The rf control and detection system for *PACO* the parametric converter detector

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Abstract

In this technical note the rf control and detection system for a detector of small harmonic displacements based on two coupled microwave cavities (*PACO*) is presented. The basic idea underlying this detector is the principle of parametric power conversion between two resonant modes of the system, stimulated by the (small) harmonic modulation of one system parameter. In this experiment we change the cavity length applying an harmonic voltage to a piezo-electric crystal. The system can achieve a great sensitivity to small harmonic displacements and can be an interesting candidate for the detection of small, mechanically coupled, interactions (e.g. high frequency gravitational waves).

1 Introduction

In this technical note we describe the rf control and detection system for a detector of small harmonic displacements based on two coupled microwave cavities (*PACO*). This experimental configuration was initially proposed by Bernard, Pegoraro, Picasso and Radicati [1], [2] and later put in practice by Reece, Reiner and Melissinos [3], [4], and has been discussed in some detail

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in previous papers [5], [6], [7], [8]. Here we just remind the basic principles underlying the detector operation.

The detector consists of an electromagnetic resonator with two resonant frequencies ω_s and ω_a both much higher than the characteristic frequency of the harmonic perturbation Ω, and which satisfy the resonance condition $|\omega_s - \omega_a| = \Omega$. In the scheme proposed in [1], [2] the two resonant modes are obtained coupling two identical resonators; ω_s is the frequency of the symmetric (even) resonant mode, while ω_a is the frequency of the antisymmetric (odd) one. If some energy is initially stored in the symmetric mode, an harmonic perturbation can induce the transition of some energy to the initially empty level that can be extracted at frequency ω_a. The electromagnetic power in the antisymmetric mode is proportional to the square of the amplitude of the perturbation.

In this experiment we use two cylindrical cavities coupled trough an axial iris and we change the cavity length applying an harmonic voltage to a piezo-electric crystal. To increase the sensitivity of the detector a resonator geometry and field configuration with high geometrical factor and high quality factor Q are preferred. To avoid electron field emission from the cavity surface rf modes with vanishing electric field at the surface are mandatory. For these reasons we have chosen TE mode (TE_{011}) superconducting cavities. The choice of frequency was imposed by the maximum dimension of the resonator that can be housed in a standard vertical cyostat; in our case the inner diameter is 300 mm, giving us enough room for a 3 GHz, TE_{011} resonator.

The system can achieve a great sensitivity to small harmonic displacements (up to $\delta l/l \approx 10^{-20}$ @ Ω =1 MHz, and can be an interesting candidate for the detection of small, mechanically coupled, interactions (e.g. high frequency gravitational waves).

2 *PACO* rf system

The three main functions of rf control and measurement system of the *PACO* experiment are shown in figure 1.

The system performs three separate functions listed in the following.

1. The first task of the system is to lock the rf frequency of the master oscillator to the resonant frequency of the even mode of the cavity and to keep constant the energy stored in the mode.

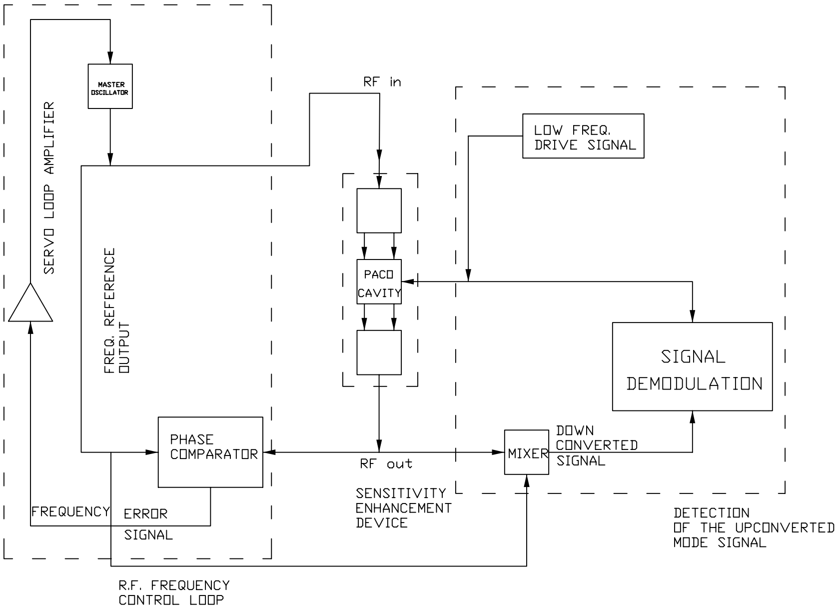

Figure 1: *PACO* rf system

2. The second task is to increase the detector's sensitivity by driving the coupled resonators purely in the even mode and receiving only the rf power up- converted to the odd mode by the perturbation of the cavity walls. This goal can be obtained by rejecting the signal at the even mode frequency tacking advantage of the symmetries in the field distribution of the two modes.

3. The third task is the detection of the up-converted signal pushing the detector's sensitivity to the limit set by the contribution of the noise sources at the operating frequency. The various noise sources have been described and discussed in a previous paper [6].

3 The rf control loop

The output of the master oscillator (HP4422B) is fed into the cavity trough a TWT amplifier giving a saturated output of 20 Watt in the frequency range of 2-4 GHz. The stored energy in the cavity is adjusted at the operating level by controlling the output of the master oscillator via the built in variable attenuator.

The output signal is sampled via a 3 dB power splitter. The A output of the splitter is sent to the TWT amplifier, the B output is sent, trough the phase shifter (PS), to the local oscillator (LO) input of a rf mixer acting as a phase detector (PD). The output of the rf power amplifier is fed to the resonant cavity trough a double directional coupler, and a 180° hybrid ring acting as a magic tee. The rf power enters the magic tee via the sum arm, Σ, and is split in two signals of same amplitude and zero relative phase, coming out the tee co-linear arms 1 and 2.

The rf signal, reflected by the input ports of the cavity, enters the magic tee trough the co-linear arms. The two signals are added at the Σ arm and sampled by the directional coupler to give information about the energy stored in cavity allowing for the measurement of the coupling factor, quality factor, stored energy. While driving the cavity on the even mode no reflected signal is shown at the Δ port of the magic tee where the signals coming from the co-linear arms are algebraically added to zero due to the 180° phase shift between the two signals coming by the co-linear arms.

To get the maximum of the performances of the magic tee we need to have equal reflected signals (phase and amplitude) at the cavity input ports. The equal amplitude goal can be achieved by a careful design of the input couplers; a good design guarantees us also very nice phase equality at the ports. To preserve the signal integrity we use matched output lines (in phase and amplitude) inside the cryostat. The cable used for the output lines is the best rf cable (as far as attenuation and phase stability are concerned) money can buy, nevertheless no characterisation at cryogenic temperatures exists. Because the phase shift is very sensitive to temperature inhomogeneities between the two cables and the phase difference between the two co-linear arms of the magic tee gives a quite strong signal at the Δ port, we need to compensate for differential thermal contractions of the cables inside the cryostat, leading to phase unbalance in the feed lines. To do that we insert a phase shifter in one of the lines to reduce to a minimum the leakage of the unwanted modes on the two ports. As we will show in the following section, mode leakage of the even mode to the Δ port sets a limit to the system sensitivity increasing the overall noise level of the detector.

Mode leakage of the odd mode to the Σ port reduces the system sensitivity by reducing the signal level available for detection. The converted rf power (odd mode) coupled to the input Σ port is lost forever. The rf system is symmetric on both the input and output ports of the detector cavity. The output ports of the cavity are coupled for a maximum output signal on the odd mode (detection mode) and the magic tee is used to reject the rf power at the frequency of the even mode. The up-converted signal (odd mode) comes out at the Δ port of the magic tee. A fraction of the signal at the Σ port is fed to the rf input of the phase detector PD via a low noise rf amplifier. The intermediate frequency (IF) output of the phase detector PD is fed back to the rf master oscillator to lock the output signal to the resonant frequency of the resonator. The total phase shift around the loop is set trough the phase shifter PS, to have the maximum of energy stored in the detector. A carefully design of the servo loop amplifier (SLA) guarantees the stability of the system and the rejection of the residual noise of the master oscillator up to one MHz. The same fraction of the Σ output of the output magic tee is used to keep constant, to 100 ppm, the energy stored in the cavity feeding back an error signal to drive the electronically controlled output attenuator of the master oscillator.

Great deal of care is needed in tailoring the frequency response of both controls because the two loops can interact producing phase-amplitude oscillations in the rf fields stored in the cavities.

4 Sensitivity enhancement using the mode symmetry

The two modes of the detector cavity have (as in the case of two coupled pendulums) opposite symmetries of the fields. The rf detection system can greatly be improved if the mode symmetry information is used in the rf feed and detection system to reduce the residual components of the noise produced by the master oscillator line width. A further improvement is obtained tacking advantage of the very high selectivity of the resonator's modes having a quality factor $Q \approx 10^{10}$.

Using two separate sets of ports to drive the cavity and to receive the up-converted signal (at the frequency of the odd mode), the cavity acts as a very sharp filter, with an high rejection of the signal noise (coming from the master oscillator) at the frequency of the up-converted signal. The resulting attenuation is given by the shape of the cavity resonance, a Lorentz shape with a full width half maximum $FWHM = f/Q$, where f is the resonant

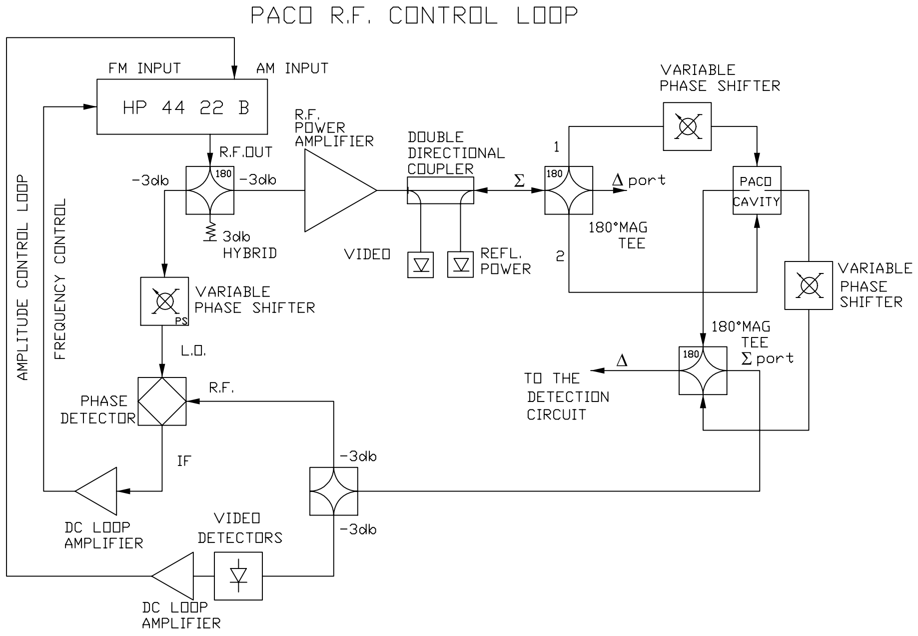

Figure 2: *PACO* rf control loop

frequency. This already low residual noise, can be even more reduced with the two magic tees. Using two equal couplers at the cavity input, driven by the two co-linear arms of a magic tee fed via the Σ port, we store energy in the cavity only on the even mode. Receiving the up-converted signal at the Δ port the of the second magic tee, rejects the even mode components from the cavity by an amount given by the magic tee insulation.

In the case of an ideal magic tee the mode rejection is infinite as the tee insulation. No even mode component is transmitted trough the system and there will be no signal at the output port if the cavity is driven purely in the even mode. In the ideal case this results is obtained also in the more simple scheme used by Melissinos and Reece [3], [4], measuring the up converted power coming out of the detector along the input lines.

Our scheme gives better sensitivity and performances in the real case. The first obvious gain is the sum of the Δ to Σ port insulation of the two tees, plus the possibility of adjusting separately the input and output lines to get better mode rejection. In a commercial magic tee the insulation is specified to be $\approx 25-35$ dB over the whole bandwidth of the device (usually

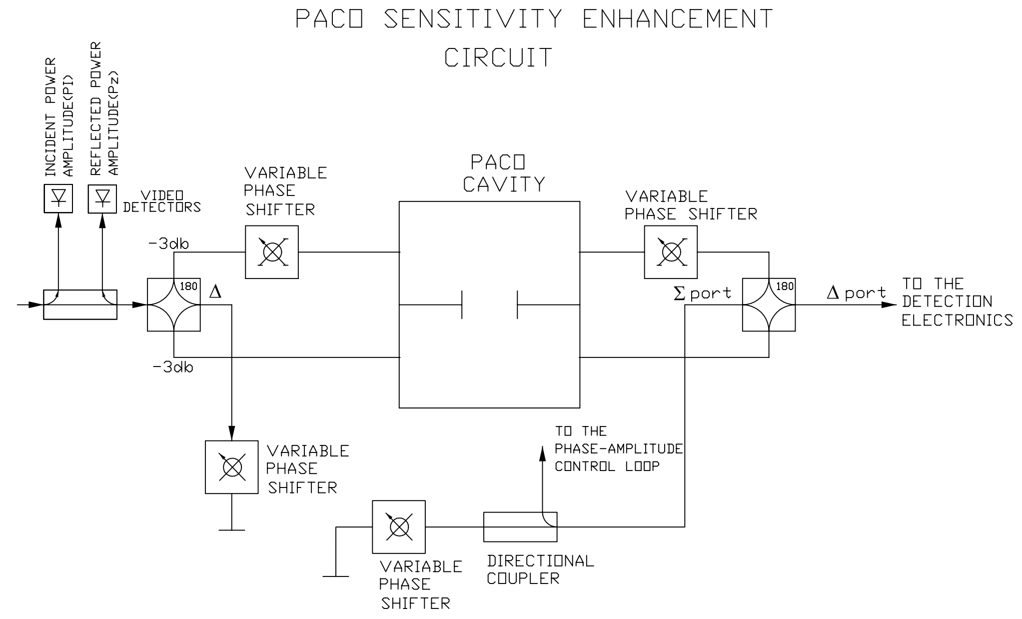

Figure 3: Sensitivity enhancement circuit

an octave in frequency). The reason for this quite low insulation is mainly due to the difficulty of balancing on a large range of frequency the phases of the signals coming from the two co-linear arms of the tee.

Consider two equal signals in phase

$$V = V_0 \cos(\omega t + \phi) \tag{1}$$

entering the co-linear arms of the magic tee; suppose a phase unbalance of α degrees between the two path to the Δ port : the resulting signals at the Δ are

$$\begin{aligned} V_\Delta = V_0 \cos(\omega t + \phi) - V_0 \cos(\omega t + \phi + \alpha) = \\ = V_0 \cos(\omega t + \phi) - V_0 \cos(\omega t + \phi) \cos\alpha + V_0 \sin(\omega t + \phi) \sin\alpha \end{aligned} \tag{2}$$

or, for a small value of α

$$V_\Delta = \alpha\, V_0 \sin(\omega t + \phi) \tag{3}$$

Now a phase unbalance as small as five degrees reduces the insulation from Δ to Σ port to only 25 dB. This fairly low insulation in magic tees comes from the difficulty of designing a 90 degrees rf broad band phase shifter. A custom designed magic tee can be optimized to give a better insulation (35-40 dB) on a more limited frequency interval. Since the bandwidth-insulation product is roughly constant, pushing the requirements to high insulation (70-80 dB) reduces too much the useful bandwidth, giving not so much flexibility in the rf system design . Any mismatch between the frequencies of the cavity modes and of the magic tees results in a very severe degradation of the noise rejection for the whole system at the detection frequency, spoiling the ultimate sensitivity of the detector.

Our electronic scheme allows for an independent compensation of the magic tee phase mismatch either at the feed frequency and at the detection frequency in a flexible way: the phase mismatch is compensated using a variable phase shifter at the input of one of the co-linear arms. Getting the optimum phase at the input side will results in a pure excitation of the even (drive) mode of the two cavity system, keeping the power at the frequency of the odd (detection) mode 70 dB (the tee insulation) below the level of the drive mode. Adjusting the phase at the output will couple to the output only the odd mode components rejecting the even mode component by 70 dB. The total even-odd mode rejection of the system is the sum of the attenuation we can obtain from the two 90^o hybrids; in the real world a practical limit is set by the cross talk effects between input and output limiting to 120-130 dB the maximum achievable insulation.

The price we pay for the mode selectivity improvement in our set-up is affordable and gives us the possibility of a separate and more controlled tuning of the input and output circuits. The use of two separated ports for the input and output (with separated magic tees) add some complication to the rf system. The coupling coefficient of the input and output port of the two cell cavity need to be critically coupled ($\beta = 1$) to the rf source and the rf detection system. In this way we will have the optimum transfer of power to the even mode (a maximum of stored energy) and to the odd mode (a maximum in the detector output). Because the frequency and field distribution of the two modes are quite close, the input and output ports will be critically coupled to both modes. For that reason 50% of the even mode signal will be coupled to the idle Σ port at the output magic tee, and symmetrically 50% of the odd mode signal is coupled to the idle Δ port at the input magic tee. The new couplings will greatly affect the detector's performances, affecting the loaded quality factor of the cavity and changing in a substantial way the way to couple the cavity to the rf source.

At the glance it is clear that closing the two idle ports with a matched load will worsen the detector performance due to the following reasons:

1. Half of the converted power at the odd mode frequency will be lost worsening of a factor 2 the detector sensitivity.
2. The loaded quality factor of the cavity is reduced by a factor roughly 2 with the same reduction in the system sensitivity.
3. The coupling to the rf generator is reduced with a reduction of the detector efficiency (due to the input mismatch the incoming power is partially reflected and more power is needed from the rf system to store the same energy in the cavity).

A detailed analysis showed that closing the idle ports with an impedance giving a reflection coefficient amplitude one and phase zero solves the problem reflecting back to the cavity all the rf power coupled to the idle ports. Using that termination of the idle ports the two rf arms at the input and output are de-coupled (at least by the mode selectivity factor equal to the sum of the Δ and Σ ports insulation of the two hybrids). An rf impedance having reflection coefficient equal to one is an open circuit or a short circuit plus a 180^{o} phase shift. For this reason we close the two idle ports using a phase shifter and a short. Again this solution gives the possibility to fine tune the system and compensate for the effect of non-ideal elements in the rf system.

At the Σ port of the detection arm we insert a directional coupler to sample a tiny amount of the even mode power coming from the cavity. This transmitted power is fed to the frequency-amplitude servo loop used to lock the frequency of the master oscillator to the cavity frequency and to keep constant the energy content in the cavity. This choice gives to our rf control system a better reliability over the Melissinos-Reece control system using the reflected power from the cavity for the frequency control loop. The effect of this sampling behaves in our system as the sampling antenna used to monitor and control the rf power fed to a superconducting cavity.

As a final remark our system, despite some complexity, guarantees the following improvement over the one used in the previous experiment:

- A better rejection of the phase amplitude noise of the master oscillator obtained using the sharp resonance of the resonator itself.
- A better insulation of the drive and detector ports obtained by using separate drive and detection arms of the rf system.

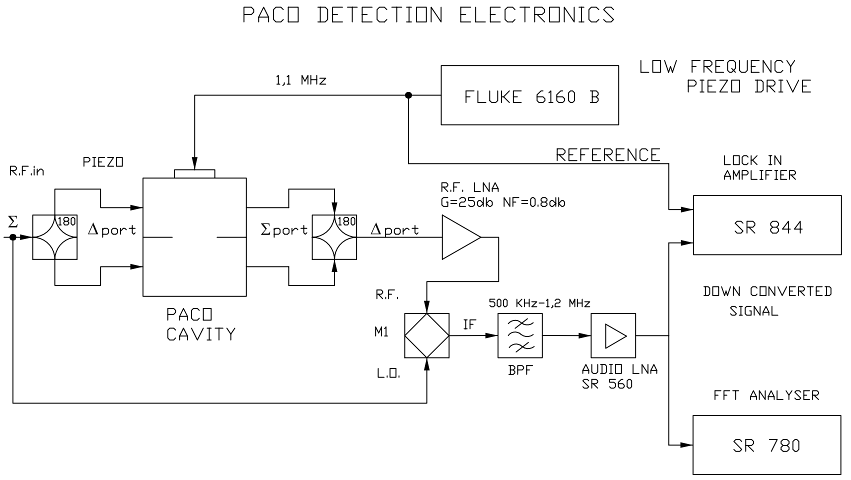

Figure 4: *PACO* detection system

- The possibility of an independent adjustment of the phase lag in the two arms giving a better magic tee insulation at the operating frequencies.
- A greater reliability for the frequency amplitude loop using the transmitted power, instead than the reflected, coming from the cavity.

5 Detection of the converted signal

The signal converted to the odd mode by the interaction between the mechanical perturbation and the rf fields is coupled to the Δ port of the detection arm of the rf system and amplified by the low noise rf amplifier LNA. Great deal of care must be used in the choice of the LNA, because the noise figure of the amplifier greatly affects the detector's sensitivity in the detection region above the mechanical resonances of the cavity (typically spanning the frequency interval 1-10 kHz) The LNA we choose is a commercial, room temperature, low noise amplifier with a 48 dB gain and quite large bandwidth of 500 MHz centered on our operating frequency of 3 GHz. The LNA Noise figure is 0.8 dB corresponding to a noise temperature of 360 K.

The converted rf signal amplified by the LNA is fed to the rf input port (RF) of a low noise double balanced mixer M1; the local oscillator port (LO) of M1 is driven by the even mode rf power (at frequency ω_s) transmitted by the cavity. The LO input level is adjusted for a minimum noise contribution from the mixer. As shown in the previous section the input spectrum to the RF port of the mixer is composed by two signals: the first at frequency ω_s coming from the rf leakage of the even mode trough the detection system (even if greatly reduced by the aforementioned double rejection of the carefully tuned magic tees); the second is the converted energy on the odd mode at frequency ω_a. Both signal are down converted by the M1 mixer giving to the IF port a DC signal proportional to the even mode leakage and the signal at frequency Ω proportional to the odd mode excitation.

The down-converted IF output is further amplified using a low noise audio preamplifier (Stanford Research SR560). The combination of tunable built in filters of the SR560 amplifier allows us to further reject (if necessary) the DC component coming from the even mode leakage. Last the output of the audio amplifier is fed to the lock in amplifier (Stanford Research SR844) used as synchronous detector driven by the low frequency synthesizer used to drive the detector cavity trough a piezoelectric ceramic at frequency Ω.

The detection electronics for the detection of harmonic, mechanically coupled interactions, at frequency Ω (as gravitational waves) is slightly different. Since the exact frequency and phase of the driving source is not known, we can't perform a synchronous detection; we need to perform a auto correlation of the detector output - or to cross-correlate the outputs of two different detectors - to detect the down converted component at Ω. The output of the SR560 preamplifier is fed to a fft signal analyzer (Stanford Research SR780) able to average the input signal on a bandwidth as small as 0.1 mHz, giving us a very comfortable margin to get the ultimate sensitivity foreseen for the feasibility study of our detector.

The outlined scheme of electronic detection gives us the non-marginal benefit of being (at the first order at least) self compensating against perturbations changing in the same way the frequency of the two modes of the detector. This type of perturbation is usually produced by changes of the cavity walls due to changes of pressure of the helium bath used to keep the cavity at the operating temperature or by the thermal contraction of the cavity walls produced by changes of the operating temperature. Furthermore similar effects are produced by the radiation pressure of the electromagnetic energy stored in the even mode; this pressure acts in a symmetrical way on the cavity walls producing equal deformation of both the detector's cells. The result of the deformation is a frequency shift of both the modes of a same amount of frequency; this kind of deformation does not affect (at the first order at least) the coupling of the cells fixing the amount of splitting between the even and odd modes. As a result the frequencies of the two modes ω_s and ω_a will change under the effect of pressure changes on the cavity walls but the difference Ω (related to the coupling) will not.

In our scheme the local oscillator (used for the down conversion of the rf signal coming from the detector) is the rf signal transmitted by our cavity at the frequency of the even mode. In this way the common mode frequency drift of the resonators is automatically cancelled (at the first order at least) and the mixer M1 output is a signal exactly at the perturbation frequency Ω. This effect gives us the possibility of using a quite simple refrigeration scheme (without any complex and cumbersome servo loop) to keep constant the helium bath temperature and pressure.

6 Experimental results

The electromagnetic properties of the cavity have been measured in a vertical cryostat after careful tuning of the two cells frequencies. The symmetric mode frequency was $\omega_s/(2\pi) = 3.03431$ GHz and the mode separation was $\Omega = 1.3817$ MHz. The unloaded quality factor at 4.2 K was $Q = 5 \cdot 10^7$, and no significant improvement was found lowering the helium bath temperature at 1.8 K. Even after a second chemical polishing, performed at CERN, which removed approximately 300 μm of niobium from the surface, no improvement was observed. We believe that this very low Q value is due to hot spots on the surface caused by welding problems occurred during the cavity fabrication. Adjusting the phase and amplitude of the rf signal entering and leaving the cavity, the arms of the two magic tees were balanced to launch the even mode at the cavity input and to pick up the odd mode at the cavity output. With 30 dBm of power at the Σ port of the first magic tee, -90 dBm were detected at the Δ port of the second one, giving an overall attenuation of the symmetric mode of 120 dB. The energy stored in the cavity with 30 dBm input power was approximately 1 mJ. The signal emerging from the Δ port of the output magic tee was amplified by the LNA and fed into a spectrum analyzer. The signal level at frequency ω_a was 120 dBm in a 1 Hz bandwidth. System sensitivity at this stage is given by

$$h_{min} \approx 6.55 \cdot 10^{-18}\,\mathrm{Hz}^{-1/2} \tag{4}$$

This value is quite far from our goal of $h_{min} = 10^{-20}$ (Hz)$^{-1/2}$. Since, to our knowledge, this is the first example of a parametric detector operated in trasmission, and since this configuration requires very careful adjustments of the input and output ports balancing, we believe that significant improvements are obtainable. Furthermore the new cavity under construction should show the high quality factor needed to reach high sensitivity.

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