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A review of the D1/D5 system and five dimensional black hole from supergravity and brane viewpoints

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ABSTRACT

We review* some aspects of the D1/D5 system of type IIB string theory and the associated five dimensional black hole. We include a pedagogical discussion of the construction of relevant classical solutions in supergravity. We discuss the gauge theory and the conformal field theory relevant to D-brane description of these systems. In order to discuss Hawking radiation we are automatically led to a discussion of near-horizon geometries and their relation to gauge theories and conformal field theories. We show how inputs from AdS/CFT correspondence resolve some earlier puzzles regarding Hawking radiation. Besides the D1/D5 system, we include a brief discussion of some nonsupersymmetric systems which show unexpected agreement between supergravity and perturbative brane/string computations. We also comment briefly on possible implications of the AdS/CFT relation for the correspondence principle and for the principle of black hole complementarity.

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1 Introduction

There are a number of theoretical reasons why black holes are important objects to study (for books discussing black holes at various levels, see, e.g. [1, 2, 3, 4]). From the point of view of classical gravity, they constitute a very interesting class of spacetimes, characterized by event horizons and (often) singularities and tied to the enigmatic no-hair theorems [5] and the laws of black hole thermodynamics [6, 7]. Each of these features embodies some challenging questions. For example,
(a) an event horizon implies a time-reversal asymmetry [6, 8]: objects can enter through the event horizon but the time-reversed process in which they would come out, is not allowed, at least classically;
(b) the presence of a spacetime singularity typically signals a breakdown of the metric description and signals some deeper underlying physics;
(c) the no-hair theorem appears to be in conflict with the existence of a black hole entropy: if for a given charge, mass and angular momentum (and possibly a few other quantum numbers) all black holes are identical as measured by an observer at asymptotic infinity then the number of *accessible* states possessing those quantum numbers should be 1, implying a zero entropy;
(d) the existence of a black hole entropy proportional to the *area* of the event horizon, coupled with Bekenstein's bound [9] on maximum entropy in a given region of spacetime, appears to necessitate a fundamental revision in our notion of degrees of freedom in the presence of gravity.

With the introduction of quantum mechanics, black holes assume an even more fundamental role. The temperature and entropy that appear in classical black hole thermodynamics are shown by Hawking [10] to refer to radiation coming out of a black hole, the value of the temperature being proportional to $\hbar$. This would appear to imply evolution of a pure state of collapsing matter to radiation, a mixed state, at least when the black hole evaporates completely. Such an evolution is contrary to unitarity and would signal incompatibility between conventional quantum mechanics and general relativity [11].

A possible resolution of the above paradox is that the radiation from a black hole is *thermal* in the same sense that radiation from burning coal is thermal, the latter being a consequence of coarse graining, *i.e* averaging over microstates. This requires knowing what the microstates of a black hole are.

The microstates should satisfy a number of properties: they should (a) account for Bekenstein-Hawking entropy, (b) explain Hawking radiation as averaging over unitary processes and (c) explain why the time-reversed process of absorption infinitely dominates over emission in the classical limit. Furthermore, the microstates should (d) be indistinguishable from one another to the asymptotic observer in the domain of validity of the no-hair theorems and (e) hopefully give us a hint regarding the nature of degrees of freedom in gravity which leads to entropy $\propto$ area.

As is well-known by now, there has been considerable progress in string theory in recent years on most of the above issues. In these lectures, we will mostly focus on the example of the five dimensional black hole [12, 13]. In Sec. 2 we will discuss how the classical solution is arrived at. In order to have the discussion reasonably self-contained we will include some techniques of constructing classical solutions in M/string theory. In Sec. 3 we will discuss the D-brane picture of the five dimensional black hole. This will include the description of microstates in terms of a conformal field theory and its relation to the world-volume gauge theory of the D5 branes. In Sec. 4 we will discuss dynamical questions like absorption and Hawking radiation. We will first present the semiclassical calculation and then the derivation in the D-brane picture. In Sec. 5 we will discuss some open questions.

2 Construction of the classical solution

The aim of this section will be to construct the classical solution representing the five-dimensional black hole in [13]. Rather than presenting the solution and showing that it solves the low energy equations of type II superstring, we will describe some aspects of the art of solution-building. There are many excellent reviews of this area (see, for example, [14, 15, 16, 17, 18], other general reviews on black holes in string/M theories include [19, 20, 21]), so we shall be brief. The method of construction of various classical solutions, we will see, will throw light on the microscopic configurations corresponding to these solutions.

Two widely used methods for construction of classical solutions are

(a) the method of harmonic superposition
(b) $O(d,d)$ transformations

We will mainly concentrate on the first one below.

As is well-known by now, classical solutions of type II string theories can be obtained from those of M-theory [22, 23] through suitable compactification and dualities. We will accordingly start with classical solutions of M-theory, or alternatively, of 11-dimensional supergravity.

We should note two important points:

(a)For classical supergravity description of these solutions to be valid, we need the curvature to be small (in the scale of the 11-dimensional Planck length l_{11} for solutions of M-theory, or of the string length l_s for string theories)
(b) Since various superstring theories are defined (through perturbation theory) only in the (respective) weakly coupled regimes, in order to meaningfully talk about classical solutions of various string theories meaningfully we need the string coupling also to be small.

For the RR charged type II solutions (charge Q) that we will describe below, both the above conditions can be met if $Q \gg 1/g_{\rm st} \gg 1$ (that is, $g_{\rm st} Q \gg 1, g_{\rm st} \ll 1$).

2.1 Classical solutions of M-theory

The massless modes are the 11-dimensional metric G_{MN}, the gravitino ψ_M and a three-form $A_{MNP}, M = 0, 1, \ldots, 10$.

The classical action is

$$S_{11} = \frac{1}{2\kappa_{11}^2}\int d^{11}x[\sqrt{-G}(R - \frac{1}{48}(dA)^2) - \frac{1}{6}A\wedge dA\wedge dA] + \text{fermions} \tag{1}$$

There are two important classical solutions of this Lagrangian, the M2 and M5 branes, whose intersections account for most stable supersymmetric solutions of M-theory [25, 26, 16].

The 2-brane of M-theory: M2[24]

We will discuss only this case in some detail.

Statement of the problem: we want to find (a) a relativistic 2-brane solution of (1) (say stretching along $x^{1,2}$) with (b) some number of unbroken supersymmetries.

Condition (a) implies that the solution must have a $SO(2,1)_{0,1,2}\times SO(8)_{3,4,5,6,7,8,9,10}$ symmetry, together with translational symmetries along $x^{0,1,2}$. The subscripts denote which directions are acted on by the SO groups.

This uniquely leads to

$$\begin{aligned} ds_{11}^2 &= e^{2A_1(r)}dx^\mu dx_\mu + e^{2A_2(r)}dx^m dx_m \\ A &= e^{A_3(r)}dt\wedge dx^1\wedge dx^2 \end{aligned} \tag{2}$$

where $\mu = 0, 1, 2$ denote directions parallel to the world-volume and $m = 3, \ldots, 9, 10$ denote the transverse directions. $r^2 \equiv x^m x_m$.

Condition (b) implies that there should exist a non-empty set of supersymmetry transformations ϵ preserving the solution (2); in particular the gravitino variation

$$\begin{aligned} \delta_\epsilon\psi_M &= D_M\epsilon + \frac{1}{288}(\Gamma_M^{NPQR} - 8\delta_M^N\Gamma^{PQR})F_{NPQR}\epsilon = 0 \\ D_M\epsilon &= \left(\partial_M + \frac{1}{4}\omega_A^{BC}\Gamma_{BC}\right)\epsilon \end{aligned} \tag{3}$$

must vanish for some ϵ's.

It is straightforward to see that Eqn. (3) vanishes for $M = \mu$ (world volume directions) if

$$\begin{aligned} \partial_\mu\epsilon &= 0, \\ A_3 &= 3A_1 \end{aligned} \tag{4}$$

and

$$\Gamma^{\hat{0}\hat{1}\hat{2}}\epsilon = \epsilon \tag{5}$$

where the caret $\hat{}$ denotes local Lorenz indices. (Flipping the sign of A would correspond to $-\epsilon$ on the right hand side of (5): this would correspond to an anti-brane solution in our convention.)

The $M = m$ (transverse) components of (3) give rise to the further conditions

$$\begin{aligned} A_1 &= -2A_2 \\ \epsilon &= e^{A_3/6}\epsilon_0 \end{aligned} \tag{6}$$

Harmonic equation

The equations (4) through (6) fix the three functions A_i in (2) in terms of just one function, say A_3. It is easy to determine it by looking at the equation of motion of the three-form potential:

$$\partial_M(\sqrt{-g}F^{MNPQ}) + \frac{1}{2.(4!)^2}\epsilon^{NPQABCDEFGH}F_{ABCD}F_{EFGH} = 0 \tag{7}$$

The second term is clearly zero for our ansatz (2) for A. The first term, evaluated for $(P, Q, R) = (0, 1, 2)$ gives to

$$\partial_m\partial_m(e^{-A_3}) = 0 \tag{8}$$

Thus, the full M2 solution is given by

$$\begin{aligned} ds^2_{11} &= H^{1/3}[H^{-1}dx^\mu dx_\mu + dx^m dx_m] \\ A &= H^{-1}dt \wedge dx^1 \wedge dx^2 \end{aligned} \tag{9}$$

where $H = H(r)$ satisfies the harmonic equation in the transverse coordinates

$$\partial_m\partial_m H = 0 \tag{10}$$

The simplest solution for H, in an asymptotically flat space, is given by

$$H = 1 + k/r^6 \tag{11}$$

Clearly, *multi-centred solutions* are also allowed:

$$H = 1 + \sum_i \frac{k_i}{|\vec{x} - \vec{x_i}|^6} \tag{12}$$

where $\vec{x}$ denotes the transverse directions x^m.

We note that, the constant, 1, in (11) is essentially an integration constant. Clearly, it can also be zero; such choices have led to M/string theory solutions involving AdS spaces. The point of this remark is to emphasize that the near-horizon geometry ($r \to 0$), important in the context of AdS/CFT correspondence [29, 67, 68], in which $H = k/r^6$, corresponds to a complete solution in its own right without the appendage of the asymptotically flat regions. We will return to the AdS/CFT correspondence several times in these lectures.

ADM mass

The integration constant k in (11) affects the asymptotic fall-off of the metric as well as of the field strength, and is related to the ADM mass (per unit area of the 2-brane) M and to the gauge charge (per unit area) q. Using the standard definitions of these quantities, namely,

$$M = \int_{S^7} d^7\Sigma^m (\partial^n h_{mn} - \partial_m h) \tag{13}$$

$$q = \int_{S^7} d^7\Sigma^m F_{m012} \tag{14}$$

we get

$$M = 6k\Omega_7 = q \tag{15}$$

Here S^7 represents the sphere at $r^2 = x^m x_m = \infty$,[1] $h_{MN} \equiv g_{MN} - \eta_{MN}$, $h \equiv \sum_{M=1}^{10} h_{MM}$, and $\Omega_n \equiv 2\pi^[(n+1)/2]/\Gamma(\frac{n+1}{2})$ is the volume of the unit sphere S^n.

BPS solution

The mass-charge equality in the last equation (15) is characteristic of a "BPS solution". We provide a very brief introduction below. The 11-dimensional supersymmetry algebra [27] is

$$\{Q, Q\} = C(\Gamma^M P_M + \Gamma^{MN} U_{MN} + \Gamma^{MNPQR} V_{MNPQR}), \tag{16}$$

where C is the charge conjugation matrix and P, U and V are various central terms. When (16) is evaluated [28] for the above M2 solution, we get

$$\frac{1}{V_2}\{Q_\alpha, Q_\beta\} = (C\Gamma^{\hat{0}})_{\alpha\beta} M + (C\Gamma^{\hat{1}\hat{2}})_{\alpha\beta} Q \tag{17}$$

using the notation

$$P_{\hat{0}} = V_2 M, \quad U_{\hat{1}\hat{2}} = V_2 q \tag{18}$$

where V_2 is the spatial volume of the 2-brane (assumed compactified on a large T^2).

Now, the positivity of the Q^2 operator implies that

$$M \geq q \tag{19}$$

[1] The total ADM mass, which diverges, includes integrals over $x^{1,2}$ as well; we ignore them here since we are interested in the mass per unit area. Similar remarks apply to the charge.

where the inequality is saturated when the right hand side of (17) has a zero eigenvector. For our solution (9), we see from (5) that the unbroken supersymmetry transformation parameter satisfies

$$(1-\Gamma^{\hat{0}\hat{1}\hat{2}})\epsilon = 0 \tag{20}$$

This clearly leads to $M = q$. This is a typical example of how classical solutions with (partially) unbroken supersymmetries satisfy the extremality condition mass= charge.

"Black brane"

The M2-brane itself has a black hole geometry. If we compactify the directions $1, 2$ on a 2-torus, we have a black hole solution in the remaining nine extended dimensions. The compactified solution is constructed by placing the multiple centres $\vec{x_i}$ in (12) at the sites of a lattice defining the 2-torus. The horizon is situated at $r = 0$. The detailed geometry has been discussed in many places, *e.g* in [14]. Since our main object of interest is the five-dimensional black hole, and we will use the M2-brane as essentially a building block for that solution, we defer the geometrical discussion till we discuss the latter.

Without compactification too, the above solution is "black", but it has extensions in 1,2 directions and is called a black 2-brane.

Broken supersymmetries reappear in the near-horizon limit

The remaining half of the supersymmetry transformations, leaving the ones in (20), are non-linearly realized in the M2 geometry and can be regarded as spontaneously broken supersymmetries. Interestingly, the supersymmetry variations under these transformations vanish in the near-horizon limit which has the geometry [29]

$$AdS_4 \times S^7 \tag{21}$$

As a result the broken supersymmetry transformations reemerge as unbroken, leading to an enhancement of the number of supersymmetry charges $16 \rightarrow 32$ in the near-horizon limit.

Intersecting M2-branes ($M2 \perp M2$)

We will now use the above solution as a building block to construct more complicated solutions corresponding to intersecting branes.

We consider first two orthogonal M2 branes, along $x^{1,2}$ and $x^{3,4}$ respectively. The geometry of the solution corresponds to a spacetime symmetry consisting of rotations $SO(2)_{1,2} \times SO(2)_{3,4} \times SO(6)_{5,6,7,8,9,10}$ plus Killing vectors $(\partial_t, \partial_1, \ldots, \partial_4)$. This leads to

$$ds^2_{11} \;=\; e^{2A_1}(-dt^2) + e^{2A_2}(dx_1^2 + dx_2^2) + e^{2A_3}(dx_3^2 + dx_4^2) + e^{2A_4} dx_i dx_i$$

$$A = e^{A_5} dt \wedge dx^1 \wedge dx^2 + e^{A_6} dt \wedge dx^3 \wedge dx^4 \tag{22}$$

— o —

Delocalized nature of the solution

We note that the ansatz above represents a "delocalized solution". A localized $M2 \perp M2$ intersection would destroy translational symmetries along the spatial world-sheet of both the 2-branes. The subject of localized intersection is interesting in its own right (see, e.g. [30] which is especially relevant to the D1/D5 system), although we do not have space to discuss them here. The delocalization here involves "smearing" the first M2 solution along the directions 3,4 (by using a continuous superposition in (12), see e.g. [16]), and "smearing" the second M2 solution along 1,2.

— o —

Now, as before, the desire to have a BPS solution leads to existence of unbroken supersymmetry, or $\delta_\epsilon \psi_M = 0$. This now yields four different type of equations, depending on whether the index M is $0, \{1,2\}, \{3,4\}$ or the rest. These express the six functions above in terms of two independent functions H_1, H_2. These functions turn out to harmonic in the common transverse directions when one imposes closure of SUSY algebra or equation of motion. The solution ultimately is

$$\begin{aligned} ds^2_{11} &= (H_1 H_2)^{1/3} [-\frac{dt^2}{H_1 H_2} + \frac{dx_1^2 + dx_2^2}{H_1} + \frac{dx_3^2 + dx_4^2}{H_2} + dx_i dx_i] \\ A &= \frac{1}{H_1} dt \wedge dx^1 \wedge dx^2 + \frac{1}{H_2} dt \wedge dx^3 \wedge dx^4 \end{aligned} \tag{23}$$

The above is an example of "harmonic superposition of branes". (see, e.g., [26]).

M2 $\perp$ M2 $\perp$ M2

Extending the above method, we get the following supergravity solution for three orthogonal M2-branes, extending respectively along $x^{1,2}, x^{3,4}$ and $x^{5,6}$:

$$\begin{aligned} ds^2_{11} &= (H_1 H_2 H_3)^{1/3} [(H_1 H_2 H_3)^{-1}(-dt^2) + H_1^{-1}(dx_1^2 + dx_2^2) \\ &+ H_2^{-1}(dx_3^2 + dx_4^2) + H_3^{-1}(dx_5^2 + dx_6^2) + dx_i dx_i] \\ A &= H_1^{-1} dt \wedge dx^1 \wedge dx^2 + H_2^{-1} dt \wedge dx^3 \wedge dx^4 \\ &\quad + H_3^{-1} dt \wedge dx^5 \wedge dx^6 \end{aligned} \tag{24}$$

2.2 The 6D black string solution of IIB on T^4

In the following we will construct solutions of type II string theories using the above M-theory solutions by using various duality relations which we will describe as we go along. For an early account of black p-brane solutions in string theory, see [31].

We apply the transformation $T_{567}R_{10}$ to the $M2 \perp M2$ solution:

M-theory $\stackrel{R_{10}}{\rightarrow}$	IIA $\stackrel{T_{567}}{\rightarrow}$	IIB
M2 (8,9)	D2 (8,9)	D5 (5,6,7,8,9)
M2 (6,7)	D2 (6,7)	D1 (5)

The first transformation R_{10} denotes the reduction from M-theory to type IIA. To do this, one first needs to compactify the $M2 \perp M2$ solution along x^{10} (by using the multi-centred harmonic functions, with centres separated by a distance $2\pi R_{10}$ along x^{10}). Essentially, at transverse distances large compared to R_{10}, this amounts to replacement of the harmonic function $1/r^4$ by $1/r^3$ and a suitable modification of the integration constant to reflect the appropriate quantization conditions. At this stage, one still has 11-dimensional fields. To get to IIA fields, we use the reduction formula

$$\begin{aligned} ds_{11}^2 &= \exp[-2\phi/3]ds_{10}^2 + \exp[4\phi/3](dx^{10} + C_\mu^{(1)}dx^\mu)^2 \\ A &= B \wedge dx^{10} + C^{(3)} \end{aligned} \tag{25}$$

It is instructive to verify at this stage that the classical D2 solutions do come out of the M2-brane after these transformations. We use the notation $C^{(n)}$ for the n-Form Ramond-Ramond (RR) potentials in type II theories.

The second transformation T_{567} involves a sequence of T-dualities (for a recent account of T-duality transformations involving RR fields, see [32]). We denote by T_m T-duality along the direction x^m. T_{567} denotes $T_5T_6T_7$.

The final transformation, not explicitly written in the above table, is to wrap $x^{6,7,8,9}$ on T^4. We will denote the volume of the T^4 by

$$V_{T^4} \equiv \alpha'^2(2\pi)^4\tilde{v} \tag{26}$$

Assuming the number of the two orthogonal sets of M2-branes to be Q_5, Q_1 respectively, the final result is: Q_5 strings from wrapping D5 on T^4 and Q_1 D-strings. This is the D1/D5 system in IIB supergravity, characterized by the following solution:

$$ds_{10}^2 = f_1^{-1/2}f_5^{-1/2}(-dt^2 + dx_5^2) + f_1^{1/2}f_5^{1/2}dx_idx^i + f_1^{1/2}f_5^{-1/2}dx_adx^a$$

$$
\begin{aligned}
f_{1,5} &= (1 + r_{1,5}^2/r^2) \\
r_1^2 &= g_{\rm st} Q_1 \alpha'/\tilde{v}, \quad r_5^2 = g_{\rm st} Q_5 \alpha' \\
B'_{05} &= -\frac{1}{2}(f_1^{-1} - 1) \\
dB'_{ijk} &= \epsilon_{ijkl} \partial_l f_5 \\
e^{-2\phi} &= f_5 f_1^{-1}
\end{aligned} \tag{27}
$$

Here $B' \equiv C^{(2)}$, the 2-form RR gauge potential of type IIB string theory.

2.3 The extremal 5D black hole solution

Let us now compactify x^5 along a circle of radius R_5 and wrap the above solution along x^5 to get a 0-brane in five dimensions. Let us also "add" gravitational waves (denoted W) moving to the left along x^5. This gives us the BPS version [12, 13] of the five-dimensional black hole. Adding such a wave can be achieved by augmenting the $M2 \perp M2$ solution by a third, transverse, set of M2-branes along $x^{5,10}$ and passing through the same sequence of transformations as above. Alternatively, one can apply the Garfinkle-Vachaspati transformation [33] to the above solution, with the same final result. In the first method, one should start from the $M2 \perp M2 \perp M2$ solution and use a transformation table like the above which has an additional line

M2 (5,10) $\rightarrow$ NS1 (5) $\rightarrow$ W (5)

The last transformation essentially reflects the fact that T-duality changes winding modes to momentum modes. (W denotes a gravitational wave and not a winding mode.)

The final configuration corresponds to D5 branes along $x^{5,6,7,8,9}$ and D1 branes along x^5, with a non-zero amount of (left-moving) momentum. If the number of the three sets of M2 branes are Q_1, Q_5 and N respectively, then these will correspond to the numbers of D1-, D5-branes and the quantized left-moving momentum respectively.

The final solution is given by

$$
\begin{aligned}
ds_{10}^2 &= f_1^{-1/2} f_5^{-1/2}(-dudv + (f_n - 1)du^2) \\
&\quad + f_1^{1/2} f_5^{1/2} dx_i dx^i + f_1^{1/2} f_5^{-1/2} dx_a dx^a \\
f_{1,5,n} &= (1 + r_{1,5,n}^2/r^2) \\
B'_{05} &= -\frac{1}{2}(f_1^{-1} - 1) \\
dB'_{ijk} &= \epsilon_{ijkl} \partial_l f_5 \\
e^{-2\phi} &= f_5 f_1^{-1}
\end{aligned} \tag{28}
$$

The spacetime symmetry $\mathcal{S}$ of the above solution is:

$$\mathcal{S} = SO(1,1) \times SO(4)_E \times `SO(4)'_I \tag{29}$$

where $SO(1,1)$ refers to directions $0,5$, $SO(4)_E$ to directions $1,2,3,4$ (E for external) '$SO(4)_I$' to directions $6,7,8,9$ (I stands for internal; the quotes signify that the symmetry is broken by wrapping the directions on a four-torus although for low energies compared to the inverse radii it remains a symmetry of the supergravity solution).

The unbroken supersymmetry can be read off either by recalling those of the M-theory solution and following the dualities or by solving the Killing spinor equations (analogous to (3)). The result is:

$$\begin{aligned}
\Gamma^{056789}\epsilon_L &= \epsilon_R \\
\Gamma^{05}\epsilon_L &= \epsilon_R \\
\Gamma^{05}\epsilon_{L,R} &= \epsilon_{L,R}
\end{aligned} \tag{30}$$

The first line corresponds to the unbroken supersymmetry appropriate for the D5-brane (extending in 5,6,7,8,9 directions). The second line refers to the D1-brane. The last line corresponds to unbroken supersymmetries in the presence of the gravitational wave. (The superscripts in $\Gamma^{ab..}$ denote local Lorenz indices like in (3), although we have dropped the carets.)

The parameters $r^2_{1,5,n}$ in (28) are related to the integer-quantized charges $Q_{1,5}$ and momentum N by

$$\begin{aligned}
r^2_{1,5} &= c_{1,5}Q_{1,5} \\
r^2_n &= c_n N
\end{aligned} \tag{31}$$

where

$$\begin{aligned}
c_1 &= \frac{4G^5_N R_5}{\pi\alpha'} \\
c_5 &= g_{\rm st}\alpha' \\
c_n &= \frac{4G^5_N}{\pi R_5} \\
G^5_N &= G^{10}_N/(2\pi R_5 V_{T^4}), \quad G^{10}_N = 8\pi^6 g^2_{\rm st}\alpha'^4
\end{aligned} \tag{32}$$

For a detailed discussion of quantization conditions like (31), see, e.g. [34, 17, 14]. Here G^d_N denotes the d-dimensional Newton's constant.

We defer the discussion of the geometry and the Bekenstein-Hawking entropy till the next section where we describe the non-extremal version.

2.4 Non-extremal five-dimensional black hole

We have explained above how to construct from first principles the BPS (hence extremal) version of the 5D black hole solution. We will now present an algorithm (without proof and specialized to intersections of M2) of how to generalize these constructions to their non-extremal (nonsupersymmetric) versions [35]:

Rule 1: In the transverse part of the metric (including time) make the following substitution:

$$\begin{aligned} dt^2 &\rightarrow h(r)dt^2, \quad dx^i dx_i \rightarrow h^{-1}(r)dr^2 + r^2 d\Omega_{d-1}^2 \\ h(r) &= 1 - \mu/r^{d-2} \end{aligned}$$

with the harmonic function now defined as

$$H(r) = 1 + \tilde{Q}/r^{d-2} \tag{33}$$

where $\tilde{Q}$ is a combination of the non-extremality parameter μ and some "boost" angles:

$$\tilde{Q} = \mu \sinh^2 \delta \tag{34}$$

(for multicentred solutions, $\tilde{Q}_i = \mu \sinh^2 \delta_i$ etc.)

Rule 2: In the expression for $F_4 = dA$, make the substitution

$$\begin{aligned} H &\rightarrow \tilde{H}(r) = 1 + \frac{\bar{Q}}{r^{d-2} + \tilde{Q} - \bar{Q}} = \left(1 - \frac{\bar{Q}}{r^{d-2}} H^{-1}\right)^{-1}, \\ \bar{Q} &= \mu \sinh\delta \cosh\delta \end{aligned} \tag{35}$$

A heuristic motivation for the algorithm

We present a brief, heuristic, motivation for the above algorithm. Suppose we view a static Schwarzschild black hole, of ADM mass m, from the five-dimensional Kaluza-Klein viewpoint. The 5-momenta $(p_0, \vec{p}, p_5)$ will be given by

$$p_0 = m, \quad p_5 \propto \text{charge} = 0, \quad \vec{p} = 0 \tag{36}$$

The second equation follows because the Schwarzschild black hole is neutral.

A way of generating charged solutions is to consider the above solution in five non-compact dimensions and perform a boost in the 0-5 plane. The momenta transform as

$$p_0' \equiv M = m\cosh\delta, \quad p_5' \equiv \tilde{Q} = m \sinh\delta, \quad \vec{p}' = 0 \tag{37}$$

We can now wrap the fifth dimension to get a charged black hole in four non-compact dimensions. The extremal limit ($M = Q$) can be attained by

$$\delta \to \infty, m \to 0, me^{\delta} \to \text{constant} \tag{38}$$

so that

$$\tilde{Q} \to M = me^{\delta}/2, \quad p'_R \equiv p'_0 - p'_5 \to 0 \tag{39}$$

Near-extremal limit

The near-extremal limit is obtained by keeping the leading corrections in $e^{-\delta}$. Thus,

$$\tilde{Q}/M = \tanh \delta \simeq 1 - m^2/(2\tilde{Q}^2), \qquad p'_R \ll p'_L \tag{40}$$

In terms of these parameters, the four-dimensional metric for a near-extremal charged (RN) black hole is given by

$$\begin{aligned} ds_4^2 &= -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2^2 \\ f &\equiv 1 - 2M/r + \tilde{Q}^2/r^2 = f_{\text{ext}} h(r) \\ f_{\text{ext}} &\equiv (1 - \tilde{Q}/r)^2, \quad h(r) = (1 - \mu/r) \\ \mu &= m^2/\tilde{Q} \end{aligned} \tag{41}$$

The last equality implies

$$\tilde{Q} = \mu \sinh^2 \delta, \tag{42}$$

same as (34) above. Also, the second equality agrees with Rule 1 for relating the non-extremal g_{tt}, g_{rr} to their extremal counterparts.

Of course, we have considered here only the near-extremal case. The remarkable thing about the algorithm mentioned above is that it works for arbitrary deviations from extremality.

Applying this rule to the $M2 \perp M2 \perp M2$ case, we get

$$\begin{aligned} ds_{11}^2 &= (H_1 H_2 H_3)^{-1/3} [-H_1 H_2 H_3 h dt^2 + H_1 (dy_1^2 + dy_2^2) \\ &+ H_2 (dy_3^2 + dy_4^2) + H_3 (dy_5^2 + dy_6^2) + h^{-1} dr^2 \\ &+ r^2 d\Omega_{d-1}] \end{aligned} \tag{43}$$

The rest of the story is similar to the BPS case. We first reduce the M-theory solution to IIA, wrap the solution on a T^5 and then T-dualize to IIB.

Under the reduction from M-theory to type IIA in ten dimensions, we get

$$\begin{aligned} e^{-2\phi} &= F_1 F_5^{-1} \\ ds_{10}^2 &= F_1^{-1/2} F_5^{-1/2} [-dt^2 + dx_5^2 \\ &+ (1 - h)(\cosh \alpha_n dt + \sinh \alpha_n dx_5)^2] \\ &+ F_1^{1/2} F_5^{1/2} (\frac{dr^2}{h} + r^2 d\Omega_3^2) \\ &+ F_1^{1/2} F_5^{-1/2} dx_a dx^a \end{aligned} \tag{44}$$

where $a = 6, .., 9$, (r, Ω_3) are polar coordinates for $x^{1,2,3,4}$ and

$$\begin{aligned} F_s &= 1 + \frac{r_s^2}{r_0^2},\ r_s^2 = r_0^2 \sinh^2 \alpha_s,\ s = 1, 5 \\ h &= (1 - \frac{r_0^2}{r^2}) \end{aligned} \tag{45}$$

The parameter r_0^2 is the same as the non-extremality parameter μ of (33), while $\alpha_{1,5,n}$ are related to the boost angle of (34).

It is easy to compactify the directions $x^{5,6,7,8,9}$ on a T^5 (radii $R_{5,6,7,8,9}$, $V \equiv R_6 R_7 R_8 R_9$), using the reduction formula

$$ds_{10}^2 = e^{2\chi} dx_a dx^a e^{2\psi} (dx_5 + A_\mu dx^\mu)^2 + e^{-(8\chi + 2\psi + \phi)/3} ds_5^2 \tag{46}$$

(the first two exponential factors are simply the definitions of the scalars χ, ψ; the factor in front of ds_5^2 can be found easily by demanding that ds_5^2 is the five-dimensional Einstein metric). Here $\mu = 1, 2, 3, 4$.

This is still a IIA solution. In order to get the IIB version, we have to apply the sequence T_{567}. We omit the details here which are fairly straightforward. At the end we get the following five-dimensional Einstein metric [36]:

$$\begin{aligned} ds_5^2 &= -h f^{-2/3} dt^2 + f^{1/3} (\frac{dr^2}{h} + r^2 d\Omega_3^2) \\ h &= 1 - r_0^2/r^2 \\ f &= F_1 F_5 (1 + r_n^2/r^2) \\ r_n^2 &= r_0^2 \sinh^2 \alpha_n \end{aligned} \tag{47}$$

There are six independent parameters of the metric $\alpha_{1,5,n}$, $r_0, R_5, \tilde{v} \equiv V_{T^4}/(2\pi l_s)^4$ ($l_s = \sqrt{\alpha'}$). The boost angles and the non-extremality parameters are related to the three charges and the mass M as follows: $(H \equiv dB')$

$$\begin{aligned} Q_1 &= \frac{V}{4\pi^2 g} \int e^\phi * H = \frac{\tilde{v} r_0^2}{2\alpha' g_{\rm st}} \sinh 2\alpha_1 \\ Q_5 &= \frac{1}{4\pi^2 g_{\rm st}} \int H = \frac{r_0^2}{2\alpha g_{\rm st}} \sinh 2\alpha_5 \\ N &= \frac{R_5^2 \tilde{v} r_0^2}{2\alpha g_{\rm st}^2} \sinh 2\alpha_n \\ M &= \frac{R_9 \tilde{v} r_0^2}{2 l_s^3 g_{\rm st}^2} (\cosh 2\alpha_1 + \cosh 2\alpha_5 + \cosh 2\alpha_n) \end{aligned} \tag{48}$$

There is another very interesting representation of the above-mentioned six parameters in terms of what appears as brane-, antibrane-numbers and left-,right-moving momenta:

$$N_{1,\bar{1}} = \frac{\tilde{v} r_0^2}{4\alpha' g_{\rm st}} e^{\pm 2\alpha_1}$$

$$N_{5,\bar{5}} = \frac{r_0^2}{4\alpha' g_{\rm st}} e^{\pm 2\alpha_5}$$
$$N_{L,R} = \frac{R_5^2 \tilde{v} r_0^2}{4\alpha'^2 g_{\rm st}^2} e^{\pm 2\alpha_n} \tag{49}$$

Clearly $N_1 - N_{\bar{1}} = Q_1, N_5 - N_{\bar{5}} = Q_5$ and $N_L - N_R = N$. The extremal limit corresponds to taking $r_0 \to 0, \alpha_i \to \infty$ keeping the charges $Q_{1,5}, N$ finite.

Geometry

It is easy to see that the above solution is a five-dimensional black hole, with horizon at $r = r_0$. The horizon has a finite area A_h, given by

$$\begin{aligned} A_h &= 2\pi^2 r_0^3 \cosh\alpha_1 \cosh\alpha_5 \cosh\alpha_n \\ &= 8\pi G_N^5 (\sqrt{N_1} + \sqrt{N_{\bar{1}}})(\sqrt{N_5} + \sqrt{N_{\bar{5}}})(\sqrt{N_L} + \sqrt{N_R}) \end{aligned} \tag{50}$$

Here we have used (48) and the value of the five-dimensional Newton's constant G_N^5 in (32).

The fact that the horizon has a finite area indicates that the singularity lies "inside" $r = r_0$. It is not at $r = 0$, however, which corresponds to the inner horizon (where light-cones "flip" the second time as one travels in). To locate the singularity one needs to use other coordinate patches which extend the manifold further "inside". The singularity is time-like and the Carter-Penrose diagram (Fig 1) is similar to that of the non-extremal Reissner-Nordstrom metric.

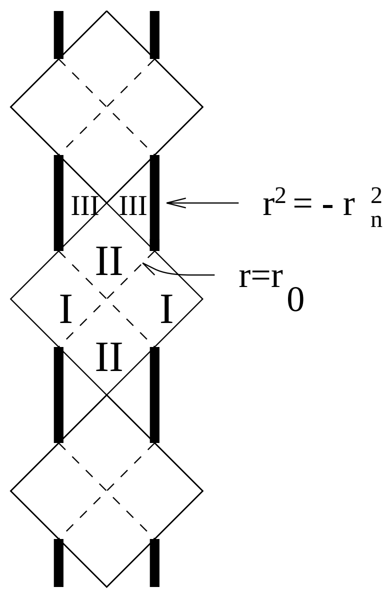

Figure 1: Carter-Penrose diagram for the non-extremal 5D black hole

Bekenstein-Hawking entropy

By using the formula $S = \frac{1}{4G_N^5} A_h$, we get

$$S_{\rm BH} = 2\pi(\sqrt{N}_1 + \sqrt{N}_{\bar{1}})(\sqrt{N}_5 + \sqrt{N}_{\bar{5}})(\sqrt{N}_L + \sqrt{N}_R) \tag{51}$$

The extremal entropy is given by

$$S_{\rm BH} = 2\pi\sqrt{Q_1 Q_5 N} \tag{52}$$

Both the above formulae are U-duality invariant, in the following sense. Consider an $S(3)$ subgroup of the U-duality group of type IIB on T^5, which permutes the three charges Q_1, Q_5 and N. Such an $S(3)$ is generated by (a) T_{6789} which sends $Q_1 \to Q_5, Q_5 \to Q_1, N \to N$, and (b) $T_{9876}ST_{65}$ which sends $Q_1 \to Q_5, Q_5 \to N, N \to Q_1$.
The entropy formula (52) remains invariant under these permutations. Since the "anti"-objects are also permuted among each other by these U-duality transformations, we can say that the entropy formula (51) is also U-duality invariant.

3 D-brane picture

D-branes are solitonic configurations of superstring theories which, by definition, are characterized by low energy excitations that are open strings ending on them. For a review, see, e.g. [37]. We will provide a very brief introduction here.

Dp branes

We will discuss a single Dp-brane first.

Open string excitations of a single Dp-brane, extending in directions $x^{1,2,..,p}$, obey the boundary conditions

$$\begin{aligned} x^M(z) &= R^M_N x^N(\bar{z}) \\ \psi^M(z) &= R^M_N \psi^N(\bar{z}) \\ R &= \mathrm{diag}[1,1,..,1,-1,-1,..,-1] \\ S^\alpha(z) &= \mathcal{R}^\alpha_\beta S^\beta(\bar{z}) \\ \mathcal{R} &= \Gamma^{01...p} \end{aligned} \tag{53}$$

at $z = \bar{z}$. The coordinates $z, \bar{z}$ refer to the upper half plane which is related by a conformal transformation to the disc geometry of tree-level open-string world-sheet (see, e.g., [38] for more details). M and α are vector and spinor indices in ten dimensions. The positive eigenvalues of the matrix R correspond to the longitudinal directions $x^\mu, \mu = 0, 1, .., p$ (Neumann boundary condition) and the negative eigenvalues correspond to the transverse directions $x^i, i = p+1, .., 9$ (Dirichlet boundary conditions).

Spacetime symmetry:
Clearly these boundary conditions reflect a $SO(p, 1) \times SO(d)$ symmetry as well as translational symmetries along $x^{0,1,\ldots,p}$.

Supersymmetry:
The boundary condition for the spin fields implies that the supersymmetry transformation parameters must satisfy

$$\epsilon_L = \Gamma^{01\ldots p} \epsilon_R \tag{54}$$

We see that only half of the supersymmetries are preserved, implying the 1/2-BPS nature of Dp-branes. It is easy to see the correspondence to the classical brane solutions described earlier, both in terms of the spacetime symmetry and supersymmetry (*cf.* Eqns. (29),(30)).

Multiple coincident Dp branes:

Spacetime symmetries and supersymmetries remain the same as for the single brane. The main additional ingredient is that open strings can now begin and end on different branes. This attaches an additional degree of freedom to the end-points of open strings, which can be identified with *Chan-Paton factors*. Thus, an open string beginning on the i-th brane and ending on the j-th brane gets labelled by a matrix $\Lambda^{(ij)}$ whose (i, j) element is 1 and rest are zero. These matrices generate the $U(N)$ algebra [37]. The massless open string excitations correspond to a supersymmetric $U(N)$ Yang-Mills theory in $p + 1$ dimensions, which can be regarded as a dimensional reduction of $\mathcal{N} = 1$ SYM in 10 dimensions [39].

Since the supergravity fields cannot couple to non-singlets of the SYM theory, the signature of multiple branes is only in the ADM mass and the total RR-charge in (13) and (14).

3.1 Microstates corresponding to the five-dimensional black hole

In section 2 we have presented the classical solution of the five-dimensional black hole. The construction of the extremal hole suggests that we should look for a bound state of Q_1 D1 branes and Q_5 D5 branes with total left-moving momentum equal to N (in units of $1/R_5$).

In order to find the low energy excitations of such a system, we begin with the D1/D5 system (which corresponds to the 6 dimensional black string) in which x^5 is still non-compact and there is no momentum mode yet. The low energy excitations correspond to open strings which can begin on a D1-brane or a D5-brane and end on a D1-brane or a D5-brane. We call such open strings (1,1), (1,5), (5,1) or (5,5) where (p, q) denotes an open string beginning on a

Dp- and ending on a Dq-brane.

The (1,1) strings obey the following boundary conditions:

DD boundary conditions along directions 1,2,3,4,6,7,8,9
NN boundary conditions along directions 0,5

(By DD we mean that the open strings satisfy Dirichlet boundary conditions at both ends. The notations ND and NN are likewise defined)

For the (5,5) strings the boundary conditions are:

DD: 1,2,3,4
NN: 0,5,6,7,8,9

For the (1,5) (and (5,1)) strings the boundary conditions are:

DD: 1,2,3,4
NN: 0,5
ND: 6,7,8,9

It is clear that the open string boundary conditions correspond to the same spacetime symmetry $SO(1,1) \times SO(4)_E \times$ '$SO(4)_I$' as in (29) which characterizes the classical solution. It is also easy to see that these boundary conditions lead to the same supersymmetries as in (30).

Massless modes

In order to see the massless degrees of freedom [17, 40, 41, 42] let us look at the following table of zero-point energies (i.e., L_0 values for the Fock space vacua of the oscillators x, ψ_R, ψ_{NS}). The table can be constructed simply from the moding of various fields in their normal mode expansion for the appropriate boundary condition.

Table of zero-point energies

	NN	DD	ND
X	$-1/24$ (P)	$-1/24$ (P)	$1/48$ (AP)
ψ_R	$1/24$ (P)	$1/24$ (P)	$-1/48$ (AP)
ψ_{NS}	$-1/48$ (AP)	$-1/48$ (AP)	$1/24$ (P)

(P=periodic, AP=anti-periodic)

(1,1) strings: In the light-cone gauge (where we choose x^0, x^5, both non-compact, as the light cone directions), there are 8 DD directions, leading to a total zero-point energy $L_0 = -1/2$ in the NS sector and $L_0 = 0$ in the R sector. We assume that the radii in the $6, 7, 8, 9$ directions are of the order of the string length; so at low energies we can ignore any winding modes in these directions. The massless spectrum of (1,1) strings, then, is that of a supersymmetric $U(Q_1)$ gauge theory in 1+1 dimensions, as mentioned above in the context of a single set of Dp-branes. The field content can be organized into the vector multiplet and the hypermultiplet of an $\mathcal{N} = 2$ theory in four-dimensions (or those of $\mathcal{N} = 1$ in six dimensions $x^{0,1,2,3,4,5}$)

$$\begin{aligned} \text{Vector multiplet} \quad &: \quad A_0^{(1)}, A_5^{(1)}, \phi_1^{(1)}, \phi_2^{(1)}, \phi_3^{(1)}, \phi_4^{(1)} \\ \text{Hypermultiplet} \quad &: \quad Y_6^{(1)}, Y_7^{(1)}, Y_8^{(1)}, Y_9^{(1)} \end{aligned} \tag{55}$$

The $A_0^{(1)}, A_5^{(1)}$ are the $U(Q_1)$ gauge fields in the non-compact directions. The $Y^{(1)}$'s and $\phi^{(1)}$'s are gauge fields in the compact directions of the $\mathcal{N} = 1$ super Yang-Mills in ten-dimensions. All the fields transform as adjoints of $U(Q_1)$. The hypermultiplet of $\mathcal{N} = 2$ supersymmetry are doublets of the $SU(2)_R$ symmetry of the theory. The $Y^{(1)}$'s can be arranged as doublets under $SU(2)_R$ as

$$N_{a\bar{a}}^{(1)} = \begin{pmatrix} N_{1(a\bar{a})}^{(1)} \\ N_{2(a\bar{a})}^{(1)\dagger} \end{pmatrix} = \begin{pmatrix} Y_{9(a\bar{a})}^{(1)} + iY_{8(a\bar{a})}^{(1)} \\ Y_{7(a\bar{a})}^{(1)} - iY_{6(a\bar{a})}^{(1)} \end{pmatrix} \tag{56}$$

where $a, \bar{a}$ runs from $1, \ldots, Q_1$.

(5,5) strings: These 4 DD and 4 NN directions. The massless spectrum can be found again from the table of zero-point energies. Ignoring the momentum modes along the T^4, we again have a $U(Q_5)$ theory in 1+1 dimensions. The field content is exactly similar to those of the (1,1) strings:

$$\begin{aligned} \text{Vector multiplet} \quad &: \quad A_0^{(5)}, A_5^{(5)}, \phi_1^{(5)}, \phi_2^{(5)}, \phi_3^{(5)}, \phi_4^{(5)} \\ \text{Hypermultiplet} \quad &: \quad Y_6^{(5)}, Y_7^{(5)}, Y_8^{(5)}, Y_9^{(5)} \end{aligned} \tag{57}$$

The superscript indicates that the fields correspond to the (5,5) strings and transform as adjoint of $U(Q_5)$.

(1,5) and (5,1) strings: These have 4 ND directions (we do not differentiate between ND and DN here) and 4 DD directions. The zero point energies *vanish* in both the NS and R sectors! From the fact that ψ_{NS} are periodic for the ND directions, one sees that the massless mode is a boson transforming as a spinor of $SO(4)_I$. This gives four bosons. The GSO projection projects out half of these which reduces the number of bosons to 2. The two bosons of the $(1,5)$ strings and the $(5,1)$ strings combine to form a complex doublet transforming under the diagonal $SU(2)$ of the $SO(4)_I$. As the hypermultiplets of $\mathcal{N}=2$ theory transform as doublets under $SU(2)_R$, the diagonal $SU(2)$ of $SO(4)_I$ can be identified with the $SU(2)_R$ of the gauge theory. The Chan-Paton factors show that they transform as bi-fundamentals $(Q_1, \bar{Q_5})$ of $U(Q_1) \times U(Q_5)$. We arrange these hypermultiplets as doublets of the $SU(2)_R$ symmetry of the theory in the form

$$\chi_{a\bar{b}} = \left(\begin{array}{c} A_{a\bar{b}} \\ B^{\dagger}_{a\bar{b}} \end{array} \right) \tag{58}$$

We note that the fermionic superpartners of these hypermultiplets which arise from the Ramond sector of the massless excitations of $(1,5)$ and $(5,1)$ strings carry spinorial indices under $SO(4)_E$ and they are singlets under $SO(4)_I$.

The gauge theory of the D1/D5 system, therefore, is a $1+1$ dimensional $(4,4)$ supersymmetric gauge theory with gauge group $U(Q_1) \times U(Q_5)$. The matter content of this theory consists of hypermultiplets $Y^{(1)}$'s, $Y^{(5)}$'s transforming as adjoints of $U(Q_1)$ and $U(Q_5)$ respectively. It also has the hypermultiplets χ's which transform as bi-fundamentals of $U(Q_1) \times U(Q_5)$.

Counting of degrees of freedom:

We will now show that this gauge theory has the required degrees of freedom to describe the entropy of the extremal D1/D5 black hole. The D1/D5 bound state is described by the Higgs branch of this gauge theory. The Higgs branch is obtained by giving expectation values to the hypers. This makes the vector multiplets massive. For a supersymmetric vacuum the hypers take values over the surface which is given by setting the superpotential of the gauge theory to zero. Setting the superpotential to zero imposes two sets of D-flatness conditions corresponding to each of the gauge groups $U(Q_1)$ and $U(Q_5)$. The D-terms for the gauge group $U(Q_1)$ are given by [41, 43, 44][2]

$$\begin{aligned} A_{a\bar{b}}A^*_{a'\bar{b}} - B_{b\bar{a}'}B^*_{b\bar{a}} + [N_1^{(1)}, N_1^{\dagger}]_{a\bar{a}'} - [N_2^{(1)}, N_2^{(1)}]_{a\bar{a}'} &= 0 \\ A_{a\bar{b}}B_{b\bar{a}'} + [N_1^{(1)}, N_2^{(1)\dagger}]_{a\bar{a}'} &= 0 \end{aligned} \tag{59}$$

[2]We ignore here Fayet-Iliopoulos terms and the related issue of singularity at the origin of the Higgs branch (see, *e.g.* [54, 56, 57]).

while the D-terms of the gauge group $U(Q_5)$ are

$$A_{a\bar{b}'}A^*_{a\bar{b}} - B_{b\bar{a}}B^*_{b'\bar{a}} + [N^{(5)}_1, N^\dagger_5]_{b\bar{b}'} - [N^{(5)}_2, N^{(5)}_2]_{b\bar{b}'} = 0 \tag{60}$$
$$A_{a\bar{b}'}B_{b\bar{a}} + [N^{(5)}_1, N^{(5)\dagger}_2{}_{b\bar{b}'}]=0$$

Here a, a' run from $1, \ldots, Q_1$ and b, b' run from $1, \ldots, Q_5$.

The total number of bosonic degrees of freedom from all the hypermultiplets $(Y^{(1)}_{a\bar{a}}Y^{(5)}_{b\bar{b}}, A_{a\bar{b}}, B_{\bar{a}b})$ is

$$4Q_1^2 + 4Q_5^2 + 4Q_1Q_5 \tag{61}$$

The first equation in (59) is real while the second equation in (59) is complex. The total number of constraints imposed by (59) is $3Q_1^2$. Similarly the set of D-term equations in (60) imposes $3Q_5^2$ constraints. Equations (59) and (60) have the same trace parts corresponding to the vanishing of $U(1)$ D-terms, namely,

$$\begin{aligned} A_{a\bar{b}}A^*_{a\bar{b}} - B_{b\bar{a}}B^*_{b\bar{a}} &= 0 \\ A_{a\bar{b}}B_{a\bar{b}} &= 0 \end{aligned} \tag{62}$$

which are three real equations. Therefore, the vanishing of D-terms imposes $3Q_1^2 + 3Q_5^2 - 3$ constraints on the fields. One can use the gauge symmetry $U(Q_1)$ and $U(Q_5)$ to remove another $Q_1^2+Q_5^2-1$ degrees of freedom. The -1 reflects the fact that all the hypermultiplets are invariant under the diagonal $U(1)$ of $U(Q_1) \times U(Q_5)$. After gauge fixing, the number of gauge invariant bosonic degrees of freedom to parameterize the moduli space is $4(Q_1Q_5 + 1)$.

We are interested in low energy black hole processes so it is sufficient to study the SCFT of the Higgs branch. The SCFT will have $\mathcal{N} = (4, 4)$ SUSY with central charge $6(Q_1Q_5 + 1)$ on some target space $\mathcal{M}$. (In general, the number of "degrees of freedom" need not simply translate to central charge owing to interactions; however, with the extent of supersymmetry present here, the manifold $\mathcal{M}$ must be hyperKahler for which the above claim for the central charge is true.) To find the microstates corresponding to the extremal D1/D5 black hole we look for states with $L_0 = N$ and $\bar{L}_0 = 0$. The asymptotic number of distinct states of this SCFT given by Cardy's formula [45]

$$\Omega = \exp(2\pi\sqrt{cL_0/6} = \exp(2\pi\sqrt{Q_1Q_5N}) \tag{63}$$

From the Boltzmann formula one obtains

$$S = 2\pi\sqrt{Q_1Q_5N} \tag{64}$$

This exactly reproduces the Bekenstein-Hawking entropy (52) of the extremal D1/D5 black hole. We will remark about the non-extremal black hole shortly.

3.2 Instanton moduli space

We found above that the Higgs branch of the gauge theory of the D1/D5 system flows in the infrared to an $\mathcal{N} = (4, 4)$ SCFT on a target space $\mathcal{M}$ with central charge $6Q_1Q_5$. For black hole processes like Hawking radiation it is important to know the target space $\mathcal{M}$. In this section we review the arguments which show that the target space $\mathcal{M}$ is a resolution of the orbifold $(\tilde{T}^4)^{Q_1 Q_5}/S(Q_1 Q_5)$. ($\tilde{T}^4$ can be different from the compactification torus T^4.)

We first note that the world-volume theory of Q_5 coincident D5 branes is a 5+1 dimensional U(Q_5) SYM theory. One way to understand D1 branes bound to these D5-branes is to represent the D1-branes as solitons [46] of this SYM theory. The simplest way to see this is to note the Chern-Simons coupling in the world-volume action of a D5 brane [37]:

$$\mu_5 \int C \mathrm{Str} e^F \Rightarrow \mu_5 \int d^6x [C^{(2)} \wedge F \wedge F] \tag{65}$$

which shows that non-zero values of F_{67}, F_{89} can act as a source term for $C^{(2)}_{05}$. The latter corresponds to a D1-brane stretching in the 5 direction. In the above equation

$$C \equiv \oplus_n C^{(n)} \tag{66}$$

and Str represents symmetrized trace.

The above observation leads us to look for non-trivial solutions of six-dimensional SYM theory.

We shall look for solutions which satisfy two conditions:

(a) the $U(Q_5)$ gauge field should be independent of $x^{0,5}$, with $A_0 = A_5 = 0$. (This corresponds to the fact that $x^{0,5}$ are Killing vectors in the supergravity solution.)

(b) the solutions should preserve 1/2 of the supersymmetries, again imitating the corresponding statement in supergravity.

In other words, we are looking for static, stringy solitons of SYM_6 satisfying the BPS property.

Conditions (a) and (b) applied to SYM_6, leads to

$$\delta_\epsilon \lambda \propto \Gamma_{ab} F^{ab} \epsilon = 0 \tag{67}$$

where a, b run over $6, 7, 8, 9$. It is easy to see that this is equivalent to

$$F_{ab} = \epsilon_{abcd} F^{cd}, \;\; a, b, .. = 6, .., 9 \tag{68}$$

where

$$\Gamma_{6789} \epsilon = \epsilon \tag{69}$$

These are nothing but instanton solutions of Euclidean SYM_4.

Note that equation (69) represents the only choice of unbroken SUSY consistent with condition (a).

We now note the following points to bring out in more detail the connection between these instantons and D1-brane embedded in D5-branes.

(1) D1-branes break by half the sixteen unbroken supersymmetries of D5-branes. To be precise, the supersymmetry breaking goes as

$$((2,2)+(2',2'))_+ + ((2,2')+(2',2))_- \rightarrow (2,2)_+ + (2,2')_- \tag{70}$$

where the representation labels correspond to the spacetime symmetry group $SO(4)_{6789} \times SO(4)_{1234} \times SO(1,1)_{05}$. The instanton solution, it can be checked, precisely breaks the same supersymmetries as the D1-brane (recall (69)).

(2) The coupling in (65), together with the solution (68), imply that the source of the field $C^{(2)}$ is the Chern class of the four-dimensional gauge field F_{ab}. Indeed the integral property of $F \wedge F$ exactly corresponds to the quantization of D1-brane charge. In other words, one can easily see that the instanton action for a Q_1-instanton solution is Q_1/g^2_{YM}. This agrees with the tension of Q_1 D1-branes, namely $Q_1/g_{\rm st}$. (Recall that $g_{\rm st} = g^2_{YM}$.)

(3) The ADHM construction [47] of a k-instanton moduli space for SU(N) Yang-Mills in R^4 is closely connected with hypermultiplets in $SU(N) \times SU(k)$: namely, the ADHM equations correspond to D-flatness conditions for the hypermultiplet fields [46]. The solutions to the D-flatness condition of course correspond to space of vacua, or moduli space, of the hypermultiplets.

—o—

Structure of moduli space:

We now discuss the structure of the k-instanton moduli space of $U(Q_5)$ SYM theory on T^4. To begin with, note that Q_1 D1 branes (along x^5) on Q_5 D5 branes (x^{56789}) can be T-dualized to Q_1 D0 branes on Q_5 D4 branes (x^{6789}). For $Q_5 = 1$, the collective coordinates of the T-dualized system would correspond to translation of Q_1 points on the T^4 along x^{6789}. Since the D0-branes are *identical*, (this corresponds to Weyl symmetry of $U(Q_1)$) the Q_1 points should be unordered. The instanton moduli space in this case should then be given by [48, 49, 51] the space of Q_1 unordered points on T^4 is given by

$$\mathcal{M} = \frac{(T^4)^{Q_1}}{S(Q_1)} \tag{71}$$

For $Q_5 > 1$, taking clue from the case $Q_5 = 2$ [50], one arrives at the guess [50, 52]

$$\mathcal{M} = [\frac{(T^4)^{Q_1 Q_5}}{S(Q_1 Q_5)}] \tag{72}$$

The notation [ ] implies an appropriate resolution of the orbifold and possible additional factors corresponding to overall centre-of-mass motions. For more details, see [53, 54, 55, 42].

—o—

(4) One of the most quantitative evidences in favour of (72) is that the cohomology of the $\mathcal{M}$ agrees with a U-dual [48] version of the cohomology of the D1-D5 system. The argument goes as follows.

A fundamental string with winding number w_6 and momentum p_6 along the circle x^6 (say), can be mapped by using the sequence of dualities $T_7 S T_{6789} S$ to w_6 D2-branes along x^{67} and p_6 D2-branes along x^{89}. For our purpose here, we should choose $w_6 = Q_1, p_6 = Q_5$ and make a further T_{567} transformation to get Q_1 D1-branes along x^5 and Q_5 D5-branes along x^{56789}.

Now, BPS states of such a fundamental string simply correspond to oscillator numbers $N_L = 0, N_R = Q_1 Q_5$. The number of states $d(n)$ with $N_L = 0, N_R = n$ is given by the standard partition function formula (no $\bar{q}$ since $N_L = 0$)

$$\sum_n d(n) q^n = 256 \prod_n \left(\frac{1+q^n}{1-q^n}\right)^8 \tag{73}$$

Here 256 reflects the ground state degeneracy. The dimension $d(Q_1 Q_5)$, from the above formula then gives the number of BPS states U-dual to the ground state of the D1-D5 system.

If (72) is true, then its BPS states should reproduce the same number. The supersymmetric states of the moduli space corresponds to its cohomology vector space $H^*(\mathcal{M})$. The dimension of this space indeed equals $d(Q_1 Q_5)/256$ as required by U-duality (the factor 256 arises because the hypermultiplet moduli space is based on gauge group $U(Q_1) \times U(Q_5)$ rather than the SU groups; it is the latter that is dual to the instanton moduli space).

More direct evidence has been provided for the case of $Q_5 = 1$ in which case the five-dimensional gauge theory is trivial and low energy degrees of freedom indeed correspond to (71).

The ansatz about the moduli space leads to the prediction that the low energy excitations are given by a conformal field theory based on the manifold $\mathcal{M}$, as the sigma-model flows in the infra-red to CFT. The supersymmetry $\mathcal{N} = (4,4)$ of the sigma-model enhances in this limit to $\mathcal{N} = (4,4)$ superconformal symmetry.

Comment on resolution of orbifold, Fayet-Iliopoulos parameters and noncommutative geometry

The discussion in this section has not included for lack of space a number of rather interesting issues related to resolutions of the orbifold and marginal deformations of the SCFT. More thorough discussions can be found in the original references mentioned earlier or in [42]. We have also not discussed the general issue of stability of the bound state; it turns out that turning on the marginal operators involved in the resolution of the orbifold corresponds to turning on Fayet-Iliopoulos parameters in the D-term equations (59) and (60). The latter is known to remove the singularity associated with the origin of the Higgs branch which can be interpreted [54] as a D1-brane splitting off from the bound system. In supergravity, on the other hand, a truly bound state of D1 and D5 branes requires the introduction of a NS B-field (without it, the mass of the bound state is the sum of the rest masses, and a D1-brane can split off at no cost of energy). Using the connection between NS B-field through noncommutative geometry (see [56] and references therein) between NS B-field and Fayet-Iliopoulos terms, one can complete the picture (see [57] and references therein) of the stabilized D1/D5 system in supergravity and gauge theory, including the moduli space of deformations around them.

Description of the SCFT degrees of freedom

The basic fields of the above-mentioned SCFT are $x^i_A(z,\bar{z})$, $\psi^{a\alpha}_A(z)$ and $\bar{\psi}^{\dot{a}\dot{\alpha}}_A(\bar{z})$. $A = 1, 2, \ldots Q_1Q_5$ denotes which-th copy of T^4. $i = 6, 7, 8, 9$ denote coordinate labels on T^4. $a, \dot{a}$ denote spinor labels of $SO(4)_I \equiv SO(4)_{6789}$ and $\alpha, \dot{\alpha}$ denote spinor labels of $SO(4)_E$. The Lagrangian is given by

$$S_0 = \int d^2z [\frac{1}{2}\partial x^i_A \bar{\partial} x^i_A + \frac{1}{2}\psi^{a\alpha}(z)\bar{\partial}\psi_{a\alpha} + \frac{1}{2}\psi^{\dot{a}\dot{\alpha}}(\bar{z})\partial\psi_{\dot{a}\dot{\alpha}}] \tag{74}$$

The orbifold in (72) corresponds to various twisted sectors corresponding to conjugacy classes of elements g of the permutation group. It is well-known that the distinct conjugacy classes of $S(m)$ are given by the number distribution n_i of cycles of various lengths l_i. These numbers satisfy

$$\sum_i n_i l_i = m \tag{75}$$

In our case $m = Q_1Q_5$.

Thus, for $S(3)$ the distinct conjugacy classes can be represented as $(1)(2)(3), (12)(3), (123)$.

The fields $x^i_A, \psi^{a\alpha}_A$ in a sector (n_i, l_i) appear as a collection of n_1 strings of length l_1 plus n_2 strings of length l_2, etc. [58] For example, in the maximally twisted sector, corresponding to the maximal cycle of length m, the boundary condition on the bosonic coordinate is

$$\begin{aligned} g : x^i_A(\sigma) \quad &\rightarrow \quad x^{(g),i}_A(\sigma) \equiv x^i_{A+1}(\sigma) && (76)\\ &= \quad x^i_A(\sigma + 2\pi) && (77) \end{aligned}$$

where for $A = m$ we define $A + 1 = 1$.

It is clear that the fields x^i_A satisfying the above boundary condition can be sewn together to form a single field $\tilde{x}^i$ with periodicity $2\pi m$ (corresponds to m times the length of the original circle), defined by

$$\tilde{x}^i(\sigma + 2\pi(A-1), t) \equiv x^i_A(\sigma, t), \sigma \in [0, 2\pi) \tag{78}$$

The sewn field will have a normal mode expansion:

$$\begin{aligned}\tilde{x}^i(\sigma,t) &= (4\pi)^{-1/2}\sum_{n>0}[(\frac{a^i_n}{\sqrt{n}}e^{in(-t+\sigma)/Q_1Q_5} \\ &+ \frac{\tilde{a}^i_n}{\sqrt{n}}e^{in(-t-\sigma)/Q_1Q_5}) + \text{h.c.}]\end{aligned} \tag{79}$$

The twist (76) acts on these oscillators as

$$\begin{aligned} g : a^i_n &\rightarrow a^i_n e^{2\pi i n/Q_1Q_5} \\ g : \tilde{a}^i_n &\rightarrow \tilde{a}^i_n e^{-2\pi i n/Q_1Q_5}\end{aligned} \tag{80}$$

Definition of the microstates

Let us now concentrate on the maximally twisted sector alone. The states $|i\rangle$ are now defined as [59, 60]

$$|i\rangle = \prod_{n=1}^{\infty}\prod_i C(n,i)(a^{i\dagger}_n)^{N^i_{L,n}}(\tilde{a}^{i\dagger}_n)^{N^i_{R,n}}|0\rangle \tag{81}$$

where $C(n,i)$ are normalization constants ensuring unit norm of the state. $|0\rangle$ represents the NS ground state. The present discussion is also valid in the Ramond sector, in which case the ground state will have an additional spinor index but will not affect the S-matrix. We have also suppressed for simplicity the fermion creation operators which can be trivially incorporated.

Gauge invariance

The permutation group $S(m)$ arises as residual gauge symmetry of the Yang-Mills theory, and the microstates $|i\rangle$ above should be invariant under it. Below we show how.

It is clear that the creation operators create KK (Kaluza-Klein) momentum along S^1 (parametrized by x^5). The total left (right) moving KK momentum of (81) (in units of $1/\tilde{R}$, $\tilde{R} \equiv Q_1Q_5R_5$, R_5 being the radius of the S^1) is N_L (N_R), where

$$N_L = \sum_{n,i} nN^i_{L,n}, \quad N_R = \sum_{n,i} nN^i_{R,n} \tag{82}$$

From (80) and (81), we see that

$$g : |i\rangle \to \exp[\frac{2\pi i}{Q_1 Q_5}(N_L - N_R)|i\rangle \tag{83}$$

Now, the total KK momentum carried by $|i\rangle$ is $p_5 = (N_L - N_R)/(Q_1 Q_5 R_5)$. Quantization of the KK charge requires that $p_5 = \text{integer}/R_5$, which implies that

$$(N_L - N_R)/(Q_1 Q_5) = \text{integer} \tag{84}$$

Thus, using (83) and the above equation, we find that the states $|i\rangle$ representing microstates of the black hole are gauge invariant[60][3].

Entropy

It can be argued (see, e.g. [61]; also see below) that for entropy counting, the maximally twisted sector dominates. Thus we are, roughly speaking, left with a free gas of bosons and fermions moving in a large circle (of $Q_1 Q_5$ times the original size).

Now we know that for a free gas of N_B species of bosons and N_F species of fermions, all moving to the left, in a one-dimensional box of length $\tilde{R}$, the total energy is

$$E_L \equiv \frac{N_L}{R_5} \equiv \frac{\tilde{N}_L}{\tilde{R}} = \frac{\pi^2}{6}\frac{\tilde{R}}{\beta_L^2}(N_B + \frac{1}{2}N_F) = \pi^2 \frac{\tilde{R}}{\beta_L^2} \tag{85}$$

$\tilde{R} = Q_1 Q_5 R, \tilde{N} = Q_1 Q_5 N$. Using

$$\partial S_L / \partial E_L = \beta_L(E_L) \tag{86}$$

we get

$$S_L = 2\pi\sqrt{E_L \tilde{R}} = 2\pi\sqrt{N Q_1 Q_5} \tag{87}$$

reproducing the Bekenstein-Hawking result from free 1D gas!

Note that the entropy could also be computed by Cardy's formula, as in (63). The fact that (87) gives the same result provides additional *a posteriori* justification for considering contribution only from the maximally twisted sector!

The above could appear to imply that the physics of the D1/D5 system, at least at low enough energies, could be entirely described by a "long string" picture [62, 63]. We shall see later that this expectation is not right: twisted sectors other than maximal play a crucial role

[3] In (82) we have written down only the bosonic contribution to the KK momentum. If we wrote the total contribution of bosons and fermions, then taking into account the fact that an identical gauge transformation property holds for the fermionic oscillators as in (80), we would arrive at the same conclusion as above, *viz*, that the state $|i\rangle$ is gauge invariant.

and the assumption of a long D-string gives wrong coupling to bulk fields (Section 4). It is essential to go back to the full orbifold SCFT and its deformations for a precise understanding of the D1/D5 system.

Density matrix interpretation

The above derivation of the entropy is based on the assumption that the quantum black hole is represented by a density matrix

$$\rho = \frac{1}{\Omega} \sum_i |i\rangle\langle i| \tag{88}$$

where the sum is over microstates which correspond to the given macroscopic charges Q_1, Q_5, N that are reflected in the geometry. $\Omega =$ the total number of such microstates.

The density matrix reflects an averaging over microstates exactly as we do in statistical mechanics. The deeper issue (on the lines of ergodic theory) of how the density matrix may appear naturally in classical time scales of observation (long compared to some typical mixing times between states) merits detailed investigation.

We will see later that the density matrix ansatz correctly reproduces Hawking radiation from the black hole.

Non-BPS entropy

For the non-extremal black hole (47) with no "anti"-branes (i.e., $\alpha_{1,5} \to \infty, N_{\bar{1},\bar{5}} = 0, \alpha_n < \infty$), we have both left and right moving gravitational waves in the classical solution. This in the CFT should correspond to exciting both L_0 and $\bar{L}_0$ (in the near-extremal case $L_0 \equiv N \gg \bar{L}_0 \equiv \bar{N}$). If we assume that the left and right moving oscillators do not interact, the total degeneracy would be given by

$$\Omega = \Omega_L(c, N) \times \Omega_R(c, \bar{N}) \tag{89}$$

so that

$$S = \log \Omega = 2\pi\sqrt{Q_1 Q_5}(\sqrt{N} + \sqrt{\bar{N}}) \tag{90}$$

which is indeed the Bekenstein-Hawking entropy of the non-extremal black hole as well! This is a surprise, since there is no obvious non-renormalization theorem for these systems which could protect the density of states as the coupling constant is varied from the D-brane regime to the supergravity regime (see below the corresponding discussion for BPS states).

For later use, we note that the left- and right-movers are separately represented by canonical ensembles characterized by temperatures β_L and β_R. By (85) we see that

$$\beta_{L,R} = \pi R_5 \sqrt{\frac{Q_1 Q_5}{N_{L,R}}} \tag{91}$$

We will find that the Hawking temperature is given by

$$\beta_H = \frac{1}{2}(\beta_L + \beta_R) \tag{92}$$

Weak and Strong couplings: BPS property

As has been indicated before, for the D1/D5 system the supergravity description is reliable for $g_{st} \to 0, Q_1, Q_5, N \to \infty$, such that the scaled charges

$$g_{\rm st} Q_1, g_{\rm st} Q_5, g_{\rm st}^2 N \gg 1 \tag{93}$$

On the other hand, the D-brane description is valid when

$$g_{\rm st} Q_1, g_{\rm st} Q_5, g_{\rm st}^2 N \gg 1 \tag{94}$$

Since the masses of BPS states do not change as we change the coupling, the counting of states that we did using the D-brane picture should continue to remain valid when $g_{\rm st}$ is increased from the range of values (94) to (93). It is in this sense that we claim that we have a derivation of the Bekenstein-Hawking formula.

What is surprising, however, is that we have agreement for non-BPS entropy (as mentioned above) and, as we will see, for Hawking radiation and absorption by near-extremal black holes. We will analyze Hawking radiation and absorption now.

4 Absorption/Emission

We saw in the last section that string theory provides an understanding of microstates of the five dimensional black hole. In this section we will come back to some of the questions raised in the introduction, and see if they can be addressed now that we have a microscopic model of black holes.

We will first address the issue of absorption by a black hole. As we remarked, classically the black hole only absorbs and does not emit. We would like to see how this is interpreted in the microscopic model. This would correspond to *an* explanation of a crucial aspect of the event horizon.

Next we will turn on quantum mechanics and would like to see how standard scattering processes described in terms of the microscopic model gives rise to Hawking radiation.

Before we get to the microscopic understanding, let us briefly review the (semi)classical calculations of absorption/emission.

4.1 Semiclassics of absorption/emission

In this section we will show how to obtain black hole absorption cross-section of various particles of type IIB supergravity.

We will mainly consider scalars. These can be minimal, namely if they couple only to the five-dimensional Einstein metric and nothing else. These are the simplest to discuss and we will consider them first. There are other scalars which couple, in addition, to dilaton, Ramond-Ramond potentials etc. and are non-minimal; we will consider later a sub-class of these called fixed scalars.

Minimal scalar

The absorption cross-section and emission rate of a particular field depend on how the field propagates and backscatters from the geometry of the black hole. We will therefore look at the equation of propagation of scalar fluctuations.

Consider type IIB Lagrangian compactified on T^5 [64, 65]:

$$\begin{aligned} S_5 &= \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}[R - \frac{4}{3}(\partial_\mu\phi_5)^2 \\ &\quad -\frac{1}{4}G^{ab}G^{cd}(\partial_\mu G_{ac}\partial^\mu G_{bd} + e^{2\phi_5}\sqrt{G}\partial_\mu B'_{ac}\partial^\mu B'_{bd}) \\ &\quad -\frac{e^{-4\phi_5/3}}{4}G_{ab}F^{(K),a}_{\mu\nu}F^{(K),b\mu\nu} - \frac{e^{2\phi_5/3}}{4}\sqrt{G}G^{ab}H_{\mu\nu a}H^{\mu\nu}_b - \frac{e^{(4\phi_5/3)}}{12}\sqrt{G}H^2_{\mu\nu\lambda}] \end{aligned} \tag{95}$$

Here $a, b, \ldots = 5, \ldots, 9$, $\mu, \nu, \ldots = 0, 1, \ldots, 4$

$$\begin{aligned} \phi_5 &= \phi_{10} - (1/4)\ln[\det_{ab}G_{ab}] \\ H_{\mu\nu a} &= (dA_a)_{\mu\nu} + \ldots,\ A_{a\mu} = B'_{\mu a} + B'_{ab}A^{(K)b}_\mu \\ H_{\mu\nu\lambda} &= (d\tilde{B})_{\mu\nu\lambda} + \ldots,\ \tilde{B}_{\mu\nu} = B'_{\mu\nu} + A^{(K)a}_{[\mu}A_{\nu]a} - A^{(K)a}_\mu B'_{ab}A^{(K)b}_\nu \\ F^{(K)a} &= dA^{(K)a} \end{aligned} \tag{96}$$

$A^{(K)a}$ are the KK vector fields. The terms denoted by ... represent "shifts" in the field strengths which are not important for either the background geometries or fluctuations we are going to consider.

This Lagrangian can be obtained from type IIB Lagrangian in 10 dimensions simply by repeating compactification on a circle several times [64] (can also be obtained by duality on 11 dimensional supergravity).

We will start with an example of a simple minimal scalar.

The near-extremal metric of the five-dimensional black hole is a classical solution of the above Lagrangian. Consider the 10-dimensional form of the metric. Let us consider an off-diagonal metric fluctuation on T^4 viz.

$$ds^2|_{T^4} = f_1^{1/2} f_5^{-1/2} dx^a dx^b (\delta_{ab} + \kappa_5 h_{ab}) \tag{97}$$

where h_{ab} has only off-diagonal entries, say h_{89}.

With this rescaling, the field h_{ab} is canonically normalized; that is, the quadratic part of the Lagrangian involving h_{ab} is

$$S = \frac{1}{2} \int d^5x \sqrt{-g} \partial_\mu h_{ab} \partial^\mu h_{ab} \tag{98}$$

It couples only to the five-dimensional metric, and is hence a minimal scalar.

For the semiclassical analysis [59, 63, 66], all we need is the equation for propagation of the fluctuation $\varphi \equiv h_{ab}$ on the black hole metric, namely

$$D_\mu \partial^\mu \varphi = 0 \tag{99}$$

For the five-dimensional black hole metric discussed earlier the equation becomes for the s-wave mode:

$$[\frac{h}{r^3} \frac{d}{dr} (h r^3 \frac{d}{dr}) + f w^2] R_w(r) = 0 \tag{100}$$

where

$$\varphi = R_w(r) \exp[-iwt] \tag{101}$$

The idea behind the absorption calculation is very simple. In terms of $\psi = r^{3/2} R$ the above equation becomes

$$[-\frac{d^2}{dr_*^2} + V_w(r_*)] \psi = 0 \tag{102}$$

where

$$V_w(r_*) = -w^2 f + \frac{3}{4r^2}(1 + 2r_0^2/r^2 - 3r_0^4/r^4) \tag{103}$$

The shape of the potential is given by (Fig 2). Absorption is caused by the tunnelling of an

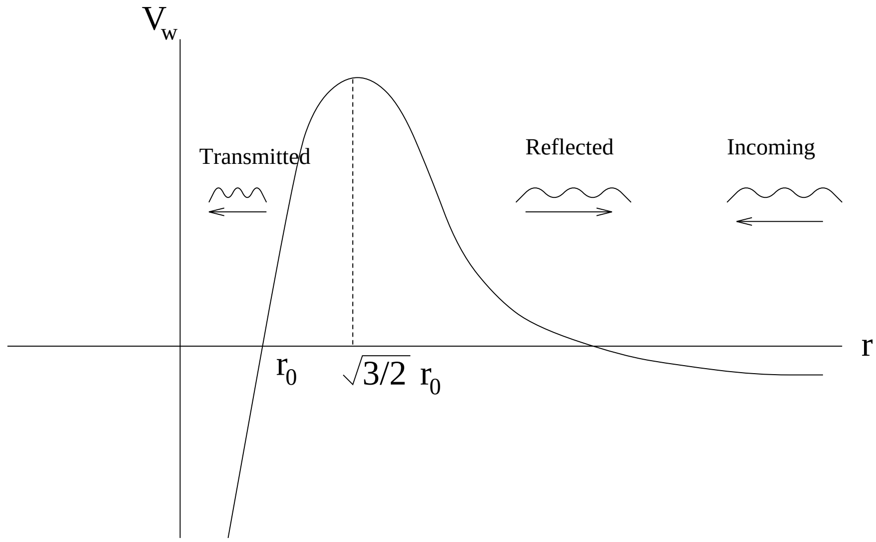

Figure 2: Potential for minimal scalar

incoming wave into the "pit of the potential".

Near and Far solutions:

It is not possible to solve the wave equation exactly. However, we can devise near and far zones where the potential simplifies enough to admit known solutions. If the zones have an overlap region then matching the near and far wave-functions and their radial derivatives will provide the solution for our purpose. We will work in the following range of frequency and parameters [66]

$$\begin{aligned} r_0, r_n &\ll r_1, r_5 \\ w r_5 &\ll 1 \\ w/T_{R,L}, r_1/r_5, r_0/r_n & \quad O(1) \end{aligned} \tag{104}$$

$T_{R,L} = 1/\beta_{R,L}$ have been defined in (91). The far and near solutions will be matched at a point r_m such that

$$r_0, r_n \ll r_m \ll r_1, r_5, \qquad w r_1 \ll r_m/r_1 \tag{105}$$

Far zone ($r > r_m$):

Here the potential V_w becomes (in terms of $\rho = wr$)

$$V_w(\rho) = -w^2(1 - \frac{3}{4\rho^2}) \tag{106}$$

This gives a Bessel equation, so that

$$\begin{aligned} \psi &= \alpha F(\rho) + \beta G(\rho) \\ F(\rho) &= \sqrt{\pi\rho/2} J_1(\rho), \quad G(\rho) = \sqrt{\pi\rho/2} N_1(\rho) \end{aligned} \tag{107}$$

For $\rho \to \infty$ it is easy to read off the parts proportional to $e^{\pm iwr}$.

Near zone ($r < r_m$):

Here we have

$$\frac{h}{r^3}\frac{d}{dr}(hr^3\frac{d}{dr}R) + [\frac{(wr_nr_1r_5)^2}{r^6} + \frac{w^2r_1^2r_5^2}{r^4}]R_w(r) = 0 \tag{108}$$

which is a Hypergeometric equation, with solution

$$\begin{aligned} R &= A\tilde{F} + B\tilde{G} \\ \tilde{F} &= z^{-i(a+b)/2}F(-ia, -ib, 1 - ia - ib, z) \\ \tilde{G} &= z^{i(a+b)/2}F(-ia, -ib, 1 - ia - ib, z) \\ z &= (1 - r_0^2/r^2) \\ a &= w/(4\pi T_R), b = w/(4\pi T_L) \end{aligned} \tag{109}$$

The temperatures $T_{R,L}$ are given by

$$T_{L,R} = \frac{r_0}{2\pi r_1 r_5}e^{\pm\alpha_n} \tag{110}$$

These expressions agree with (91).

Now we impose the condition on the near solution that the wave at the horizon should not have any outcoming component: it should be purely ingoing (no "white hole"). This requires $B = 0$.

Matching

We now match R and $\frac{d}{dr}R$ between the near and far regions at some point r_m in the overlap region.

This gives

$$\begin{aligned} \sqrt{\pi/2}w^{3/2}\alpha/2 &= Ae_1 \\ e_1 &\equiv \frac{\Gamma(1-ia-ib)}{\Gamma(1-ib)\Gamma(1-ia)} \\ \beta/\alpha &\ll 1 \end{aligned} \tag{111}$$

<u>Fluxes</u>

The equation for R implies

$$\frac{d}{dr}\mathcal{F} = 0 \tag{112}$$

where

$$\mathcal{F}(r) = \frac{1}{2i}[R^* hr^3 dR/dr - \text{c.c.}] \tag{113}$$

In order to find out what fraction of the flux gets absorbed at the horizon, we compute the ratio

$$R_1 = \mathcal{F}(r_0)/\mathcal{F}^{in}(\infty) = r_0^2 \frac{a+b}{w|e_1|^2} w^3 \pi/2 \tag{114}$$

where the superscript "in" indicates the flux calculated from the "ingoing" part of the wave at infinity.

Absorption Cross-section:

In order to define absorption cross-section in the standard way, one has to consider plane waves and not s-waves. Using the fact that

$$e^{-iwz} = (4\pi/w^3)e^{-iwr}Z_{000} + \text{other partial waves} \tag{115}$$

and the standard definition of absorption cross-section we get [66]

$$\begin{aligned} \sigma_{abs} &= (4\pi/w^3)R_1 \\ &= 2\pi^2 r_1^2 r_5^2 \frac{\pi w}{2} \frac{\exp(w/T_H)-1}{(\exp(w/2T_R)-1)(\exp(w/2T_L)-1)} \end{aligned} \tag{116}$$

In the $w \to 0$ limit, one gets [59]

$$\sigma_{abs} = A_h \tag{117}$$

where A_h denotes the area of the event horizon.

Hawking radiation:

The semiclassical calculation of Hawking radiation is performed through the standard route of finding Bogoliubov coefficients representing mixing of negative and positive frequency modes due to evolution from "in" to "out" vacua, relevant for Minkowski observers existing in the asymptotically flat regions at $t = -\infty$ and $t = +\infty$ respectively [10]. The rate of radiation is given by

$$\Gamma_H = \sigma_{abs}(e^{w/T_H} - 1)^{-1} \frac{d^4k}{(2\pi)^4} \tag{118}$$

We will see below how Γ_H and σ_{abs} are reproduced in the D-brane picture.

4.2 D brane description

In this section we will discuss the case of minimal scalars.

We have seen that the entropy of the five-dimensional black hole can be reproduced in the weak coupling regime by representing it as a density matrix

$$\rho = \frac{1}{\Omega} \sum_i |i\rangle\langle i| \tag{119}$$

where $|i\rangle$ represent the microstates.

The picture of absorption in the microscopic description [59] is as follows. Consider throwing in a closed string mode, say a minimal scalar φ_n, towards the D1-D5 configuration. A microstate $|i\rangle$ will couple to such a fluctuation in a certain way through some interaction

$$S_{int} = \int d^2z \varphi_n|_B O_n(z, \bar{z}) \tag{120}$$

and will get excited to a different state $|f\rangle$ with the amplitude

$$S_{if} = \langle f|H_{int}(|i\rangle|\psi_c\rangle) \tag{121}$$

where $|\psi_c\rangle$ represents the closed string mode. $\varphi_n|_B$ means here the value of the supergravity mode on the brane.

Since we started from a density matrix description for the initial state rather than individual microstates, the probability of the process would be given by the "unpolarized" or "inclusive" expression:

$$\text{Prob}_{abs} = \frac{1}{\Omega} \sum_i \sum_f |S_{if}|^2 \tag{122}$$

Note that the "unpolarized" transition probability corresponds to *averaging* over initial states and *summing* over final states. Ω is the total number of initial microstates representing the macroscopic charges of the black hole.

The most crucial ingredient in this calculation is to figure out what H_{int} (or S_{int}) is. In particular, given a particular supergravity mode, what the corresponding operator $O(z, \bar{z})$ is. This is the issue we discuss next.

Coupling supergravity modes to D-branes[60]

We will work with the picture of microstates obtained from the instanton moduli space described in the last section.

As we remarked above, the coupling of a minimal scalar h_{ij} to the D-brane degrees of freedom is given in the form of the interaction

$$S_{\text{int}} = \mu \int d^2 z h^{ij}|_B O_{ij}(z, \bar{z}) \tag{123}$$

Here $h_{ij}|_B$ denotes the restriction of h_{ij} to the location of the SCFT, and μ is a number denoting the strength of the coupling. (the indices i, j in this section will take values 6,7,8,9)

The question is, what is the SCFT operator $O_{ij}(z, \bar{z})$?

One way to determine it, of course, would be to reanalyze the instanton moduli space or the hypermultiplet moduli space with the metric of the T^4 deformed by h_{ij}. With the present level of technology this is not very feasible.

A simpler but more elegant approach is to appeal to *symmetries*. The steps involved are:
(a) find the symmetries $\mathcal{S}$ of the bulk,
(b) find how (all, or a part of) these symmetries appear in D-brane world-volume and consequently how they act on the variables of the SCFT,
(c) find how h_{ij} transforms under $\mathcal{S}$, and
(d) demand that $\mathcal{O}_{ij}$ should transform under the *same* representation of the symmetry group $\mathcal{S}$ when it acts on the SCFT.

The last step arises from the fact that $h_{ij}(\bar{z})$ in (123) is a source for $\mathcal{O}_{ij}$.

We have already indicated that the symmetries $\mathcal{S} = SO(4)^I \times SO(4)^E$ of the bulk theory (tangent group of the 4-torus and rotation in the transverse space respectively) appear naturally in the SCFT of the D-brane world volume as well. The $SO(4)^I$ part is obvious; $SO(4)^E$ appears as the R-parity group.

Let us now apply steps (c) and (d) above in the context of this symmetry group $\mathcal{S}$.

The field h_{ij} (symmetric, traceless) transforms as $(\mathbf{3}, \mathbf{3})$ under $SO(4)^I \equiv SU(2)^I \times SU(2)^I$ and as $(\mathbf{1}, \mathbf{1})$ under $SO(4)^E \equiv SU(2)^E \times SU(2)^E$.

Now there are at least three possible SCFT operators which belong to the above representation of $\mathcal{S}$:

$$\begin{aligned}
\mathcal{O}_{ij} &= \partial x^i_A \bar{\partial} x^j_A \\
\mathcal{O}'_{ij} &= \psi^{\alpha}_{aA}(z)\sigma^{a\dot{a}}_i \bar{\psi}^{\dot{\alpha}}_{\dot{a}B}(\bar{z})\psi_{\alpha,bA}(z)\sigma^{b\dot{b}}_j \bar{\psi}_{\dot{\alpha},\dot{b}B}(\bar{z}) \\
\mathcal{O}''_{ij} &= \psi^{\alpha}_{aA}(z)\sigma^{a\dot{a}}_i \bar{\psi}^{\dot{\alpha}}_{\dot{a}A}(\bar{z})\psi_{\alpha,bB}(z)\sigma^{b\dot{b}}_j \bar{\psi}_{\dot{\alpha},\dot{b}B}(\bar{z})
\end{aligned} \tag{124}$$

The spinor labels are raised/lowered above using the $\epsilon^{\alpha\beta}, \epsilon_{\alpha\beta}$ symbol. The σ_i's denote the matrices : $(1, i\vec{\tau})$. The last two operators differ only in the way the $S(Q_1Q_5)$ labels are contracted. All the three operators should be symmetrized (i, j) and projected onto the traceless part.

The complete list of operators with the same transformation property under $\mathcal{S}$ contains, in addition, those obtained by multiplying any of the above by singlets. These would necessarily be irrelevant operators, but cannot be ruled out purely by the above symmetries.

It might seem 'obvious' that the operator $\mathcal{O}_{ij}$ should be the right one to couple to the bulk field h_{ij}. However, the simplest guesses can sometimes lead to wrong answers, as we will see later for fixed scalars, where it will turn out that the operator $\partial x^i_A \bar{\partial} x^i_A$ is far from being the right one to couple to h_{ii} (trace). We proceed, therefore, to look more closely into symmetries to lift the degeneracy of the operators.

Fixing the operator using near-horizon Symmetry

It has been conjectured recently that if one takes the large gQ $(g = g_{\rm st}, Q = Q_1, Q_5)$ limit (which corresponds to the near-horizon limit in supergravity and the large 'tHooft coupling limit in the gauge-theory of the brane world-volume), a powerful correspondence [29, 67, 68] can be built between the physics of the bulk and the physics of the branes at the boundary (for a review, see [69]). For the scaling of various quantities in this limit, see the references just mentioned.

Let us consider for a moment the supergravity solution for the D1/D5 system (27). In the limit $gQ_{1,5} \gg 1$, we have $r_{1,5} \gg l_s$. It is easy to see that the metric becomes

$$ds^2_{10} = ds^2_{AdS_3} + ds^2_{S_3} + ds^2_{T^4} \tag{125}$$

where

$$\begin{aligned}
ds^2_{AdS_3} &= \frac{r^2}{R^2}(-dudv) + \frac{R^2}{r^2}dr^2 \\
ds^2_{S_3} &= R^2 d\Omega_3^2 \\
ds^2_{T^4} &= \frac{r_1}{r_5}(dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2)
\end{aligned} \tag{126}$$

In the above, $R^2 \equiv r_1 r_5$.

In order to arrive at the black hole solution (28),(47), we have to first compactify x^5. In terms of the near-horizon limit, this turns [70] AdS$_3$ into a BTZ black hole (with zero mass and angular momentum; turning on momentum modes along x^5 corresponds to mass and angular momentum of the BTZ solution). In the SCFT, this corresponds to switching from the NS sector to the Ramond sector. For our purposes here it will be enough to continue to work with the AdS$_3$ description since the local operators of interest here can be shown to have the same symmetry properties irrespective of whether they belong to the Ramond or the NS sector. The ground state of the Ramond sector is degenerate, as against that of the NS sector; this degeneracy would be reflected in our construction of the black hole state — however, this would not affect the S-matrix relevant for absorption and emission.

It is easy to see that the symmetry group of the solution (125) is enhanced to

$$\mathcal{S} \to \mathcal{S}' = SO(4)^I \times SO(4)^E \times SU(1,1|2) \times SU(1,1|2) \tag{127}$$

The factor $SO(4)^I$ appears as before. The group $SO(4)^E$ corresponds now to the isometry group of S^3. The bosonic part of $SU(1,1|2) \times SU(1,1|2)$ arises as the isometry group of AdS_3 (which is the $SL(2,R)$ group manifold). The $SU(2)$ part which transform the fermions among themselves, and the "off-diagonal" part of $SU(1,1|2)$, are a consequence of $\mathcal{N} = (4,4)$ supersymmetry of this compactification.

On the SCFT side, the $SO(4)_{I,E}$ groups have actions as explained before. The $SU(1,1|2)$ is identified with the subgroup of the superconformal algebra generated by $L_{\pm 1,0}, G^{a\alpha}_{\pm 1/2}$ (the other $SU(1,1|2)$ involves $\bar{L}, \bar{G}$;. indices have been explained above (74)).

Let us now apply steps (c) and (d) of Method 2 to this enhanced symmetry group $\mathcal{S}$. How does h_{ij} transform under $SU(1,1|2)$?

States	j	L_0	degeneracy
$\|\rangle$	h	h	$2h+1$
$\bar{G}^1_{-\frac{1}{2}}\|\rangle, G^2_{-\frac{1}{2}}\|\rangle$	$h-\frac{1}{2}$	$h+\frac{1}{2}$	$2h+2h$
$\bar{G}^1_{-\frac{1}{2}}G^2_{-\frac{1}{2}}\|\rangle$	$h-1$	$h+1$	$2h-1$

Here j denotes the j-value of R-parity group $SU(2)$ of $SU(1,1|2)$. Since our minimal scalar h_{ij} is a singlet of $SO(4)_E$, it has $j=0$. The fact that it is a massless supergravity mode, leads to $L_0=1$. From the table, such a multiplet corresponds to $h=\frac{1}{2}$ and the field h_{ij} fits into the middle row. For $h=\frac{1}{2}$, the middle row corresponds to the "top" component of the supermultiplet, since it is annihilated by the raising operators $\bar{G}^1_{-\frac{1}{2}}, G^2_{-\frac{1}{2}}$.

From our list of candidate operators $\mathcal{O}_{ij}, \mathcal{O}'_{ij}$ and $\mathcal{O}''_{ij}$:
only $\mathcal{O}_{ij}=\partial x^i_A \bar{\partial} x^j_A$ *belongs to this representation.*

For a more detailed matching of various supergravity moduli with short multiplets of the SCFT, see [71].

S-matrix

We have found above that

$$S_{\rm int} = \frac{\mu}{2}\int d^2z\ \left[h_{ij}\partial_z x^i_A \partial_{\bar{z}} x^j_A\right] \tag{128}$$

We have omitted a factor of effective string tension appearing in front of both $S_{\rm int}$ and S_0 (in (74)), since the factor cancels in the S-matrix between the interaction Lagrangian and the external leg factors. However, the value of μ *is important* to determine since the absorption cross-section and Hawking radiation rates calculated from the SCFT depend on it. We do not have space to detail the argument but the quantitative version of Maldacena conjecture demands that $\mu=1$.

Let us now restrict our attention to the maximally twisted sector of the orbifold Hilbert space, as in the case of the entropy calculation. This would imply, as before, that the fields x, ψ live on a large circle, of radius

$$\tilde{R} = Q_1 Q_5 R_5 \tag{129}$$

Using the interaction Hamiltonian obtained above, and considering the example of h_{89}, we get for the process

$$h_{89}(w,0) \to x^8_L(w/2,-w/2) + x^9_R((w/2,w/2) \tag{130}$$

(the numbers in parenthesis denote (k_0, k_5))

$$S_{if} = \frac{\sqrt{2}\kappa_5 w_1 w_2 \tilde{R}\delta_{n_1,n_2} 2\pi\delta(w-w_1-w_2)}{\sqrt{w_1\tilde{R}w_2\tilde{R}wV_4}}\sqrt{N^8_{L,n_1}}\sqrt{\tilde{N}^9_{R,n_2}} \tag{131}$$

$V_4 =$ volume of the noncompact space (box normalization). The notation $N^i_{L,n}$ and $N^i_{R,n}$ denotes number distribution of oscillators with left- and right-moving momentum n respectively (see (81)). The factors of $\sqrt{N}$ appear from the identity

$$\langle N-1|a|N\rangle = \sqrt{N} \tag{132}$$

where $|N\rangle = (a^\dagger)^N/\sqrt{N!}|0\rangle$, $[a,a^\dagger]=1$.

—o—

Seeing Hair: Eqn. (131) shows that the S-matrix describing emission (or absorption) of waves of a given frequency w contain information about the number distribution of quanta of frequencies $w/2$. By repeating this for all w (subject to the overall condition that the frequencies are not too high), we can get the (limited) information $\sqrt{N_L(w/2)N_R(w/2)}$ for all such w. If, however, we are interested in the "inclusive" processes (like in the case of unpolarized cross-sections for standard particle physics experiments), then one computes probabilities by taking a modulus-square of the S-matrix, *sum* over the final states and and *average* over the initial states.

—o—

Thus, the probability of absorption of the quantum of frequency w is

$$\begin{aligned} \mathrm{Prob_{abs}} &= \Omega^{-1}\sum_{i,f}|S_{if}|^2 \\ &= \frac{\tilde{R}T}{V_4}\kappa_5^2 w\langle N_L(w/2)\rangle\,\langle N_R(w/2)\rangle \end{aligned} \tag{133}$$

Here T is the length of the time-direction.

The decay probability is obtained by considering the reverse process. One gets

$$\mathrm{Prob_{decay}} = \frac{1}{\Omega'}\sum_{i,f}|S_{if}|^2$$

$$= \frac{\tilde{R}T}{V_4}\kappa_5^2 w\langle N'_L(w/2)\rangle\,\langle N'_R(w/2)\rangle \tag{134}$$

where the primed number distribution in the last line refers to the final state. Ω' = total number of final microstates.

For the mode $w/2 = n_1/\tilde{R}$, $N_{L,R}(n_1) = N'_{L,R}(n_1) + 1$. The classical absorption probability should be compared with $\mathrm{Prob}_{\mathrm{abs}} - \mathrm{Prob}_{\mathrm{decay}}$ of the string calculation.

σ_{abs} is defined by

$$\begin{aligned} \sigma_{abs} \times \text{speed of particles} \quad &\times \quad \text{particles/volume} \equiv \text{Rate} \\ &\equiv \quad \mathrm{Prob}_{\mathrm{abs}}/T \end{aligned} \tag{135}$$

Here, particles have speed = 1. Also number of particles particles per unit volume $= 1/V_4$ (box normalization).

Putting in all the above, we get

$$\sigma_{abs} = 2\pi^2 r_1^2 r_5^2 \frac{\pi w}{2} \frac{\exp(w/T_H) - 1}{(\exp(w/2T_R) - 1)(\exp(w/2T_L) - 1)} \tag{136}$$

which is the same expression as we had obtained semiclassically.

The decay rate is given by

$$\Gamma = \mathrm{Prob}_{\mathrm{decay}} \frac{V_4}{\tilde{R}T} \frac{d^4k}{(2\pi)^4} \tag{137}$$

giving

$$\Gamma_H = \sigma_{abs}(e^{w/T_H} - 1)^{-1} \frac{d^4k}{(2\pi)^4} \tag{138}$$

which exactly reproduces the semiclassical result.

We summarize this section by making the following comments:

1. **The black hole is a black body:** This *nearly* completes the derivation that so far as the Hawking radiation of minimal scalars is concerned, the emission from the black hole under discussion is really that from a black body. The reason we say "nearly" is that the strength of the interaction Hamiltonian is still not determined from first principles, but rather by using the postulate of the AdS/CFT correspondence [60].

2. **The black hole arrow of time is the same as the thermodynamic arrow of time**: In the limit $\hbar \to 0$, the enhancement factor Ω'/Ω, representing the ratio of absorption versus

emission probabilities, blows up and the decay rate goes to zero in precisely such a way that the absorption cross-section is finite and reproduces the classical calculation precisely. This is how a black hole classically only absorbs and does not emit. As we see, the explanation comes merely from the way we calculate the inclusive processes here, by summing over the final states and averaging over the initial states; this is the assumption of randomization or ergodization at the quantum mechanical level, giving rise to thermodynamics.

3. **Randomization**: As emphasized above, we have represented the state of a black hole by a density matrix in stead of in terms of any specific microstate. The assumption made here is that of randomization standard in any thermodynamic system. In most simple thermodynamic systems, we have a mechanism in mind how some interaction, characterized by some time scale, between the microstates causes a hopping between them, leading ultimately to density matrices. In the case of the black hole microstates, as long as we are in the conformal field theory description, the microstates described above are strict eigenstates and there is no mixing between them. In order to see the mixing, we need to go away from the infra-red fixed point and go back to the effective sigma-model describing the gauge theory of the D1/D5 system. It will be extremely interesting to estimate the time scale of the mixing from this and to understand how a semiclassical calculation of Hawking radiation bypasses all this and anticipates randomization in an in-built way.

4.3 Fixed scalar

The above example describes the emission/absorption of any scalar that couples only to the five-dimensional Einstein metric. It is clear from the type II Lagrangian in five dimensions (95) that not all scalars are that way. For example, consider scalar fluctuations described by

$$(ds_{10}^2)_{T^5} = e^{2\nu_5} dx_5^2 + e^{2\nu}(dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2) \tag{139}$$

The field ν appearing above, and

$$\lambda = \frac{3}{4}\nu_5 - \frac{1}{2}\phi_5 \tag{140}$$

are examples of "fixed" scalars, which couple to the KK vector field strengths and to the RR B'-field.

The wave equations are fairly complicated. The solutions are obtained [73, 72, 74] by matching behaviours of solutions that are valid in a near region, intermediate region and a far region. The low energy absorption cross-section that follows vanishes as $w \to 0$.

D-brane picture

The first attempt at calculating the absorption/emission in the D-brane picture was made [65] by guessing the following form of $S_{\rm int}$:

$$S_{\rm int} = \int d^2z[\lambda|_B(\mathcal{O}_{3,1} + \mathcal{O}_{1,3} + \mathcal{O}_{2,2}) + \nu|_B(\mathcal{O}'_{3,1} + \mathcal{O}'_{1,3} + \mathcal{O}'_{2,2})] \tag{141}$$

The subscripts refer to values of $h, \bar{h}$. This form of the interaction was guessed by imagining the degrees of freedom of the D1-D5 system to be those of a D-string and coupling it to supergravity through a Dirac-Born-Infeld action.

The absorption/emission rates obtained thus were at variance with the semiclassical calculation.

By applying the method we described above in the context of the minimal scalars, namely by using the near-horizon symmetry, we find that only the (2,2) operators are allowed. Since the earlier discrepancy was caused by the coupling to (1,3) and (3,1) operators, we get agreement between semiclassical calculation and D-brane picture [60].

5 Discussion

We have touched on several open problems in the course of this review. We have not had time to go into some others which I find rather exciting. I will end this discourse with a short description of some of these.

5.1 Correspondence principle

In our previous discussion we described our understanding of the physics of the D1/D5 system and the five-dimensional black hole in terms of D-brane microstates. It is clear that supersymmetry has played an important role in the entire discussion. The question that naturally arises is: how essential is the role of supersymmetry? Another related point is that the understanding is rather too detailed for comfort and any universality, if at all, is fairly non-obvious.

An alternative way to address the question is to ask how generic is the fact that black holes can be understood as states in a string theory. Does a very massive string state always give rise to a black hole irrespective of whether or not supersymmetry is present? To answer this, let us consider an elementary string state of mass M and increase the coupling from the string regime ($r_h \ll l_s$) to the supergravity regime $r_h \gg l_s$ (by r_h we mean the radius of the horizon). It was emphasized by Susskind [75, 76] that for the 4D Schwarzschild black hole, the entropy formula (as a function of mass) given by the string theoretic expression

$$S_{\rm string} \sim \sqrt{N} \sim \sqrt{\alpha'} M \tag{142}$$

has a different functional form from the Bekenstein formula

$$S_{\rm BH} \sim G_N M^2, \tag{143}$$

A straightforward identification of the black hole as a state in a string theory therefore does not seem to be feasible. Susskind argued, however, that the string theoretic entropy is calculated at string perturbation theory (in fact at the tree level) and such a calculation need not be valid in the supergravity regime. It should not come as a surprise, therefore, if the density of states undergoes a change in form through renormalization.

Horowitz and Polchinski [77] argued that a crucial test of this idea (whether an elementary string collapses into a black hole or not) is to find a region of overlap between the regions of validity of the string description and the (super)gravity description and to see if the formulae agree there. They found (upto a constant of order one) a correspondence point $g = g_c$ below which the string description should be valid and above which the gravity description should be valid. They found in a wide variety of cases that the two expressions for entropy match at the correspondence point, except possibly upto a numerical constant. This establishes a correspondence principle for the scenario of a string collapsing into a black hole.

We quote below the simplest case considered in [77], that of the 4-D Schwarzschild black hole. The functional forms of the entropy, in the string and in the gravity regime, are different, as mentioned above. Let us find the respective regions of validity of the string and the gravity pictures. Clearly, for the gravity picture to be reliable, the Schwarzschild radius r_h should be large compared to the string length $l_s = \sqrt{\alpha'}$, that is

$$G_N M \sim l_s^2 g_{\rm st}^2 M \gg l_s \Rightarrow M \gg M_c \equiv \frac{1}{g_{\rm st}^2 l_s} \tag{144}$$

The string description, on the other hand, should be valid at weak coupling $g_{\rm st}$ where the mass of the string will be smaller than M_c. The transition point is given by

$$M = M_c = \frac{1}{g_{\rm st}^2 l_s} \Rightarrow g_{\rm st} = (l_s M)^{-1/2} \tag{145}$$

If we evaluate the string entropy and the Bekenstein-Hawking entropy at this point we get

$$S_{\rm string} \sim l_s M_c \sim 1/g_{\rm st}^2 \tag{146}$$

and

$$S_{\rm BH} \sim l_s^2 g_{\rm st}^2 M_c^2 \sim 1/g_{\rm st}^2 \tag{147}$$

Thus the two distinct entropy formulae match at the transition point upto a possible numerical constant. The numerical constant cannot be fixed more accurately since the correspondence point itself is known only upto a numerical constant.

Having seen that the entropies (142) and (143) go over to each other at the transition point, one naturally wonders about the *mechanism* of the transition between the two behaviours. In particular, whether it is possible to understand (143) by including the effect of interactions in the string picture. Such an attempt has been made in [78, 79]. The idea in these references is that it is the gravitational self-interaction of the strings that renormalizes the density of states to convert it into the one appropriate for the black hole.

It is very important to understand in detail the physics of the above transition since it may provide some universal clues to gravitational collapse into a generic black hole. An interesting direction of investigation would be to try to understand such a collapse in the context of D-brane black holes (extremal as well as non-extremal)[4]. Two of these black holes have been discussed extensively in the literature: the five-dimensional black hole described in this review and the seven dimensional black hole whose BPS limit is the (wrapped) D3-brane. These systems (more precisely their near-horizon counterparts, related to $AdS_3 \times S^3 \times T^4$ and $AdS_5 \times S_5$ respectively) have a dual description: in terms of (a) string theory/supergravity and (b) gauge theory/conformal field theory. The discussion of a string-black hole transition would seem to appear naturally in the string/supergravity description, but should have interesting correspondence with the phase structure of the gauge theory. In the context of D3-branes, the confinement/deconfinement transition of the world-volume gauge theory has been related [83] to a Hawking-Page [84] phase transition in AdS_5 gravity. It would be interesting to compare this transition with a possible string-black hole transition in this system. The Hawking-Page transition occurs as one varies the temperature whereas the string-black hole transition occurs as one varies $g_{\rm st}$. Both cases involve, in a sense, collapse of strings into black holes: in the thermal case, strings in empty AdS space condense to form a black hole whereas in the other case strings in flat space condense to form a black hole. Thermal phase transitions have also been described [70, 85] in the $\mathrm{AdS}_3/\mathrm{CFT}_2$ framework which arises in the context of the D1/D5 system. The possibility of a connection with a string-black hole transition would be interesting to explore in this case as well.

Another possible insight into the nature of the string-black hole transition might be obtained from a study of stable non-BPS states [86]. Typically, generic massive string states that are non-BPS have a decay width which increases as $g_{\rm st}$ increases. These states therefore get mixed up with string states at other mass levels, thus complicating the issue of how to "identify" these states as $g_{\rm st}$ goes up. For the stable non-BPS states, this problem is not there, and in principle they can be identified at strong coupling as well. In a number of cases they have been identified and their masses at strong coupling have been obtained exactly using duality arguments [86]. These therefore provide examples of exact mass renormalizations and it may be instructive to compare with the mass renormalizations envisaged in the string-black hole transition.

[4]It should be emphasized that even in the BPS cases where the density of states is not renormalized (or in some non-BPS cases where, as mentioned in the next subsection, the renormalization is only by a numerical factor) there are non-trivial features of collapse into a black hole which bear investigation. For example, a string state at weak coupling is unlikely to satisfy no-hair theorems (see, e.g., [80, 81]) whereas in the domain of validity of classical supergravity, black holes are expected to satisfy no-hair theorems (see [82], though, for possible counterexamples). Such change of behaviour, described possibly by an order parameter related to some "hair", may be gradual or in the form of a phase transition. We use below the phrase "string-black hole transition" keeping such possibilities in mind.

5.2 Obedient non-supersymmetric black holes

A rather important aspect of nonsupersymmetric black holes is the existence of examples where string/D-brane quantities agree with results from (super)gravity without the help of any obvious supersymmetry non-renormalization theorems. These are important to study since they may provide us with a principle other than supersymmetry to understand these black holes. We mention briefly a few examples (these are not exhaustive):

- (1) We have already mentioned in the context of the non-extremal black hole in five dimensions that the D-brane entropy formula (90) agrees with the semiclassical formula although the system is non-BPS.

- (2) The agreement for the absorption cross-section and Hawking rate between the D-brane calculation and supergravity is also surprising since they refer to near-extremal black holes. It is not entirely clear for what range of energies above extremality (and for what precise reason) it should be possible to extend BPS non-renormalization arguments.

- (3) There are examples of elementary heterotic string states [87] which are non-BPS (although extremal), but whose mass renormalizations are bound, to all loop order, by $1/M$ (rather than by M as one would expect normally). This explains why the string entropy of these states matches, in the sense of [88], the semiclassical entropy of the stretched horizon of the corresponding black hole solution [89]. The mechanism for the existence of the bound, however, is rather technical and very stringy, and merits a simpler understanding.

- (4) There is a type I black hole [90], closely related to the 5D black hole discussed in these lectures, but with no supersymmetry, whose entropy agrees with the Bekenstein-Hawking formula. It has been argued [91] that there appears to be a large N (where N refers to Q_1, Q_5) understanding of this phenomenon.

- (5) In the context of scattering involving a system of D0- and D2-branes [92] or D0- and D6-branes [93], one finds agreement between a matrix theory calculation and supergravity calculation in the limit when the D0- charge is much larger than the D2- (respectively D6-) charge (equivalent to a large boost along x^{10} of M-theory). Ordinarily, one could say that this is an example of effective restoration of supersymmetry by a large boost (this is similar to the fact that in (39) extremality is equivalent to a large boost δ). However, the agreement in question here is in the static potential between branes which does not even exist for BPS branes. The agreement here, therefore, does not automatically follow from supersymmetry.

- (6) The entropy of the near-extremal D3-brane, as calcaluted from the world-volume gauge theory, matches [94], upto a factor 4/3, the entropy calculated from supergravity. This is related to the demonstration in the context of AdS/CFT correspondence, that the entropy of an AdS-Schwarzschild black hole in five dimensions is exactly reproduced from SYM theory at finite temperature, upto a constant of proportionality [83]. Although the agreement is not exact, it is quite remarkable that, unlike in the Schwarzschild case mentioned in the last section, the functional form of the entropy is the same at both ends.

- (7) The leading term in the high temperature partition function of a BTZ black hole agrees [85] with the partition function of the two-dimensional CFT of the brane world-volume. The interesting point is that the agreement includes the case of zero angular momentum which is a completely nonsupersymmetric configuration. The agreement is exact and there are no factors.

It is clear that there would be considerable progress towards understanding nonsupersymmetric black holes and nonsupersymmetric gauge theory if one can tie up the above (and similar other) examples alongwith the observations in the last section on the phase transition of a string state into a black hole.

5.3 The AdS/CFT conjecture and Complementarity

One of the important aspects of black hole physics is the issue of complementarity, which says, very roughly speaking, that the physics outside the horizon can encode information of "stuff" inside that "makes" the black hole. In a way the AdS/CFT correspondence [29, 67, 68] captures some of the essence of complementarity. To elaborate, the Hilbert space of the gauge theory/CFT on the brane-world-volume contains information about the microstates of the black hole. Normally one does not suppose supergravity states outside the horizon to carry information about these microstates. However, supergravity states in the near-horizon AdS geometry, by virtue of the AdS/CFT correspondence, are in one-to-one correspondence with the spectrum of operators of the gauge theory/CFT. This implies that physics outside the horizon can, after all, encode complete information about gauge theory fluctuations which may in a sense be regarded as degrees of freedom inside the black hole. It is important to mention, however, that the holographic correspondence has some crucial nonlocal features in it. For example, local (delta-function) boundary fluctuations on the brane, propagated to the bulk by means of the boundary-bulk Green's function [67], have nonlocal support. Similarly, local degrees of freedom in the bulk get holographically projected to nonlocal fluctuations on the boundary. This is also reflected in the nonlocality of the operator algebra [95] on the boundary. Such nonlocality accords with the expectation [96] that in a completely local field theory one cannot have complementarity.

We would like to mention in this context that the AdS/CFT correspondence appears to necessitate a closer look at Hawking's original derivation of thermal radiation from a black hole [10]. One of the crucial assumptions in [10] is that the Hilbert spaces of observables on the future null infinity $\mathcal{I}_+$ and on the horizon $\mathcal{H}_+$ (see Fig 3) are independent of each other, implying thereby that the observables belonging to these Hilbert spaces commute. Now, recall that these two sets of observables correspond, respectively, to (a) objects outside the event horizon that

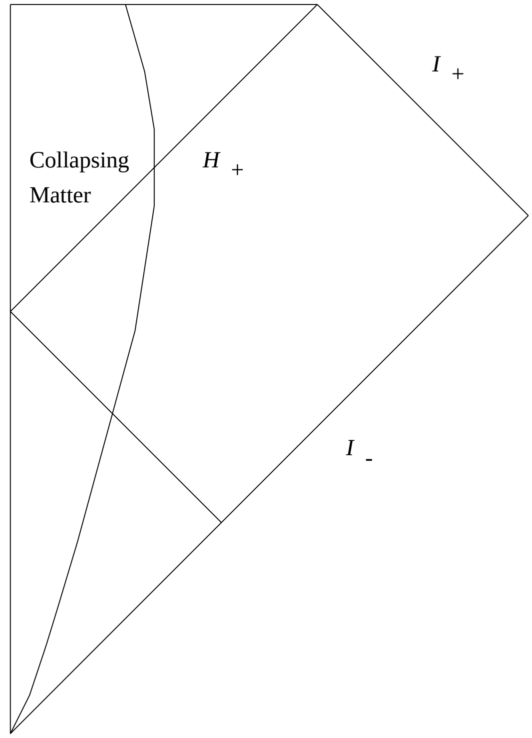

Figure 3: Collapsing black hole

escape to infinity and (b) infalling matter. The discussion in the previous paragraph suggests that the observables outside and inside the horizon are not quite independent and should not be regarded as mutually commuting sets. Although that discussion was in the context of near-horizon (anti-de Sitter) geometries and does not include the asymptotically flat part, it already makes the assumption in [10] of mutually commuting observables far from obvious.

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