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TWO-LOOP DIMENSIONAL REDUCTION AND EFFECTIVE POTENTIAL WITHOUT TEMPERATURE EXPANSIONS

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Abstract

In many extensions of the Standard Model, finite temperature computations are complicated by a hierarchy of zero temperature mass scales, in addition to the usual thermal mass scales. We extend the standard thermal resummations to such a situation, and discuss the 2-loop computations of the Higgs effective potential, and an effective 3d field theory for the electroweak phase transition, without carrying out high or low temperature expansions for the heavy masses. We also estimate the accuracy of the temperature expansions previously used for the MSSM electroweak phase transition in the presence of a heavy left-handed stop. We find that the low temperature limit of dealing with the left-handed stop is accurate up to surprisingly high temperatures.

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1 Introduction

Electroweak baryogenesis in the Minimal Supersymmetric Standard Model (MSSM) is a viable option for explaining the matter-antimatter asymmetry observed in the present Universe, provided that there is a mild hierarchy between the right and left-handed stop masses [1]–[6]. The dominantly right-handed stop should be lighter than the top in order to make a strong transition, yet the left-handed stop should be rather heavy, ~ 1 TeV, in order to raise the Higgs mass upper bound to ~ 110 GeV. Various details of the electroweak phase transition in this regime are constantly being investigated [7].

Here we will be concerned with the thermodynamics of the phase transition. In the perturbative approach, this problem is approached by computing the effective potential for the Higgs field to some order in the loop expansion. In general, such a computation in a weakly coupled gauge theory faces two problems: (i) The system has a hierarchy of mass scales $(2\pi T, gT, g^2T)$, which spoils a straightforward perturbative computation. Historically, this was observed by finding large "linear terms" at 2-loop level [8], which were then shown to be absent after an appropriate resummation [9]. (ii) At momenta of the order of the lowest of the mass scales (g^2T), the system is also inherently non-perturbative [10].

The resummations needed at 2-loop level for dealing with the heavy scale $2\pi T$ were discussed in detail by Arnold and Espinosa [11]. However, the problem can also be dealt with in another way, namely by constructing a sequence of effective field theories by integrating out, to a given order in perturbation theory, the scales $2\pi T, gT$ [12, 13]. This construction is highly accurate in the Standard Model [14, 15]. The final theory is three-dimensional (3d), purely bosonic, and contains only the momentum scale g^2T. A perturbative analysis of the 3d theory automatically reproduces the results of the resummed 4d effective potential, but the theory can also be studied efficiently with relatively simple lattice simulations [16], to account for the non-perturbative part.

The problem we consider here is the observation that the hierarchy of mass scales can be even more severe in extensions of the Standard Model such as the MSSM. Indeed, there one tends to have new mass parameters that are not related to the temperature in the same way as m_H is in the Standard Model, where $m_H \sim gT_c$. In particular, as mentioned above, one prefers rather large left-handed squark mass parameters, say $m_Q \sim 1$ TeV. Previously, the effects of m_Q have been considered (on the 2-loop or non-perturbative level) only in the high temperature expansion, or in the extreme limit $m_Q \gg 2\pi T$ where the finite temperature effects decouple completely.

Our objective here is to treat in some detail the general situation $m_Q \sim 2\pi T$. First of all, we discuss how the resummations used previously need to be changed in such a situation (Sec. 3). We then show with a simple example how the full resummed 2-loop effective potential could be computed without any temperature expansions related to m_Q, and how the result can be used for a 2-loop computation of the mass parameter of an effective 3d field theory (Sec. 4). Finally we consider a particular observable sensitive to m_Q, the critical temperature of the electroweak phase transition, and estimate the accuracy of the high and low temperature expansions employed earlier on (Sec. 5). We conclude in Sec. 6 and discuss several possible extensions of the computations presented in this paper. The expressions used for the 1-loop tadpole and bubble, as well as 2-loop sunset graphs are discussed in the appendices.

2 Parametric conventions

In order to be explicit yet concise, we illustrate the situation with a simple model reminiscent of the scalar sector of the MSSM. We take

$$\begin{aligned}\mathcal{L} &= m_H^2 H^\dagger H + m_U^2 U_\alpha^* U_\alpha + m_Q^2 Q_\alpha^\dagger Q_\alpha \\ &+ h_1^2 H^\dagger H U_\alpha^* U_\alpha + h_2^2 H^\dagger Q_\alpha Q_\alpha^\dagger H + h_3 \Big(A H^\dagger Q_\alpha U_\alpha + \text{H.c.} \Big) + \ldots . \end{aligned} \tag{2.1}$$

Here H is an SU(2) doublet, U an SU(3) (anti-)triplet, while Q changes under both groups. We ignore gauge interactions for the moment. We assume that $h_1 \sim h_2 \sim h_3 \sim g$ are small couplings, and $m_H^2, m_U^2 \sim (gT)^2$. It is also important to specify the order of magnitude of the dimensionful parameter A in Eq. (2.1). In this paper we work under the assumption that

$$|\hat{A}|^2 \equiv \frac{|A|^2}{m_Q^2} \sim g^2, \tag{2.2}$$

which simplifies the procedure considerably.

In the imaginary time formalism, the fields in Eq. (2.1) can be divided into Matsubara modes. We assume that the only *light modes* are the zero Matsubara modes of H, U. The non-zero Matsubara modes of H, U have effectively a mass parameter $\geq (2\pi T)^2$. For the field Q, we assume that m_Q itself is large, $m_Q \sim 2\pi T$, so that even the zero Matsubara mode is heavy. If $m_Q \sim gT$, then the zero Matsubara mode of Q is light as well and the procedure is the one described in [17]. If $m_Q \sim 2\pi T/g$, on the other hand, Q can be integrated out at $T = 0$ with exponentially small corrections.

The issue of resummation can now be formulated as follows. Due to the presence of the heavy mass scales, the $n = 0$ modes of H, U can receive radiative corrections as large as the tree-level terms, $\delta m_{H,U}^2 \sim g^2(m_Q^2, T^2)$. Such corrections have to be resummed. In fact, close to the phase transition point, the effective mass parameters $m_{H\text{eff},\text{Ueff}}^2$ can be even smaller, of the non-perturbative magnitude $\sim (g^2 T)^2$. Then resummation has to be extended to the 2-loop level. Non-zero Matsubara modes, or the field Q, on the other hand, do not require resummation [11], since the mass corrections $g^2 T^2, g^2 m_Q^2$ are according to our convention small compared with the tree-level terms.

This statement can be formulated more precisely as follows. Let us write down the effective Lagrangian obtained after integrating out all the heavy modes. The light fields

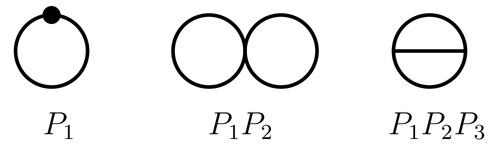


Figure 1: The 2-loop graphs considered. The blob means a counterterm. Different particles are denoted by $P_i = H, U, Q$.

being the $n = 0$ modes of H, U, the form of the Lagrangian is

$$\mathcal{L}_{\rm eff} = m^2_{H\rm eff} H^\dagger H + m^2_{U\rm eff} U^*_\alpha U_\alpha + h^2_{1\rm eff} H^\dagger H U^*_\alpha U_\alpha + \ldots . \tag{2.3}$$

Our aim is now to compute expressions of the form[3]

$$m^2_{H\rm eff} = m^2_H + \#g^2(m^2_H, m^2_U) + \#g^2 m^2_Q(1+\#g^2) + \#g^2T^2(1+\#g^2). \tag{2.4}$$

3 Leading order resummation

In order to carry out the resummation explicitly, let us consider the effective potential of the theory. To illustrate the procedure, it is enough to consider only the field H, keeping the expectation value of U at zero. Introducing $\langle H \rangle = (0\,,\phi)^T/\sqrt{2}$ changes the mass spectrum of the system, $m \to m_\phi = m + \delta_\phi m$. We are interested in a certain range of $\phi \sim 0...T$. Then, in the case of heavy modes, $\delta_\phi m \ll m$, and we can expand in $\delta_\phi m$, while in the case of light modes we cannot. For the purpose of illustration, let us suppress m for the light modes here.

Then, in the standard thermal case, the 1-loop and 2-loop contributions to the effective potential behave at small ϕ as

$$\delta V_{\text{1-loop}} \sim T^2(\delta m_\phi)^2 + T(\delta m_\phi)^3 + ..., \tag{3.1}$$

$$\delta V_{\text{2-loop}} \sim g^2T^3\delta m_\phi + g^2T^2(\delta m_\phi)^2 + \ldots . \tag{3.2}$$

The statement of resummation is now that the dominant 2-loop terms, the "linear" ones $\sim g^2T^3\delta m_\phi$, arise from a badly convergent series which can be resummed into a better convergent one [9]. The way the resummation proceeds is obvious from Eqs. (3.1), (3.2): the non-analytic 1-loop and 2-loop terms combine to

$$T(\delta m_\phi)^3 + g^2T^3\delta m_\phi \to T(g^2T^2 + \delta m^2_\phi)^{3/2}. \tag{3.3}$$

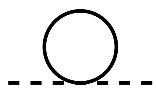
This corresponds simply to the corrections of order g^2T^2 in Eq. (2.4). The extension we make here is that when $m_Q \sim 2\pi T$, the contribution to be resummed goes to a non-trivial function $g^2T^2 f(m_Q/T, |\hat{A}|^2)$.

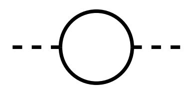
[3] In a gauge theory there are also corrections of order g^3T^2.

In order to proceed systematically, we write the mass parameters related to the light modes as

$$m_H^2 = m_{H\text{eff}}^2 - \delta_r m_H^2, \quad m_U^2 = m_{U\text{eff}}^2 - \delta_r m_U^2, \tag{3.4}$$

where $m_{H\text{eff}}^2, m_{U\text{eff}}^2$ appear in the propagators, and $\delta_r m_H^2, \delta_r m_U^2$ are treated as interactions. Denoting the heavy modes by solid lines and the light modes by dashed lines, the graphs





suggest that

$$\begin{aligned}
\delta_r m_H^2 &= 3h_1^2 I_{n\neq0}(m_U) + 3h_2^2 I(m_Q) + 3h_3^2|\hat{A}|^2\Big[I(m_Q) - I_{n\neq0}(0)\Big], &(3.5)\\
\delta_r m_U^2 &= 2h_1^2 I_{n\neq0}(m_H) + 2h_3^2|\hat{A}|^2\Big[I(m_Q) - I_{n\neq0}(0)\Big]. &(3.6)
\end{aligned}$$

Here $I, I_{n\neq0}$ are tadpole integrals defined in Eqs. (A.1), (A.11), and we have made use of $m_H^2, m_U^2 \ll m_Q^2$. Note that the fact that $|\hat{A}|^2$ is small, Eq. (2.2), implies that wave function corrections need not be considered, since their effect would be of order $\sim h_3^2|\hat{A}|^2 m_H^2 \sim g^4 m_H^2$, beyond Eq. (2.4). For the same reason, we have dropped any m_H^2, m_U^2 dependence in the terms proportional to $|\hat{A}|^2$.

In addition to the mass parameters of the scalar fields, resummation of course also affects the zero components of the gauge fields. In fact, as is well known [8, 9], in the Standard Model the latter effect is more important for physical observables such as the strength of the phase transition, while the former is important particularly for the critical temperature. We do not discuss infrared dominated observables such as the strength of the phase transition, nor gauge fields, to any length in this paper, but let us nevertheless note that the contributions of H, U, Q to the Debye masses of the SU(2) and SU(3) fields A_0, C_0 are, in the presence of $m_Q \sim 2\pi T$,

$$\begin{aligned}
\delta_r m_{A_0}^2 &= g^2 T \frac{d}{dT}\Big(I_{n\neq0}(m_H) + 3I_T(m_Q)\Big), &(3.7)\\
\delta_r m_{C_0}^2 &= g_S^2 T \frac{d}{dT}\Big(I_{n\neq0}(m_U) + 2I_T(m_Q)\Big), &(3.8)
\end{aligned}$$

where g_S is the SU(3) gauge coupling. In addition to these terms, the Debye masses of course contain the usual gauge and fermion contributions.

In order to now show that the procedure introduced in Eqs. (3.4), (3.5), (3.6) is a consistent one, we need to demonstrate that all "linear terms" at 2-loop level cancel, and the remainder is quadratic in δm_ϕ. Recalling that we have set the quartic Higgs self-coupling to zero (at tree-level) for the purpose of simplicity, we get for the shifts in the mass parameters ($Q_{1(2)}$ denote the upper (lower) SU(2) component of Q)

$$\delta_\phi m_{H\text{eff}}^2 = 0, \qquad \delta_\phi m_{U\text{eff}}^2 = \frac{1}{2}h_1^2\phi^2, \tag{3.9}$$

$$\delta_\phi m^2_{Q_1} = 0, \qquad \delta_\phi m^2_{Q_2} = \frac{1}{2} h_2^2 \phi^2. \tag{3.10}$$

Note that due to the assumption $|\hat{A}|^2 \sim g^2$ we can ignore all corrections involving $\hat{A}$ here, since the corresponding 2-loop contributions are at most of order $\sim h_i^2 h_3^2 |\hat{A}|^2 \sim g^6$. We will denote $(m^\phi_{H\text{eff}})^2 = m^2_{H\text{eff}} + \delta_\phi m^2_{H\text{eff}}$, etc.

Linear terms in the effective potential arise from graphs of the types (H), (U), (HU), (HQ), (HUQ) in the notation of Fig. 1. Denoting by $I_{3\text{d}}$ the 3d tadpole in Eq. (A.12) and by H the bosonic sunset integral in Eq. (C.1), we obtain

$$(H) + (U) = -2\delta_r m_H^2 I_{3\text{d}}(m^\phi_{H\text{eff}}) - 3\delta_r m_U^2 I_{3\text{d}}(m^\phi_{U\text{eff}}), \tag{3.11}$$

$$(HU) + (HQ) = 6h_1^2 I(m^\phi_{H\text{eff}}) I(m^\phi_{U\text{eff}}) + 3h_2^2 I(m^\phi_{H\text{eff}}) \Big[I(m^\phi_{Q_1}) + I(m^\phi_{Q_2}) \Big], \tag{3.12}$$

$$(HUQ) = -3h_3^2 |A|^2 \Big[H(m^\phi_{Q1}, m^\phi_{H\text{eff}}, m^\phi_{U\text{eff}}) + H(m^\phi_{Q2}, m^\phi_{H\text{eff}}, m^\phi_{U\text{eff}}) \Big]. \tag{3.13}$$

We then expand these contributions in $\delta_\phi m$. Employing the expansions

$$I(m^\phi_Q) = I(m_Q) - \delta_\phi m_Q^2 D(m_Q) + \mathcal{O}(\delta_\phi m_Q)^4, \tag{3.14}$$

$$I(m^\phi_{\text{eff}}) = I_{n\neq 0}(m_{\text{eff}}) + I_{3\text{d}}(m^\phi_{\text{eff}}) - \delta_\phi m^2_{\text{eff}} D_{n\neq 0}(m_{\text{eff}}) + \mathcal{O}(\delta_\phi m_{\text{eff}})^4, \tag{3.15}$$

$$\begin{aligned} H(m^\phi_Q, m^\phi_{H\text{eff}}, m^\phi_{U\text{eff}}) = \frac{1}{m_Q^2} \Big[& \Big(I_{3\text{d}}(m^\phi_{H\text{eff}}) + I_{3\text{d}}(m^\phi_{U\text{eff}}) \Big) \Big(-I(m_Q) + I_{n\neq 0}(0) \Big) \\ & + I_{3\text{d}}(m^\phi_{H\text{eff}}) I_{3\text{d}}(m^\phi_{U\text{eff}}) \Big] + \mathcal{O}(\delta_\phi m)^2, \end{aligned} \tag{3.16}$$

where $D, D_{n\neq 0}$ are from Eqs. (B.5), (B.10) and we have used Eq. (C.17), we find that:

- there are *linear terms* $\propto I_{3\text{d}}(m^\phi_{H\text{eff}}), I_{3\text{d}}(m^\phi_{U\text{eff}})$ which are however all cancelled, when the choice in Eqs. (3.5), (3.6) is made for $\delta_r m_H^2, \delta_r m_U^2$.

- there is an *infrared* sensitive contribution, quadratic in the masses, from the Matsubara zero modes in the graphs (HU), (HUQ):

$$(HU) + (HUQ)|_{\text{IR}} = 6(h_1^2 - h_3^2|\hat{A}|^2) I_{3\text{d}}(m^\phi_{H\text{eff}}) I_{3\text{d}}(m^\phi_{U\text{eff}}). \tag{3.17}$$

  The appearance of $h_1^2 - h_3^2|\hat{A}|^2$ corresponds to coupling constant resummation which we however do not discuss in any detail here, since the corresponding effects are in principle beyond the accuracy of Eq. (2.4). Similarly, the graph (HUQ) also produces terms of order $\sim h^4|\hat{A}|^2\phi^2$, again beyond Eq. (2.4).

- finally, there are *ultraviolet* sensitive (not from the zero modes) quadratic terms from the graphs (HU), (HQ):

$$\begin{aligned} (HU) + (HQ)|_{\text{UV}} = & -6h_1^2 \delta_\phi m^2_{U\text{eff}} I_{n\neq 0}(m_H) D_{n\neq 0}(m_U) \\ & -3h_2^2 \delta_\phi m^2_{Q_2} I_{n\neq 0}(m_H) D(m_Q) + \mathcal{O}(\delta_\phi m)^4. \end{aligned} \tag{3.18}$$

To summarize, we have observed that the linear terms are cancelled when the thermal counterterms are chosen according to Eq. (3.11). The remainder involves quadratic terms, which can either come from the ultraviolet or the infrared.

4 Next-to-leading order

We next evaluate the 2-loop contributions from the remaining graphs, and expand them again in $\delta_\phi m$; however, these graphs do not involve contributions linear in $\delta_\phi m$. The graphs left are the sunsets $(HQQ), (HUU)$, as well as the 1-loop graphs $(H), (U), (Q)$, where the blobs are now the bilinears obtained from the coupling constant counterterms after the shift of H [4].

After an expansion in $\delta_\phi m$, we obtain

$$
\begin{aligned}
(H)+(U)+(Q) &= \frac{\phi^2}{2}\frac{1}{(4\pi)^2}\frac{1}{\epsilon}\Big[9(h_1^4+h_2^4)I_{n\neq 0}(m_H)+6h_1^4 I_{n\neq 0}(m_U) \\
&\quad +12h_2^4 I(m_Q)\Big]+\mathcal{O}(\delta_\phi m)^3, &(4.1)\\
(HQQ) &= -3h_2^4\phi^2 H(m_Q,m_Q,0)+\mathcal{O}(\delta_\phi m)^3, &(4.2)\\
(HUU) &= -\frac{3}{2}h_1^4\phi^2 H(m^\phi_{H\mathrm{eff}},m^\phi_{U\mathrm{eff}},m^\phi_{U\mathrm{eff}}). &(4.3)
\end{aligned}
$$

The numerical expression of $H(m_Q,m_Q,0)$ is discussed in appendix C. As to the graph (HUU), on the other hand, we recall that it arises completely from the zero Matsubara modes ($H=H_{3\mathrm{d}}+\mathcal{O}(m/T)$), and is thus a purely IR quantity [21].

Adding all terms together from Eqs. (3.18), (4.1), (4.2) and using Eqs. (A.11), (B.5), (B.10), (C.20), we obtain the 2-loop ultraviolet contribution to the 3d mass parameter,

$$
\begin{aligned}
\delta^{\mathrm{UV}}_{\text{2-loop}} m^2_{H\mathrm{eff}} = & h_2^4\Big[-6H_{\mathrm{vac}}(m_Q,m_Q,0)+12\frac{1}{(4\pi)^2\epsilon}I_{\mathrm{vac}}(m_Q)\Big] \\
& +\frac{T^2}{(4\pi)^2}\Big\{h_1^4\Big[\frac{3}{4}\frac{1}{\epsilon}-\frac{5}{4}\ln\frac{\bar\mu^2}{\bar\mu_T^2}-3\Big(\ln\frac{3T}{\bar\mu}+c\Big)\Big] \\
& -h_2^4\Big[\frac{3}{4}\ln\frac{\bar\mu^2}{m_Q^2}+6\mathcal{I}_1\Big(\frac{m_Q}{T}\Big)\Big(\ln\frac{\bar\mu^2}{m_Q^2}+2\Big)+\frac{1}{4}\mathcal{D}\Big(\frac{m_Q}{T}\Big)+6\mathcal{H}\Big(\frac{m_Q}{T}\Big)\Big]\Big\}. \quad (4.4)
\end{aligned}
$$

Here the first line is a 2-loop vacuum renormalization correction of order $g^4 m_Q^2$,

$$
\bar\mu_T = 4\pi e^{-\gamma_E}T \approx 7.0555T, \qquad c=\frac{1}{2}\Big[\ln\frac{8\pi}{9}+\frac{\zeta'(2)}{\zeta(2)}-2\gamma_E\Big]\approx -0.34872274, \quad (4.5)
$$

and $\mathcal{I}_1$, $\mathcal{D}$, $\mathcal{H}$ are functions defined in Eqs. (A.7), (B.8), (C.21). The IR sensitive part of the effective potential is, from Eqs. (3.17), (4.3),

$$
\delta^{\mathrm{IR}}_{\text{2-loop}} V = 6(h_1^2-h_3^2|\hat A|^2)I_{3\mathrm{d}}(m^\phi_{H\mathrm{eff}})I_{3\mathrm{d}}(m^\phi_{U\mathrm{eff}})-\frac{3}{2}h_1^4\phi^2 H_{3\mathrm{d}}(m^\phi_{H\mathrm{eff}},m^\phi_{U\mathrm{eff}},m^\phi_{U\mathrm{eff}}). \quad (4.6)
$$

[4]Mass counterterms do not contribute at the present order; terms proportional to m_Q^2 in them would, had we included self-interactions of the type $\sim (H^\dagger H)^2, (U^*_\alpha U_\alpha)^2$ in Eq. (2.1).

The divergence in $H_{3\rm d}$ (Eq. (C.2)) cancels against that from Eq. (4.4), $m^2_{H\rm eff}\phi^2/2$.

Including also the 1-loop terms in Eq. (3.5), we can now write down the complete mass parameter $m^2_{H\rm eff}$ with accuracy $g^4 m_Q^2, g^4T^2$. In order to do so, let us first note that 1-loop radiative corrections generate couplings other than those in Eq. (2.1), viz.

$$\delta\mathcal{L} = h_4^2 H^\dagger H Q^\dagger_\alpha Q_\alpha + \lambda(H^\dagger H)^2 + ..., \tag{4.7}$$

which we have to include in the discussion for a moment. The corresponding contribution in Eq. (3.5) is $\delta_r m_H^2 = 6h_4^2 I(m_Q) + 6\lambda I_{n\neq 0}(m_H^2)$. Furthermore, in order to cancel spurious $\bar\mu$-dependences, we should express the $\overline{\rm MS}$ parameters in terms of physical observables as in [12]. In this paper we will not consider actual physical pole masses etc, but simply some finite physical scale independent parameters $()_{\rm phys}$ which, dropping all terms beyond the accuracy of Eq. (2.4), we define through the following relations:

$$\begin{aligned}
m_H^2(\bar\mu) &= m^2_{H\rm phys} + \frac{3}{(4\pi)^2}\Big[h_1^2 m_U^2 + (h^2_{2\rm phys} + h_3^2|\hat A|^2)m_Q^2\Big]\Big(\ln\frac{\bar\mu^2}{m_Q^2} + 1\Big) \\
&\quad + h_2^4\Big[6H_{\rm vac}(m_Q,m_Q,0) - 12 D_{\rm vac}(m_Q)I_{\rm vac}(m_Q)\Big]_{\text{finite part}}, && (4.8)\\
h_1^2(\bar\mu) &= h^2_{1\rm phys} + h_1^4\frac{2}{(4\pi)^2}\ln\frac{\bar\mu^2}{m_Q^2}, \quad h_2^2(\bar\mu) = h^2_{2\rm phys} + h_2^4\frac{2}{(4\pi)^2}\ln\frac{\bar\mu^2}{m_Q^2}, && (4.9)\\
h_4^2(\bar\mu) &= h^2_{4\rm phys} + h_2^4\frac{1}{(4\pi)^2}\ln\frac{\bar\mu^2}{m_Q^2}, \quad \lambda(\bar\mu) = \lambda_{\rm phys} + \frac{3}{2}(h_1^4 + h_2^4)\frac{1}{(4\pi)^2}\ln\frac{\bar\mu^2}{m_Q^2}. && (4.10)
\end{aligned}$$

Moreover, let us now declare $h^2_{4\rm phys}, \lambda_{\rm phys} \sim 0$. We then obtain the final expression for the effective (bare) mass parameter $m^2_{H\rm eff}$ in the theory of Eq. (2.1):

$$\begin{aligned}
m^2_{H\rm eff} &= m^2_{H\rm phys} - \frac{3}{(4\pi)^2}h_1^2 m_U^2\Big(\ln\frac{m_Q^2}{\bar\mu_T^2} - 1\Big) \\
&+ T^2\bigg\{\frac{1}{4}(h^2_{1\rm phys} - h_3^2|\hat A|^2) + \frac{3}{2}(h^2_{2\rm phys} + h_3^2|\hat A|^2)\mathcal{I}_1\Big(\frac{m_Q}{T}\Big) \\
&+ \frac{1}{(4\pi)^2}\bigg[h_1^4\Big(\frac{5}{4}\ln\frac{\bar\mu_T^2}{m_Q^2} + \frac{3}{4}\frac{1}{\epsilon} - 3\Big(\ln\frac{3T}{\bar\mu} + c\Big)\Big) \\
&\quad - h_2^4\Big(12\mathcal{I}_1\Big(\frac{m_Q}{T}\Big) + \frac{1}{4}\mathcal{D}\Big(\frac{m_Q}{T}\Big) + 6\mathcal{H}\Big(\frac{m_Q}{T}\Big)\Big)\bigg]\bigg\}. \qquad (4.11)
\end{aligned}$$

5 High-T and low-T expansions

We now wish to employ Eq. (4.11) to estimate in a non-trivial physical context the accuracy of the high and low temperature expansions in m_Q/T. We can do this by inspecting the critical temperature T_c of the phase transition. Let us recall that the leading (and next-to-leading in a gauge theory) terms in T_c are perturbative [18],

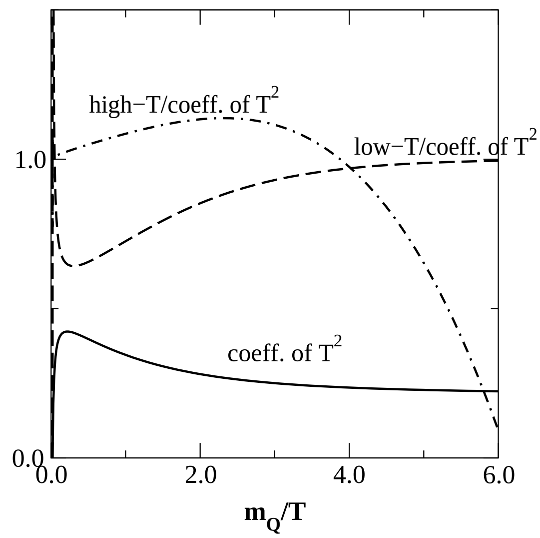


Figure 2: The coefficient of T^2 from Eq. (4.11), compared with the high-T and low-T limits. The parameters have been chosen as explained in Sec. 5.

thus ultraviolet dominated and particularly sensitive to m_Q/T. Most of the physical characteristics of the phase transition, on the contrary, are infrared dominated and less sensitive to m_Q/T. We may also remind that in the MSSM the determination of T_c is physically more important than in the Standard Model, since one has to address the question of whether other phase transitions could take place before the electroweak one, in particular a transition to the dangerous U-direction [1]–[6].

The transition will take place when $m_{H\text{eff}}^2(\bar{\mu} = g^2T) = \# g^4T^2$, where # is some non-perturbative coefficient, to be determined with lattice simulations. We shall keep the physical parameters $()_{\text{phys}}$ fixed and vary m_Q/T. It is then clear that the perturbative contribution to T_c can equivalently be inspected by considering the finite part of the coefficient of T^2 in Eq. (4.11). We choose for simplicity $h_{1\text{phys}}, h_{2\text{phys}} \sim 1$, $h_3^2|\hat{A}|^2 \sim 0$. The high and low temperature limits of $\mathcal{I}_1, \mathcal{D}, \mathcal{H}$ are given in Eqs. (A.9), (B.9), (C.22). To be in accordance with the limiting procedures usually applied in the literature, we keep in the high temperature expansions terms up to logarithmic order, whereas in the low temperature expansions we simply replace the exponentially small corrections in Eqs. (A.9), (B.9), (C.22) with zero.

The numerically evaluated full expression for the coefficient of T^2, as well as a comparison of the high and low temperature expanded versions thereof with the full result, are plotted in Fig. 2. We observe that the high temperature expansion gives typically too large a coefficient of the T^2-term, leading to too small a T_c [5]. With the

[5]Numerically the relative effect is larger here than in the realistic MSSM, since in that case there are other terms in the coefficient of T^2 (such as gauge bosons) for which the high temperature expansion should work perfectly.

low-temperature expansion, on the other hand, T_c is slightly too large. Furthermore, we observe that while naively one might have expected the crossover between the high and low temperature regimes to be close to the first non-zero Matsubara frequency at $m_Q \sim 2\pi T$, the low temperature expansion is in fact perfectly sufficient already at $m_Q \gtrsim 3T$, which is the case for realistic values $m_Q \gtrsim 300$ GeV. The fact that the high temperature expansion converges relatively poorly as early as at $m_Q/T \sim 2$ is due particularly to the 2-loop function $\mathcal{H}$, whose behaviour is shown in Fig. 4.

6 Conclusions

In this paper, we have pointed out that standard thermal resummations should be extended in two different ways, when one goes from the Standard Model to a general MSSM. First of all, the left-handed stop m_Q is typically of the order of magnitude $\sim 2\pi T$. Then it cannot, a priori, be treated either in the high or in the low temperature expansion, but a more general function appears. Second, the presence of dimensionful trilinear couplings leads to the emergence of new "linear" terms coming from the scalar sunset diagrams. The results for the scalar thermal counterterms including both of these effects in the model of Eq. (2.1) are shown in Eqs. (3.5), (3.6), while the scalar contributions to the Debye masses are shown in Eqs. (3.7), (3.8). In an effective theory approach such as the one followed in [19] (in [19] it was assumed that $m_Q \gg 2\pi T$, but the procedure can be extended to $m_Q \sim 2\pi T$ in a straightforward way), all these effects of course arise automatically, whereas in a direct computation of the 2-loop effective potential they should be explicitly taken into account.

In the framework of effective field theories, we have also extended the resummation for the Higgs mass parameter to the next order beyond the effects described above. The mass parameter thus determined, including corrections of order $\sim g^4T^2$, could be used for a precise estimation of the critical temperature of the corresponding electroweak phase transition using 3d lattice Monte Carlo simulations. Let us stress that the only change with respect to previous effective 3d theories is in the expressions for the effective parameters, not in the functional form of the theory.

Using these results, we have estimated the accuracy of the high and low temperature expansions used previously in the literature. Inspection of the critical temperature suggests that the low temperature expansion, whereby all finite temperature contributions from heavy particles are simply left out, works well already at $m \gtrsim 3T$ for bosonic particles. Thus it should be completely clear that for the values $m_Q \sim 1$ TeV of interest for obtaining a strong phase transition with experimentally allowed Higgs masses in the MSSM, the Q-field can simply be left out in all finite temperature contributions.

The present results could clearly be extended in many directions. First of all, the restricted model we have employed here can be extended to the full MSSM with gauge fields and fermions in a straightforward way. Second, we have shown that the evaluation of the integrals appearing in the perturbative 2-loop effective potential is numerically feasible without any further temperature expansions — thus the complete 2-loop potential of the MSSM could in principle be computed, extending thus the results of [1, 2], [4]–[6], [20]. Third, we have here considered explicitly only the effects of a heavy m_Q, while in the MSSM many other mass parameters could be heavy as well. In particular, M_2, μ related to the gaugino and Higgsino mass matrices and also relevant for providing sources of CP violation can have values for which neither high nor low temperature expansions are applicable. In the effective theory approach M_2, μ can be easily included at 1-loop level without any temperature expansions [19], but this could now be extended to the 2-loop level. The accuracy of previous approximations with respect to the contributions from the second Higgs doublet with $m_A \gtrsim 100$ GeV could also be explicitly checked. Finally, we assumed that the trilinear couplings are not exceedingly large, $|\hat{A}|^2 \lesssim g^2$, an assumption which could be relaxed.

We believe that as long as the existence of a Higgs particle with MSSM type couplings lighter than about 110 GeV is not experimentally excluded, these are worthwhile questions to consider.

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A The tadpole

For completeness, let us review here some properties of the tadpole integral,

$$I(m) = T\sum_n \int \frac{d^{3-2\epsilon}p}{(2\pi)^{3-2\epsilon}} \frac{1}{p_0^2 + p^2 + m^2}, \tag{A.1}$$

where $p_0 = p_b \equiv 2\pi nT$ for bosons, $p_0 = p_f \equiv \pi T(2n+1)$ for fermions. We will need two types of subdivisions of $I(m)$. In the first case, relevant for all fermions and heavy bosons, we write $I(m) = I_{\rm vac}(m) + I_T(m)$, where I_T vanishes at $T = 0$. In the second case, relevant for light bosons ($m^2 \sim (gT)^2$), we separate the contribution from the Matsubara zero mode into $I_{\rm 3d}(m)$, writing $I(m) = I_{\rm 3d}(m) + I_{n\neq 0}(m)$. We denote

$$n_b(\omega) = \frac{1}{e^{\beta\omega} - 1}, \quad n_f(\omega) = \frac{1}{e^{\beta\omega} + 1}, \tag{A.2}$$

$$\omega_{p,i} = (p^2 + m_i^2)^{1/2}, \quad \hat{\omega}_p = (p^2 + (m/T)^2)^{1/2}, \tag{A.3}$$

$$I_{T,b(f)}(m_i) = \int \frac{d^{3-2\epsilon}p}{(2\pi)^{3-2\epsilon}} \frac{n_{b(f)}(\omega_{p,i})}{\omega_{p,i}}. \tag{A.4}$$

When the superscript $()_{b,f}$ is left out from I, we assume the bosonic case.

A heavy mass in the loop. Writing $I_b(m) = I_{\rm vac}(m) + I_T(m)$, we get

$$I_{\rm vac}(m) = -\mu^{-2\epsilon}\frac{m^2}{(4\pi)^2}\Big(\frac{1}{\epsilon}+\ln\frac{\bar\mu^2}{m^2}+1\Big), \tag{A.5}$$

$$I_T(m) = \frac{1}{2}\mu^{-2\epsilon}T^2\Big\{\Big[1+\epsilon\Big(2-2\ln 2+\ln\frac{\bar\mu^2}{T^2}\Big)\Big]\mathcal{I}_1\Big(\frac{m}{T}\Big)-\epsilon\mathcal{I}_2\Big(\frac{m}{T}\Big)\Big\}, \tag{A.6}$$

$$\mathcal{I}_1\Big(\frac{m}{T}\Big) = \frac{1}{\pi^2}\int_0^\infty dp\, p^2\frac{n_b(\hat\omega_p)}{\hat\omega_p}, \tag{A.7}$$

$$\mathcal{I}_2\Big(\frac{m}{T}\Big) = \frac{1}{\pi^2}\int_0^\infty dp\, p^2\ln p^2\frac{n_b(\hat\omega_p)}{\hat\omega_p}. \tag{A.8}$$

The limiting values are

$$\mathcal{I}_1(y) \overset{y\lll 1}{=} \frac{1}{6}-\frac{y}{2\pi}+\frac{y^2}{8\pi^2}\Big(1+2\ln\frac{4\pi}{y}-2\gamma_E\Big), \qquad \overset{y\ggg 1}{=} \sqrt{\frac{y}{2\pi}}\frac{e^{-y}}{\pi}, \tag{A.9}$$

$$\mathcal{I}_2(y) \overset{y\lll 1}{=} \frac{1}{3}\Big[1-\gamma_E+\frac{\zeta'(2)}{\zeta(2)}\Big]-\frac{y}{\pi}\ln y, \quad \overset{y\ggg 1}{=} \sqrt{\frac{y}{2\pi}}\frac{e^{-y}}{\pi}\Big(\ln y+2-\gamma_E-\ln 2\Big), \tag{A.10}$$

where $\gamma_E = 0.57721566, \zeta'(2)/\zeta(2) = -0.56996099$.

A light mass in the loop. Writing $I(m) = I_{n\neq 0}(m) + I_{\rm 3d}(m)$, we obtain

$$I_{n\neq 0}(m) = \mu^{-2\epsilon}\frac{T^2}{12}(1+\epsilon\imath_\epsilon)-\mu^{-2\epsilon}\frac{m^2}{(4\pi)^2}\Big(\frac{1}{\epsilon}+\ln\frac{\bar\mu^2}{\bar\mu_T^2}\Big)+\mathcal{O}(\epsilon^2,\epsilon m^2,m^4), \tag{A.11}$$

$$I_{\rm 3d}(m) = -\mu^{-2\epsilon}\frac{mT}{4\pi}\Big[1+\epsilon\Big(\ln\frac{\bar\mu^2}{m^2}+2-2\ln 2\Big)\Big]+\mathcal{O}(\epsilon^2). \tag{A.12}$$

Here $\bar\mu_T$ is from Eq. (4.5) and [11]

$$\imath_\epsilon = \ln\frac{\bar\mu^2}{T^2}+2\gamma_E-2\ln 2-2\frac{\zeta'(2)}{\zeta(2)}. \tag{A.13}$$

The fermionic tadpole. Using the standard trick, the fermionic tadpole can be expressed in terms of the bosonic tadpole:

$$I_f(m) = I_{\rm vac}(m)+I_{T,f}(m), \tag{A.14}$$

$$I_{T,f}(m) = 2I_{T/2,b}(m)-I_{T,b}(m) = 2^{-1+2\epsilon}I_{T,b}(2m)-I_{T,b}(m). \tag{A.15}$$

Defining $\mathcal{I}_{1,f},\mathcal{I}_{2,f}$ as in Eqs. (A.7), (A.8) but with n_f instead of n_b, we obtain

$$I_{T,f}(m) = \frac{1}{2}\mu^{-2\epsilon}T^2\Big\{-\Big[1+\epsilon\Big(2-2\ln 2+\ln\frac{\bar\mu^2}{T^2}\Big)\Big]\mathcal{I}_{1,f}\Big(\frac{m}{T}\Big)+\epsilon\mathcal{I}_{2,f}\Big(\frac{m}{T}\Big)\Big\}. \tag{A.16}$$

The high and low temperature expansions of $\mathcal{I}_{1,f}, \mathcal{I}_{2,f}$ can be obtained from those of $\mathcal{I}_1, \mathcal{I}_2$ by noting that

$$\mathcal{I}_{1,f}\Big(\frac{m}{T}\Big) = \mathcal{I}_1\Big(\frac{m}{T}\Big) - \frac{1}{2}\mathcal{I}_1\Big(\frac{2m}{T}\Big), \tag{A.17}$$

$$\mathcal{I}_{2,f}\Big(\frac{m}{T}\Big) = \mathcal{I}_2\Big(\frac{m}{T}\Big) - \frac{1}{2}\mathcal{I}_2\Big(\frac{2m}{T}\Big) + \ln 2\, \mathcal{I}_1\Big(\frac{2m}{T}\Big). \tag{A.18}$$

B The bubble

Let us then consider the 1-loop "bubble" diagram with two propagators,

$$D_{b(f)}(m_1, m_2) = \sum\!\!\!\!\!\!\!\int_{P_{b(f)}} \frac{1}{P^2 + m_1^2} \frac{1}{P^2 + m_2^2} = \frac{1}{m_1^2 - m_2^2}\Big[I_{b(f)}(m_2) - I_{b(f)}(m_1)\Big], \tag{B.1}$$

where $P_{b(f)} = (p_{b(f)}, \mathbf{p})$ and we have taken the external momentum to zero. We can again write

$$D_{b(f)}(m_1, m_2) = D_{\rm vac}(m_1, m_2) + D_{T,b(f)}(m_1, m_2), \tag{B.2}$$

$$D_{\rm vac}(m_1, m_2) = \frac{\mu^{-2\epsilon}}{(4\pi^2)}\Big(\frac{1}{\epsilon} + \ln\frac{\bar\mu^2}{m_1 m_2} + 1 - \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2}\ln\frac{m_1}{m_2}\Big), \tag{B.3}$$

$$D_{T,b(f)}(m_1, m_2) = \underset{(-)}{+}\, \mu^{-2\epsilon}\frac{T^2}{2}\frac{1}{m_1^2 - m_2^2}\Big[\mathcal{I}_{1,b(f)}\Big(\frac{m_2}{T}\Big) - \mathcal{I}_{1,b(f)}\Big(\frac{m_1}{T}\Big)\Big] + \mathcal{O}(\epsilon). \tag{B.4}$$

The special case $m_1 = m_2$ gives the derivative of $I(m)$ with respect to m^2:

$$D(m) \equiv -\frac{dI(m)}{dm^2} = D_{\rm vac}(m) + D_T(m), \tag{B.5}$$

$$D_{\rm vac}(m) = \frac{\mu^{-2\epsilon}}{(4\pi)^2}\Big(\frac{1}{\epsilon} + \ln\frac{\bar\mu^2}{m^2}\Big), \tag{B.6}$$

$$D_T(m) = \frac{\mu^{-2\epsilon}}{(4\pi)^2}\mathcal{D}\Big(\frac{m}{T}\Big) + \mathcal{O}(\epsilon), \tag{B.7}$$

$$\mathcal{D}\Big(\frac{m}{T}\Big) = 4\int_0^\infty dp \frac{n_b(\hat\omega_p)}{\hat\omega_p}, \tag{B.8}$$

with

$$\mathcal{D}(y) \overset{y \lll 1}{=} \frac{2\pi}{y} + 2\ln\frac{y}{4\pi} + 2\gamma_E, \qquad \overset{y \ggg 1}{=} 2\sqrt{\frac{2\pi}{y}}e^{-y}. \tag{B.9}$$

We also need the derivative of $I_{n\neq 0}(m)$ with respect to m^2:

$$D_{n\neq 0}(m) \equiv -\frac{dI_{n\neq 0}(m)}{dm^2} = \frac{\mu^{-2\epsilon}}{(4\pi)^2}\Big(\frac{1}{\epsilon} + \ln\frac{\bar\mu^2}{\bar\mu_T^2}\Big) + \mathcal{O}(\epsilon, m^2). \tag{B.10}$$

C The sunset

Let us then consider the bosonic and fermionic 2-loop sunset diagrams

$$H_{b(f)}(m_1,m_2,m_3) \;=\; \SumInt_{P_{b(f)}} \SumInt_{Q_{b(f)}} \frac{1}{P^2+m_1^2}\frac{1}{Q^2+m_2^2}\frac{1}{(P+Q)^2+m_3^2}. \qquad \text{(C.1)}$$

In the limit $m_i/T \ll 1$, it is known that [11, 21]

$$H_b(m_1,m_2,m_3) \;=\; \mu^{-4\epsilon}\frac{T^2}{(4\pi)^2}\left(\frac{1}{4\epsilon}+\ln\frac{\bar\mu}{m_1+m_2+m_3}+\frac{1}{2}\right)+\mathcal{O}(m_iT), \qquad \text{(C.2)}$$

$$H_f(m_1,m_2,m_3) \;=\; \mathcal{O}(m_iT). \qquad \text{(C.3)}$$

Our objective here is to compute these diagrams in the case of general m_i. We are aware of previous results in this direction in [22, 23].

General case. The method we employ for evaluating H_b, H_f follows the standard procedure (see, e.g., [22, 23, 24]). The twofold sum over the Matsubara modes is first written as a threefold sum with a Kronecker delta, and the delta is then written as $\delta(p_0) = T\int_0^\beta dx\,\exp(ip_0x)$. The sums can now be performed,

$$T\sum_{p_{b(f)}} \frac{e^{ip_{b(f)}x}}{p_{b(f)}^2+\omega_i^2} = \frac{n_{b(f)}(\omega_i)}{2\omega_i}\Big[e^{(\beta-x)\omega_i} \underset{(-)}{+} e^{x\omega_i}\Big]. \qquad \text{(C.4)}$$

The integral over x is then very simple. The outcome can be organized in a transparent form, when different types of contributions are identified with known expressions; the same result could also have been obtained from the rules of the real time formalism, as noted for the 3-loop bosonic basketball diagram in [25]. In the remaining integral over the spatial vectors $\mathbf{p},\mathbf{q}$, we can perform at least the integration over $z=\mathbf{p}\cdot\mathbf{q}/(|\mathbf{p}||\mathbf{q}|)$, leaving for numerics at most a rapidly convergent 2d integral over $p\equiv|\mathbf{p}|$, $q\equiv|\mathbf{q}|$.

Let us denote

$$\Pi(Q^2;m_1^2,m_2^2) \;=\; \int\frac{d^{4-2\epsilon}P}{(2\pi)^{4-2\epsilon}}\frac{1}{[P^2+m_1^2][(P+Q)^2+m_2^2]}, \qquad \text{(C.5)}$$

$$f_{p,q}(m_1,m_2;m_3) \;=\; \ln\left|\frac{4(p^2+m_1^2)(q^2+m_2^2)-(m_1^2+m_2^2-m_3^2-2pq)^2}{4(p^2+m_1^2)(q^2+m_2^2)-(m_1^2+m_2^2-m_3^2+2pq)^2}\right|. \qquad \text{(C.6)}$$

The explicit expression for $\Pi(Q^2;m_1^2,m_2^2)$, often denoted by B_0, is well known:

$$\Pi(Q^2;m_1^2,m_2^2) = \frac{\mu^{-2\epsilon}}{(4\pi)^2}\left[\frac{1}{\epsilon}+\ln\frac{\bar\mu^2}{m_1m_2}+1-\frac{m_1^2+m_2^2}{m_1^2-m_2^2}\ln\frac{m_1}{m_2}+F_E(Q^2;m_1^2,m_2^2)\right], \qquad \text{(C.7)}$$

$$F_E(Q^2;m_1^2,m_2^2) = 1+\frac{m_1^2+m_2^2}{m_1^2-m_2^2}\ln\frac{m_1}{m_2}+\frac{m_1^2-m_2^2}{Q^2}\ln\frac{m_1}{m_2}$$
$$+\frac{1}{Q^2}\sqrt{(m_1+m_2)^2+Q^2}\sqrt{(m_1-m_2)^2+Q^2}\,\ln\frac{1-\sqrt{\frac{(m_1-m_2)^2+Q^2}{(m_1+m_2)^2+Q^2}}}{1+\sqrt{\frac{(m_1-m_2)^2+Q^2}{(m_1+m_2)^2+Q^2}}}. \qquad \text{(C.8)}$$

The absolute value inside the logarithm in $f_{p,q}$ in Eq. (C.6) means that we take the real part of the expression; the imaginary part would anyway cancel against $\mathrm{Im}\,\Pi$.

With this notation, we obtain

$$\begin{aligned} H_b(m_1,m_2,m_3) &= H_{\rm vac}(m_1,m_2,m_3) \\ &+ \sum_{i\neq j\neq k} I_{T,b}(m_i)\mathrm{Re}\,\Pi(-m_i^2;m_j^2,m_k^2) \\ &+ \sum_{i\neq j\neq k} \frac{\mu^{-4\epsilon}}{32\pi^4}\int_0^\infty dp\,p\int_0^\infty dq\,q\frac{n_b(\omega_{p,i})}{\omega_{p,i}}\frac{n_b(\omega_{q,j})}{\omega_{q,j}}f_{p,q}(m_i,m_j;m_k), \end{aligned} \tag{C.9}$$

$$\begin{aligned} H_f(m_1,m_2,m_3) &= H_{\rm vac}(m_1,m_2,m_3) \\ &+ I_{T,b}(m_3)\mathrm{Re}\,\Pi(-m_3^2;m_1^2,m_2^2) - \sum_{i\neq j} I_{T,f}(m_i)\mathrm{Re}\,\Pi(-m_i^2;m_j^2,m_3^2) \\ &+ \frac{\mu^{-4\epsilon}}{32\pi^4}\int_0^\infty dp\,p\int_0^\infty dq\,q\frac{n_f(\omega_{p,1})}{\omega_{p,1}}\frac{n_f(\omega_{q,2})}{\omega_{q,2}}f_{p,q}(m_1,m_2;m_3) \\ &- \sum_{i\neq j} \frac{\mu^{-4\epsilon}}{32\pi^4}\int_0^\infty dp\,p\int_0^\infty dq\,q\frac{n_b(\omega_{p,3})}{\omega_{p,3}}\frac{n_f(\omega_{q,i})}{\omega_{q,i}}f_{p,q}(m_3,m_i;m_j), \end{aligned} \tag{C.10}$$

where $\sum_{i\neq j\neq k} \equiv \sum_{(i,j,k)=(1,2,3),(2,3,1),(3,1,2)}$, $\sum_{i\neq j} \equiv \sum_{(i,j)=(1,2),(2,1)}$, and the zero temperature contribution is

$$H_{\rm vac}(m_1,m_2,m_3) = \int \frac{d^{4-2\epsilon}P}{(2\pi)^{4-2\epsilon}}\int \frac{d^{4-2\epsilon}Q}{(2\pi)^{4-2\epsilon}}\frac{1}{P^2+m_1^2}\frac{1}{Q^2+m_2^2}\frac{1}{(P+Q)^2+m_3^2}. \tag{C.11}$$

The only ultraviolet divergences are in $H_{\rm vac}(m_1,m_2,m_3)$, which has $1/\epsilon^2, 1/\epsilon$ poles, and in the Π's, which have the pole $\mu^{-2\epsilon}/(16\pi^2\epsilon)$, cf. Eq. (C.7).

Let us also mention a few words about the numerical evaluation of the 2d integrals involving $f_{p,q}(m_1,m_2;m_3)$, left to be carried out in Eqs. (C.9), (C.10). These integrals are of course well-defined and finite. However, if $m_3 < |m_1-m_2|$ or $m_3 > m_1+m_2$, they involve integration over logarithmic singularities. In our numerics, we found that the integration is more effective if we factorise out the singularities explicitly. Suppose we, for instance, first perform the integral over q. If $m_1 \neq 0$, we write

$$a = \frac{p}{2m_1^2}(m_3^2-m_1^2-m_2^2), \tag{C.12}$$

$$b = \frac{1}{2m_1^2}(p^2+m_1^2)^{1/2}\Big(m_3^4-2m_3^2(m_1^2+m_2^2)+(m_1^2-m_2^2)^2\Big)^{1/2}, \tag{C.13}$$

$$f_{p,q}(m_1,m_2;m_3) = \ln\left|\frac{(q-a-b)(q+b-a)}{(q+a+b)(q+a-b)}\right|, \tag{C.14}$$

and there are then singularities at $q=|a+b|,|a-b|$. If $m_1=0$, we write

$$q_0 = \frac{4m_2^2p^2-(m_3^2-m_2^2)^2}{4p(m_3^2-m_2^2)}, \quad f_{p,q}(0,m_2;m_3) = \ln\left|\frac{q-q_0}{q+q_0}\right|, \tag{C.15}$$

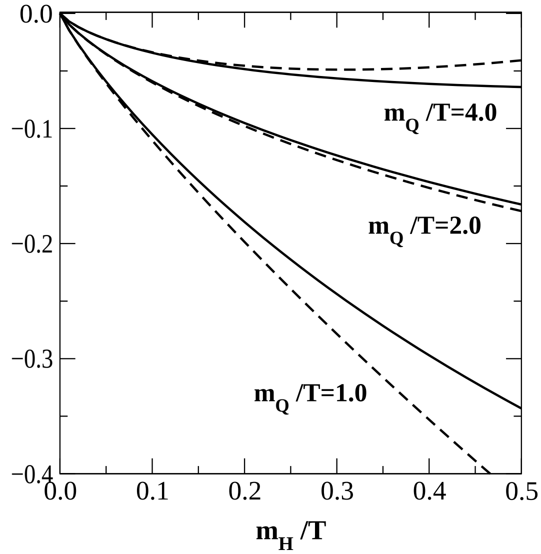

Figure 3: The finite part of $H_b(m_H, m_Q, 0) - H_b(0, m_Q, 0)$ (the zero temperature contribution has been subtracted) with solid lines, compared with the "linear term" in Eq. (C.17) with dashed lines. The results have been divided by $T^2/(4\pi)^2$.

and there is a singularity at $q = |q_0|$. If $m_2 = m_3$, $f_{p,q}(0, m; m) = 0$.

One heavy, one light mass. Let us now consider in more detail some special cases of $H_b(m_1, m_2, m_3)$ needed in the main part of this paper. For the consideration in Sec. 3, we need to know how $H_b(m_H, m_U, m_Q)$ behaves for small m_H, m_U. We claim that there is a linear term $\propto m_H, m_U$ (modulo logarithms). Since the result is symmetric in m_H, m_U, it is enough to consider $m_H \ll m_Q$.

Non-analytic terms can only arise from a zero Matsubara mode. Thus,

$$H_b \overset{m_H \ll m_Q}{\sim} \int \frac{d^{3-2\epsilon}p}{(2\pi)^{3-2\epsilon}} \frac{1}{p^2 + m_H^2} \sum\!\!\!\!\!\!\!\int_Q \frac{1}{q_0^2 + (p+q)^2 + m_U^2} \frac{1}{q_0^2 + q^2 + m_Q^2}. \tag{C.16}$$

Let us denote the latter integral by $\Pi(p)$. It is then obvious that the leading behaviour must be

$$\begin{aligned} H_b \overset{m_H \ll m_Q}{\sim} & \int \frac{d^{3-2\epsilon}p}{(2\pi)^{3-2\epsilon}} \frac{1}{p^2 + m_H^2} \Pi(0) = I_{\rm 3d}(m_H) \frac{1}{m_Q^2 - m_U^2} \Big[I(m_U) - I(m_Q) \Big] \\ = & \frac{1}{m_Q^2} I_{\rm 3d}(m_H) \Big[-I(m_Q) + I_{\rm 3d}(m_U) + I_{n\neq 0}(0) \Big] \bigg(1 + \mathcal{O}\Big(\frac{m_U^2}{m_Q^2}, \frac{m_U^2}{T^2} \Big) \bigg). \end{aligned} \tag{C.17}$$

Indeed, the remainder,

$$\sim \int \frac{d^{3-2\epsilon}p}{(2\pi)^{3-2\epsilon}} \frac{1}{p^2 + m_H^2} \Big[\Pi(p) - \Pi(0) \Big], \tag{C.18}$$

behaves at small p as $\int dp\, p^4/(p^2+m_H^2)$. This is IR finite even after expanding in m_H^2, therefore there cannot be any further linear contributions.

In order to verify this behaviour explicitly, we set $m_U = 0$ and compare the finite part of Eq. (C.17) with a numerical evaluation of the finite part of $H_b(m_H, m_Q, 0)$ in Eq. (C.9). We fix $\bar\mu = T$, and choose $m_Q/T = 1.0, 2.0, 4.0$. The result is shown in Fig. 3. Note that in Eq. (C.17), one must include a contribution $\sim m_H \ln m_H$, arising when the $\mathcal{O}(\epsilon)$ part of $I_{\rm 3d}(m_H)$ (cf. Eq. (A.12)) combines with the $1/\epsilon$ pole in $I(m_Q)$. From the perfect agreement at small m_H/T in Fig. 3, we conclude that for $m_H/T \ll 1$ the behaviour is indeed according to Eq. (C.17).

Two equal heavy masses. Finally, let us consider the special case needed in Sec. 4, $H_b(m,m,0)$ (cf. Eq. (4.2)). The $T=0$ part in Eq. (C.11), related to 2-loop vacuum renormalization of $m_H^2(\bar\mu)$, is

$$H_{\rm vac}(m,m,0) = -\mu^{-4\epsilon}\frac{m^2}{(4\pi)^4}\left(\frac{\bar\mu^2}{m^2}\right)^{2\epsilon}\left(\frac{1}{\epsilon^2}+\frac{3}{\epsilon}+7+\frac{\pi^2}{6}+\mathcal{O}(\epsilon)\right). \tag{C.19}$$

Using Eq. (A.6) as well as the simple expressions for $\Pi(0;m^2,m^2), \Pi(-m^2;m^2,0)$ obtained from Eqs. (C.7), (C.8), we then get from Eq. (C.9)

$$\begin{aligned} H_b(m,m,0) = H_{\rm vac}(m,m,0) + \mu^{-4\epsilon}\frac{T^2}{(4\pi)^2}\bigg[\Big(\frac{1}{12}+\mathcal{I}_1\Big(\frac{m}{T}\Big)\Big)\Big(\frac{1}{\epsilon}+\ln\frac{\bar\mu^2}{T^2}+\ln\frac{\bar\mu^2}{m^2}\Big) \\ +(4-2\ln 2)\mathcal{I}_1\Big(\frac{m}{T}\Big) - \mathcal{I}_2\Big(\frac{m}{T}\Big) + \frac{1}{6}\Big(\gamma_E - \ln 2 - \frac{\zeta'(2)}{\zeta(2)}\Big) + \mathcal{H}\Big(\frac{m}{T}\Big)\bigg], \end{aligned} \tag{C.20}$$

where $\mathcal{I}_1, \mathcal{I}_2$ are from Eq. (A.8), and

$$\mathcal{H}\Big(\frac{m}{T}\Big) = \frac{2}{\pi^2}\int_0^\infty dp\, p\int_0^p dq\, q \frac{n_b(\hat\omega_p)}{\hat\omega_p}\frac{n_b(\hat\omega_q)}{\hat\omega_q}\ln\frac{p+q}{p-q}. \tag{C.21}$$

The function $\mathcal{H}(m/T)$, numerically very easily evaluated, is plotted in Fig. 4, together with a comparison with the limiting values

$$\mathcal{H}\Big(\frac{m}{T}\Big) \overset{m\ll T}{=} -\frac{1}{2}\Big(\ln\frac{2m}{T}-\frac{1}{3}+\gamma_E-\frac{\zeta'(2)}{\zeta(2)}\Big), \qquad \overset{m\gg T}{=} \frac{1}{2\pi}e^{-2m/T}. \tag{C.22}$$

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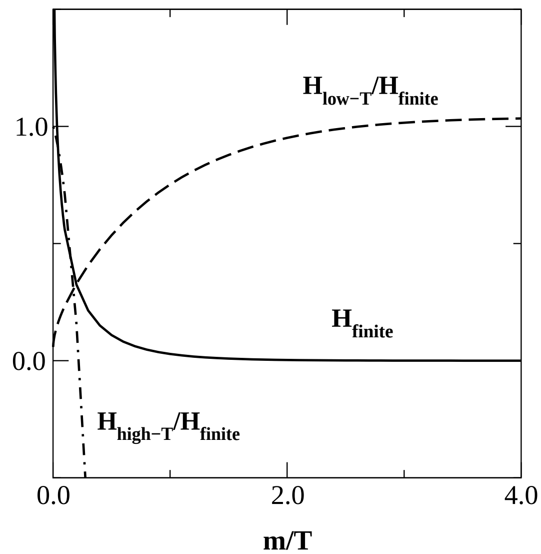

Figure 4: The function $\mathcal{H} \equiv H_{\rm finite}$ in Eq. (C.21), compared with the high-T and low-T limits in Eq. (C.22).

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