ requires the dualization of some of the vector fields to tensor fields in the magical theories as well as in the theories with $SU(N)$ isometries.

We should note that the triplet of vector fields transforming in the adjoint representation of $SU(2)_G$ cannot include the graviphoton. This is also expected from the fact that $SU(2)_G$ is a subgroup of the compact part of the isometry group of $\mathcal{M}$, under which the graviphoton is inert. This shows that the $U(1)_R$ gaugings obtained by restricting oneself to a $U(1)$ subgroup of $SU(2)_R$ is not the most general $U(1)_R$ gauging possible. For the most general $U(1)_R$ gauging, we can choose an arbitrary linear combination $A_\mu^I V_I$ of all the vector fields as was done in [13], including the graviphoton.

For the theories of the Jordan family, it was shown that the generic $U(1)_R$ gauging either leads to a flat potential with Minkowski ground states whenever V_I corresponds to an idempotent of the Jordan algebra, or an Anti-de Sitter groundstate whenever V_I lies in the "domain of positivity" of the Jordan algebra. Looking at the $U(1)_R$ restrictions of the $SU(2)_R$ gaugings in the Jordan family, however, one finds that the V_I are neither idempotents nor do they lie in the domain of positivity. This already suggests that, at least in the Jordan family, the $SU(2)_R$ gauging leads to theories without critical points. We will verify this statement for all theories for which the scalar manifold $\mathcal{M}$ is a symmetric space.

These can be divided into three families: the generic Jordan, the magical Jordan and the generic non-Jordan family.

The generic Jordan family corresponds to the scalar manifolds of the form $\mathcal{M} = SO(1,1)\times SO(\tilde{n}-1,1)/SO(\tilde{n}-1)$. The latter can be described as the hypersurface $N(\xi)=1$ of the cubic polynomial

$$\begin{aligned} N(\xi) &= \left(\frac{2}{3}\right)^{\frac{3}{2}} C_{\tilde{I}\tilde{J}\tilde{K}}\xi^{\tilde{I}}\xi^{\tilde{J}}\xi^{\tilde{K}} \\ &= \sqrt{2}\xi^0\left[(\xi^1)^2-(\xi^2)^2-\ldots-(\xi^{\tilde{n}})^2\right] \end{aligned} \tag{2.27}$$

The isometry group of this space is $SO(1,1)\times SO(\tilde{n}-1,1)$. For $SU(2)\sim SO(3)$ to be a subgroup, one obviously needs $\tilde{n}\geq 4$, as we will assume from now on.

The constraint $N(\xi)=1$ can be solved by

$$\begin{aligned} \xi^0 &= \frac{1}{2\|\varphi\|^2} \\ \xi^1 &= \varphi^1 \\ &\vdots \\ \xi^{\tilde{n}} &= \varphi^{\tilde{n}}, \end{aligned} \tag{2.28}$$

where $\|\varphi\|^2 \equiv (\varphi^1)^2-(\varphi^2)^2-\ldots(\varphi^{\tilde{n}})^2$ has been introduced. As explained in [19], the scalar field metric $g_{\tilde{x}\tilde{y}}$ and the vector field metric $\overset{\circ}{a}_{\tilde{I}\tilde{J}}$ are positive definite only for $\|\varphi\|^2>0$.

Without loss of generality, we choose A^2_μ, A^3_μ, A^4_μ, as the $SO(3)$ gauge fields. Then using [11]

$$C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{K}} = h_{\tilde{I}}h_{\tilde{J}} - \frac{1}{2}h^{\tilde{a}}_{\tilde{I}}h^{\tilde{a}}_{\tilde{J}},$$

the scalar potential $P^{(R)}$ (eq. (2.17)) can always be brought to the form

$$P^{(R)} = -C^{AB\tilde{I}}h_{\tilde{I}}\delta_{AB}, \tag{2.29}$$

where we have defined

$$C^{\tilde{I}\tilde{J}\tilde{K}} \equiv \overset{\circ}{a}{}^{\tilde{I}\tilde{I}'}\overset{\circ}{a}{}^{\tilde{J}\tilde{J}'}\overset{\circ}{a}{}^{\tilde{K}\tilde{K}'}C_{\tilde{I}'\tilde{J}'\tilde{K}'}$$

with $\overset{\circ}{a}{}^{\tilde{I}\tilde{J}}$ being the inverse of $\overset{\circ}{a}_{\tilde{I}\tilde{J}}$.

For the Jordan cases, one has $C_{\tilde{I}\tilde{J}\tilde{K}} = C^{\tilde{I}\tilde{J}\tilde{K}} =$ const. (componentwise). Using $h_{\tilde{I}} = \frac{1}{\sqrt{6}}\frac{\partial}{\partial\xi^{\tilde{I}}}N|_{N=1}$, one finally obtains

$$P^{(R)} = \frac{3}{2}\|\varphi\|^2. \tag{2.30}$$

It is easy to see that this scalar potential does *not* admit any critical points in the physically relevant region $\|\varphi\|^2>0$.

This situation doesn't change, when an additional $SO(2)$ subgroup of G is gauged as in [19]. For this, one needs at least $\tilde{n} \geq 6$. Choosing ξ^5 and ξ^6 as transforming as an $SO(2)$ doublet, the corresponding vector fields A_μ^5 and A_μ^6 have to be dualized to tensor fields. this gives rise to the additional potential term [19]

$$P = \frac{1}{8}\frac{\left[(\varphi^5)^2 + (\varphi^6)^2\right]}{\|\varphi\|^6}. \tag{2.31}$$

It is easy to verify that the combined potential $P_{tot} = P^{(R)} + P$ does not have any ground states either.

We now turn to the magical Jordan family [11]. The simplest example in which $SU(2)_R$ can be gauged, is provided by the model with the scalar manifold $\mathcal{M} = SL(3,\mathbb{C})/SU(3)$. This theory contains eight vector multiplets (i.e. eight scalar fields and nine vector fields). $\mathcal{M}$ can be described as the hypersurface $N(\xi) = 1$ of the cubic polynomial

$$N(\xi) = \sqrt{2}\xi^4\eta_{\alpha\beta}\xi^\alpha\xi^\beta + \gamma_{\alpha MN}\xi^\alpha\xi^M\xi^N, \tag{2.32}$$

where

$$\begin{aligned}
\alpha, \beta, \ldots &= 0,1,2,3 \\
M, N, \ldots &= 5,6,7,8 \\
\eta_{\alpha\beta} &= \mathrm{diag}(+,-,-,-) \\
\gamma_0 &= -\mathbf{1}_4 \\
\gamma_1 &= \mathbf{1}_2 \otimes \sigma_1 \\
\gamma_2 &= -\sigma_2 \otimes \sigma_2 \\
\gamma_3 &= \mathbf{1}_2 \otimes \sigma_3.
\end{aligned}$$

It is easy to show that the vector field metric becomes degenerate, when $\eta_{\alpha\beta}\xi^\alpha\xi^\beta = 0$. We therefore can restrict ourselves to the region $\eta_{\alpha\beta}\xi^\alpha\xi^\beta \neq 0$, where the constraint $N(\xi) = 1$ can be solved by

$$\begin{aligned}
\xi^\alpha &= \varphi^\alpha =: x^\alpha \\
\xi^4 &= \frac{1 - b^T\bar{x}b}{\sqrt{2}\|x\|^2} \\
\xi^M &= \varphi^M =: b^M,
\end{aligned}$$

where $b^T\bar{x}b \equiv b^M x^\alpha \bar{x}_{MN} b^N$ with $\bar{x}_{MN} \equiv x^\alpha\gamma_{\alpha MN}$ and $\|x\|^2 \equiv \eta_{\alpha\beta}x^\alpha x^\beta$.

In the above model, one can gauge a $(U(1) \times SU(2))$-subgroup of the isometry group $SL(3,\mathbb{C})$. The vector field A_μ^0 corresponds to the $U(1)$ gauge field, whereas the vector fields A_μ^1, A_μ^2, A_μ^3 act as the $SU(2)$ gauge fields. The vector fields A_μ^M are charged under

$(U(1)\times SU(2))$ and have to be dualized to tensor fields. The vector field A^4_μ is a spectator vector field. The introduction of the tensor fields leads to a non-trivial potential P, which turns out to be

$$P = -\frac{1}{8} b^T (\bar{x})^3 b. \tag{2.33}$$

As described earlier, the $SU(2)_G$ gauge fields A^1_μ, A^2_μ, A^3_μ can be used to simultaneously gauge $SU(2)_R$. This leads to an additional potential

$$P^{(R)} = \frac{3}{2}\|\varphi\|^2. \tag{2.34}$$

Taking into account that $\det((\bar{x})^3) = [\|x\|^2]^6$, it easy to verify that the total potential $P_{tot} = P + P^{(R)}$ does *not* have any critical points in the physically relevant region where $\|x\|^2 \neq 0$.

The other magical theories corresponding to $\mathcal{M} = SU{*}(6)/Usp(6)$ and $\mathcal{M} = E_{6(-26)}/F_4$, which also allow the gauging of $SU(2)_R$ have a very similar structure to the above, and one doesn't expect to find any critical points either.

This leaves us with the theories of the generic non-Jordan family. They are given by $\mathcal{M} = SO(1,\tilde{n})/SO(\tilde{n})$, which can be described as the hypersurface $N(\xi)=1$ of

$$N(\xi) = \sqrt{2}\xi^0(\xi^1)^2 - \xi^1\left[(\xi^2)^2 + \ldots + (\xi^{\tilde{n}})^2\right] \tag{2.35}$$

The constraint $N=1$ can be solved by

$$\begin{aligned}
\xi^0 &= \frac{1}{\sqrt{2}(\varphi^1)^2} + \frac{1}{\sqrt{2}}\varphi^1\left[(\varphi^2)^2 + \ldots (\varphi^{\tilde{n}})^2\right] \\
\xi^1 &= \varphi^1 \\
\xi^2 &= \varphi^1\varphi^2 \\
&\vdots \\
\xi^{\tilde{n}} &= \varphi^1\varphi^{\tilde{n}}
\end{aligned}$$

In contrast to the Jordan families, one now has $C_{\tilde{I}\tilde{J}\tilde{K}} \neq C^{\tilde{I}\tilde{J}\tilde{K}} \neq$ const.. What makes the calculation of the scalar potential nevertheless feasible, however, is that the scalar field metric $g_{\tilde{x}\tilde{y}}$ becomes diagonal and therefore easily invertible in the above coordinate system. To be specific, one obtains

$$g_{\tilde{x}\tilde{y}} = \mathrm{diag}[3/(\varphi^1)^2, (\varphi^1)^3, \ldots (\varphi^1)^3] \tag{2.36}$$

In order to gauge a $SO(3) \sim SU(2)$ subgroup of the isometry group of $\mathcal{M}$, one obviously needs at least $\tilde{n} \geq 4$, as we will assume from now on. We choose A^2_μ, A^3_μ, A^4_μ as the $SU(2)_G$ gauge fields. Inspection of N above shows that this group rotates ξ^2, ξ^3, ξ^4 into each other, but leaves the other ξ unchanged. Thus, no tensor fields have to be introduced. The resulting scalar potential turns out to be

$$P^{(R)} = -\frac{1}{2}(\varphi^1)^2[(\varphi^2)^2 + (\varphi^3)^2 + (\varphi^4)^2] + \frac{3}{2}\frac{1}{\varphi^1} \tag{2.37}$$

which does not admit any ground states either for the physically interesting region $\varphi^1 > 0$. Similar conclusions hold true when one also gauges an additional $SO(2)$ for $\tilde{n} > 5$. Thus, all the $SU(2)_R$ gaugings with *symmetric* scalar manifold $\mathcal{M}$ admit no critical points.

One also notes that the gauge coupling g_R for $SU(2)_R$ has to be the same as g, which is, of course, a consequence of the fact that we are gauging a diagonal subgroup of $SU(2)_R \times SU(2)_G$. This implies, however, that one cannot tune the relative coupling constants as in the gaugings of $U(1)_R \times K$ in order to change the properties of critical points of the scalar potential $P_{tot} = P + P^{(R)}$ [12, 19]. Hence, $SU(2)_R$-gauged supergravity theories are much more rigid than their $U(1)_R$-gauged relatives.

Note added: After this work was completed, we saw the paper hep-th/0004111 [20] in the archive on the general $\mathcal{N} = 2$ $d = 5$ supergravity including hypermultiplets and $SU(2)_R$ gauging. Although this work has clearly some overlap with our work, it is not quite clear whether our theories can be obtained from the ones in [20] by turning off the hypermultiplets. As the authors of [20] state, turning off their hypermultiplets, breaks $SU(2)_R$ gauge symmetry to $U(1)_R$.

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