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Neutrino mass matrix suppression by Abelian charges with see-saw mechanism

H. B. Nielsen[a,b,‡] and Y. Takanishi[b,§]

[a] *Theory Division, CERN,*
CH-1211 Geneva 23, Switzerland

[b] *The Niels Bohr Institute,*
Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Abstract

We have investigated a neutrino mass matrix model without supersymmetry including three see-saw right-handed neutrinos around order 10^{12} GeV masses, aiming at a picture with all small numbers explained as being due to approximately conserved gauge charges. The prediction of the solar neutrino mixing angle is given by $\sin^2 2\theta_\odot = 3\,^{+3}_{-2} \times 10^{-2}$; in fact, the solar mixing angle is, apart from detailed order unity corrections, equal to the Cabibbo angle. Furthermore the ratio of the solar neutrino mass square difference to that for the atmospheric neutrino oscillation is predicted to $6\,^{+11}_{-4} \times 10^{-4}$ and is given by the same Cabibbo angle related parameter ξ as $6\,\xi^4$.

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[‡] E-mail: holger.bech.nielsen@cern.ch; hbech@nbi.dk
[§] E-mail: yasutaka@nbi.dk

1 Introduction

1.1 Neutrino data

According to several experiments [1, 2, 3, 4, 5] it now seems that neutrino oscillations have to be taken seriously, and in all likelihood we should interpret such neutrino oscillations as being due to neutrino masses. We consider it relatively hopeless to incorporate the neutrino oscillations observed by LSDN [6] which are partly excluded by Karmen 2 [7], especially insofar as in the present article we adopt the philosophy that only particles which are mass protected by the Standard Model (SM) gauge symmetries are sufficiently light to be observed at present. Thus we do not have sterile neutrinos in the model, which we will explain.

The numbers characterising the positively observed neutrino oscillations are concentrated in the fits of the atmospheric neutrino oscillations from Super-Kamiokande and Kamiokande (and neglecting the LSDN) [1, 2], leading to the presumed mixing of the ν_μ with the ν_τ corresponding to a mass square difference between the two mainly involved eigenmass neutrinos of

$$\Delta m^2_{\rm atm} \approx (2-6)\times 10^{-3}\ {\rm eV}^2 \tag{1}$$

and a mixing angle

$$\sin^2 2\theta_{\rm atm} \geq 0.82\ . \tag{2}$$

On the other hand, the solar neutrino problems can be solved through either vacuum or matter-enhanced Mikhyev-Smirnov-Wolfenstein (MSW) oscillation [8]. The combination of the various solar neutrino experiments [9] allows - if taken seriously - three different regions of fitting of mass square difference and mixing angle:

(i) a small mixing angle (SMA):

$$\Delta m^2_\odot \approx (4-10)\times 10^{-6}\ {\rm eV}^2 \tag{3}$$
$$\sin^2 2\theta_\odot \approx (0.1-1.0)\times 10^{-2} \tag{4}$$

(ii) a large mixing angle (LMA):

$$\Delta m^2_\odot \approx (2-20)\times 10^{-5}\ {\rm eV}^2 \tag{5}$$
$$\sin^2 2\theta_\odot \approx (0.65-0.97) \tag{6}$$

(iii) vacuum oscillations (VO):

$$\Delta m^2_\odot \approx (0.5-5)\times 10^{-10}\ {\rm eV}^2 \tag{7}$$
$$\sin^2 2\theta_\odot \geq 0.67 \tag{8}$$

Clearly these data do not match what would have been the most simple philosophy: namely, that there be only the SM scales and the Planck scale. Putting see-saw particles [10] only with Planck masses and assuming all couplings of order unity could not give neutrino masses corresponding to the observed oscillations. Rather there is a call for a new scale; either some new Higgs field giving neutrino masses directly, or a new scale for see-saw particles at 10^{12} GeV.

We are basically forced to take this new scale, say, the see-saw scale, as a parameter which is just fitted to neutrino masses and their mixing angles. Thus we can, in fact, only hope to make predictions for the ratio of the two mass differences observed:

$$\frac{\Delta m_\odot^2}{\Delta m_{\rm atm}^2} = \begin{cases} 0.67 - 5.0 \times 10^{-3} & \text{for small mixing angle} \\ 0.33 - 10 \times 10^{-2} & \text{for large mixing angle} \\ 0.08 - 2.5 \times 10^{-7} & \text{for vacuum oscillation} \end{cases} \tag{9}$$

1.2 Content of present article

Our goal is to understand the orders of magnitude - only orders of magnitude - of masses and mixing angles using a model with all small numbers coming from Higgs vacuum expectation values (VEV) which are small relative to, say, the Planck scale. In the whole paper we do not assume supersymmetry (SUSY), but much of our work would not be disturbed by it. However, our work is really an extension of an earlier model which we call the "old" Anti-GUT model - put forward by C.D. Froggatt and one of us, mainly fitting the quark and charged lepton mass matrices - and in these fits the same field and its Hermitian conjugate field are both used, and it goes into the predictive power that they have the same VEV numerically. In SUSY this causes problems of the sort one would have in introducing a $\tan\beta$, which would be a new parameter and thus take away some predictive power out of the fits, unless $\tan\beta \approx 1$.

Since, however, the neutrino masses only come from the up-type Weinberg-Salam Higgs field in SUSY models the neutrino mass relations would not be disturbed by SUSY.

The fitting mentioned as the "old" Anti-GUT of the charged quark and lepton masses and quark mixing angles already presents a rather good fit, order-of-magnitude-wise.

However, extending it has met with some difficulty: at first, a philosophy of having in addition to the weak and the strong scale only the Planck scale would never give neutrinos masses in the observed range. So, either lower than Planck mass scale see-saw particles are needed, or a new, very low VEV Higgs field must be included in the model. The latter is the approach of M. Gibson *et al.* [11, 12, 13], who introduce a Higgs field triplet under the SM $SU(2)$ with weak hypercharge $y/2 = 1$ to deliver masses for the neutrinos.

The best working model of this type achieves the large atmospheric neutrino oscillation mixing angle $\theta_{\rm atm}$ by letting it come dominantly from the transformation diagonalising the charged lepton mass matrix. A viable scheme is obtained by introducing a couple of extra Higgs fields.

It is the purpose of the present article to seek to extend and rescue the "old" Anti-GUT model from being falsified by neutrino oscillations along the first-mentioned road: introduction of the see-saw particles much lighter than the Planck mass; to be more precise, in the range around 10^{12} GeV.

Since this needed scale for the see-saw particles is a rather "new" scale - in the middle of the desert we shall in the present article - where all small numbers should be explained by Higgs field VEVs - just "shamelessly" introduce a new Higgs field ϕ_{B-L} which has a VEV of this size, just fitting it.

Our point is therefore not to explain the overall size of neutrino masses, but rather ratio(s) and mixing angles. So the "holy grail" for this article becomes the ratio of the mass square difference describing respectively the solar neutrino oscillations and the $\nu_\mu - \nu_\tau$ atmospheric oscillations (9).

In the following section, we review briefly the Anti-GUT model and its extension to the calculation of neutrino mass square differences and mixing angles. Then, in the next section, we shall write down the no-anomaly constraints on the possible charge assignments for the quarks and leptons, and present the charge assignments for both fermions and Higgs fields in the extended Anti-GUT model. In section 4 we put forward the mass matrices and in section 5 we define the mixing angles. Then in section 6 we calculate in the model the neutrino oscillation parameters, both crudely and numerically, using "random order unity factors". Section 7 contains our resumé and conclusion.

2 Anti-Grand Unification Theory and its extension

In this section we review the Anti-GUT model and its extension describing neutrino masses and mixing angles.

The Anti-GUT model [12, 13, 14, 15, 16, 17, 18] has been put forward by one of us and his collaborators over many years, with several motivations, first of all justified by a very promising series of experimental agreements by fitting many of the SM parameters with rather few parameters, in an impressive way even though most predictions are only order of magnitude wise. The Anti-GUT model deserves its name in as far as its gauge group $SMG^3 \times U(1)_f$ which, so to speak, replaces the often-used GUT gauge groups such as $SU(5)$, $SO(10)$ *etc.* by one that can be specified by requiring that:

1. It should only contain transformations which change the known 45 (= 3 generations of 15 Weyl particles each) Weyl fermions - counted as left-handed, say, - into each other unitarily, (*i.e.* it must be a subgroup of $U(45)$).

2. It should be anomaly-free even without using the Green-Schwarz [19] anomaly cancellation mechanism.

3. It should NOT unify the irreducible representations under the SM gauge group, called here $SMG = SU(3) \times SU(2) \times U(1)$.

4. It should be as big as possible under the foregoing assumptions.

In the present article we shall, however, allow for see-saw neutrinos - essentially right-handed neutrinos - whereby we want to extend the number of particles to be transformed under the group being specified to also include the right-handed neutrinos, even though they have not been directly "seen".

The group which shall be used as the model gauge group replacing the unifying groups could be specified by a similar set of assumptions like that used by C.D. Froggatt and one of us to specify the "old" Anti-GUT, by replacing assumption 1 by a slightly modified assumption excluding only already observed fermions in the system to only exclude the unobserved fermions when they have nontrivial quantum numbers under the SM group, so that they are mass-protected. The particles that are mass protected under the SM would namely be rather light and would likely have been seen. But see-saw neutrinos with zero SM quantum numbers could not be mass protected by the SM and could easily be so heavy as not to have been "seen".

The model which we have in mind as the extended Anti-GUT model, that should inherit the successes of the "old" Anti-GUT model and in addition have see-saw neutrinos, is proposed to have the gauge group $SMG^3 \times U(1)_f \times U(1)_{\mathrm{B-L},1} \times U(1)_{\mathrm{B-L},23} \ni SU(3)^3 \times SU(2)^3 \times U(1)^6$, and it is assumed to couple in the following way:

The three SM groups $SMG = SU(3) \times SU(2) \times U(1)$ are supposed to be one for each generation. That is to say, there is, *e.g.*, a first generation SMG among the three; all the fermions in the second and third generations are in the trivial representations, and with zero charge, while the first generation particles couple to this first generation SMG as if in the same representations (same charges too) as they are under the SM. For example, the left proto-electron and the proto-electron neutrino form a doublet under the $SU(2)_1$ belonging to the first generation (while they are in singlets w.r.t. the other $SU(2)$'s) and have weak hypercharge w.r.t. the first generation $U(1)_1$, the charge $y_1/2 = -1/2$ analogous to the SM weak hypercharge being $y/2 = -1/2$ for left-handed leptons.

The $U(1)_f$-charge is assigned in a slightly complicated way which is, however, largely the only one allowed modulo various permutations and rewritings from the no-anomaly requirements. It is zero for all first-generation and for all particles usually called left-handed, and the charge values are the opposite on a "right-handed" particle in the second generation and the corresponding one in the third generation. See Table 1 for the detailed assignment.

The two last $U(1)$-groups, $U(1)_{\mathrm{B-L},1}$ and $U(1)_{\mathrm{B-L},23}$ in our model have charge assignments corresponding to the quantum number $B-L$ (= baryon number minus lepton number) though in such a way that the charges of $U(1)_{\mathrm{B-L},1}$ are zero for the second and third generations, and only non-zero for the first generation, for which they then coincide with the baryon number minus the lepton number. Analogously the $U(1)_{\mathrm{B-L},23}$-charge assignments are zero on the first-generation quarks and leptons, while they coincide with the baryon number minus the lepton number for second and third generations. We will discuss in the next section the anomaly cancellation in the extended Anti-GUT model.

It is then further part of our model that this large gauge group is broken down spontaneously to the SM group, lying as the diagonal subgroup of the SMG^3 part of the group by means of a series of Higgs fields, the quantum numbers of which have been selected mainly from the criteria of fitting the the masses and mixing angles w.r.t. order of magnitude. The quantum numbers of the "old" Anti-GUT Higgs fields proposed were:

$$S: \quad (\frac{1}{6}, -\frac{1}{6}, 0, -1) \tag{10}$$

$$W: \quad (0, -\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}) \tag{11}$$

$$T: \quad (0, -\frac{1}{6}, \frac{1}{6}, -\frac{2}{3}) \tag{12}$$

$$\xi: \quad (\frac{1}{6}, -\frac{1}{6}, 0, 0) \tag{13}$$

for the four Higgs fields supposed to have VEVs of the order of between a twentieth and unity compared to the fundamental scale supposed to be the Planck scale. In addition there was then the Higgs field under the Anti-GUT-group which should take the role of finally breaking the SM gauge group down to $U(3) = SU(3) \times U(1)_{em}$, *i.e.*, play the role of the Weinberg-Salam Higgs field

$$\phi_{WS}: \quad (0, \frac{2}{3}, -\frac{1}{6}, 1). \tag{14}$$

Here the quantum numbers were presented in the order of first giving the three different weak hypercharges corresponding to the three generations $SU(2)_i$, $SU(3)_i$ and $y_i/2\,(i = 1, 2, 3)$, and then the $U(1)_f$-charge.

In reference [12] we fitted the parameters being Higgs fields VEVs to masses and mixing angles for charged fermions and the values are as follows:

$$\langle S\rangle = 1 \ , \ \langle W\rangle = 0.179 \ , \ \langle \xi\rangle = 0.099 \ , \ \langle T\rangle = 0.071 \tag{15}$$

In the following we shall often abbreviate by deleting the $\langle\cdots\rangle$ around these Higgs fields, mostly with the understanding that S, W, ... then mean the VEV "measured in fundamental" units.

In the Anti-GUT model, old as well as new, it is assumed at some "fundamental scale" that particles which can play the role of see-saw with whatever quantum numbers are needed, exist. The fitted "suppression factors" are the VEVs in units of the "fundamental scale" see-saw particles.

It has to be checked that extending the group to have the $U(1)_{\mathrm{B-L},1}$ and $U(1)_{\mathrm{B-L},23}$ does not disturb the model already functioning rather well, and it can be done by only giving the field ξ and S non-zero charges under these "new" $U(1)$ groups, so as to get:

$$S: \qquad (\frac{1}{6}, -\frac{1}{6}, 0, -1, -\frac{2}{3}, \frac{2}{3}) \tag{16}$$

$$\xi: \qquad (\frac{1}{6}, -\frac{1}{6}, 0, 0, \frac{1}{3}, -\frac{1}{3}) \tag{17}$$

But now we also want to introduce two new Higgs fields ϕ_{B-L} and χ into the model: the first, ϕ_{B-L}, is a Higgs field to fit the new scale that comes in by neutrino oscillations giving the scale of the see-saw particle masses. When the left-right-transition mass matrix is of the same order as the usual charge fermion mass matrices, this scale is of the order 10^{12} GeV.

We use in our model the gauged $B - L$, in fact the total one, because we break $U(1)_{B-L,1} \times U(1)_{B-L,23} \supseteq U(1)_{B-L,\mathrm{total}}$ at a much higher scale (near Planck scale), to mass-protect the right-handed neutrinos meant to function as see-saw particles, so they can be sufficiently light to give by the see-saw mechanism the "observed" left-handed neutrino masses. The breaking of the $U(1)_{B-L,\mathrm{total}}$ and thereby the setting of the see-saw scale is then caused by our "new" Higgs field called ϕ_{B-L}.

In order to get viable neutrino spectra we shall choose the quantum numbers of ϕ_{B-L} so that it is the effective $\overline{\nu_{\tau_R}}\, C\, \overline{\nu_{e_R}}^t + h.c.$ which gets the direct contribution and thus is not further suppressed.

This is the way to avoid "factorised mass matrices" - *i.e.* of the form

$$\begin{pmatrix} \phi_1^2 & \phi_1\phi_2 & \phi_1\phi_3 \\ \phi_1\phi_2 & \phi_2^2 & \phi_2\phi_3 \\ \phi_1\phi_3 & \phi_2\phi_3 & \phi_3^2 \end{pmatrix} \tag{18}$$

with different order unity factors, though on different terms. Such factorised matrices are rather difficult to avoid otherwise. If we get such a "factorised matrix" and, as in our model, have mainly diagonal elements in the ν-Dirac matrix, M_ν^D, the prediction comes out that

$$\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} \approx (\sin\theta_{\rm atm})^4 \quad , \tag{19}$$

which is not true experimentally. Therefore we choose ϕ_{B-L} to have the quantum numbers of $\overline{\nu_{\tau_R}}$ plus those of $\overline{\nu_{e_R}}$:

$$\begin{aligned} Q_{\phi_{B-L}} &= Q_{\bar\nu_{\tau_R}} + Q_{\bar\nu_{e_R}} \\ &= (0,0,0,0,1,0) + (0,0,0,1,0,1) \\ &= (0,0,0,1,1,1) \quad . \end{aligned}$$

The other "new" relative to "old" Anti-GUT Higgs field we call χ and one of its roles is to help the $\langle\phi_{B-L}\rangle$ to give non-zero effective mass terms for the see-saw neutrinos by providing a transition between ν_{τ_R} and ν_{μ_R}. It also turns out to play a role in fitting the atmospheric mixing angle (to be of order unity). Its quantum numbers are therefore postulated to be the difference of those of these two see-saw particles

$$\begin{aligned} Q_\chi &= Q_{\bar\nu_{\mu_R}} - Q_{\bar\nu_{\tau_R}} \\ &= (0,0,0,1,0,-1) - (0,0,0,-1,0,-1) \\ &= (0,0,0,2,0,0) \quad . \end{aligned}$$

3 Anomaly

We should introduce here an anomaly-free Abelian extension of the SM which we shall discuss below to obtain the neutrino mass spectra and their mixing angles. The "new" Anti-GUT gauge group is

$$SMG^3 \times U(1)_f \times U(1)_{B-L,1} \times U(1)_{B-L,23} \tag{20}$$

and is broken by a set of Higgs fields S, W, T, ξ, χ and ϕ_{B-L} down to the SM gauge groups. The SMG will be broken down by the field ϕ_{WS} playing the role of Weinberg-Salam Higgs field into $SU(3)\times U(1)_{em}$.

The requirement that all anomalies involving $U(1)_f$, $U(1)_{\rm B-L,1}$ and $U(1)_{\rm B-L,23}$ then vanish strongly constrains the possible fermion charges (denoting the $U(1)_f$ charges by $Q_f(t_R)\equiv t_R$ *etc.* and the $U(1)_{\rm B-L}$ charges by $Q_{\rm B-L,1}(u_R)\equiv \bar u_R$, $Q_{\rm B-L,23}(t_R)\equiv \tilde t_R$ *etc.* respectively).

The anomaly cancellation conditions then constrain the fermion $U(1)_f$ and $U(1)_{B-L,1}$ and also $U(1)_{\rm B-L,23}$ charges to satisfy the following equations:

$$\begin{aligned}
{\rm Tr}\,[{\rm SU_1(3)^2U(1)_f}] &= 2u_L - u_R - d_R = 0 \\
{\rm Tr}\,[{\rm SU_2(3)^2U(1)_f}] &= 2c_L - c_R - s_R = 0 \\
{\rm Tr}\,[{\rm SU_3(3)^2U(1)_f}] &= 2t_L - t_R - b_R = 0 \\
{\rm Tr}\,[{\rm SU_1(2)^2U(1)_f}] &= 3u_L + e_L = 0 \\
{\rm Tr}\,[{\rm SU_2(2)^2U(1)_f}] &= 3c_L + \mu_L = 0 \\
{\rm Tr}\,[{\rm SU_3(2)^2U(1)_f}] &= 3t_L + \tau_L = 0 \\
{\rm Tr}\,[{\rm U_1(1)^2U(1)_f}] &= u_L - 8u_R - 2d_R + 3e_L - 6e_R = 0
\end{aligned}$$

$$
\begin{aligned}
\mathrm{Tr}\ [\mathrm{U}_2(1)^2\mathrm{U}(1)_\mathrm{f}] &= c_L - 8c_R - 2s_R + 3\mu_L - 6\mu_R = 0 \\
\mathrm{Tr}\ [\mathrm{U}_3(1)^2\mathrm{U}(1)_\mathrm{f}] &= t_L - 8t_R - 2b_R + 3\tau_L - 6\tau_R = 0 \\
\mathrm{Tr}\ [\mathrm{U}_1(1)\mathrm{U}(1)_\mathrm{f}^2] &= u_L^2 - 2u_R^2 + d_R^2 - e_L^2 + e_R^2 = 0 \\
\mathrm{Tr}\ [\mathrm{U}_2(1)\mathrm{U}(1)_\mathrm{f}^2] &= c_L^2 - 2c_R^2 + s_R^2 - \mu_L^2 + \mu_R^2 = 0 \\
\mathrm{Tr}\ [\mathrm{U}_3(1)\mathrm{U}(1)_\mathrm{f}^2] &= t_L^2 - 2t_R^2 + b_R^2 - \tau_L^2 + \tau_R^2 = 0 \\
\mathrm{Tr}\ [\mathrm{U}(1)_\mathrm{f}^3] &= 6u_L^3 + 6c_L^3 + 6t_L^3 - 3u_R^3 - 3c_R^3 - 3t_R^3 - 3d_R^3 - 3s_R^3 \\
&\quad -3b_R^3 + 2e_L^3 + 2\mu_L^3 + 2\tau_L^3 - e_R^3 - \mu_R^3 - \tau_R^3 \\
&\quad -\nu_{e_R}^3 - \nu_{\mu_R}^3 - \nu_{\tau_R}^3 = 0 \\
\mathrm{Tr}\ [(\mathrm{graviton})^2\mathrm{U}(1)_\mathrm{f}] &= 6u_L + 6c_L + 6t_L - 3u_R - 3c_R - 3t_R - 3d_R - 3s_R \\
&\quad -3b_R + 2e_L + 2\mu_L + 2\tau_L - e_R - \mu_R - \tau_R \\
&\quad -\nu_{e_R} - \nu_{\mu_R} - \nu_{\tau_R} = 0
\end{aligned}
$$

So they should be obeyed both by the $U(1)_{B-L,1}$, and $U(1)_{B-L,23}$, replacing the t_R, b_R, $\ldots$ by $\tilde{t}_R$, $\tilde{b}_R$, $\ldots$:

$$
\begin{aligned}
\mathrm{Tr}\ [\mathrm{SU}_1(3)^2\mathrm{U}(1)_{\mathrm{B-L,1}}] &= 2\bar{u}_L - \bar{u}_R - \bar{d}_R = 0 \\
\mathrm{Tr}\ [\mathrm{SU}_2(3)^2\mathrm{U}(1)_{\mathrm{B-L,23}}] &= 2\tilde{c}_L - \tilde{c}_R - \tilde{s}_R = 0 \\
\mathrm{Tr}\ [\mathrm{SU}_3(3)^2\mathrm{U}(1)_{\mathrm{B-L,23}}] &= 2\tilde{t}_L - \tilde{t}_R - \tilde{b}_R = 0 \\
\mathrm{Tr}\ [\mathrm{SU}_1(2)^2\mathrm{U}(1)_{\mathrm{B-L,1}}] &= 3\bar{u}_L + \bar{e}_L = 0 \\
\mathrm{Tr}\ [\mathrm{SU}_2(2)^2\mathrm{U}(1)_{\mathrm{B-L,23}}] &= 3\tilde{c}_L + \tilde{\mu}_L = 0 \\
\mathrm{Tr}\ [\mathrm{SU}_3(2)^2\mathrm{U}(1)_{\mathrm{B-L,23}}] &= 3\tilde{t}_L + \tilde{\tau}_L = 0
\end{aligned}
$$

But with several $U(1)$s there will in addition be anomaly conditions for combinations between the different ones. Taking it that $U(1)_{B-L,1}$ charges are zero for all second- and third-generation fermions while $U(1)_{B-L,23}$ charges are zero for the first generation, the further conditions are:

$$
\begin{aligned}
\mathrm{Tr}\ [\mathrm{U}_1(1)^2\mathrm{U}(1)_{\mathrm{B-L,1}}] &= \bar{u}_L - 8\bar{u}_R - 2\bar{d}_R + 3\bar{e}_L - 6\bar{e}_R = 0 \\
\mathrm{Tr}\ [\mathrm{U}_2(1)^2\mathrm{U}(1)_{\mathrm{B-L,23}}] &= \tilde{c}_L - 8\tilde{c}_R - 2\tilde{s}_R + 3\tilde{\mu}_L - 6\tilde{\mu}_R = 0 \\
\mathrm{Tr}\ [\mathrm{U}_3(1)^2\mathrm{U}(1)_{\mathrm{B-L,23}}] &= \tilde{t}_L - 8\tilde{t}_R - 2\tilde{b}_R + 3\tilde{\tau}_L - 6\tilde{\tau}_R = 0 \\
\mathrm{Tr}\ [\mathrm{U}_1(1)\mathrm{U}(1)_{\mathrm{B-L,1}}^2] &= \bar{u}_L^2 - 2\bar{u}_R^2 + \bar{d}_R^2 - \bar{e}_L^2 + \bar{e}_R^2 = 0 \\
\mathrm{Tr}\ [\mathrm{U}_2(1)\mathrm{U}(1)_{\mathrm{B-L,23}}^2] &= \tilde{c}_L^2 - 2\tilde{c}_R^2 + \tilde{s}_R^2 - \tilde{\mu}_L^2 + \tilde{\mu}_R^2 = 0 \\
\mathrm{Tr}\ [\mathrm{U}_3(1)\mathrm{U}(1)_{\mathrm{B-L,23}}^2] &= \tilde{t}_L^2 - 2\tilde{t}_R^2 + \tilde{b}_R^2 - \tilde{\tau}_L^2 + \tilde{\tau}_R^2 = 0 \\
\mathrm{Tr}\ [\mathrm{U}(1)_{\mathrm{B-L,1}}^3] &= 6\bar{u}_L^3 - 3\bar{u}_R^3 - 3\bar{d}_R^3 + 2\bar{e}_L^3 - \bar{e}_R^3 - \bar{\nu}_{e_R}^3 = 0 \\
\mathrm{Tr}\ [\mathrm{U}(1)_{\mathrm{B-L,23}}^3] &= 6\tilde{c}_L^3 + 6\tilde{t}_L^3 - 3\tilde{c}_R^3 - 3\tilde{t}_R^3 - 3\tilde{s}_R^3 - 3\tilde{b}_R^3 \\
&\quad +2\tilde{\mu}_L^3 + 2\tilde{\tau}_L^3 - \tilde{\mu}_R^3 - \tilde{\tau}_R^3 - \tilde{\nu}_{\mu_R}^3 - \tilde{\nu}_{\tau_R}^3 = 0 \\
\mathrm{Tr}\ [(\mathrm{graviton})^2\mathrm{U}(1)_{\mathrm{B-L,1}}] &= 6\bar{u}_L - 3\bar{u}_R - 3\bar{d}_R + 2\bar{e}_L - \bar{e}_R - \bar{\nu}_{e_R} = 0 \\
\mathrm{Tr}\ [(\mathrm{graviton})^2\mathrm{U}(1)_{\mathrm{B-L,23}}] &= 6\tilde{c}_L + 6\tilde{t}_L - 3\tilde{c}_R - 3\tilde{t}_R - 3\tilde{s}_R - 3\tilde{b}_R \\
&\quad +2\tilde{\mu}_L + 2\tilde{\tau}_L - \tilde{\mu}_R - \tilde{\tau}_R - \tilde{\nu}_{\mu_R} - \tilde{\nu}_{\tau_R} = 0 \\
\mathrm{Tr}\ [\mathrm{U}(1)_\mathrm{f}^2\mathrm{U}(1)_{\mathrm{B-L,1}}] &= 6u_L^2\bar{u}_L - 3u_R^2\bar{u}_R - 3d_R^2\bar{d}_R + 2e_L^2\bar{e}_L - e_R^2\bar{e}_R - \nu_{e_R}^2\bar{\nu}_{e_R} = 0 \\
\mathrm{Tr}\ [\mathrm{U}(1)_\mathrm{f}^2\mathrm{U}(1)_{\mathrm{B-L,23}}] &= 6c_L^2\tilde{c}_L - 3c_R^2\tilde{c}_R - 3s_R^2\tilde{s}_R + 2\mu_L^2\tilde{\mu}_L - \mu_R^2\tilde{\mu}_R - \nu_{\mu_R}^2\tilde{\nu}_{\mu_R} \\
&\quad +6t_L^2\tilde{t}_L - 3t_R^2\tilde{t}_R - 3b_R^2\tilde{b}_R + 2\tau_L^2\tilde{\tau}_L - \tau_R^2\tilde{\tau}_R - \nu_{\tau_R}^2\tilde{\nu}_{\tau_R} = 0
\end{aligned}
$$

Table 1: All $U(1)$ quantum charges in extended Anti-GUT model

	SMG_1	SMG_2	SMG_3	$U(1)_f$	$U_{B-L,1}$	$U_{B-L,23}$
u_L, d_L	$\frac{1}{6}$	0	0	0	$\frac{1}{3}$	0
u_R	$\frac{2}{3}$	0	0	0	$\frac{1}{3}$	0
d_R	$-\frac{1}{3}$	0	0	0	$\frac{1}{3}$	0
e_L, ν_{e_L}	$-\frac{1}{2}$	0	0	0	-1	0
e_R	-1	0	0	0	-1	0
ν_{e_R}	0	0	0	0	-1	0
c_L, s_L	0	$\frac{1}{6}$	0	0	0	$\frac{1}{3}$
c_R	0	$\frac{2}{3}$	0	1	0	$\frac{1}{3}$
s_R	0	$-\frac{1}{3}$	0	-1	0	$\frac{1}{3}$
μ_L, ν_{μ_L}	0	$-\frac{1}{2}$	0	0	0	-1
μ_R	0	-1	0	-1	0	-1
ν_{μ_R}	0	0	0	1	0	-1
t_L, b_L	0	0	$\frac{1}{6}$	0	0	$\frac{1}{3}$
t_R	0	0	$\frac{2}{3}$	-1	0	$\frac{1}{3}$
b_R	0	0	$-\frac{1}{3}$	1	0	$\frac{1}{3}$
τ_L, ν_{τ_L}	0	0	$-\frac{1}{2}$	0	0	-1
τ_R	0	0	-1	1	0	-1
ν_{τ_R}	0	0	0	-1	0	-1
ϕ_{WS}	0	$\frac{2}{3}$	$-\frac{1}{6}$	1	0	0
S	$\frac{1}{6}$	$-\frac{1}{6}$	0	-1	$-\frac{2}{3}$	$\frac{2}{3}$
W	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{4}{3}$	0	0
ξ	$\frac{1}{6}$	$-\frac{1}{6}$	0	0	$\frac{1}{3}$	$-\frac{1}{3}$
T	0	$-\frac{1}{6}$	$\frac{1}{6}$	$-\frac{2}{3}$	0	0
χ	0	0	0	2	0	0
ϕ_{B-L}	0	0	0	1	1	1

$$\begin{aligned}
\mathrm{Tr}\,[\mathrm{U(1)_f U(1)^2_{B-L,1}}] &= 6u_L\bar{u}_L^2 - 3u_R\bar{u}_R^2 - 3d_R\bar{d}_R^2 + 2e_L\bar{e}_L^2 - e_R\bar{e}_R^2 - \nu_{e_R}\bar{\nu}_{e_R}^2 = 0 \\
\mathrm{Tr}\,[\mathrm{U(1)_f U(1)^2_{B-L,23}}] &= 6c_L\tilde{c}_L^2 - 3c_R\tilde{c}_R^2 - 3s_R\tilde{s}_R^2 + 2\mu_L\tilde{\mu}_L^2 - \mu_R\tilde{\mu}_R^2 - \nu_{\mu_R}\tilde{\nu}_{\mu_R}^2 \\
&\quad + 6t_L\tilde{t}_L^2 - 3t_R\tilde{t}_R^2 - 3b_R\tilde{b}_R^2 + 2\tau_L\tilde{\tau}_L^2 - \tau_R\tilde{\tau}_R^2 - \nu_{\tau_R}\tilde{\nu}_{\tau_R}^2 = 0 \\
\mathrm{Tr}\,[\mathrm{U(1)_1 U(1)_f U(1)_{B-L,1}}] &= u_L\bar{u}_L - 2u_R\bar{u}_R + d_R\bar{d}_R - e_L\bar{e}_L + e_R\bar{e}_R = 0 \\
\mathrm{Tr}\,[\mathrm{U(1)_2 U(1)_f U(1)_{B-L,23}}] &= c_L\tilde{c}_L - 2c_R\tilde{c}_R + s_R\tilde{s}_R - \mu_L\tilde{\mu}_L + \mu_R\tilde{\mu}_R = 0 \\
\mathrm{Tr}\,[\mathrm{U(1)_3 U(1)_f U(1)_{B-L,23}}] &= t_L\tilde{t}_L - 2t_R\tilde{t}_R + b_R\tilde{b}_R - \tau_L\tilde{\tau}_L + \tau_R\tilde{\tau}_R = 0
\end{aligned}$$

From these equations we can get the following solutions:

$$\begin{aligned}
(u_L, u_R, d_R, e_L, e_R, \nu_{e_R}) &= (0, 0, 0, 0, 0, 0) \\
(c_L, c_R, s_R, \mu_L, \mu_R, \nu_{\mu_R}) &= (0, 1, -1, 0, -1, 1) \\
(t_L, t_R, b_R, \tau_L, \tau_R, \nu_{\tau_R}) &= (0, -1, 1, 0, 1, -1) \\
(\bar{u}_L, \bar{u}_R, \bar{d}_L, \bar{d}_R, \bar{e}_L, \bar{e}_R, \bar{\nu}_{e_L}, \bar{\nu}_{e_R}) &= (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1, -1, -1, -1)
\end{aligned}$$

$$
\begin{aligned}
(\tilde{c}_L, \tilde{c}_R, \tilde{s}_L, \tilde{s}_R, \tilde{b}_L, \tilde{b}_R, \tilde{t}_L, \tilde{t}_R) &= (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}) \\
(\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R, \tilde{\nu}_{\mu_L}, \tilde{\nu}_{\mu R}, \tilde{\nu}_{\tau_L}, \tilde{\nu}_{\tau_R}) &= (-1, -1, -1, -1, -1, -1, -1, -1)
\end{aligned}
$$

We summarise the Abelian gauge quantum numbers of our model for fermions and scalars in Table 1. However, the following three points should be kept in mind; then the information in Table 1 and these three points describe our whole model:

1. We have only presented here the six $U(1)$-charges in our model. The non-Abelian quantum charge numbers are to be derived from the following rule: find in the table $y_i/2$ ($i = 1, 2, 3$ is the generation number), then find that Weyl particle in the SM for which the SM weak hypercharge divided by two is $y/2 = y_i/2$ and use its $SU(2)$ and $SU(3)$ representation for the particle considered in the table. But now use it for $SU(2)_i$ and $SU(3)_i$.
2. Remember that we imagine that at the "fundamental" scale ($\simeq$ presumed to be the Planck scale) we have essentially all particles that can be imagined with couplings of order unity. But we do not want to be specific about these very heavy particles in order not to decrease the enormous likelihood of our model being right. We are only specific about the particles in the table and the gauge fields.
3. The 39 gauge bosons correspond to the group (equation (20)) and are also not written in the table.

4 Mass matrices within the Anti-GUT model

In the "old" Anti-GUT model we have only the usual SM fermions at low energies, but in our "new" version we assume that there exist very heavy right-handed neutrinos, all of them having already decayed and not being observable in our world. They shall function as see-saw particles and thus give rise to an effective Majorana-type mass matrix for the left-handed particles. These three "right-handed"neutrinos get masses from the VEV of ϕ_{B-L} (10^{12} GeV), ξ and also χ Higgs fields (the latter in order of Planck unit, about 1/10).

The effective mass matrix elements, left-left, for the left-handed neutrinos - the ones we "see" experimentally - then come about using the ν_R see-saw propagator surrounded by left-right transition neutrino mass matrices. The latter are rather analogous to the charged lepton and quark mass matrices, which are proportional to the VEV of the Weinberg-Salam Higgs field in our model, being components of ϕ_{WS} (with VEV ~ 173 GeV).

Both $B - L$ quantum gauge groups are violated by $\phi_{\rm B-L}$, thus the effective Majorana mass terms are added into the Lagrange density using the Higgs field ϕ_{B-L}. The part of the effective Lagrangian we have to consider is:

$$
\begin{aligned}
-\mathcal{L}_{\rm lepton-mass} &= \bar{\nu}_L\, M_\nu^D\, \nu_R + \frac{1}{2}(\overline{\nu_L})^c\, M_L\, \nu_L + \frac{1}{2}(\overline{\nu_R})^c\, M_R\, \nu_R + h.c. \\
&= \frac{1}{2}(\overline{n_L})^c\, M\, n_L + h.c. \qquad (21)
\end{aligned}
$$

where

$$
n_L \equiv \begin{pmatrix} \nu_L \\ (\nu_L)^c \end{pmatrix} \quad , \quad M \equiv \begin{pmatrix} M_L & M_\nu^D \\ M_\nu^D & M_R \end{pmatrix} \quad ; \qquad (22)
$$

M_ν^D is the standard $SU(2)\times U(1)$ breaking Dirac mass term, and and M_L and M_R are the isosinglet Majorana mass terms of left-handed and right-handed neutrinos, respectively.

Supposing that the left-handed Majorana mass M_L terms are comparatively negligible, because of SM gauge symmetry protection, a naturally small effective Majorana mass for the light neutrinos (predominantly ν_L) can be generated by mixing with the heavy states (predominantly ν_R) of mass M_{ν_R}. The Dirac mass matrix of neutrinos is similar to the up-type quark mass matrix [20] and therefore has similar magnitude. For no left-left term, the light eigenvalues of the matrix M are

$$M_{\rm eff} \approx M_\nu^D\, M_R^{-1}\, (M_\nu^D)^t \ . \tag{23}$$

This result is the well-known see-saw mechanism [10]: the light neutrino masses are quadratic in the Dirac masses and inversely proportional to the large ν_R Majorana masses. Notice that if some ν_R are massless or light they would not be integrated away but simply added to the light neutrinos.

We have already given the quantum charges of the Higgs fields, S, W, T, ξ, ϕ_{WS}, ϕ_{B-L} and χ in Table 1. With this quantum number choice of Higgs fields the mass matrices are given by the uct-quarks:

$$M_U \simeq \frac{\langle\phi_{\rm ws}\rangle}{\sqrt{2}} \begin{pmatrix} S^\dagger W^\dagger T^2(\xi^\dagger)^2 & W^\dagger T^2 \xi & (W^\dagger)^2 T\xi \\ S^\dagger W^\dagger T^2(\xi^\dagger)^3 & W^\dagger T^2 & (W^\dagger)^2 T \\ S^\dagger(\xi^\dagger)^3 & 1 & W^\dagger T^\dagger \end{pmatrix} \tag{24}$$

the dsb-quarks:

$$M_D \simeq \frac{\langle\phi_{\rm ws}\rangle}{\sqrt{2}} \begin{pmatrix} SW(T^\dagger)^2\xi^2 & W(T^\dagger)^2\xi & T^3\xi \\ SW(T^\dagger)^2\xi & W(T^\dagger)^2 & T^3 \\ SW^2(T^\dagger)^4\xi & W^2(T^\dagger)^4 & WT \end{pmatrix} \tag{25}$$

the charged leptons:

$$M_E \simeq \frac{\langle\phi_{\rm ws}\rangle}{\sqrt{2}} \begin{pmatrix} SW(T^\dagger)^2\xi^2 & W(T^\dagger)^2(\xi^\dagger)^3 & WT^4(\xi^\dagger)^3\chi \\ SW(T^\dagger)^2\xi^5 & W(T^\dagger)^2 & WT^4\chi \\ S(W^\dagger)^2T^4\xi^5 & (W^\dagger)^2T^4 & WT \end{pmatrix} \tag{26}$$

the Dirac neutrinos:

$$M_\nu^D \simeq \frac{\langle\phi_{\rm ws}\rangle}{\sqrt{2}} \begin{pmatrix} S^\dagger W^\dagger T^2(\xi^\dagger)^2 & W^\dagger T^2(\xi^\dagger)^3 & (W^\dagger)T^2(\xi^\dagger)^3\chi \\ S^\dagger W^\dagger T^2\xi & W^\dagger T^2 & (W^\dagger)T^2\chi \\ S^\dagger W^\dagger T^\dagger \xi\chi^\dagger & W^\dagger T^\dagger\chi^\dagger & W^\dagger T^\dagger \end{pmatrix} \tag{27}$$

and the Majorana neutrinos:

$$M_R \simeq \langle\phi_{\rm B-L}\rangle \begin{pmatrix} S^\dagger\chi^\dagger\xi & \chi^\dagger & 1 \\ \chi^\dagger & S\chi^\dagger\xi^\dagger & S\xi^\dagger \\ 1 & S\xi^\dagger & S\chi\xi^\dagger \end{pmatrix} \tag{28}$$

Note that the random complex order of unity and factorial factors which are supposed to multiply all the mass matrix elements are not represented here. We will discuss these factors in section 6.

The matrices for the quarks M_U and M_D happen not to have been changed at all by the introduction of the "new" Higgs fields χ (and ϕ_{B-L}, but that has so little VEV compared to the Planck scale that it could never compete), and even in the charged lepton mass matrix the appearance of χ occurs on off-diagonal matrix elements which are already small and remain so small as to have no significance for the charge lepton mass predictions as long as χ is of the order $\langle\chi\rangle \approx 0.07$ as we need for fitting $\theta_{\rm atm}$.

Therefore all the fits of the "old" Anti-GUT model are valid and we can still use the parameter values obtained by these earlier fits to S, W, T, ξ, presented above in equation (15).

5 Mixing Angles in extended Anti-GUT

The neutrino flavour eigenstates ν_α are related to the mass eigenstates ν_i in the vacuum by a unitary matrix U,

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}\,|\nu_i\rangle \quad . \tag{29}$$

We will investigate in this paper a three-neutrino-generation model, so the Maki-Nakagawa-Sakata (MNS)[21] mixing matrix becomes a 3×3 matrix and is parametrised by

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta_{13}} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta_{13}} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta_{13}} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta_{13}} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta_{13}} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{30}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ and δ_{13} is the CP-violating phase. Here two CP-violation Majorana phases α, β are also included.

Since in the parametrisation equation (30) of the mixing matrix the CP-violating phase δ_{13} is associated with $\sin\theta_{13}$, it is clear that CP-violation is negligible in the lepton sector if the mixing angle θ_{13} is small and for our estimations we should not consider them, since we will not discuss CP-violation in this paper.

Since in our model it turns out that the main mixing of leptons comes from the rotation of the (left-handed) neutrino eigenstates relative to the protoflavour eigenstates (by "protoflavour eigenstates" is understood states of definite $U(1)_f, y_i/2, (B-L)_{1,\rm gen}, (B-L)_{2,\rm gen}$ charges) rather than from the rotations of the charged mass eigenstates relative to the (charged) protoflavour eigenstate (leptons), it is preferable to choose a parametrisation such that the mixing angle $\sin\theta_{13}$ becomes small in this situation. It is the matrix element representing the overlap of the heaviest left neutrino mass eigenstate with the ν_{e_L}, the state which couples to $W^\pm$ bosons, which is small, of order ξ^3; in fact we get $\sqrt{10^{-4}} \approx 10^{-2}$. So we shall take the parametrisation in which this matrix element is just $\sin\theta_{13}\,e^{-i\delta_{13}}$ and not $\sin\theta_{23}\sin\theta_{12} - \sin\theta_{13}\cos\theta_{23}\cos\theta_{12}\,e^{i\delta_{13}}$, as comes out in an alternative parametrisation. But for phenomenology today, the most crucial mixing angles are only the

$$\sin^2 2\theta_\odot \;=\; \sin^2 2\theta_{12} \tag{31}$$
$$\sin^2 2\theta_{\rm atm} \;=\; \sin^2 2\theta_{23} \;, \tag{32}$$

defining

$$U_E M_E M_E^\dagger U_E^\dagger \;=\; {\rm diag}(m_e^2, m_\mu^2, m_\tau^2) \tag{33}$$

$$U_{\text{eff}} M_{\text{eff}} M_{\text{eff}}^\dagger U_{\text{eff}}^\dagger \;=\; \text{diag}(m_{\nu_e}^2, m_{\nu_\mu}^2, m_{\nu_\tau}^2)\ . \tag{34}$$

The $(1,3)$-component of the $U_{\text{eff}}\, U_E^\dagger$ in our model turns out to be of the order of magnitude of product $\sin\theta_{12}\sin\theta_{23}$, while the $(3,1)$-component is smaller and it is natural to choose the parametrisation corresponding to putting

$$U = U_E\, U_{\text{eff}}^\dagger \;=\; (U^\dagger)_{i\alpha}\ , \tag{35}$$

where U has a very small $\sin\theta_{13}$ achieved when U is parametrised according to equation (30).

6 Calculation of M_{eff}

We calculate in this section the effective neutrino mass matrix for left-handed components. Since, strictly speaking, our model only predicts orders of magnitude, a crude calculation is in principle justified. This calculation is presented in the first subsection, and then in the next subsection we make "statistical calculations" with random order-one factors and "factorial factors".

6.1 Crude calculation

From equation (28) we see to the first approximation that there are one massless and two degenerate right-handed neutrinos coming from the VEV of the $B-L$ breaking Higgs field, $\langle\phi_{\text{B-L}}\rangle$.

The splitting between the two almost degenerate see-saw neutrinos would be $M_{31}\,\langle S\rangle\,\langle\chi\rangle\,\langle\xi\rangle$, where $M_{31}\approx\langle\phi_{\text{B-L}}\rangle$ is the approximately common mass of the two heaviest see-saw neutrinos. The third lightest see-saw neutrino is dominantly "proto second generation" and has the mass $\langle\phi_{\text{B-L}}\rangle\,\langle\chi\rangle\,\langle\xi\rangle$.

For the left-handed neutrinos to the first approximation we get the effective mass matrix as follows:

$$M_{\text{eff}} \approx \frac{W^2T^2\,\langle\phi_{WS}\rangle^2}{2\,\langle\phi_{\text{B-L}}\rangle}\begin{pmatrix} \frac{T^2\xi^5}{\chi} & \frac{T^2\xi^2}{\chi} & T\xi^2 \\ \frac{T^2\xi^2}{\chi} & \frac{T}{\xi} & \frac{T}{\xi} \\ T\xi^2 & \frac{T}{\xi} & \frac{\chi}{\xi} \end{pmatrix}\ , \tag{36}$$

But we have to emphasise here that this approximation is *not good enough to calculate only the heaviest left-handed neutrino*, because all the mass matrix elements to this approximation come from the propagator contribution of the lightest see-saw particle, so that they really form a degenerate matrix of rank one. Using this contribution only would lead to two left-handed massless neutrinos and one massive. But we can still obtain the mixing angles θ_{13} and θ_{23} and the heaviest mass from M_{eff}:

$$\theta_{13} \;=\; \theta_{e,\text{heavy}} \approx \frac{T}{\chi}\xi^3 \tag{37}$$

$$\theta_{23} \;=\; \theta_{\mu,\text{heavy}} \approx \begin{cases} \frac{T}{\chi} & \text{when } \chi \gtrsim T \\ 1 & \text{when } \chi \lesssim T \end{cases} \tag{38}$$

$$M_{\nu_L\text{heavy}} \;\approx\; \begin{cases} \frac{W^2T^2\langle\phi_{WS}\rangle^2}{2\langle\phi_{\text{B-L}}\rangle}\frac{\chi}{\xi} & \text{when } \chi \gtrsim T \\ \frac{W^2T^2\langle\phi_{WS}\rangle^2}{2\langle\phi_{\text{B-L}}\rangle}\frac{T}{\xi} & \text{when } \chi \lesssim T \end{cases} \tag{39}$$

From these equations we can restrict the region of χ comparing with Super-Kamiokande experimental data; χ must be almost of the same order as T. Thus we know the mixing angle of the first and third generations must be of the order of ξ^3.

However, to get the much lower masses we cannot use the contribution from the lightest see-saw propagator, but we have to use the propagator terms from the two approximately equally heavy see-saw particles. This contribution to the propagator matrix is

$$M_R^{-1}\big|_{\substack{\text{heavy}\\\text{see-saws}}} \approx \frac{1}{\langle\phi_{\mathrm{B-L}}\rangle}\begin{pmatrix}\chi\xi & \xi & 1\\ \xi & \chi\xi & \chi\\ 1 & \chi & \chi\xi\end{pmatrix} \tag{40}$$

where the $\chi\xi/\langle\phi_{B-L}\rangle$ comes from the mass difference of the almost degenerate see-saw particles.

Surrounding this propagator contribution with the “Dirac ν”-mass matrix we get

$$\begin{aligned} M_{\text{eff}}\big|_{\substack{\text{heavy}\\\text{see-saws}}} &\approx M_\nu^D\, M_R^{-1}\big|_{\substack{\text{heavy}\\\text{see-saws}}} (M_\nu^D)^t \\ &\approx \frac{W^2T^2\langle\phi_{WS}\rangle^2}{2\langle\phi_{\mathrm{B-L}}\rangle}\begin{pmatrix}T^2\xi^6 & T\xi^3 & T\xi^2\\ T\xi^3 & T^2\chi\xi & T\chi\\ T\xi^2 & T\chi & \chi\xi\end{pmatrix} \quad . \end{aligned} \tag{41}$$

It is from this contribution that the two lightest left-handed neutrino masses and their mixing angle, θ_{12}, should be obtained:

$$M_{\nu_L\text{medium}} \approx \frac{W^2T^2\chi\xi\langle\phi_{WS}\rangle^2}{2\langle\phi_{\mathrm{B-L}}\rangle} \tag{42}$$

$$\theta_{12} = \theta_{e,\text{medium}} \approx \begin{cases} \frac{T}{\chi}\xi & \text{when } \chi \gtrsim T \\ \xi & \text{when } \chi \lesssim T \quad . \end{cases} \tag{43}$$

Note that the lightest mass is quite dominantly the ν_{e_L} neutrino and the small mixing angle goes mainly to the medium mass neutrino ($\theta_{e,\text{medium}}/\theta_{e,\text{heavy}} \approx \xi^{-2} \gg 1$). So we should identify approximately the solar oscillation mixing angle with the mixing to the medium heavy neutrino:

$$\theta_\odot \simeq \theta_{e,\text{medium}} \approx \xi \tag{44}$$

and the solar mass square difference

$$\Delta m_\odot^2 \approx M_{\nu_L\text{medium}}^2 \approx \frac{W^4T^4\chi^2\xi^2\langle\phi_{WS}\rangle^4}{4\langle\phi_{\mathrm{B-L}}\rangle^2} \quad . \tag{45}$$

The atmospheric mixing angle goes between the heaviest and the medium one:

$$\begin{aligned} \Delta m_{\text{atm}}^2 &\approx M_{\nu_L\text{heavy}}^2 - M_{\nu_L\text{medium}}^2 \\ &\approx \frac{W^4T^4\chi^2\langle\phi_{WS}\rangle^4}{4\langle\phi_{\mathrm{B-L}}\rangle^2\xi^2} \end{aligned} \tag{46}$$

From equations (45) and (46) we find that the ratio of solar and atmospheric mass square differences must be of the order of ξ^4, say, about 10^{-4}.

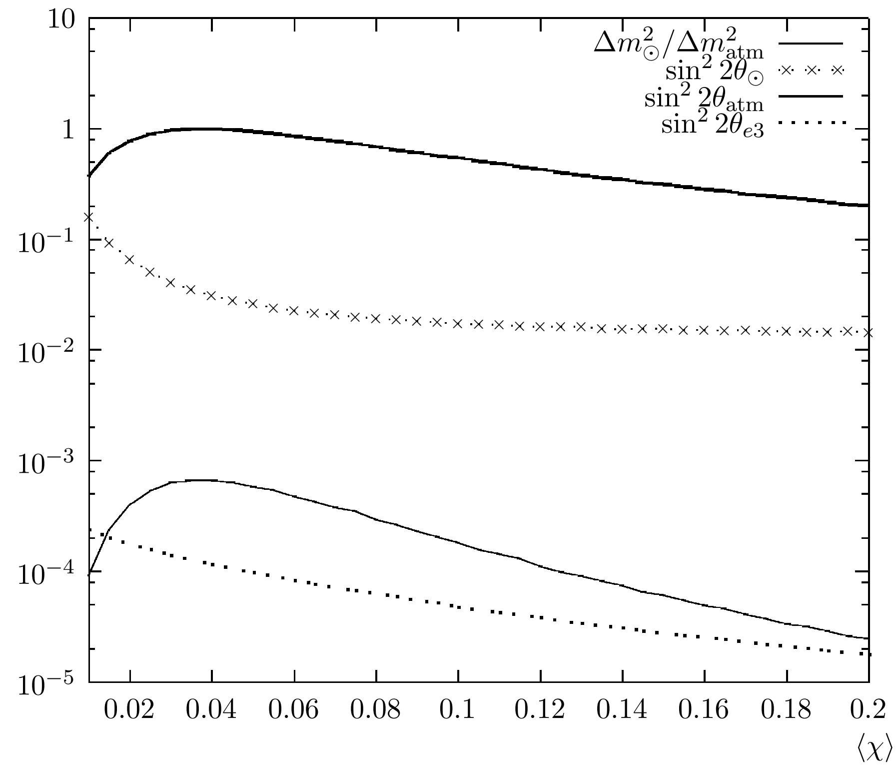

Figure 1: The numerical results of the ratio of the solar neutrino mass square difference to that for the atmospheric neutrino oscillation, and the squared sine of the double of the solar neutrino mixing angle, the atmospheric neutrino mixing angle and the mixing angle θ_{e3}.

6.2 Statistical calculation using random order unity factors

In this subsection we will discuss the numerical calculation. The elements of the mass matrices are determined up to factors of order one to be a product of several Higgs VEVs measured in units of the fundamental scale $M_{\rm Planck}$ - the Planck scale.

We imagine that the mass matrix elements for, *e.g.*, the right-handed neutrino masses or the mass matrix M^D_ν, are given by chain diagrams consisting of a backbone of fermion propagators for fermions with fundamental masses, with side ribs (branches) symbolising a Yukawa coupling to one of the Higgs field VEVs.

We know neither the Yukawa couplings nor the precise masses of the fundamental mass fermions, but it is a basic assumption of the naturalness of our model that these couplings are of order unity and that the masses, also deviate from the Planck mass by factors of order unity. In the numerical evaluation of the consequences of the model we explicitly take into account these uncertain factors of order unity by providing each matrix element with an explicit random number λ_{ij} - with a distribution so that its average $\langle \log \lambda_{ij}\rangle \approx 0$ and its spreading is a factor two.

Then the calculation is performed with these numbers time after time with different random number λ_{ij}-values and the results averaged in logarithms. A crude realisation of the distribution of these λ_{ij} could be a flat distribution between -2 and $+2$, then provided also with a random phase (with flat distribution).

Another "detail" is the use of a factor $\sqrt{\#\text{diagrams}}$ multiplying the matrix elements, to take into account that, due to the possibility of permuting the Higgs field attachments in the chain-diagram, the number of different diagrams is roughly proportional to the number of such permutations #diagrams. This is the correction introduced and studied for the charged mass matrices by C.D. Froggatt, D. Smith and one of us [22]. In the philosophy of each diagram coming with a random order unity factor, the sum of #diagrams get of the order $\sqrt{\#\text{diagrams}}$ bigger than a single diagram of that sort. But we counted these permutations ignoring the field S. If we allowed both S and $S^\dagger$ in the same diagram, the $\sqrt{\#\text{diagrams}}$ factor could give arbitrarily large numbers. It turns out that these factors are especially important for some elements involving the electron-neutrino in the matrix M_ν^D, which are suppressed by several factors, as then many permutations can be made.

Yet another detail is that the symmetric mass matrices - occurring for the Majorana neutrinos - give rise to the same off-diagonal term twice in the right-handed neutrino matrix in the effective Lagrangian, so we must multiply off-diagonal elements with a factor 1/2. But in the M_ν^D-matrix, columns and rows are related to completely different Weyl fields and of course a similar factor 1/2 should not be introduced.

Concerning the $\sqrt{\#\text{diagrams}}$ factor for the diagonal mass matrix terms in the symmetric matrices, *e.g.* M_R, we shall remember that, contrary to what we shall do in non-symmetric matrices such as M_ν^D, and the charged lepton ones. We must count diagrams with the Higgs fields attachment assigned in opposite order as only one diagram. The backbone in the diagram has no orientation and we shall count diagrams obtained from each other by inverting the sequence of the attached Higgs fields as only ONE diagram. Thus the diagonal elements will tend to have only half as many diagrams.

We give in Figure 1 results obtained with $50,000$ random combinations averaged as a function of the small VEV $\langle\chi\rangle$ of the new Higgs field χ.

In order to get the atmospheric mixing angle of the order of unity the range for $\langle\chi\rangle$ around the "old" Anti-GUT VEV $\langle T\rangle \approx 0.07$ is suggested, so only this range is presented.

7 Conclusion

In this article we have made an extension of the Anti-GUT model to neutrinos by including see-saw ν_R at a scale of mass around 10^{12} GeV. By this extension we introduced two more parameters, namely the vacuum expectation values of two additional Higgs fields, ϕ_{B-L} and χ. But from the neutrino oscillation data one extracts two mixing angles $\theta_\odot$ and θ_{atm} and two mass square differences $\Delta m^2_\odot$ and Δm^2_{atm}, so in this sense we have two predictions:

$$\sin^2 2\theta_\odot \approx 3\times 10^{-2} \tag{47}$$

$$\frac{\Delta m^2_\odot}{\Delta m^2_{\text{atm}}} \approx 6\times 10^{-4} \tag{48}$$

These results are *only order of magnitude* estimates, and we shall count something like an uncertainty of 50% for mixing angles and masses and thus for the square, $\sin^2 2\theta$, 100% and *i.e.* a factor 2 up or down and for the $\Delta m^2_\odot/\Delta m^2_{\rm atm}$, $\sqrt{2}\cdot 100\%$ meaning roughly a factor 3 up or down,

$$\sin^2 2\theta_\odot = (3\,^{+3}_{-2}) \times 10^{-2} \tag{49}$$

$$\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} = (6\,^{+11}_{-4}) \times 10^{-4} \ . \tag{50}$$

These two small numbers both come from the parameter ξ - the VEV in "fundamental units" of one of the 7 Higgs fields in our model - which has already been fitted to the charged fermions in earlier works and which is essentially the Cabibbo angle measuring strange to up-quark weak transitions ($\xi \simeq 0.1$ essentially $\sin\theta_c \simeq 0.22$). But it is also important for the success of our model that there has been room to put in the χ field, with which we could fix the atmospheric mixing angle to be of order unity (by taking $\chi \sim T$), as well as a parameter ϕ_{B-L}, the Higgs field VEV for breaking the gauged $B-L$ charge to fit the overall scale of observed neutrino masses. These factors in front of equations (51), (52) and (53) are results of our rather arbitrary averaging over random order unity factors and inclusion of diagram counting square root factors as put forward in reference [22]. But in principle the factors in front are just of order unity:

$$\sin^2 2\theta_\odot = 3\,\xi^2 \tag{51}$$

$$\sin\theta_c = 1.8\,\xi \tag{52}$$

$$\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} = 6\,\xi^4 \tag{53}$$

We want to emphasise here that our model - extended Anti-GUT as well as "old" Anti-GUT - is itself a good model in the sense that all coupling constants are order of unity except for Higgs fields VEVs, and thereby also Higgs masses giving rise to these VEVs. In the SM the most remarkable non-natural feature is the tremendously small Weinberg-Salam Higgs VEV compared to Planck or realistic GUT scales. If somehow we have to accept that there must be a mechanism in nature for making the Weinberg-Salam Higgs VEV very small, we also should admit that the other Higgs VEVs could be very small. In our model we manage to interpret the second non-natural feature of most Yukawa couplings, namely, that they are very small to be also due to small Higgs field VEVs. In this way all small numbers come from VEVs in our model; the rest is put to unity in Planck units. In this sense it is "natural" by the fact that it has only one source of small numbers, VEVs. Even the gauge couplings are interpretable as

Table 2: Number of parameters

	"Yukawa"	"Neutrino"	# of parameters	# of predictions
Standard Model	13	4	17	—
"Old" Anti-GUT	4	—*	4†	9
"New" Anti-GUT	6		6	11

* The "old"Anti-GUT cannot predict the neutrino oscillation.
† Here we have not counted the neutrino oscillation parameters.

being of order of unity, if we follow our assumption, MPP [23, 24], which goes extremely well together with the present model.

It should, however, be stressed that to bring the neutrino oscillation into the model we have now two VEVs which are *extremely* small, namely not only the usual Weinberg-Salam Higgs VEV $\langle\phi_{WS}\rangle \approx 173$ GeV but also $\langle\phi_{B-L}\rangle \approx 10^{12}$ GeV. In this sense the hierarchy problem is doubled. But this is not so much a special problem for our model; the problem is rather that neutrino oscillations point to a completely new scale model independently.

In the column "Yukawa" in Table 2 we write the numbers of parameters with which the observed Yukawa couplings are fitted in the three models mentioned in first column, while "Neutrino" stands for the number of the neutrino oscillation fitting parameters.

The column "# of parameters" is the number of these two classes of parameters taken together. The last column "# of predictions" gives the number constraints predicted among measured parameters in SM and neutrino oscillations.

Including the old fits of the charge mass matrices we can say that we now fit, order-of-magnitude-wise, 17 quantities (11 observed fermion masses or mass square differences, 5 mixing angles and CP-violating phase of quarks) with 6 parameters - the Higgs field VEVs. We can find from Table 2 that we have used in the present article two parameters to fit four more quantities, thus gaining two predictions (the solar mixing angle and the ratio of the neutrino oscillation masses).

We should present here the order of magnitude of the right-handed neutrino masses and the mixing angle θ_{e3} (see in Figure 1):

$$\begin{aligned} M_{R_{\nu_1}} &\approx 10^{11} \ \text{GeV} \ , && (54)\\ M_{R_{\nu_2}} &\approx 10^{13} \ \text{GeV} \ , && (55)\\ M_{R_{\nu_3}} &\approx 10^{13} \ \text{GeV} \ , && (56)\\ \sin^2 2\theta_{e3} &\approx 10^{-4} \ . && (57)\end{aligned}$$

Note that our model is very successful in describing neutrino oscillations and their mixing angles, but this model does not have any good candidate for dark matter; the monopoles could be such a candidate. We will study this problem in a forthcoming article.

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