

CERN-TH/2000-086
hep-th/0003174

Dilatonic p -Branes and Brane Worlds

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Abstract

We study a general dilatonic p -brane solution in arbitrary dimensions in relation to the Randall-Sundrum scenario. When the p -brane is fully localized along its transverse directions, the Kaluza-Klein zero mode of bulk graviton is not normalizable. When the p -brane is delocalized along its transverse directions except one, the Kaluza-Klein zero mode of bulk graviton is normalizable if the warp factor is chosen to increase, in which case there are singularities at finite distance away from the p -brane. Such delocalized p -brane can be regarded as a dilatonic domain wall as seen in higher dimensions. This unusual property of the warp factor allows one to avoid a problem of dilatonic domain wall with decreasing warp factor that free massive particles are repelled from the domain wall and hit singularities, since massive particles with finite energy are trapped around delocalized p -branes with increasing warp factor by gravitational force and can never reach the singularities.

CERN-TH/2000-086
March, 2000

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1 Introduction

According to the brane world scenario [1, 2], the extra spatial dimensions can be as large as a millimeter without contradicting the current experimental observations, if the fields of Standard Model are confined within a brane. More recently proposed scenario by Randall and Sundrum (RS) [3, 4, 5] even allows infinite extra space because of the special property of warped spacetime that leads to localization of gravity around domain wall. So, the brane world scenarios open up the possibility of probing the extra dimensions in the near future. Furthermore, the brane world scenarios provide with a new framework for solving the hierarchy problem of particle physics.

In order for the RS brane world scenario to describe our world, it has to be derivable from well-established theories. The previous studies attempt to embed the RS domain wall (with the exponentially decreasing warp factor) into supergravity theories [6, 7, 8, 9, 10, 11, 12, 13, 14, 15] or domain walls which localize gravity into string theories [16, 17]. Also, a main motivation for the embedding into supergravity theories is that the fine-tuned value of domain wall tension in terms of the bulk cosmological constant is required by supersymmetry through the BPS condition, if such embedding is possible.

In this paper, we study the Kaluza-Klein (KK) zero mode of graviton in the bulk of a general dilatonic p -brane in arbitrary dimensions for the purpose of seeing any relevance to the RS type scenario. Since the brane world scenarios assume that fields of Standard Model are identified as the worldvolume fields of p -branes in string theories, it is natural to embed domain walls of the RS type models into p -branes in higher dimensions. So, it appears that the embedding into p -branes in string theories is a natural way of realizing the RS type scenario within string theories.

We find in general that when a p -brane is fully localized along its transverse directions, the KK zero mode of bulk graviton cannot be normalized, thereby the $(p + 1)$ -dimensional gravity in the worldvolume of the p -brane cannot be realized through the KK zero mode. However, if the p -brane is delocalized along its transverse directions except one, the KK zero mode of bulk graviton is normalizable. This implies that if our four-dimensional world is embedded in the worldvolume of some (intersecting) p -brane of string theories through the RS type scenario, then some of the transverse directions of the brane has to be compact or the range of the radial coordinate of the transverse space has to be within a finite interval. Unlike the case of domain walls (with codimension one), the KK zero mode of bulk graviton is normalizable if the warp factor increases, in which case there are singularities at finite distance from the brane. Due to the increasing warp factor, free massive particles are attracted towards the brane, never hitting the singularities, in contrast to the case of domain walls with decreasing warp factor, which repels free massive particles [18, 19]. So, a delocalized p -brane with increasing warp factor does not suffer from the problem of a domain wall with decreas-

ing warp factor that massive matter escapes into the extra spatial dimensions due to the gravitational repulsion by the wall.

The paper is organized as follows. In section 2, we summarize the dilatonic domain wall solution in relation to the RS scenario. We study the KK zero mode of graviton in the bulk of fully localized dilatonic p -brane in section 3 and in the bulk of delocalized dilatonic p -brane in section 4.

2 Dilatonic Domain Wall Solution

In this section, we summarize a general dilatonic domain wall solution in arbitrary dimensions for the purpose of reference in the later sections, where we will make comparison of the properties of such domain wall solutions to those of p -brane solutions. The Einstein frame action for the domain wall solution has the following form:

$$S_{\text{DW}} = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-g} \left[\mathcal{R}_g - \frac{4}{D-2} \partial_\mu \phi \partial^\mu \phi + e^{-2a\phi} \Lambda \right]. \quad (1)$$

The extreme dilatonic domain wall solution is given by:

$$\begin{aligned} g_{\mu\nu} dx^\mu dx^\nu &= H^{\frac{4}{(D-2)\Delta}} \left[-dt^2 + dx_1^2 + \cdots + dx_{D-2}^2 \right] + H^{\frac{4(D-1)}{(D-2)\Delta}} dx_{D-1}^2, \\ e^{2\phi} &= H^{\frac{(D-2)a}{\Delta}}, \quad \Delta \equiv \frac{(D-2)a^2}{2} - \frac{2(D-1)}{D-2}, \end{aligned} \quad (2)$$

where the harmonic function H for the domain wall located at $x_{D-1} = 0$ is

$$H = 1 + Q|x_{D-1}| = 1 \pm \sqrt{-\frac{\Delta\Lambda}{2}} |x_{D-1}|, \quad (3)$$

where the invariance under the \mathbf{Z}_2 transformation $x_{D-1} \rightarrow -x_{D-1}$ is imposed. By redefining the transverse coordinate x_{D-1} , one can put the domain wall metric into the following standard form of the RS brane world scenario:

$$g_{\mu\nu} dx^\mu dx^\nu = e^{2A(y)} \left[-dt^2 + dx_1^2 + \cdots + dx_{D-2}^2 \right] + dy^2, \quad (4)$$

where the warp factor e^{2A} and the dilaton ϕ are given by

$$e^{2A} = (K|y| + 1)^{\frac{8}{(D-2)^2 a^2}}, \quad \phi = \frac{1}{a} \ln(K|y| + 1), \quad (5)$$

where

$$K = \pm \frac{(D-2)a^2}{2} \sqrt{-\frac{\Lambda}{2\Delta}}. \quad (6)$$

It is observed [17, 20, 21, 22] that when the sign \pm in the above expression for K is chosen so that the warp factor e^{2A} has a root at a finite non-zero value of y on both

sides of the domain wall, there exists the normalizable KK zero mode graviton bound state, which can be identified as a massless graviton in one lower dimensions. In such case, the warp factor decreases on both sides of the wall, so the RS type scenario can be realized. However, a problem with such domain wall is that the roots of the warp factor correspond to naked singularities, which are undesirable unless significant physical meanings are associated with them. The resolution or the physical interpretation of such naked singularities is still an open question. With different choice of sign \pm in Eq. (6), there is no singularity at finite nonzero y , but the normalizable graviton KK zero mode does not exist.

3 Fully Localized p -Branes

We begin by summarizing a general dilatonic p -brane solution in D spacetime dimensions, where $p \geq 3$ and $D > 5$. The reason for considering such solution is that it covers all the possible single charged p -branes in string theories. In this paper, we regard the p -brane as a solitonic brane magnetically charged under the field strength F_n of the rank $n \equiv D - p - 2$, for the reason which will become clear in the following. The action has the following form:

$$S_p = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-G} \left[\mathcal{R}_G - \frac{4}{D-2} (\partial\phi)^2 - \frac{1}{2 \cdot n!} e^{-2a_p \phi} F_n^2 \right], \quad (7)$$

where once again $D = p + n + 2$. The solution to the equations of motion of this action for the extreme dilatonic p -brane with the longitudinal coordinates $\mathbf{x} = (x_1, \dots, x_p)$ and the transverse coordinates $\mathbf{y} = (y_1, \dots, y_{n+1})$, located at $\mathbf{y} = \mathbf{0}$, has the following form:

$$\begin{aligned} G_{MN} dx^M dx^N &= H_p^{-\frac{4(n-1)}{(p+n)\Delta_p}} \left[-dt^2 + d\mathbf{x} \cdot d\mathbf{x} \right] + H_p^{\frac{4(p+1)}{(p+n)\Delta_p}} d\mathbf{y} \cdot d\mathbf{y}, \\ e^{2\phi} &= H_p^{\frac{(p+n)a_p}{\Delta_p}}, \quad F_n = \star(dH_p \wedge dt \wedge dx_1 \wedge \dots \wedge dx_p), \end{aligned} \quad (8)$$

where

$$H_p = 1 + \frac{Q_p}{|\mathbf{y}|^{n-1}}, \quad \Delta_p = \frac{(p+n)a_p^2}{2} + \frac{2(p+1)(n-1)}{p+n}. \quad (9)$$

We will consider p -branes with asymptotically flat spacetime, only, i.e. the $n > 1$ case. So, Δ_p defined in the above is always positive.

To study the KK modes of graviton in the p -brane background, we consider the leading order Einstein's equations satisfied by the small fluctuations $h_{\mu\nu}(x^\mu, \mathbf{y})$ around the p -brane metric. In general, the linearized Einstein's equations $\delta\mathcal{G}_{MN} = \kappa_D^2 \delta T_{MN}$ for the following form of the metric fluctuations

$$G_{MN} dx^M dx^N = e^{B(y^k)} \left[\eta_{\mu\nu} + h_{\mu\nu}(x^\rho, y^k) \right] dx^\mu dx^\nu + g_{ij}(y^k) dy^i dy^j, \quad (10)$$

is given, in the transverse traceless gauge $h^\mu{}_\mu = 0 = \partial^\mu h_{\mu\nu}$, by (Cf. Ref. [23])

$$e^{-B} \eta^{\rho\sigma} \partial_\rho \partial_\sigma h_{\mu\nu} + e^{-\frac{p+1}{2}B} \frac{1}{\sqrt{g}} \partial_i \left[e^{\frac{p+1}{2}B} \sqrt{g} g^{ij} \partial_j h_{\mu\nu} \right] = 0, \quad (11)$$

if the stress tensor T_{MN} satisfies the following condition:

$$\delta T_{MN} = T_M^P h_{PN}. \quad (12)$$

Here, $\delta \mathcal{G}_{MN}$ and δT_{MN} denote the variation of the Einstein tensor \mathcal{G}_{MN} and the stress tensor T_{MN} with respect to the metric perturbation $\eta_{\mu\nu} \rightarrow \eta_{\mu\nu} + h_{\mu\nu}$. Of course, in Eq. (12) it is assumed that $h_{y^i y^j} = 0$.

It can be easily shown that the stress tensor condition (12) is satisfied for the p -brane solution in Eq. (8) by using the fact that the dilaton field ϕ depends only on the transverse coordinates \mathbf{y} and only the transverse components of the form field strength F_n are nonzero. So, the linearized Einstein's equations in the transverse traceless gauge satisfied by the metric perturbation $h_{\mu\nu}$ around the p -brane metric (8) of the following form:

$$G_{MN} dx^M dx^N = H_p^{-\frac{4(n-1)}{(p+n)\Delta_p}} [\eta_{\mu\nu} + h_{\mu\nu}] dx^\mu dx^\nu + H_p^{\frac{4(p+1)}{(p+n)\Delta_p}} d\mathbf{y} \cdot d\mathbf{y} \quad (13)$$

is given by

$$\eta^{\rho\sigma} \partial_\rho \partial_\sigma h_{\mu\nu} + H_p^{-\frac{4}{\Delta_p}} \delta^{ij} \partial_i \partial_j h_{\mu\nu} = 0. \quad (14)$$

To consider the graviton KK mode of mass m_α , we decompose the metric perturbation $h_{\mu\nu}$ as $h_{\mu\nu}(x^\rho, y^k) = \hat{h}_{\mu\nu}^{(\alpha)}(x^\rho) f_\alpha(y^k)$ and require $\hat{h}_{\mu\nu}^{(\alpha)}$ to satisfy $\square_x \hat{h}_{\mu\nu}^{(\alpha)} = m_\alpha^2 \hat{h}_{\mu\nu}^{(\alpha)}$, where $\square_x \equiv \eta^{\rho\sigma} \partial_\rho \partial_\sigma$. Then, the linearized Einstein's equations (14) take the following form:

$$\nabla_y^2 f_\alpha + m_\alpha^2 H_p^{\frac{4}{\Delta_p}} f_\alpha = 0, \quad (15)$$

where $\nabla_y^2 \equiv \delta^{ij} \partial_i \partial_j$. We decompose the KK mode $f_\alpha(\mathbf{y})$ into the radial and the angular parts as $f_\alpha(\mathbf{y}) = g_\alpha(y) Y_\ell(\Omega_n)$, where $y \equiv |\mathbf{y}|$ and Ω_n collectively denotes the angular coordinates of the unit n -sphere S^n . The n -dimensional spherical harmonics Y_ℓ satisfies $\nabla_y^2 Y_\ell = \frac{\ell(\ell+n-1)}{y^2} Y_\ell$. So, Eq. (15) reduces to the following form:

$$\partial_y [y^n \partial_y g_\alpha] + \ell(\ell+n-1) y^{n-2} g_\alpha + m_\alpha^2 y^n H_p^{\frac{4}{\Delta_p}} g_\alpha = 0, \quad (16)$$

from which we see that the KK modes g_α with different masses m_α are orthogonalized with respect to the weighting function $w(y) = y^n H_p^{\frac{4}{\Delta_p}}$. From Eq. (15), one can see that $f_0(\mathbf{y}) = \text{constant}$ is the KK zero mode ($m_0 = 0$). The normalization integral for the KK zero mode $g_0(y) = \text{constant}$ is as follows:

$$\int_0^\infty dy y^n H_p^{\frac{4}{\Delta_p}} g_0^2 = \frac{g_0^2 Q_p^{\frac{4}{\Delta_p}}}{(n-1)\frac{4}{\Delta_p} - (n+1)} y^{n+1-(n-1)\frac{4}{\Delta_p}}$$

$$\times {}_2F_1\left[\frac{n+1}{n-1} - \frac{4}{\Delta_p}, -\frac{4}{\Delta_p}; \frac{2n}{n-1} - \frac{4}{\Delta_p}; -\frac{y^{n-1}}{Q_p}\right] \Bigg|_{y=0}^{y=\infty}, \quad (17)$$

where ${}_2F_1$ is the hypergeometric function defined in terms of series as

$${}_2F_1[a, b; c; z] = \sum_{l=0}^{\infty} \frac{(a)_l (b)_l}{(c)_l} \frac{z^l}{l!} = 1 + \frac{ab}{c} \frac{z}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)} \frac{z^2}{2!} + \dots \quad (18)$$

Note, this graviton KK zero mode normalization integral is equivalent to the normalization condition $\int d^{n+1}\mathbf{y} G^{tt} \sqrt{-G} = V_n \int dy y^n H_p^{\frac{4}{\Delta_p}} < \infty$, where V_n is the volume of S^n , obtained in Ref. [23]. The above expression (17) for the normalization integral is valid and well-defined, only for the case when $\frac{n+1}{n-1} \neq \frac{4}{\Delta_p}$ and $\frac{2n}{n-1} - \frac{4}{\Delta_p}$ is neither zero nor a negative integer. These conditions are satisfied by Δ_p defined in Eq. (9) with $n \geq 2$ and $p \geq 3$. The normalization integral (17) does not have a diverging contribution from $y = 0$, where the p -brane is located, if $\frac{n+1}{n-1} > \frac{4}{\Delta_p}$. This condition is also satisfied by Δ_p defined in Eq. (9) with $n \geq 2$ and $p \geq 3$. Note, this condition is essential in normalization of the KK zero mode, since we are not allowed to truncate the transverse space to exclude the $y = 0$ region, where the p -brane is located. However, the normalization integral (17) always has a diverging contribution from $y = \infty$ for any values of n and Δ_p . So, one has to truncate the transverse space (namely, take the integration interval in the normalization integral to be $0 \leq y \leq y_0$ with $y_0 < \infty$), if one wishes to normalize the KK zero mode. A possible scenario with such truncated integration interval is the one proposed in Ref. [24], where the *jump brane* (identifiable as the *Planck brane*) at a finite distance from the p -brane is introduced through the T-dualization of the transverse space.

Since the dilatonic p -brane solution (8) is asymptotically flat, it may be possible to reproduce the $(p+1)$ -dimensional gravity in the intermediate distance region on the worldvolume of the p -brane through the massive KK modes by applying the mechanism proposed in Ref. [25]. However, we shall not pursue this direction, since its validity for describing our world is rather controversial [26, 27, 28, 29, 30, 31]. Also, study of massive graviton KK modes in the bulk of a p -brane is a lot more complicated than the domain wall case. The coordinate transformation that brings the $(p+2)$ -dimensional part of the p -brane metric (with the coordinates $(x^\mu, |\mathbf{y}|)$) into the conformally flat form, in which the equation satisfied by the metric fluctuation takes the Schrödinger equation form, involves a special function of the coordinate $|\mathbf{y}|$, which cannot be inverted to re-express the metric in new coordinates.

4 Delocalized p -Branes

In the previous section, we saw that the KK zero mode of graviton in the bulk of a fully localized dilatonic p -brane (with asymptotically flat spacetime) is not normalizable, if

the transverse space of the p -brane is of infinite size. In this section, we attempt to normalize the graviton KK zero mode by delocalizing the p -brane along its transverse directions except one. Delocalization is achieved by constructing dense periodically arrayed multi-center p -branes along the transverse directions to be delocalized. The delocalization process can also be regarded as first placing p -branes at equivalent points of the compactification lattices and then taking the limit of very small compactification manifold. Therefore, the spacetime of such delocalized p -brane can also be thought of as the product of a $(p + 1)$ -dimensional domain wall spacetime and an n -dimensional compact space.

The delocalized p -brane solution has the following form:

$$G_{MN}dx^M dx^N = H_p^{-\frac{4(n-1)}{(p+n)\Delta_p}} \left[-dt^2 + d\mathbf{x} \cdot d\mathbf{x} \right] + H_p^{\frac{4(p+1)}{(p+n)\Delta_p}} \left[d\tilde{y}^2 + d\tilde{s}_n^2 \right], \quad (19)$$

where \tilde{y} is one of the transverse coordinates \mathbf{y} and $d\tilde{s}_n^2$ is the metric on an n -dimensional compact manifold \mathcal{K}_n upon which the p -brane is delocalized and the harmonic function is given by $H_p = 1 + \tilde{Q}_p |\tilde{y}|$. If $\tilde{Q}_p < 0$, the location $|\tilde{y}| = -\tilde{Q}_p^{-1}$, where the harmonic function H_p vanishes, corresponds to the curvature singularity. When $\tilde{Q}_p > 0$, there is no singularity except at $y = 0$, where the delocalized p -brane is located.

For the purpose of studying the delocalized p -brane solution in relation to the RS type scenario, it is convenient to transform the transverse coordinate \tilde{y} so that the $(p + 2)$ -dimensional part of the metric (with the coordinates (x^μ, \tilde{y})) takes the standard form of the RS model with the warp factor \mathcal{W} or the conformally flat form with the conformal factor \mathcal{C} . The solutions in the new coordinates are given by

$$G_{MN}dx^M dx^N = \mathcal{W} \left[-dt^2 + d\mathbf{x} \cdot d\mathbf{x} \right] + dy^2 + \mathcal{W}^{-\frac{p+1}{n-1}} d\tilde{s}_n^2, \quad (20)$$

where the warp factor is given by

$$\begin{aligned} \mathcal{W}(y) &= \left[1 \pm \frac{2(p+1) + (p+n)\Delta_p}{(p+n)\Delta_p} \tilde{Q}_p |y| \right]^{-\frac{4(n-1)}{2(p+1) + (p+n)\Delta_p}} \\ &= \left[1 \pm \frac{(p+n)^2 a_p^2 + 4(p+1)n}{(p+n)^2 a_p^2 + 4(p+1)(n-1)} \tilde{Q}_p |y| \right]^{-\frac{8(n-1)}{(p+n)^2 a_p^2 + 4(p+1)n}}, \end{aligned} \quad (21)$$

and

$$G_{MN}dx^M dx^N = \mathcal{C} \left[-dt^2 + d\mathbf{x} \cdot d\mathbf{x} + dz^2 \right] + \mathcal{C}^{-\frac{p+1}{n-1}} d\tilde{s}_n^2, \quad (22)$$

where the conformal factor is given by

$$\mathcal{C}(z) = \left[1 \pm \frac{\Delta_p + 2}{\Delta_p} \tilde{Q}_p |z| \right]^{-\frac{4(n-1)}{(p+n)(\Delta_p+2)}}. \quad (23)$$

The expressions for dilaton ϕ and the n -form field strength F_n in terms of the warp factor or the conformal factor can be obtained by using the relation $\mathcal{W}(y) = H_p^{-\frac{4(n-1)}{(p+n)\Delta_p}}(\tilde{y}) = \mathcal{C}(z)$.

We notice the following important difference of the warp factor (21) for the delocalized p -brane from that (5) of the dilatonic domain wall (of codimension one). In the case of the dilatonic domain walls, the exponent in the warp factor (5) is always positive. So, one has to choose the sign \pm in Eq. (6) such that the warp factor has zeros at finite nonzero y on both sides of the wall, if one wants to have the decreasing warp factor. The side-effect of such choice of sign is the naked singularities at the positions where the warp factor vanishes. On the other hand, in the case of the delocalized p -branes, the exponent in the warp factor (21) is always negative. So, in order to have a decreasing warp factor on both sides $y > 0$ and $y < 0$, one has to choose the sign \pm in Eq. (21) such that the term in the square bracket does not have a root at finite nonzero y . With such choice of the sign, the delocalized p -brane solution does not have problematic naked singularities. With a choice of the sign \pm such that the term in the square bracket has a root at a finite nonzero y on both sides of the brane, the warp factor \mathcal{W} increases, asymptotically approaching infinity as the roots are reached. Such roots correspond to the curvature singularities. However, as we will see in the following, unlike the case of dilatonic domain walls of codimension one, the graviton KK zero mode in the bulk of delocalized p -brane is normalizable when the warp factor \mathcal{W} *increases* on both sides of the p -brane.

To study the KK modes of the bulk graviton, we consider the following small fluctuation around the delocalized p -brane metric:

$$G_{MN}dx^M dx^N = \mathcal{C} \left[(\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu + dz^2 \right] + \mathcal{C}^{-\frac{p+1}{n-1}} d\tilde{s}_n^2, \quad (24)$$

where the metric perturbation $h_{\mu\nu}(x^\rho, z)$, which is taken to be independent of the coordinates of the n -dimensional compact space \mathcal{K}_n with the metric $d\tilde{s}_n^2$, is assumed to satisfy the transverse traceless gauge condition $h^\mu{}_\mu = 0 = \partial^\mu h_{\mu\nu}$. Then, the (μ, ν) -component of the Einstein's equations is approximated, to the first order in $h_{\mu\nu}$, to

$$\left[\square_x + \partial_z^2 - \frac{p+n}{2(n-1)} \frac{\partial_z \mathcal{C}}{\mathcal{C}} \partial_z \right] h_{\mu\nu} = 0, \quad (25)$$

where $\square_x \equiv \eta^{\mu\nu} \partial_\mu \partial_\nu$. To consider the KK mode with mass m_α , only, we decompose $h_{\mu\nu}$ as $h_{\mu\nu}(x^\rho, z) = \hat{h}_{\mu\nu}^{(\alpha)}(x^\rho) f_\alpha(z)$ and require $\hat{h}_{\mu\nu}^{(\alpha)}$ to satisfy $\square_x \hat{h}_{\mu\nu}^{(\alpha)} = m_\alpha^2 \hat{h}_{\mu\nu}^{(\alpha)}$. Then, the linearized Einstein's equations (25) reduce to the following form:

$$\left[\partial_z^2 - \frac{p+n}{2(n-1)} \frac{\partial_z \mathcal{C}}{\mathcal{C}} \partial_z + m_\alpha^2 \right] f_\alpha(z) = 0. \quad (26)$$

This equation can be brought to the following form of the Sturm-Liouville equation:

$$\partial_z \left[\mathcal{C}^{-\frac{p+n}{2(n-1)}} \partial_z f_\alpha \right] + m_\alpha^2 \mathcal{C}^{-\frac{p+n}{2(n-1)}} f_\alpha = 0, \quad (27)$$

from which we see that the KK modes $f_\alpha(z)$ with different masses m_α are orthogonalized with respect to the weighting function $w(z) = \mathcal{C}^{-\frac{p+n}{2(n-1)}}$. Had we used the transverse coordinate y , instead of z , the equation satisfied by the KK mode f_α would have taken the following form:

$$\partial_y \left[\mathcal{W}^{-\frac{p+1}{2(n-1)}} \partial_y f_\alpha \right] + m_\alpha^2 \mathcal{W}^{-\frac{p+2n-1}{2(n-1)}} f_\alpha = 0, \quad (28)$$

from which we know that the KK modes $f_\alpha(y)$ are orthogonalized with respect to the weighting function $w(y) = \mathcal{W}^{-\frac{p+2n-1}{2(n-1)}}$.

In terms of a new z -dependent function defined as $\tilde{f}_\alpha \equiv \mathcal{C}^{-\frac{p+n}{4(n-1)}} f_\alpha$, Eq. (27) takes the following form of the Schrödinger equation:

$$-\frac{d^2 \tilde{f}_\alpha}{dz^2} + V(z) \tilde{f}_\alpha = m_\alpha^2 \tilde{f}_\alpha, \quad (29)$$

with the potential

$$\begin{aligned} V(z) &= \frac{p+n}{16(n-1)^2} \left[(p+5n-4) \left(\frac{\mathcal{C}'}{\mathcal{C}} \right)^2 - 4(n-1) \frac{\mathcal{C}''}{\mathcal{C}} \right] \\ &= -\frac{(\Delta_p+1)\tilde{Q}_p^2}{\Delta_p^2} \frac{1}{(1 \pm \frac{\Delta_p+2}{\Delta_p} \tilde{Q}_p |z|)^2} \pm \frac{2\tilde{Q}_p}{\Delta_p} \delta(z), \end{aligned} \quad (30)$$

where the order in the sign \pm is the same as that in Eq. (23). The zero mode solution \tilde{f}_0 (corresponding to the zero KK mass $m_0 = 0$) to the Schrödinger equation satisfying the boundary condition $\tilde{f}'_0(0^+) - \tilde{f}'_0(0^-) = \pm \frac{2\tilde{Q}_p}{\Delta_p} \tilde{f}_0(0)$ is $\tilde{f}_0 \sim (1 \pm \frac{\Delta_p+2}{\Delta_p} \tilde{Q}_p |z|)^{\frac{1}{\Delta_p+2}}$. So, the KK zero mode is independent of z : $f_0(z) = \mathcal{C}^{\frac{p+n}{4(n-1)}} \tilde{f}_0 = \text{constant}$. By calculating the normalization integration $\int dy f_0^2 w(y) = \int dy f_0^2 \mathcal{W}^{-\frac{p+2n-1}{2(n-1)}}$, one can see that the KK zero mode f_0 is normalizable when the sign \pm in the warp factor (21) is chosen so that the warp factor *increases* on both sides of the p -brane, in which case there are curvature singularities at finite non-zero y . With another choice of the sign, even if the warp factor decreases, the graviton KK zero mode is not normalizable.

Recently, it is observed [18, 19] that free massive particles in the bulk of (non-dilatonic) domain wall with exponentially decreasing warp factor of RS are repelled by the domain wall into the extra spatial direction, whereas those in the domain wall with exponentially increasing warp factor are attracted towards the domain wall. This undesirable feature of the RS domain wall calls for need to find mechanism of trapping

massive matter within the domain wall so that matter in our world is not lost into the extra dimension.

Generally, one can show that free massive particles in the bulk of domain wall with decreasing [increasing] warp factor are repelled from [attracted towards] the domain wall, as follows. We consider the following general form of metric with the warp factor $\mathcal{W}(y)$:

$$g_{\mu\nu}dx^\mu dx^\nu = \mathcal{W}(y) \left[-dt^2 + d\mathbf{x} \cdot d\mathbf{x} \right] + dy^2. \quad (31)$$

By contracting the Killing vectors $\partial/\partial t$ and $\partial/\partial x^i$ of the metric (31) with the velocity $U^\mu = dx^\mu/d\lambda$ of a free test particle along its geodesic path $x^\mu(\lambda)$ parameterized by an affine parameter λ , one obtains the following constants of motion for the test particle:

$$\begin{aligned} E &= -g_{\mu\nu} \left(\frac{\partial}{\partial t} \right)^\mu U^\nu = -g_{tt} \frac{dt}{d\lambda} = \mathcal{W}(y) \frac{dt}{d\lambda}, \\ p^i &= g_{\mu\nu} \left(\frac{\partial}{\partial x^i} \right)^\mu U^\nu = g_{x^i x^i} \frac{dx^i}{d\lambda} = \mathcal{W}(y) \frac{dx^i}{d\lambda}, \end{aligned} \quad (32)$$

which can be thought of as the energy and linear momentum for massless particles and the energy and linear momentum per unit mass for massive particles. In addition, metric compatibility for the geodesic motion implies

$$\epsilon = -g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = \mathcal{W} \left[\left(\frac{dt}{d\lambda} \right)^2 - \frac{d\mathbf{x}}{d\lambda} \cdot \frac{d\mathbf{x}}{d\lambda} \right] - \left(\frac{dy}{d\lambda} \right)^2, \quad (33)$$

where $\epsilon = 1, 0$ respectively for a massive and a massless test particle. By making use of Eq. (32), one can bring Eq. (33) into the following form:

$$\epsilon = (E^2 - \mathbf{p} \cdot \mathbf{p}) \mathcal{W}^{-1} - \left(\frac{dy}{d\lambda} \right)^2, \quad (34)$$

where $\mathbf{p} = (p^i)$. Note, the test particles do not feel any force along the \mathbf{x} -direction, because the metric (31) does not depend on \mathbf{x} . So, it is possible to consider the geodesic motion with $d\mathbf{x}/d\lambda = 0$, or one can just move to a frame in which a massive test particle moves along the y -direction by using the boost invariance of the metric along the \mathbf{x} -direction. In this case, the velocity of a massive test particle along the y -direction is given by

$$\frac{dy}{d\lambda} = \pm \sqrt{\frac{E^2}{\mathcal{W}} - 1}. \quad (35)$$

Note, this equation is valid also for the motion of a massive test particle along the y -direction in the spacetime with the metric (20), since the massive test particle does not feel any force along the isometry directions of the n -dimensional compact manifold \mathcal{K}_n if we, for example, take $\mathcal{K}_n = T^n$. For a decreasing warp factor \mathcal{W} , the velocity

$dy/d\lambda$ of a free massive particle ($\epsilon = 1$) along the y -direction increases as it moves away from the domain wall, implying that the domain wall repels a massive particle. In the case of a massless test particle ($\epsilon = 0$), as can be seen from Eq. (34), its motion can be confined within the worldvolume directions (since $E^2 - \mathbf{p} \cdot \mathbf{p} = 0$ for such motion), but it will also be repelled by the domain wall once it has non-zero velocity along the y -direction. For a increasing warp factor, the velocity of a free massive particle ($\epsilon = 1$) along the y -direction decreases as it moves away from the domain wall and then the particle reflects back to the domain wall at the turning point $y = y_0$, given by $\mathcal{W}(y_0) = E^2$, implying that the domain wall attracts the massive particle. However, the massless particle ($\epsilon = 0$) can move along the worldvolume directions, but it will continue to move away from the domain wall with its velocity asymptotically approaching zero, if $\mathcal{W}(y) \rightarrow \infty$ as one moves away from the domain wall, once its velocity has nonzero component along the y -direction. Note, for massless particles it is $dy/d\lambda$ that is changing, whereas the speed of light remains constant.

It is therefore quite problematic for the dilatonic domain wall (4) with decreasing warp factor, because massive test particles will be repelled away from the domain wall and hit the singularity at $|y| = -K^{-1}$. On the other hand, in the bulk background of the delocalized p -brane (20) with increasing warp factor (21), a massive test particle with a finite energy E will always be reflected back to the domain wall before it reaches the singularities, since the warp factor \mathcal{W} is monotonically increasing, approaching infinity as the singularities are reached. So, even if the delocalized p -brane with increasing warp factor suffers from singularities at finite non-zero y , massive matter is trapped within the p -brane by gravitational force and can never reach the singularities. One can think of a dilatonic domain wall as being compactified from a (intersecting) p -brane in higher dimensions. Namely, starting from a dilatonic p -brane solution (8), constructing a dense periodic array of parallel p -branes along n transverse directions to obtain the delocalized p -brane (19), and then dimensionally reducing along the n delocalized transverse directions, one obtains the dilatonic domain wall (2) in $D = p+2$ dimensions with the following dilaton coupling parameter:

$$a^2 = \frac{(p+n)a_p^2}{p} + \frac{4(p+1)^2 n}{p^2(p+n)}. \quad (36)$$

So, if we take the spacetime to be the product of the domain wall spacetime and a compact manifold \mathcal{K}_n , instead of taking it as just the spacetime of the domain wall, we do not face the problem of test particles repelled to the extra spatial direction and hitting naked singularities of the dilatonic domain wall with decreasing warp factor.

From this picture of dilatonic domain walls, we see that the naked singularities of dilatonic domain walls with the decreasing warp factor can be regarded as a consequence of dimensionally reducing a non-BPS p -brane. Namely, for the BPS case, the parameter Q_p of the p -brane solution (8) is positive and its corresponding delocalized

p -brane solution (19), therefore, has positive \tilde{Q}_p . So, the corresponding dimensionally reduced dilatonic domain wall solution (3) will have positive Q , thereby having no naked singularities at finite nonzero x_{D-1} . And the same has to be true for the domain wall solution (4) in different coordinates ². In the case of a non-BPS p -brane, $Q_p < 0$ and therefore $Q < 0$ in the corresponding dilatonic domain wall solution, so there are naked singularities away from the wall.

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²This requirement of the similar singularity structure in different coordinates can be used to fix the sign ambiguity in the coordinate transformation from x_{D-1} to y , which brings the metric (2) to the form (4). This sign ambiguity in the coordinate transformation (resulting from $H^{\frac{2(D-1)}{(D-2)\Delta}} dx_{D-1} = \pm dy$) appears as the ambiguity of the choice of sign \pm in Eq. (6).

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