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RADIATIVE HIGGS-SECTOR CP VIOLATION IN THE MSSM

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We briefly review the phenomenological implications of the minimal supersymmetric standard model (MSSM) with explicit radiative breaking of CP invariance in the Higgs sector for the LEP2 and Tevatron colliders.

It has recently been shown [1] that the tree-level CP invariance of the MSSM Higgs potential can sizeably be broken at the one-loop level by large soft CP-violating trilinear couplings of the Higgs bosons to stop and sbottom squarks. Several recent studies [2,3,4,5,6] have been devoted to analyze in more detail the effective Higgs potential of the MSSM with explicit radiative breaking of CP invariance. We shall briefly review the main phenomenological implications of this rather rich and very predictive theoretical framework of the MSSM for the LEP2 and Tevatron colliders.

In the $\overline{\rm MS}$ scheme, the one-loop CP-violating effective potential of the MSSM is given by

$$-\mathcal{L}_V \;=\; -\mathcal{L}_V^0 + \frac{3}{32\pi^2}\sum_{q=t,b}\left[\,\sum_{i=1,2}\widetilde{m}^4_{q_i}\left(\ln\frac{\widetilde{m}^2_{q_i}}{Q^2}-\frac{3}{2}\right) - 2\bar{m}^4_q\left(\ln\frac{\bar{m}^2_q}{Q^2}-\frac{3}{2}\right)\right], \tag{1}$$

where $\mathcal{L}_V^0$ is the tree-level Lagrangian of the MSSM Higgs potential, and $\bar{m}_q$ and $\widetilde{m}_{q_i}$ are the field-dependent quark and squark masses of the third generation. Here, we adopt the notation of Refs. [1,2,5]. The minimization of the CP-violating effective potential differs from that in the CP-conserving case by the presence of a non-trivial CP-odd tadpole condition [1] of the would-be CP-odd scalar a. In the $\overline{\rm MS}$ scheme, the CP-odd tadpole condition reads [1,7]

$$T_A \;=\; -v\,{\rm Im}(m^2_{12}e^{i\xi}) = -\,\frac{3}{16\pi^2}\sum_{q=t,b}\frac{s_{2q}}{s_\beta}\,{\rm Im}\,h^q_1\;\Delta m^2_{\tilde{q}}\;B^{\rm fin}_0(0,m^2_{\tilde{q}_1},m^2_{\tilde{q}_2})\,, \tag{2}$$

where $t_\beta = s_\beta/c_\beta = v_2/v_1$, $s_{2q} = 2\sin\theta_q\cos\theta_q$, $\Delta m^2_{\tilde{q}} = m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1}$, $h^t_1 = -m_t\mu^* e^{-i\delta_t}/(s_\beta v)$, $h^b_1 = m_b A^*_b e^{i\delta_b}/(c_\beta v)$, $\delta_{t(b)} = \arg\,[A_{t(b)} - \mu^* t_\beta(1/t_\beta)]$ and

$$B^{\rm fin}_0(0,m^2_1,m^2_2) \;=\; -\,\ln\left(\frac{m_1m_2}{Q^2}\right) + 1 + \frac{m^2_1+m^2_2}{m^2_1-m^2_2}\,\ln\left(\frac{m_2}{m_1}\right). \tag{3}$$

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Furthermore, μ and m_{12}^2 are respectively the SUSY and soft SUSY-breaking Higgs-mixing terms, $A_{t,b}$ are the soft SUSY-breaking Yukawa couplings, and θ_q is the mixing angle between the mass and weak squark eigenstates.

To one-loop order, CP violation is introduced into the MSSM Higgs potential through the complex parameters μ and $A_{t,b}$. The CP-violating terms are proportional to the rephasing invariant combination: $\mathrm{Im}\,(m_{12}^{2*} A_{t,b}\mu)$. From this last expression, it is clear that only the relative phase between $\mu A_{t,b}$ and m_{12}^2 plays a role. Therefore, a good phase convention is to define m_{12}^2 to be real. This can always be achieved by a global U(1) rotation.[1,2] As a result, the relative phase ξ between the two Higgs vacuum expectation values v_1 and v_2 vanishes at the tree level. The phase choice $\xi = 0$ can be preserved order by order in perturbation theory by a corresponding choice of the counter-term of $\mathrm{Im}\, m_{12}^2$, exactly as is given in Eq. (2).[1,5] In fact, this perturbative resetting of the phase ξ to zero is equivalent to the general requirement that within the effective-potential formalism, v_1, $\mathrm{Re}\, v_2 = |v_2| \cos\xi$ and $\mathrm{Im}\, v_2 = |v_2| \sin\xi$ do not receive finite radiative shifts in higher orders other than those due to Higgs wave-function renormalization. The latter approach is also consistent with the one followed in the CP-conserving studies.[8]

It is known that the MSSM faces the difficulty of explaining naturally the apparent absence of electric dipole moments (EDMs) of the neutron and electron.[9,12,10,11,13,14,15] Several suggestions have been made to suppress the SUSY contributions to electron and neutron EDMs, at a level just below their present experimental upper limits. Apart from the obvious choice of suppressing the new CP-violating phases of the theory to the 10^{-3} level,[9] a more phenomenologically appealing possibility is to make the first two generations of scalar fermions as heavy as few TeV,[10] but keep the soft-breaking mass parameters of the third generation relatively small, e.g. 0.5–0.7 TeV. An interesting alternative is to arrange for partial cancellations among the different EDM contributions either at the short-distance level[11] or the non-perturbative long-distance one.[13]

In addition to the one-loop EDM contributions of the first two generations, one may have to worry that third-generation squarks do not induce observable effects on the electron and neutron EDMs through the three-gluon operator [12], through the effective coupling of the 'CP-odd' Higgs boson to the gauge bosons [14], and through two-loop gaugino/higgsino-mediated EDM graphs [15]. For low-t_β scenarios, the two-loop EDM contributions are found to be of the order of the experimental upper bounds.[14] Therefore, it not very difficult to arrange the different two-loop EDM terms to partially cancel one another,[15] and so reduce significantly their total size.

An immediate consequence of CP violation in the Higgs potential of the

MSSM is the presence of mixing-mass terms between the CP-even and CP-odd Higgs fields.[1] In the weak basis (ϕ_1, ϕ_2, a), the neutral Higgs-boson mass matrix $\mathcal{M}_N^2$ takes on the form

$$\mathcal{M}_N^2 \;=\; \begin{bmatrix} \mathcal{M}_S^2 & \mathcal{M}_{SP}^2 \\ (\mathcal{M}_{SP}^2)^T & \mathcal{M}_P^2 \end{bmatrix}\,, \tag{4}$$

where $\mathcal{M}_S^2$ and $\mathcal{M}_P^2$ describe the CP-conserving transitions between scalar and pseudoscalar particles, respectively, whereas $\mathcal{M}_{SP}^2$ describes CP-violating scalar-pseudoscalar transitions. The characteristic size of these CP-violating off-diagonal terms in the Higgs-boson mass matrix was found to be [1,2]

$$\begin{aligned} M_{SP}^2 \simeq \mathcal{O}\left(\frac{m_t^4}{v^2}\frac{|\mu||A_t|}{32\pi^2 M_{\rm SUSY}^2}\right)\sin\phi_{\rm CP} \\ \times\left(6,\ \frac{|A_t|^2}{M_{\rm SUSY}^2}\,,\ \frac{|\mu|^2}{\tan\beta\, M_{\rm SUSY}^2}\,,\ \frac{\sin 2\phi_{\rm CP}}{\sin\phi_{\rm CP}}\,\frac{|\mu||A_t|}{M_{\rm SUSY}^2}\right), \end{aligned} \tag{5}$$

where the last bracket summarizes the relative sizes of the different contributions, and $\phi_{\rm CP} = \arg(A_t\mu)$. As can be seen from Eq. (5), the CP-violating effects can become substantial if $|\mu|$ and $|A_t|$ are larger than the average of the stop masses, denoted as $M_{\rm SUSY}$. For example, the off-diagonal terms of the neutral Higgs-mass matrix may be of order $(100\ {\rm GeV})^2$, for $|\mu| \simeq |A_t| \lesssim 3M_{\rm SUSY}$, and $\phi_{\rm CP} \simeq 90^\circ$.

The main effect of Higgs-sector CP violation is the modification of the couplings of the Higgs bosons to fermions and the W and Z bosons, i.e. ffH_i, WWH_i, ZZH_i and ZH_iH_j. The modified effective Lagrangians are given by

$$\begin{aligned} \mathcal{L}_{H\bar{f}f} = & -\sum_{i=1}^{3} H_i\Big[\frac{g_w m_d}{2M_W c_\beta}\,\bar{d}\,(O_{1i} \,-\, is_\beta O_{3i}\gamma_5)\,d \\ & +\frac{g_w m_u}{2M_W s_\beta}\,\bar{u}\,(O_{2i} \,-\, ic_\beta O_{3i}\gamma_5)\,u\Big]\,, \end{aligned} \tag{6}$$

$$\begin{aligned} \mathcal{L}_{HVV} = & \ g_w M_W \sum_{i=1}^{3}(c_\beta O_{1i} \,+\, s_\beta O_{2i})\Big(H_i W_\mu^+ W^{-,\mu} \\ & +\frac{1}{2c_w^2}\,H_i Z_\mu Z^\mu\Big)\,, \end{aligned} \tag{7}$$

$$\begin{aligned} \mathcal{L}_{HHZ} = & \ \frac{g_w}{4c_w}\sum_{i,j=1}^{3}\Big[O_{3i}\,(c_\beta O_{2j} \,-\, s_\beta O_{1j}) \,-\, O_{3j}\,(c_\beta O_{2i} \,-\, s_\beta O_{1i})\Big] \\ & \times Z^\mu\,(H_i \overleftrightarrow{\partial}_\mu H_j)\,, \end{aligned} \tag{8}$$

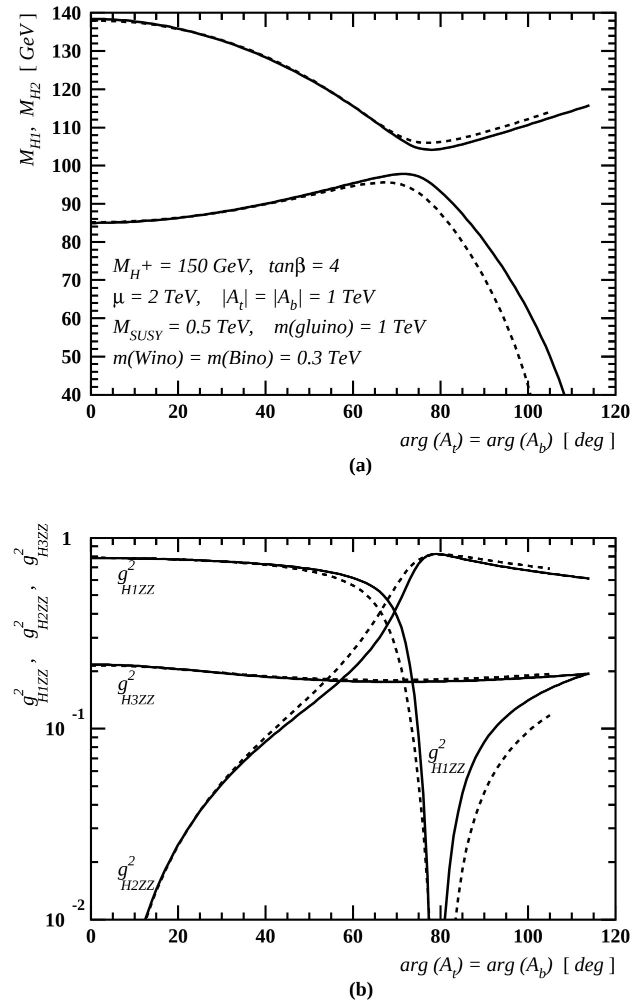

Figure 1: Numerical estimates [5] of (a) $M_{H_1} \leq M_{H_2}$ and (b) $g^2_{H_i ZZ}$ and as a function of $\arg(A_t)$.

where $c_w = M_W/M_Z$, $\overleftrightarrow{\partial}_\mu \equiv \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu$, and O is the orthogonal transformation matrix relating the weak with the mass Higgs-boson eigenstates.[2,5]

Let us now discuss a representative example demonstrating the phenomenological consequences of Higgs-sector CP violation on the LEP2 and Tevatron colliders. We consider an intermediate value for $\tan\beta = 4$, and a relatively light charged Higgs boson $M_{H^+} = 150$ GeV, with $M_{\rm SUSY} = 0.5$ TeV, $A_t = A_b = 1$ TeV and $\mu = 2$ TeV. In Fig. 1, we then find regions for which the lightest Higgs-boson mass M_{H_1} is as small as 60–70 GeV for $\arg(A_t) \approx 90°$, and the H_1ZZ coupling g_{H_1ZZ}, which is normalized to its SM value, is small enough for the H_1 boson to escape detection at the latest LEP2 run with $\sqrt{s} = 202$ GeV.[5] Moreover, the H_2 boson is too heavy to be detected through the H_2ZZ channel. In addition, either the coupling H_1H_2Z, $g_{H_1H_2Z} = g_{H_3ZZ}$, is too small or H_2 is too heavy to allow Higgs detection in the H_1H_2Z channel.[5] An upgraded Tevatron machine has the potential capabilities to close most of such experimentally open windows. The results of the recent complete RG analysis of Ref. [5] are in good qualitative agreement with earlier studies.[2,4]

In conclusion, the MSSM with explicit radiative breaking of CP invariance in the Higgs sector [1] constitutes a very rich and predictive theoretical framework, with interesting consequences on collider experiments,[1,3,2,4,5,6,16] CP asymmetries in B-meson decays, [17] and electroweak baryogenesis.[18]

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