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CP Violation and Rare Kaon Decays†

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Abstract
We summarize both the study of CP violation with K and B mesons, as well as rare decays of kaons, emphasizing recent developments. The topics discussed include the unitarity triangle, ε'/ε, $K \to \pi\nu\bar{\nu}$ and other rare K decays, T-odd asymmetries in kaon physics, theoretical aspects of CP violation in B decays, $B \to J/\Psi K_S$, $B \to \pi\pi$, $B \to \pi K$ and inclusive CP asymmetries.

1. Introduction

The intensive study of weak interaction processes and the physics of flavour has proved crucial for our present understanding of matter at the most fundamental level. The idea of strangeness as an additional quantum number opened extremely fruitful new directions in flavour physics. At the same time it laid the ground for the quark model, which in turn provided the basis for the subsequent development of QCD. The θ–τ puzzle in kaon decays suggested the violation of parity, a property now reflected in the chiral nature of the weak gauge interactions. The strong suppression of flavour-changing neutral current processes, as $K_L \to \mu^+\mu^-$ or K–$\bar{K}$ mixing, motivated the GIM mechanism and the introduction of the charm quark. Later, the hint of an unexpectedly large top-quark mass, infered from B–$\bar{B}$ mixing, replicated, in a sense, at higher energies, the story of K–$\bar{K}$ mixing and charm. Finally, the 1964 discovery of CP violation in $K \to \pi\pi$ decays already carried the seed of a three-generation Standard Model, ten years before even the charm quark was found. These examples illustrate impressively how the careful study of low energy phenomena may be sensitive to physics at scales much larger than m_K or m_B, and that profound insights can be obtained by such indirect probes.

Among the various aspects of flavour dynamics the phenomenon of CP violation is a particular focus of current research. This fundamental symmetry violation provides us with an absolute distinction between matter and antimatter, and it is one of the necessary conditions for a dynamical generation of the baryon asymmetry in the universe (see the talk by M. Shaposhnikov, these proceedings). CP violation is also a very important testing ground for flavour dynamics in general.

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Let us very briefly recall the basic mechanism of CP violation in the Standard Model. Gauge (q'_L) and mass eigenstates (q_L) of up ($q = u$) and down-type ($q = d$) quarks are related by unitary rotations U_L, D_L (of dimension n for n generations)

$$u'_L = U_L u_L \qquad d'_L = D_L d_L \tag{1}$$

Due to a mismatch of U_L and D_L, i.e. $V = U_L^\dagger D_L \neq 1$, the charged current weak interactions

$$\mathcal{L}_{cc} = \frac{g_W}{2\sqrt{2}} V_{ij}\, \bar{u}_i \gamma^\mu (1-\gamma_5) d_j\, W_\mu^\dagger + h.c. \tag{2}$$

induce transitions between different families ($i = 1, \ldots, n$). Their coupling strengths are described by the Cabibbo–Kobayashi–Maskawa (CKM) matrix V. With 1 or 2 generations, V can be chosen to be real. For 3 generations V is parametrized by 3 angles and 1 complex phase, which is responsible for breaking CP symmetry in eq. (2). The CKM matrix has the explicit form

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \tag{3}$$

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\varrho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{4}$$

where the second expression is the useful approximate representation due to Wolfenstein with the parameters λ, A, ϱ and the complex phase η.

The replication of fermion generations, quark mixing and CP violation are striking features of the theory of weak interactions. While the gauge sector of the theory is well understood and tested with high precision, the breaking of electroweak symmetry and its ramifications in flavour physics leave still many questions unanswered. Detailed and quantitative tests of the flavour sector have so far remained rather limited. On the other hand the Standard Model relates all phenomena of CP violation to a single complex phase, a constraint that enables us to put this framework to a decisive test. Additional CP violating phases occur in many Standard Model extensions [1, 2] and could lead to substantial deviations from what we expect in the Standard Model.

The present talk summarizes recent developments in the field of CP violation and rare kaon decays. Further aspects of flavour physics are discussed in the talk by M. Artuso in these proceedings. After this introduction we give, in section 2, a classification of CP violation and briefly review the status of the unitarity triangle in section 3. Section 4 is devoted to CP violation and rare decays in the kaon sector, including ε'/ε, $K \to \pi\nu\bar{\nu}$, T-odd CP asymmetries and a short overview of further rare decays of interest. In section 5 we discuss CP violation in B decays, summarizing the theoretical basis and important applications, $B \to J/\Psi K_S$, $B \to \pi^+\pi^-$, $B \to \pi K$ and the determination of γ, a selection of other strategies, and inclusive CP asymmetries. We conclude in section 6.

2. Classification of CP Violation

The complex phase in the 3-generation CKM matrix, which is present independently of quark phase conventions, violates the CP symmetry of the Standard Model Lagrangian. As a consequence weak decays that would otherwise be forbidden by CP symmetry may occur, or processes related to each other by a CP transformation may have different rates. In order for the CKM phase to manifest itself in such observable asymmetries, an interference of some sort is in general necessary to induce a physical phase *difference* between the interfering components. There are various ways in which this general condition may be realized and it is useful to introduce a classification of the possible mechanisms.

In many cases an important role is played by neutral mesons P such as $P = K^0$, B_d or B_s. Their characteristic feature is neutral meson (P–$\bar{P}$) mixing, which is described by a 2×2 Hamiltonian matrix $H = M - i\Gamma/2$.

Following common practice, three classes may then be distinguished:

a) CP violation in the mixing matrix. A typical case is the rate difference

$$\frac{\Gamma(K_L \to \pi^- l^+ \nu) - \Gamma(K_L \to \pi^+ l^- \bar{\nu})}{\Gamma(K_L \to \pi^- l^+ \nu) + \Gamma(K_L \to \pi^+ l^- \bar{\nu})} = \frac{|1+\bar{\varepsilon}|^2 - |1-\bar{\varepsilon}|^2}{|1+\bar{\varepsilon}|^2 + |1-\bar{\varepsilon}|^2} \simeq 2Re\bar{\varepsilon} \simeq \frac{1}{4}\mathrm{Im}\frac{\Gamma_{12}}{M_{12}} \quad (5)$$

where we have used ($CP \cdot K = -\bar{K}$)

$$K_L \sim (1+\bar{\varepsilon})K + (1-\bar{\varepsilon})\bar{K} \quad (6)$$

Here the lepton charge tags the flavour component (K or $\bar{K}$) in the K_L. The first equality in (5) follows immediately. Note the proportionality of the asymmetry to $\mathrm{Im}(\Gamma_{12}/M_{12})$, which characterizes the effect as originating in the mixing matrix itself. The sign of the asymmetry, $2\mathrm{Re}\bar{\varepsilon} = 3.3 \cdot 10^{-3} > 0$, allows us to give an absolute definition of positive electric charge. Similar observables can be constructed for B decays. A peculiarity of the kaon system is the large hierarchy of lifetimes between the neutral eigenstates K_L and K_S. The long lived K_L state is singled out by the time evolution itself, which provides a convenient initial state tag. Explicit initial state flavour tagging by other means is in general required for neutral B mesons.

b) CP violation in the decay amplitude. In its simplest form this mechanism can be introduced without any reference to mixing. In this case the general requirement of amplitude interference is particularly transparent. Consider the decay amplitudes

$$A(P \to f) = A_1 e^{i\delta_1} e^{i\phi_1} + A_2 e^{i\delta_2} e^{i\phi_2} \quad (7)$$

$$A(\bar{P} \to \bar{f}) = A_1 e^{i\delta_1} e^{-i\phi_1} + A_2 e^{i\delta_2} e^{-i\phi_2} \quad (8)$$

where we have assumed two different components $i = 1, 2$ with strong phases δ_i and weak (CKM) phases ϕ_i. Only the weak phases change sign in going from $P \to f$ to the CP conjugate reaction $\bar{P} \to \bar{f}$. For the rate difference one finds

$$|A(P \to f)|^2 - |A(\bar{P} \to \bar{f})|^2 \sim A_1 A_2 \sin(\delta_1 - \delta_2)\sin(\phi_1 - \phi_2) \quad (9)$$

The conditions for a CP violating asymmetry are, obviously, the presence of two components in the decay amplitude, as well as both a strong and a weak phase difference between them. This mechanism is relevant for neutral mesons, but also for charged mesons ($K^\pm$, $B^\pm$, ...) or baryons.

c) CP violation in the interference of mixing and decay. In this case the necessary interference arises in an interplay of decay and mixing. A typical, illustrative example is the CP violating amplitude

$$A(K_L \to \pi\pi(I=0)) \sim (1+\bar{\varepsilon})A_0 e^{i\delta_0} - (1-\bar{\varepsilon})A_0^* e^{i\delta_0} \sim \bar{\varepsilon} + i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \equiv \varepsilon \quad (10)$$

where δ_0 is the strong phase and the weak phase is included in A_0 (as defined after (13)). One recognizes the combination of mixing ($\bar{\varepsilon}$) and decay amplitude (A_0) in the observable ε, the well known CP violation parameter in the kaon sector. Note that ε is a physical quantity, in contrast to the components $\bar{\varepsilon}$ and $i\mathrm{Im}A_0/\mathrm{Re}A_0$, which are phase convention dependent in the CKM framework.

In analogy to (10) one can consider the decays to $\pi^+\pi^-$ and $\pi^0\pi^0$. In the CP symmetry limit K_L would be CP odd and could not decay to the CP even two-pion final states. Thus the amplitude ratios

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \qquad \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \tag{11}$$

measure CP violation. They can be parametrized as

$$\eta_{+-} = \varepsilon + \varepsilon' \qquad \eta_{00} = \varepsilon - 2\varepsilon' \tag{12}$$

with ε as in (10) and

$$\frac{\varepsilon'}{\varepsilon} \simeq \frac{\omega}{\sqrt{2}|\varepsilon|}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right) \tag{13}$$

where $A(K^0 \to \pi\pi(I = 0,2)) \equiv A_{0,2}e^{i\delta_{0,2}}$ (with strong phases $\delta_{0,2}$ factored out) and $\omega \equiv \mathrm{Re}A_2/\mathrm{Re}A_0 \simeq 0.045$. The smallness of ω reflects the empirical $\Delta I = 1/2$ rule in kaon decays.

If CP violation was only due to mixing, the ratios in (11) were independent of the final state and $\eta_{+-} = \eta_{00}$. Any difference $\eta_{+-} \neq \eta_{00}$ necessarily involves CP violation in the decay amplitudes and is measured by ε'. Since the ratio ε'/ε is real to very good accuracy, it can be experimentally determined via the double ratio of rates

$$\left|\frac{\eta_{+-}}{\eta_{00}}\right|^2 = 1 + 6\mathrm{Re}\frac{\varepsilon'}{\varepsilon} \tag{14}$$

In parallel to the classes a) – c), another terminology is in use. CP violation in the mixing matrix itself (a)) is refered to as *indirect* CP violation. In contrast, CP violation in the decay amplitude (as in b)) is called *direct* CP violation. As we have seen, class c) comprises elements of both the indirect and the direct effect. In particular, ε'/ε is a measure of direct CP violation. Completely analogous notions apply to B decays as well, although some details of the phenomenology are different. We shall come back to those applications in the B physics sections.

3. Status of the Unitarity Triangle

The CKM unitarity relation

$$V^*_{ub}V_{ud} + V^*_{cb}V_{cd} + V^*_{tb}V_{td} = 0 \tag{15}$$

defines a triangle in the complex plane. Rescaling the sides by $|V^*_{cb}V_{cd}|$, and keeping subleading terms in the Wolfenstein expansion in λ [3], the unitarity triangle is displayed in the plane of $\bar{\varrho} = \varrho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$ (fig. 1). This representation is useful since it emphasizes the least known CKM quantities $\bar{\varrho}$ and $\bar{\eta}$. Fig. 1 also defines the CKM angles α, β and γ. At present the most important constraints on ($\bar{\varrho}$, $\bar{\eta}$), the apex of the unitarity triangle, are coming from kaon CP violation (ε), semileptonic B decays ($|V_{ub}/V_{cb}|$) and $B - \bar{B}$ mixing (ΔM_d and $\Delta M_d/\Delta M_s$, constraining $|V_{td}|$ and $|V_{td}/V_{ts}|$, respectively). In principle, ε and $|V_{ub}/V_{cb}|$ are sufficient to determine the unitarity triangle, as illustrated in fig. 2. The characteristic dependence on the relevant input parameters m_t, V_{cb}, $|V_{ub}/V_{cb}|$ and the hadronic matrix element B_K is indicated in this plot. Note that the consistency of the Standard Model, i.e. intersection of the two curves, requires all four parameters to be sufficiently *large.* In particular the large value of the top-quark mass $\bar{m}_t(m_t) = 167\,\mathrm{GeV}$ is important for the – nontrivial – consistency of the Standard Model description. The theory of ε and $B - \bar{B}$ mixing at next-to-leading order in QCD is described in [4, 5]. The current status of the unitarity triangle obtained from a detailed numerical analysis is shown in fig. 3 (see also M. Artuso, these proceedings).

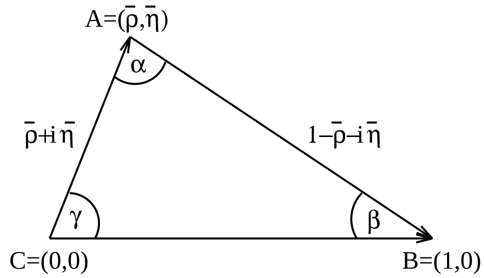

Figure 1. The unitarity triangle in the ($\bar{\varrho}$, $\bar{\eta}$) plane.

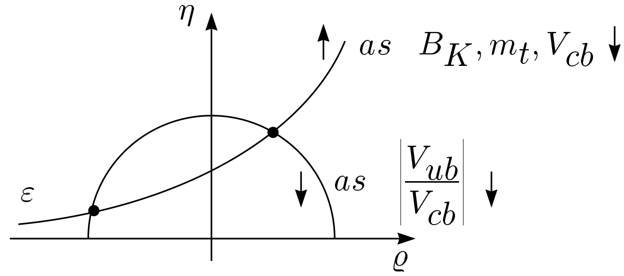

Figure 2. The ε-constraint on the unitarity triangle.

4. Kaons: CP Violation and Rare Decays

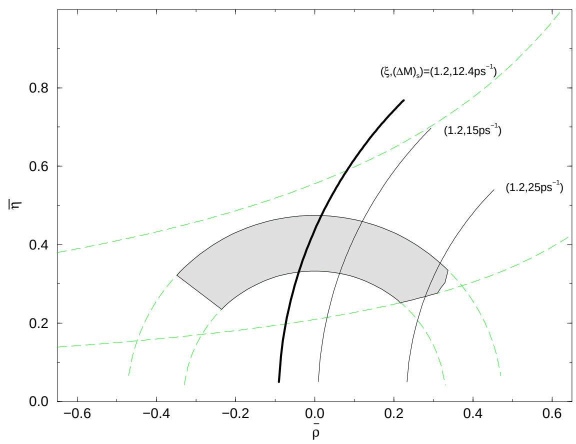

Figure 3. The allowed region (shaded) in the ($\bar{\varrho}$, $\bar{\eta}$) plane, combining information from ε, $|V_{ub}/V_{cb}|$ and including the constraint from ΔM_d. The independent constraint from the lower limit on $\Delta M_s/\Delta M_d$ excludes the region to the left of the curves labeled with ΔM_s in the plot. $\xi \simeq 1.2$ measures $SU(3)$ breaking in the hadronic matrix elements of $B_d - \bar{B}_d$ versus $B_s - \bar{B}_s$ mixing. The plot is taken from [6] where further particulars can be found. Similar analyses have been presented in [7, 8, 9, 10].

4.1. ε'/ε

Until very recently all CP violation effects ever observed in the laboratory were different manifestations of a single quantity, the parameter ε describing indirect CP violation in the neutral kaon system. This ε-effect was first seen 1964 in $K_L \to \pi^+\pi^-$ and subsequently in a few further decay channels of K_L, namely $\pi^0\pi^0$, $\pi l\nu$, $\pi^+\pi^-\gamma$ and finally $\pi^+\pi^-e^+e^-$. As we have seen, the experimental value of ε is well compatible with the Standard Model. Nevertheless it is clear that this quantity alone gives insufficient information to provide us with a decisive test. Complementary and qualitatively new insight can be gained by measuring direct CP violation in $K \to \pi\pi$, i.e. ε'/ε. In superweak scenarios [1, 11] ε' is predicted to vanish, while it is generically nonzero in the Standard Model. Considerable efforts were therefore invested over the years to determine ε'/ε in experiment, but the situation as to whether ε'/ε was zero or not had remained inconclusive. Two measurements, about ten years old, characterize the experimental status before 1999

$$\mathrm{Re}\frac{\varepsilon'}{\varepsilon} = \left\{ \begin{array}{ll} (23 \pm 6.5)\cdot 10^{-4} & \text{CERN NA31} \\ (7.4 \pm 5.9)\cdot 10^{-4} & \text{FNAL E731} \end{array} \right. \tag{16}$$

Earlier this year the successor experiments at both laboratories have announced their first new results [12, 13] (see also H. Nguyen (KTeV) and S. Palestini (NA48), these proceedings)

$$\mathrm{Re}\frac{\varepsilon'}{\varepsilon} = \left\{ \begin{array}{ll} (18.5 \pm 7.3)\cdot 10^{-4} & \text{CERN NA48} \\ (28.0 \pm 4.1)\cdot 10^{-4} & \text{FNAL KTeV} \end{array} \right. \tag{17}$$

An average of all four numbers gives

$$\mathrm{Re}\frac{\varepsilon'}{\varepsilon} = (21.2 \pm 4.6)\cdot 10^{-4} \tag{18}$$

These new experimental achievments (17) are among the major highlights of this year in particle physics. They establish direct CP violation for the first time, confirming the earlier evidence from NA31. Both NA48 and KTeV are still ongoing and a third experiment, KLOE, at the Frascati Φ-factory has just started. ε'/ε will therefore be known with still higher precision ($\sim 10^{-4}$) in the future.

A detailed theoretical interpretation of these results is a very difficult task. Although important progress has been made over the years, the calculation of ε'/ε still suffers from considerable uncertainties related to the low energy dynamics of QCD. Before presenting the theoretical status of ε'/ε, we begin by recalling the essential features of the calculational framework. The expression (13) for ε'/ε may also be written as

$$\frac{\varepsilon'}{\varepsilon} = -\frac{\omega}{\sqrt{2}|\varepsilon|\mathrm{Re}A_0}\left(\mathrm{Im}A_0 - \frac{1}{\omega}\mathrm{Im}A_2\right) \tag{19}$$

$\mathrm{Im}A_{0,2}$ are calculated from the general low energy effective Hamiltonian for $\Delta S = 1$ transitions [14]. Including electroweak penguins this Hamiltonian involves ten different operators and one has

$$\mathrm{Im}A_{0,2} = -\mathrm{Im}\lambda_t \frac{G_F}{\sqrt{2}} \sum_{i=3}^{10} y_i(\mu)\langle Q_i\rangle_{0,2} \tag{20}$$

Here y_i are Wilson coefficients, $\lambda_t = V^*_{ts}V_{td}$ and

$$\langle\pi\pi(I=0,2)|Q_i|K^0\rangle \equiv \langle Q_i\rangle_{0,2}e^{i\delta_{0,2}} \tag{21}$$

For the purpose of illustration we keep only the numerically dominant contributions and write

$$\frac{\varepsilon'}{\varepsilon} = \frac{\omega G_F}{2|\varepsilon|\mathrm{Re}A_0}\mathrm{Im}\lambda_t\left(y_6\langle Q_6\rangle_0 - \frac{1}{\omega}y_8\langle Q_8\rangle_2 + \ldots\right) \tag{22}$$

Q_6 originates from gluonic penguin diagrams and Q_8 from electroweak contributions. The matrix elements of Q_6 and Q_8 can be parametrized by bag parameters B_6 and B_8 as

$$\langle Q_6\rangle_0 = -\sqrt{24}\left[\frac{m_K}{m_s(\mu)+m_d(\mu)}\right]^2 m_K^2(f_K - f_\pi)\cdot B_6 \tag{23}$$

$$\langle Q_8\rangle_2 \simeq \sqrt{3}\left[\frac{m_K}{m_s(\mu)+m_d(\mu)}\right]^2 m_K^2 f_\pi \cdot B_8 \quad (24)$$

that is

$$\langle Q_6\rangle_0 \sim \left(\frac{m_K}{m_s}\right)^2 B_6 \qquad \langle Q_8\rangle_2 \sim \left(\frac{m_K}{m_s}\right)^2 B_8 \quad (25)$$

$B_6 = B_8 = 1$ corresponds to the factorization assumption for the matrix elements, which holds in the large N_C limit of QCD.

$y_6\langle Q_6\rangle_0$ and $y_8\langle Q_8\rangle_2$ are positive numbers. The value for ε'/ε in (22) is thus characterized by a cancellation of competing contributions. Since the second contribution is an electroweak effect, suppressed by $\sim \alpha/\alpha_s$ compared to the leading gluonic penguin $\sim \langle Q_6\rangle_0$, it could appear at first sight that it should be altogether negligible for ε'/ε. However, a number of circumstances actually conspire to systematically enhance the electroweak effect so as to render it a sizable contribution:

- Unlike Q_6, which is a pure $\Delta I = 1/2$ operator, Q_8 can give rise to the $\pi\pi(I=2)$ final state and thus yield a non-vanishing $\mathrm{Im}A_2$.
- The $\mathcal{O}(\alpha/\alpha_s)$ suppression is largely compensated by the factor $1/\omega \approx 22$ in (22), reflecting the $\Delta I = 1/2$ rule.
- $-y_8\langle Q_8\rangle_2$ gives a negative contribution to ε'/ε that strongly grows with m_t [15, 16]. For the realistic top mass value it can be substantial.

In the following we will summarize current theoretical analyses of ε'/ε, point out the major sources of uncertainty and briefly describe some recent developments.

To display the most important ingredients for theoretical predictions, it is useful to consider an approximate numerical formula for ε'/ε [17]

$$\frac{\varepsilon'}{\varepsilon} \approx 13\mathrm{Im}\lambda_t \left(\frac{\Lambda^{(4)}_{\overline{MS}}}{340\,\mathrm{MeV}}\right)\left[\frac{110\,\mathrm{MeV}}{m_s(2\,\mathrm{GeV})}\right]^2 \cdot \left[B_6(1-\Omega_{IB}) - 0.4 B_8 \left(\frac{m_t(m_t)}{165\,\mathrm{GeV}}\right)^{2.5}\right] \quad (26)$$

This expression is a convenient approximation for the purpose of illustration, but it should not be used for a detailed quantitative analysis.

The Wilson coefficients y_i in (20), (22) have been calculated at next-to-leading order [18, 19]. The short-distance part in (26) is therefore quite well under control. The largest theoretical uncertainties come from the hadronic matrix elements, in particular from $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$, which are parametrized by B_6, B_8 and m_s. Another hadronic quantity entering the analysis is Ω_{IB}, representing the effect of isospin breaking in the quark masses ($m_u \neq m_d$). An estimate of $\Omega_{IB} = 0.25 \pm 0.08$ is reported in [17] and has been used in most analyses of ε'/ε.

These quantities parametrize nonperturbative effects of QCD that are difficult to compute in practice. They are not fundamental parameters of the Standard Model, but could be expressed, in principle, in terms of the latter, that is Λ_{QCD}, m_u, m_d, m_s. (Note that m_s entering (26) does not represent the complete physical m_s-dependence of ε'/ε, as the kaon mass $m_K^2 = r m_s$ is always kept fixed; m_s is introduced indirectly as a conventional way to rewrite the value of the quark condensate $\sim r$. The condensate appears in the factorized matrix elements of $Q_{6,8}$, which are products of matrix elements of two (pseudo-) scalar currents.) In principle the dependence of $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$ on the strange-quark mass m_s need not be made explicit if the matrix elements are calculated directly within a nonperturbative approach such as lattice QCD. However, m_s enters naturally in the large-N_c limit of QCD, where the matrix elements factorize and $B_6 = B_8 = 1$ are valid exactly. Since the Wilson coefficients $y_{6,8}$ are already of $\mathcal{O}(1/N_c)$, the leading-order approximation of $B_{6,8}$ in large-N_c is formally consistent with a next-to-leading order treatment of the full decay amplitude, known to be important for the CP conserving real parts [20]. Also, the dependence of $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$ on the renormalization scale μ is carried almost entirely by the quark masses in (23), (24). B_6 and B_8 are thus practically μ-independent. This feature is convenient for comparing various analyses, where the B-factors are obtained at different scales. For these reasons the parametrization of $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$ in terms of B_6, B_8 and m_s is still a useful convention.

The value of the strange-quark mass has received considerable attention recently. Representative numbers are $m_s(2\,\mathrm{GeV}) = (110 \pm 20)$MeV from quenched lattice simulations [21] and $m_s(2\,\mathrm{GeV}) = (124 \pm 22)$MeV from QCD sum rules [22], the latter result quoted here as the average reported in [23]. Recent quenched lattice calculations obtain for $m_s(2\,\mathrm{GeV})$ the values (121 ± 13)MeV [24], (106 ± 7)MeV [25], (97 ± 4)MeV [26] and (105 ± 5)MeV [27]. Unquenching is expected to lower these values by about 15MeV ([28] and refs. therein). The tendency towards relatively small m_s had already been emphasized in [29, 30]. For a review on lattice determinations of the strange-quark mass see e.g. [28] and the contribution of H. Wittig in these proceedings. On the other hand it has been argued on the basis of dispersion relations that m_s obeys a lower bound, typically $m_s(2\,\mathrm{GeV}) \gtrsim 100\,\mathrm{MeV}$ [31].

The framework employed is similar to the one in QCD sum rule calculations, but less assumptions are used to avoid model dependence. An alternative method determines m_s from Cabibbo-suppressed hadronic τ decays, where the latest analysis finds $m_s(2\,\mathrm{GeV}) = (114 \pm 23)\,\mathrm{MeV}$ [32] (see also [33, 34] for earlier results).

Small values of the strange-quark mass, as the ones indicated particularly by lattice calculations, contribute to increasing the theoretical values for ε'/ε, at fixed B_6, B_8.

Various groups have presented analyses of ε'/ε. Their findings are summarized in table 1.

All results are based on the effective Hamiltonian with Wilson coefficients calculated in [18, 19]. There are slight variations in the choice of input parameters, as those entering the determination of $\mathrm{Im}\lambda_t$ from ε. The most important differences are in the treatment of hadronic matrix elements. All groups use values of m_s close to the representative range [23]

$$m_s(2\,\mathrm{GeV}) = (110 \pm 20)\,\mathrm{MeV} \tag{27}$$

Several approaches have been used to obtain estimates for B_6 and B_8. Unfortunately none of them can be considered fully satisfactory at present and in particular the error bars are hard to quantify with confidence.

The leading order large-N_c limit gives $B_6 = B_8 = 1$ [20]. More recent estimates based on the large-N_c approach and using a simultaneous chiral (p^2) and $1/N_c$ expansion find [39]

$$B_6 = 0.72\text{–}1.10 \qquad B_8 = 0.42\text{–}0.64 \tag{28}$$

These estimates include the corrections to the matrix elements of $\mathcal{O}(p^2)$ and $\mathcal{O}(p^0/N_c)$ (the leading term $\mathcal{O}(p^0)$ is nonvanishing only in the case of B_8). A particular term at higher order ($\mathcal{O}(p^2/N_c)$) is identified that would enhance B_6 to about 1.5 [36, 40]. In view of other contributions and systematic uncertainties of the approach such a conclusion has to be taken with caution and appears premature at present.

The Trieste group [37] employs a chiral quark model to evaluate hadronic quantities. They find

$$B_6 = 1.07\text{–}1.58 \qquad B_8 = 0.75\text{–}0.79 \tag{29}$$

The model is able to fit the $\Delta I = 1/2$ rule and leads to enhanced values for B_6, however, its relation to QCD is not entirely clear.

Ultimately, the most reliable results should come from a first-principles lattice calculation. For the matrix elements under discussion the lattice method is, however, at present still affected by large systematic uncertainties (for instance, only $\langle\pi|Q_i|K\rangle$ is simulated directly and lowest order chiral perturbation theory is used to obtain the required matrix element with two pions in the final state; also quenching introduces uncertainties that are hard to quantify). In particular no reliable results seem to be available for B_6 [35], which is plagued by large corrections from lattice perturbation theory [41]. A nonperturbative lattice-continuum matching procedure is therefore required [42]. For B_8 one finds from lattice QCD computations [43, 44, 45]

$$B_8 = 0.69\text{–}1.06 \tag{30}$$

As is evident from table 1, the uncertainties in theoretical calculations of ε'/ε are very large. Still, there is a tendency for results with bag parameters $B_{6,8}$ in the vicinity of 1 to yield estimates of ε'/ε below the experimental values.

This situation has encouraged additional theoretical efforts aimed at a better understanding of various aspects of the low-energy hadronic dynamics entering ε'/ε. Let us briefly mention some recent activities.

The authors of [46] also use a large-N_c framework, supplemented by the extended Nambu–Jona-Lasinio (ENJL) model to interpolate low-energy QCD, and quote $B_6 = 2.2 \pm 0.5$, which would comfortably accommodate the sizable experimental numbers. Unfortunately also this result is not entirely free of model-dependent input.

An interesting approach is discussed in [47], where B_8 is obtained from a dispersive analysis in the strict chiral limit. It is found that $B_8 = 1.11 \pm 0.16 \pm 0.23$ in the NDR scheme and assuming $m_s(2\,\mathrm{GeV}) + m_d(2\,\mathrm{GeV}) = 100\,\mathrm{MeV}$ (see also J. Donoghue, these proceedings). Similar ideas are presented in [48] in the context of the large-N_c limit.

A promising new method in lattice QCD employs domain-wall fermions to ensure good chiral properties on the lattice (see A. Soni, these proceedings). The Riken-BNL-Columbia collaboration [49] has performed an exploratory study finding very large and negative B_6, which translates into large and negative $\varepsilon'/\varepsilon = (-3.3 \pm 0.3(\mathrm{stat}) \pm 1.6(\mathrm{syst})) \cdot 10^{-2}\eta$. Taken literally, the central value would imply a striking disagreement with experimental results. However, the error bars are still very large and the numbers obtained have still to be considered preliminary, awaiting further scrutiny.

The effect of isospin breaking due to the m_u–m_d mass difference, parametrized by Ω_{IB}, has been reconsidered in [50]. In this paper it is argued that new sources of isospin breaking at order $\mathcal{O}(p^4)$

Table 1. Summary of recent theoretical results for ε'/ε, together with the hadronic quantities B_6 and B_8 used in the analysis. B_6 and B_8 are taken to be in the NDR scheme; they are approximately scale independent. The values for B_6 and B_8 of the Rome group are rescaled to correspond to a nominal strange-quark mass of $m_s(2\,\mathrm{GeV}) = 110\,\mathrm{MeV}$. (MC) denotes a Gaussian treatment of uncertainties for the experimental input, and (S) the more conservative scanning of all parameters within their assumed ranges (see the quoted references for more details).

B_6	B_8	$\varepsilon'/\varepsilon[10^{-4}]$	reference
1.0 ± 0.3	0.8 ± 0.2	$7.7^{+6.0}_{-3.5}$ (MC)	Munich [23]
1.0 ± 0.3	0.8 ± 0.2	$1.1 \to 28.8$ (S)	Munich [23]
0.7 ± 0.7	0.7 ± 0.1	$6.7^{+9.2}_{-8.5} \pm 0.4$ (MC)	Rome [35]
0.7 ± 0.7	0.7 ± 0.1	$-10 \to 30$ (S)	Rome [35]
0.91 ± 0.19	$B_6/1.72$	$2.1 \to 26.4$ (S)	Dortmund [36]
1.33 ± 0.25	0.77 ± 0.02	$7 \to 31$ (S)	Trieste [37]
1.0	1.0	$-3.2 \to 3.3$ (S)	Dubna [38]

in chiral perturbation theory might have a sizable impact, possibly leading even to negative Ω_{IB} and a resulting enhancement of ε'/ε. The issue was subsequently also addressed in [51], where the contribution of $\pi^0 - \eta$ mixing to Ω_{IB} is obtained to be $\Omega_{IB}^{\pi^0\eta} = 0.16 \pm 0.03$ within chiral perturbation theory to $\mathcal{O}(p^4)$.

Another mechanism that would lead to larger values of B_6 than in conventional estimates is discussed in [52]. In this paper it is proposed that the effect of the σ-resonance, specific to the $I = 0$ channel of the two-pion final state, could increase B_6 and ε'/ε considerably. This effect corresponds to a resummation of a class of two-pion rescattering diagrams.

A complete, exact calculation of the matrix elements $\langle\pi\pi(I)|Q_i|K^0\rangle$ would automatically yield the correct final state interaction (FSI) phase factors $\exp(i\delta_I)$ multiplying $\langle Q_i\rangle_I$ in (21). Current methods of estimating the matrix elements, however, do not correctly reproduce these phases, which are known experimentally. The phases obtained are either zero or much smaller than the empirical values. A general discussion of FSI in the context of ε'/ε was very recently given in [53], starting from the Omnès solution of a dispersion relation for the decay amplitudes. It was argued that FSI enhance the $I = 0$ channel, and hence B_6, and suppress $I = 2$ contributions. This would bring the central value of ε'/ε closer to the data. The framework discussed in [53] is certainly interesting. It will be important to see the same features emerge also within the context of a complete and explicit calculation of the matrix elements.

There is a general consensus that it is impossible, at present, to conclude a significant discrepancy with the Standard Model in the measurement of ε'/ε, taking into account the substantial hadronic uncertainties. Nevertheless, contributions from non-standard physics could indeed affect the value of ε'/ε, and the very sizable experimental figures for this quantity have reinforced the interest in exploring New-Physics scenarios. A brief summary of these investigations, most of which focus on supersymmetric models, and further references can be found in [17, 54] (see also Y.-L. Wu, these proceedings). Typically the most likely sources of New Physics in ε'/ε are Z-penguin and chromomagnetic penguin contributions. They correspond to operators of dimension 4 and 5, respectively, and could be quite naturally, by dimensional counting, larger than higher dimensional contributions from New Physics [55] (see [56] for an alternative scenario in supersymmetry).

Additional general discussions of the theory of ε'/ε may be found, for instance, in the recent review articles [17, 54, 57].

The clear and unambiguous experimental determination of ε'/ε has demonstrated the existence of direct CP violation. This result thus confirms the qualitative expectation of nonzero ε' characteristic of the CKM mechanism and rules out superweak scenarios where $\varepsilon'/\varepsilon = 0$. The new measurements have also reinforced theoretical efforts to gain an improved understanding of the complicated hadronic dynamics. Further progress may be anticipated, although a good accuracy of the computations is likely to remain a serious theoretical challenge in the future. The tantalizing situation of an intriguing experimental result whose interpretation is hampered by large theoretical uncertainties, lends further motivation to look for

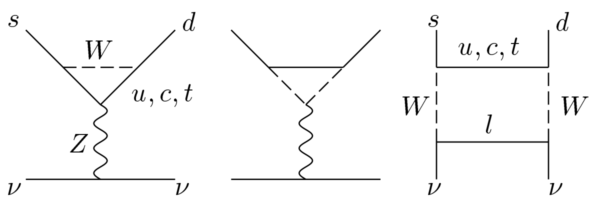

Figure 4. *Leading order electroweak diagrams contributing to $K \to \pi\nu\bar{\nu}$ in the Standard Model.*

other observables with better theoretical control. Examples are provided by certain rare decays of kaons, which will form the subject of the following section.

4.2. $K \to \pi\nu\bar{\nu}$

In this paragraph we focus on the rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, which are particularly promising. In these modes the loop-induced FCNC transition $s \to d$ is probed by a neutrino current, which couples only to heavy gauge bosons (W, Z), as shown in fig. 4. Correspondingly, the GIM pattern of the $\bar{s} \to \bar{d}\nu\bar{\nu}$ amplitude has, roughly speaking, the form

$$A(\bar{s} \to \bar{d}\nu\bar{\nu}) \sim \lambda_i m_i^2 \tag{31}$$

summed over $i = u, c, t$ ($\lambda_i = V_{is}^* V_{id}$). The power-like mass dependence strongly enhances the short-distance contributions, coming from the heavy flavours c and t. (This is to be contrasted with the logarithmic mass dependence of the photonic penguin, important for $\bar{s} \to \bar{d}e^+e^-$.) The short-distance dominance has, then, two crucial consequences. First, the transition proceeds through an effectively local $(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$ interaction. Second, because that local interaction is semileptonic, the only hadronic matrix element required, $\langle\pi|(\bar{s}d)_V|K\rangle$, can be obtained from $K^+ \to \pi^0 l^+\nu$ decay using isospin. As a result $K \to \pi\nu\bar{\nu}$ is calculable completely and with exceptional theoretical control. While $K^+ \to \pi^+\nu\bar{\nu}$ receives both top and charm contributions, $K_L \to \pi^0\nu\bar{\nu}$ probes direct CP violation [58] and is dominated entirely by the top sector.

The $K \to \pi\nu\bar{\nu}$ modes have been studied in great detail over the years to quantify the degree of theoretical precision. Important effects come from short-distance QCD corrections. These were computed at leading order in [59]. The complete next-to-leading order calculations [60, 61, 62] reduce the theoretical uncertainty in these decays to $\sim 5\%$ for $K^+ \to \pi^+\nu\bar{\nu}$ and $\sim 1\%$ for $K_L \to \pi^0\nu\bar{\nu}$. This picture is essentially unchanged when further effects are considered, including isospin breaking in the relation of $K \to \pi\nu\bar{\nu}$ to $K^+ \to \pi^0 l^+\nu$ [63], long-distance contributions [64, 65], the CP-conserving effect in $K_L \to \pi^0\nu\bar{\nu}$ in the Standard Model [64, 66] and two-loop electroweak corrections for large m_t [67]. The current Standard Model predictions for the branching ratios are [6]

$$B(K^+ \to \pi^+\nu\bar{\nu}) = (0.8 \pm 0.3) \cdot 10^{-10} \tag{32}$$

$$B(K_L \to \pi^0\nu\bar{\nu}) = (2.8 \pm 1.1) \cdot 10^{-11} \tag{33}$$

The study of $K \to \pi\nu\bar{\nu}$ can give crucial information for testing the CKM picture of flavor mixing. This information is complementary to the results expected from B physics and is much needed to provide the overdetermination of the unitarity triangle necessary for a real test. Let us briefly illustrate some specific opportunities.

$K_L \to \pi^0\nu\bar{\nu}$ is probably the best probe of the Jarlskog parameter $J_{CP} \sim \mathrm{Im}\lambda_t$, the invariant measure of CP violation in the Standard Model [68]. For example a 10% measurement $B(K_L \to \pi^0\nu\bar{\nu}) = (3.0 \pm 0.3) \cdot 10^{-11}$ would directly give $\mathrm{Im}\lambda_t = (1.37 \pm 0.07) \cdot 10^{-4}$, a remarkably precise result.

Combining 10% measurements of both $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ determines the unitarity triangle parameter $\sin 2\beta$ with an uncertainty of about ± 0.07, comparable to the precision obtainable for the same quantity from CP violation in $B \to J/\Psi K_S$ before the LHC era.

A measurement of $B(K^+ \to \pi^+\nu\bar{\nu})$ to 10% accuracy can be expected to determine $|V_{td}|$ with similar precision.

As a final example, using only information from the ratio of $B_d - \bar{B}_d$ to $B_s - \bar{B}_s$ mixing, $\Delta M_d/\Delta M_s$, one can derive a stringent and clean upper bound [62]

$$B(K^+ \to \pi^+\nu\bar{\nu}) \tag{34}$$
$$< 0.4 \cdot 10^{-10} \left[P_{charm} + A^2 X(m_t) \frac{r_{sd}}{\lambda} \sqrt{\frac{\Delta M_d}{\Delta M_s}} \right]^2$$

Note that the ε-constraint or V_{ub} with their theoretical uncertainties are not needed here. Using $V_{cb} \equiv A\lambda^2 < 0.043$, $r_{sd} < 1.4$ (describing SU(3) breaking in the ratio of B_d to B_s mixing matrix elements) and $\sqrt{\Delta M_d/\Delta M_s} < 0.2$, gives the bound $B(K^+ \to \pi^+\nu\bar{\nu}) < 1.67 \cdot 10^{-10}$, which can be confronted with future measurements of $K^+ \to \pi^+\nu\bar{\nu}$ decay. Here we have assumed

$$\Delta M_s > 12.4\,\mathrm{ps}^{-1} \tag{35}$$

corresponding to the world average presented at this conference. A future increase in this lower bound will strengthen the bound in (34) accordingly. Any violation of (34) will be a clear signal of physics beyond the Standard Model.

Indeed, the decays $K \to \pi\nu\bar{\nu}$, being highly suppressed in the Standard Model, could potentially be very sensitive to New-Physics effects. This topic has been addressed repeatedly in the recent literature [69, 70, 71, 72, 73, 74, 55]. Most discussions have focussed in particular on general supersymmetric scenarios [70, 72, 73, 74, 55]. Large effects are most likely to occur via enhanced Z-penguin contributions. This is expected because the $\bar{s}dZ$ vertex is a dimension-4 operator (allowed by the breaking of electroweak symmetry) in the low-energy effective theory, where the heavy degrees of freedom associated with the New Physics have been integrated out. The corresponding Z-penguin amplitude for $\bar{s} \to \bar{d}\nu\bar{\nu}$ will thus be $\sim 1/M_Z^2$, much larger than the New Physics contribution of dimension 6 scaling as $\sim 1/M_S^2$, if we assume that the scale of New Physics $M_S \gg M_Z$. It has been pointed out in [74] that, in a generic supersymmetric model with minimal particle content and R-parity conservation, the necessary flavour violation in the induced $\bar{s}dZ$ coupling is potentially dominated by double LR mass insertions related to squark mixing. This mechanism could lead to sizable enhancements still allowed by known constraints. An updated discussion is given in [55]. Typically, enhancements over the Standard Model branching ratios could be up to a factor of 10 (3) for $K_L \to \pi^0\nu\bar{\nu}$ ($K^+ \to \pi^+\nu\bar{\nu}$) within this framework.

In the experimental quest for $K \to \pi\nu\bar{\nu}$ an important step has been accomplished by Brookhaven experiment E787, which observed a single, but very clean candidate event for $K^+ \to \pi^+\nu\bar{\nu}$ in 1997. This event is practically background free and corresponded to a branching fraction of $B(K^+ \to \pi^+\nu\bar{\nu}) = (4.2^{+9.7}_{-3.5}) \cdot 10^{-10}$ [75]. The experiment is very challenging and requires extremely good control over possible background (see fig. 5). Charged pions from K^+ decays at rest are analyzed in momentum. The signal pions have a continuous momentum spectrum, which distinguishes them from the monochromatic spectra of the major background modes $K^+ \to \pi^+\pi^0$ (where the π^0 escapes detection) and $K^+ \to \mu^+\nu$ (where μ^+ is misidentified as π^+). Excellent photon rejection capability and distinction of π^+ versus μ^+, by requiring observation of the full $\pi^+ \to \mu^+ \to e^+$ decay sequence, are essential elements of the experiment. E787 is continuing and has very recently released an updated result, based on about 2.5 times the data underlying the previous measurement. In addition to the single, earlier event, no new signal candidates are observed, which translates into [77]

$$B(K^+ \to \pi^+\nu\bar{\nu}) = \left(1.5^{+3.5}_{-1.3}\right) \cdot 10^{-10} \qquad \text{BNL E787} \tag{36}$$

The experiment is still ongoing and will be followed by a successor experiment, E949 [78], at Brookhaven. Recently, a new experiment, CKM [79], has been proposed to measure $K^+ \to \pi^+\nu\bar{\nu}$ at the Fermilab Main Injector, studying K decays in flight. Plans to investigate this process also exist at KEK for the Japan Hadron Facility (JHF) [80].

The neutral mode, $K_L \to \pi^0\nu\bar{\nu}$, is currently pursued by KTeV. The present upper limit reads [81]

$$B(K_L \to \pi^0\nu\bar{\nu}) < 5.9 \cdot 10^{-7} \qquad \text{KTeV} \tag{37}$$

For $K_L \to \pi^0\nu\bar{\nu}$ a model independent upper bound can be infered from the experimental result on $K^+ \to \pi^+\nu\bar{\nu}$ [69]. It is given by $B(K_L \to \pi^0\nu\bar{\nu}) < 4.4 B(K^+ \to \pi^+\nu\bar{\nu}) < 2 \cdot 10^{-9}$. At least this sensitivity will have to be achieved before New Physics is constrained with $B(K_L \to \pi^0\nu\bar{\nu})$. Concerning the future of $K_L \to \pi^0\nu\bar{\nu}$ experiments, a proposal exists at Brookhaven (BNL E926) to measure this decay at the AGS with a sensitivity of $\mathcal{O}(10^{-12})$ [82]. There are furthermore plans to pursue this mode with comparable sensitivity at Fermilab [83] and KEK [84]. The prospects for $K_L \to \pi^0\nu\bar{\nu}$ at a ϕ-factory are discussed in [85].

It will be very exciting to follow the development and outcome of the ambitious projects aimed at measuring $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. Their goal is highly worthwhile.

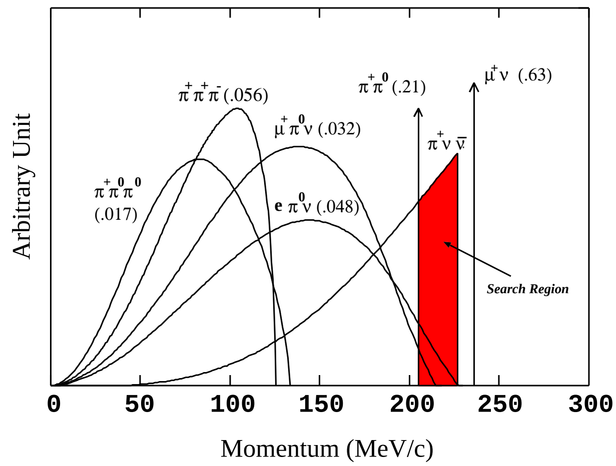

Figure 5. *Momentum spectrum of charged particles from stopped K^+ decays (from [76]).*

4.3. T-odd Asymmetries

The time reversal operation T is closely connected with the CP transformation. If CPT is conserved, which by the CPT theorem is true for quantum field theories under very general assumptions, then the known violation of CP symmetry implies T violation. It is thus of great interest to study T-odd observables and to measure those effects in the laboratory. With the first observation of such T-odd asymmetries, reported recently by CPLEAR at CERN and KTeV at Fermilab, the subject has received renewed attention.

CPLEAR studies kaons from proton-antiproton annihilation at low energies. This experiment measured the following T-odd rate asymmetry (Kabir test) [86]

$$\frac{R[\bar{K}(0) \to K(t)] - R[K(0) \to \bar{K}(t)]}{R[\bar{K}(0) \to K(t)] + R[K(0) \to \bar{K}(t)]} = (6.6 \pm 1.6) \cdot 10^{-3} \tag{38}$$

observing, e.g., $p\bar{p} \to K^+\pi^-\bar{K}^0$ and the subsequent decay via mixing $\bar{K}^0 \to K^0 \to \pi^- e^+ \nu$. The charged kaon provides the initial tag (time 0), the charged lepton the final tag (time t) of the flavour of the neutral kaon. From conventional CP violation and assuming CPT, the asymmetry is expected to be $4\mathrm{Re}\bar{\varepsilon} = (6.6 \pm 0.1) \cdot 10^{-3}$, in excellent agreement with (38).

The KTeV collaboration investigates kaon decays in a high-energy fixed-target experiment. They studied the rare decay $K_L \to \pi^+\pi^-e^+e^-$. The amplitude for this decay mode receives two contributions with different CP properties. First, a $K_L \to \pi^+\pi^-$ decay, where the charged pion legs radiate a virtual photon that produces the e^+e^- pair. This mechanism, refered to as the bremsstrahlung contribution, is clearly CP violating because of the underlying $K_L \to \pi\pi$ transition. The second component is a $M1$ direct emission amplitude, where the virtual photon and the pions originate from the same vertex. This contribution is CP conserving. There are other contributions, but these are the most important ones. The interference of the two amplitudes induces an asymmetry in the angular distribution of the decay rate, $d\Gamma/d\phi$, where ϕ is the angle between the $(\pi^+\pi^-)$- and the (e^+e^-) decay plane. The distribution can be written as

$$\frac{d\Gamma}{d\phi} = \Gamma_1 \cos^2\phi + \Gamma_2 \sin^2\phi + \Gamma_3 \sin\phi\cos\phi \tag{39}$$

The last term $\sim \Gamma_3$ describes the asymmetry. Using $\vec{z} = (\vec{p}_{\pi^+} + \vec{p}_{\pi^-})/|\vec{p}_{\pi^+} + \vec{p}_{\pi^-}|$, the unit vector in the direction of the total pion momentum, and the normals to the pion (electron) decay planes, $\vec{n}_\pi$ ($\vec{n}_e$), one has

$$\sin\phi = (\vec{n}_\pi \times \vec{n}_e) \cdot \vec{z} \tag{40}$$

Thus, $\sin\phi$ is both CP-odd and T-odd. The interference term in (39) can be isolated by forming the asymmetry

$$|\mathcal{A}| = \left| \frac{1}{\Gamma} \int d\phi \frac{d\Gamma}{d\phi} \mathrm{sign}(\sin\phi\,\cos\phi) \right| \tag{41}$$

The existence of this asymmetry was pointed out in [87] and predicted to be $|\mathcal{A}| \approx 14\%$. The subject was subsequently also discussed in [88]. It is interesting to note that a CP asymmetry of this size can be generated in K_L decay, where typical effects are at the permille level. The reason is a relative dynamical suppression of the CP conserving component that allows it to interfere more effectively with the small CP violating amplitude.

KTeV has measured the asymmetry and the branching ratio and found [89]

$$|\mathcal{A}| = 13.6 \pm 2.5 \pm 1.2\% \tag{42}$$

$$B(K_L \to \pi^+\pi^-e^+e^-) = (3.32 \pm 0.14 \pm 0.28) \cdot 10^{-7} \tag{43}$$

Both asymmetry and branching ratio are in full agreement with the predictions [87]. Preliminary results were also presented by NA48 at CERN. They likewise agree with expectations (J. Nassalski, these proceedings):

$$|\mathcal{A}| = 20 \pm 5\% \quad (\mathrm{MC}22\%) \quad [\text{with cuts}] \tag{44}$$

$$B(K_L \to \pi^+\pi^-e^+e^-) = (2.90 \pm 0.15) \cdot 10^{-7} \tag{45}$$

Under the assumption of CPT invariance, both the Kabir test and the angular asymmetry in $K_L \to \pi^+\pi^-e^+e^-$ are equivalent to the well-known indirect CP violation effect in the neutral kaon system. (The possibility to extract information on direct CP violation from $K_L \to \pi^+\pi^-e^+e^-$ has been discussed in [87, 88]. Unfortunately the expected signals are very small.) The significance of both experiments is that they demonstrate explicitly the existence of T-odd asymmetries in nature, anticipated from CP violation on the basis of the CPT theorem.

A somewhat different question is the problem of testing T violation directly, without assuming CPT symmetry. In general, the existence of a T-odd correlation in a decay process does not by itself imply T violation, which requires the interchange of initial and final states in constructing the asymmetry. This condition is, for instance, not fulfilled in $K_L \to \pi^+\pi^-e^+e^-$ decay. On the other hand, it holds apparently for the Kabir test. A potential problem for a direct test of T violation in this latter case is the need to use tagging via the decay $K \to \pi e\nu$. Independent measurments can however be used to avoid this loophole [86]. These issues are further discussed in [90, 91, 92, 93, 94].

There are other T-odd observables that have been proposed in the literature. A typical example is the transverse muon polarization

$$P^T_\mu = \langle \hat{s}_\mu \cdot \frac{(\vec{p}_\mu \times \vec{p}_\pi)}{|\vec{p}_\mu \times \vec{p}_\pi|} \rangle \tag{46}$$

in $K^+ \to \pi^0\mu^+\nu$ decay. A nonvanishing polarization could arise from the interference of the leading, standard W-exchange amplitude with a charged-higgs exchange contribution involving CP violating couplings. P^T_μ is therefore an interesting probe of New Physics [95] with conceivable effects of $P^T_\mu \sim 10^{-3}$. Planned experiments could reach a sensitivity of $P^T_\mu \sim 10^{-4}$ [96, 97]. Independently of CP or T violation a nonvanishing P^T_μ can in principle be induced by final state interactions (FSI). (Note that $P^T_\mu \neq 0$ is not forbidden by CP or T symmetry, although it can be induced when these symmetries are violated.) In the case of $K^+ \to \pi^0\mu^+\nu$ FSI phases arise only at two loops in QED and are very small ($\sim 10^{-6}$) [98]; they would be much larger in $K^0 \to \pi^-\mu^+\nu$. We remark that, in contrast, the angular asymmetry in $K_L \to \pi^+\pi^-e^+e^-$ discussed above cannot be generated by FSI alone since it is forbidden by CP invariance.

Another T-odd observable of interest is the transverse muon polarization in $K^+ \to \mu^+\nu\gamma$ (see [99] for a recent discussion). Related topics are also addressed in the contribution of C.-Q. Geng, these proceedings.

4.4. Further Results on Rare K Decays

The field of rare K decays is very rich in opportunities. They range from studies of chiral perturbation theory as a framework to describe the low-energy dynamics of QCD in weak decays ($K_S \to \gamma\gamma$, $K_L \to \pi^0\gamma\gamma$, $K_S \to \pi^0e^+e^-$, $K_L \to l^+l^-\gamma$, ...), over tests of flavour dynamics at the weak scale ($K \to \pi\nu\bar{\nu}$, $K_L \to \pi^0e^+e^-$, $K_L \to \mu^+\mu^-$, ...), to searches for exotic phenomena, such as lepton flavour violation, with potential sensitivity to scales of several 100 TeV ($K_L \to \mu e$, $K \to \pi\mu e$). A number of useful review articles exists in the literature on the subject of rare K decays and kaon CP violation [100, 101, 102, 103, 104].

Without going into details we shall summarize here some recent experimental results that are relevant to various important aspects of this large and fruitful field of research. (More information can be found in the talks of parallel session 4 at this conference.)

The updated results from KTeV include the branching ratio limits

$$\begin{aligned} B(K_L \to \pi^0 e^+e^-) &< 5.6\cdot 10^{-10} \quad (\sim 5\cdot 10^{-12}) \\ B(K_L \to \pi^0 \mu^+\mu^-) &< 3.4\cdot 10^{-10} \quad (\sim 1\cdot 10^{-12}) \end{aligned}$$

where the Standard Model expectation is shown in brackets.

NA48 presented measurements of the radiative decays

$$\begin{aligned} B(K_L \to e^+e^-\gamma) &= (1.05 \pm 0.02 \pm 0.04)\cdot 10^{-5} \\ B(K_L \to e^+e^-\gamma\gamma) &= (5.82 \pm 0.27 \pm 0.49)\cdot 10^{-7} \end{aligned}$$

The current limits on kaon decays violating lepton family number are

$$\begin{aligned} B(K_L \to \mu e) &< 4.7\cdot 10^{-12} \quad \text{BNL E871} \\ B(K^+ \to \pi^+\mu^+e^-) &< 4.8\cdot 10^{-11} \quad \text{BNL E865} \\ B(K_L \to \pi^0\mu e) &< 3.2\cdot 10^{-9} \quad \text{KTeV} \end{aligned}$$

from experiments at Brookhaven and Fermilab. As a byproduct of the search for $K_L \to \mu e$, BNL E871 has observed the mode $K_L \to e^+e^-$, finding [105]

$$B(K_L \to e^+e^-) = (8.7^{+5.7}_{-4.1})\cdot 10^{-12} \tag{47}$$

This result testifies to the extraordinary precision achievable in kaon experiments and is noteworthy as the smallest branching ratio ever observed. Theoretically this decay is determined by long distance contributions. Because those are dominated by calculable large logarithms $\ln(m_K/m_e)$, theoretical estimates [106, 107] are nevertheless relatively reliable and gave predictions in agreement with (47).

5. CP Violation in B Decays

5.1. Theoretical Basis

B meson decays offer a large array of exciting possibilities to expand our knowledge of flavour physics and to complement what we can learn from kaon studies. The large number of available channels makes this field rich and complex, but provides us at the same time with multiple opportunities to combine different pieces of information and to extract the underlying physics. Of special interest is the question of CP violation in the B sector, which is most promisingly addressed using nonleptonic modes.

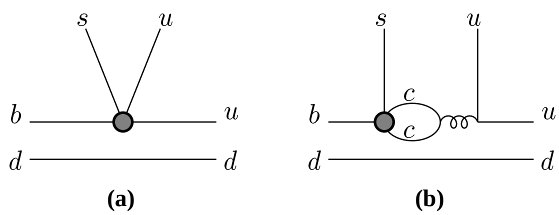

Figure 6. *Typical matrix elements of local four-quark operators for $B \to \pi^- K^+$: Leading order tree-level matrix element (a), penguin contraction involving a charm-loop (b).*

The theoretical basis for a discussion of hadronic B decays is given by the low-energy effective Hamiltonian (low-energy with respect to M_W). To be specific, let us consider strangeness-changing ($\Delta S = 1$) b decays, which describe for instance $B \to \pi K$, but also $B \to J/\Psi K_S$. In this case the effective Hamiltonian has the form

$$\mathcal{H}_{eff} = \tag{48}$$
$$\frac{G_F}{\sqrt{2}} V_{ub}^* V_{us} \left(C_1 Q_1^u + C_2 Q_2^u + \sum_p C_p Q_p \right)$$
$$+ \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} \left(C_1 Q_1^c + C_2 Q_2^c + \sum_p C_p Q_p \right)$$

Here the C_i are Wilson coefficients, containing the short-distance physics from scales $\mu > \mu_b = \mathcal{O}(m_b)$, while the Q_i are local four-quark operators, whose matrix elements comprise the contributions from scales $\mu < \mu_b$. The operators have the flavour form $Q_i^u \sim (\bar{b}u)(\bar{u}s)$, $Q_i^c \sim (\bar{b}c)(\bar{c}s)$ and for the penguin operators $Q_p \sim (\bar{b}s)(\bar{q}q)$ ($q = u,\ d,\ s,\ c,\ b$), and come in different Dirac and colour structures. The Hamiltonian for $\Delta S = 0$ transitions (describing for instance $B \to \pi\pi$) is similar to (48), with an obvious change of flavour labels in the operators and CKM quantities ($s \leftrightarrow d$).

Typical diagrams for the matrix elements are illustrated in fig. 6. In writing (48) the CKM combination $V_{tb}^* V_{ts}$ has been eliminated using CKM unitarity, so that the GIM cancellation is already manifest. The amplitudes to be derived from (48) are clearly seen to exhibit ingredients necessary for CP violation. There are two components with, in general, different weak and strong phases, for instance

$$A(B \to \pi^- K^+) = e^{i\gamma} A_T + A_P \tag{49}$$

where the term proportional to A_T (A_P) corresponds to the second (third) line in (48), and we have used standard phase conventions with $\arg(V_{ub}^* V_{us}) = \gamma$, $\arg(V_{cb}^* V_{cs}) \approx 0$. Here we have made the weak phases explicit, while strong phases are still contained in $A_{T,P}$. The components $\exp(i\gamma) A_T$ and A_P are often refered to as the tree- and the penguin amplitude, respectively.

Eq. (48) allows us already to read off some gross features for the amplitudes of a given decay. In general $C_p \ll C_{1,2}$. Also $V_{ub}^* V_{us} \sim \lambda^4 \ll V_{cb}^* V_{cs} \sim \lambda^2$. Now if, for example, the final state contains u-quarks, as $B \to \pi^- K^+$, $Q_{1,2}^u$ contribute at leading order, $Q_{1,2}^c$ only through loops. Then the second line of (48) can compete with the third line, despite the CKM suppression of the former. For $B \to J/\Psi K_S$, on the contrary, $Q_{1,2}^c$ give the largest contribution and the term with $V_{ub}^* V_{us}$ is entirely negligible.

The Wilson coefficients are well under control theoretically. They are known at next-to-leading order in renormalization group improved QCD [18, 19] (for a review see [14]). The main theoretical problem are the matrix elements, e.g. $\langle \pi K | Q_i | B \rangle$ or $\langle \pi\pi | Q_i | B \rangle$, which involve complicated nonperturbative QCD dynamics. In order to solve (or to circumvent) this problem, the following possibilities could be distinguished.

First, the hadronic matrix elements may simply cancel in the observable of interest, as is the case for the CP asymmetry in $B \to J/\Psi K_S$. Second, the uncertainties can sometimes be eliminated or reduced by combining various channels, exploiting SU(2) or SU(3) flavour symmetry. A general framework for parametrizing the hadronic amplitudes in a consistent way in terms of the possible Wick contractions has been developed in [108, 109]. This could be a useful starting point for the systematic incorporation of other approaches or approximations. Finally, one can try to compute the QCD dynamics of the decay process directly, at least in a certain limit.

We will come back to applications of the first two possibilities below and turn now to theoretical approaches towards computing explicitly the matrix elements of two-body nonleptonic B decays.

A simple approach, which has been widely used in phenomenological studies [110, 111], is the *naive factorization* of matrix elements, schematically

$$\langle \pi^+ \pi^- | (\bar{u}b)_{V-A} (\bar{d}u)_{V-A} | \bar{B} \rangle \to \tag{50}$$
$$\langle \pi^+ | (\bar{u}b)_{V-A} | \bar{B} \rangle \cdot \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle$$

for the example of $B \to \pi\pi$. The justification for this procedure has been less clear (see also [112] for applications and [113] for a critical discussion). An obvious issue is the proper scheme and scale dependence of the matrix elements of four-quark operators, which is needed to cancel the corresponding dependence in the Wilson coefficients. Clearly, this dependence is lost in naive factorization as the factorized currents are scheme independent objects. In many cases the factorization can be justified in the large-N_c limit of QCD [114], but this approximation is often too crude for a reliable phenomenology. In any case one would prefer not to rely exclusively on this argument. A different qualitative justification for factorization has been given by Bjorken [115]. It is based on the *colour transparency* of the hadronic environment for the highly energetic pion emitted in the decay of a B meson (the π^- in the above example, which is being created from the vacuum). Formally this is related to the decoupling of soft gluons from the small-size colour-singlet object that the emitted pion represents. This aspect of cancellation of soft IR divergences was also addressed in [116] in the context of $B \to D\pi$ decay employing a *large-energy effective theory (LEET)* framework. The important issues of collinear singularities and the formation of the boundstate pion were however not treated in that paper. The idea to compute decays such as $B \to \pi\pi$ in perturbative QCD has been pursued using a *hard-scattering approach* in [117]. The result of this study raised however several questions that remained open. For instance, the $B \to \pi$ transition form factor was assumed to be dominated by hard gluon exchange, leading to unrealistically small rates. The role of nonfactorizable contributions was not explicitly addressed. A hard-scattering framework was also applied by other groups [118, 119, 120, 121], but without providing a clear conceptual basis. The question of separating model-dependent from model-independent ingredients in the analysis, or issues as the scheme dependence of matrix elements were left unanswered. A further method to gain dynamical insight into exclusive hadronic matrix elements uses *QCD sum rule techniques* to estimate nonfactorizable effects. It has been first discussed for $B \to D\pi$ decays [122] (see also [123] for class II decays) and was later applied to $B \to J/\Psi K$ [124]. Although the validity of the sum rule implies certain restrictions on the applicability of this approach, it can be expected to give useful information in various cases. Ideas to address nonfactorizable effects in $B \to D\pi$ by means of an operator product expansion are discussed in [125].

A new, systematic approach, going beyond previous attempts, was recently formulated in [126]. There it was proposed that factorization can be justified within QCD to leading order in the heavy-quark limit for a large class of two-body hadronic B decays. The statement of *QCD factorization* in the case of $B \to \pi\pi$, for instance, can be schematically written

$$A(B \to \pi\pi) = \langle\pi|j_1|B\rangle\,\langle\pi|j_2|0\rangle \cdot \left[1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_B}\right)\right] \tag{51}$$

Up to corrections suppressed by Λ_{QCD}/m_B the amplitude is calculable in terms of simpler hadronic objects: It factorizes, to lowest order in α_s, into matrix elements of bilinear quark currents ($j_{1,2}$) that are expressed by form factors such as f_π and $f_+(0)$ [124, 127]. To higher order in α_s, but still to leading order in Λ_{QCD}/m_B, there are nonfactorizable corrections, which are however governed by hard gluon exchange. They are therefore again calculable in terms of few universal hadronic quantities, such as the pion wave function $\Phi_\pi(x)$. This treatment of hadronic B decays is based on the analysis of Feynman diagrams in the heavy-quark limit, utilizing consistent power counting to identify the leading contributions. The framework is very similar in spirit to more conventional applications of perturbative QCD in exclusive hadronic processes with a large momentum transfer, as the pion electromagnetic form factor [128, 129, 130]. The assumptions used, namely the demonstration of a separation of hard and soft physics to leading power in a high energy process on the basis of Feynman diagrams, is very general and underlies basically all practical applications of perturbative QCD. In this sense the approach of [126] constitutes a general, systematic and QCD-based framework to analyze nonleptonic B decays. The new approach may be viewed as a consistent formalization of Bjorken's colour transparency argument [115]. In addition the method includes, for $B \to \pi\pi$, the hard non-factorizable spectator interactions, penguin contributions and rescattering effects. As a corollary, one finds that strong rescattering phases are either of $\mathcal{O}(\alpha_s)$, and calculable, or power suppressed. In any case they vanish therefore in the heavy-quark limit. QCD factorization is valid for cases where the emitted particle (the meson created from the vacuum in the weak process, as opposed to the one that absorbs the b-quark spectator) is a small size colour-singlet object, e.g. either a fast light meson (π, ϱ, K, K^*) or a J/Ψ. For the special case of the ratio $\Gamma(B \to D^*\pi)/\Gamma(B \to D\pi)$ the perturbative corrections to naive factorization have been evaluated in [131] using a formalism similar to the one described above. Note that factorization cannot be justified in this way if the emitted particle is a heavy-light meson ($D^{(*)}$), which is not a compact object and has strong overlap with the remaining hadronic environment. An important general caveat is the question of power corrections, which is relevant for assessing the accuracy in phenomenological applications. Moreover, (51) has so far been demonstrated to one loop. Clearly, more theoretical work is necessary to establish this result at higher orders. For more details on QCD factorization in B decays see [126] and the contribution of M. Beneke in these proceedings.

5.2. $B \to J/\Psi K_S$ and $B \to \pi^+\pi^-$

$B \to J/\Psi K_S$ is the prototype decay for the study of CP violation in the b sector. It is one of the rare cases where good experimental feasibility and a clear signature are matched by exceedingly small theoretical uncertainties. For the latter property the decisive reason is the simple structure of the amplitude, which has the form

$$A(B \to \Psi K_S) = |A| e^{i\delta_s} e^{i\phi_w}(1 + \mathcal{O}(\lambda^2 \cdot \text{penguin})) \tag{52}$$

and where the small $\mathcal{O}(\lambda^2)$ contribution is negligible. The amplitude is then governed by a single weak phase ϕ_w. As a consequence there is practically no direct CP violation. Moreover, all hadronic uncertainties, related to the single factor $|A|\exp(i\delta_s)$, cancel in the time-dependent mixing induced CP asymmetry

$$\begin{aligned} \mathcal{A}_{CP} &= \frac{\Gamma(B(t) \to \Psi K_S) - \Gamma(\bar{B}(t) \to \Psi K_S)}{\Gamma(B(t) \to \Psi K_S) + \Gamma(\bar{B}(t) \to \Psi K_S)} \\ &= -\sin 2\beta \sin \Delta M t \end{aligned} \tag{53}$$

which allows for a simple and direct interpretation in terms of the CKM angle β. Current constraints on the unitarity triangle (see sect. 3) lead to the Standard Model expectation of $\sin 2\beta = 0.71 \pm 0.14$ [6]. One of the most interesting recent developments in CP violation has been the CDF measurement [132]

$$\sin 2\beta = 0.79^{+0.41}_{-0.44} \qquad \text{(CDF)} \tag{54}$$

based on the full run-I data sample consisting of about 400 (untagged) $B \to J/\Psi K_S$ decays. The result has been made possible by a careful combination of several different tagging strategies (see talk by G. Bauer, these proceedings). This 2-σ measurement provides the first evidence for CP violation with B mesons. It excludes negative values of $\sin 2\beta$ at a confidence level of 93%. Previous analyses were reported in [133]. The determination of $\sin 2\beta$ will be further pursued by several experiments (BaBar, Belle, Hera-B, CDF (run II)) and exciting new results can be expected in the near future.

A complementary source of information on CKM quantities is the decay $B \to \pi^+\pi^-$. As for $B \to J/\Psi K_S$, the final state is an eigenstate of CP and an interesting CP asymmetry can be induced by B–$\bar{B}$ mixing. However, now the amplitude has two components

$$A(B \to \pi^+\pi^-) = V^*_{ub} V_{ud} T + V^*_{cb} V_{cd} P \tag{55}$$

where both are of the same order in the Wolfenstein expansion and have different weak phases ($V^*_{ub} V_{ud} \sim \lambda^3 \exp(i\gamma)$, $V^*_{cb} V_{cd} \sim \lambda^3$). For this reason the hadronic dynamics will not cancel completely when forming the CP asymmetry, but will enter the observable in the ratio P/T, expected to be about 10 – 20% in modulus. The time dependent asymmetry is then more complicated

$$\mathcal{A}_{CP}(B \to \pi^+\pi^-) = C \cos \Delta M t + S \sin \Delta M t \tag{56}$$

Only when $P/T \to 0$ do the coefficients reduce to a simple form, $C = 0$, $S = -\sin 2\alpha$. In this case the asymmetry would be a direct measure of $\sin 2\alpha$, but the penguin amplitude P cannot a priori be neglected. Various proposals exist to deal with this complication and still make use of the information from $B \to \pi^+\pi^-$.

A well-known strategy is the isospin analysis of Gronau and London [134], a classical example for the use of flavour symmetry to eliminate hadronic uncertainties. In the case at hand this method requires the measurement of $B^\pm \to \pi^\pm\pi^0$ and $B/\bar{B} \to \pi^0\pi^0$. Because of the small expected branching ratio and the experimentally problematic final state of the latter decay, it appears to be extremely difficult to apply this idea in practice. Another option is to relate CP violation measurements in $B \to \pi^+\pi^-$ with the corresponding observables in $B_s \to K^+K^-$ and to use the fact that the hadronic dynamics of these processes is related by U-spin symmetry (the subgroup of flavour SU(3) acting on $(d,\ s)$) [135, 136]. Finally, one may try to estimate P/T theoretically, which would allow us to extract $\sin 2\alpha$ from a measurement of (56) alone ([126] and M. Beneke, these proceedings).

5.3. $B \to \pi K$ and γ

Decays of B mesons into a pair of light pseudoscalars play a central role in exploring CP violation. The recent CLEO discovery of such modes, which have small branching ratios $\sim 10^{-5}$, has therefore attracted considerable interest. In fact, experimental access to these processes opens new avenues in the search for CP violating phenomena outside the kaon system, for the determination of CKM parameters and for probing physics beyond the Standard Model. Among the results reported by the CLEO collaboration [137], the following are especially relevant for the present discussion (for additional information see the talk by M. Artuso, these proceedings)

$$B(B \to \pi^\pm K^\mp) = (18.8^{+2.8}_{-2.6} \pm 1.3) \cdot 10^{-6} \quad (57)$$

$$B(B^\pm \to \pi^\pm K) = (18.2^{+4.6}_{-4.0} \pm 1.6) \cdot 10^{-6} \quad (58)$$

$$B(B^\pm \to \pi^0 K^\pm) = (12.1^{+3.0+2.1}_{-2.8-1.4}) \cdot 10^{-6} \quad (59)$$

$$B(B \to \pi^+\pi^-) = (4.7^{+1.8}_{-1.5} \pm 0.6) \cdot 10^{-6} \quad (60)$$

$$B(B^\pm \to \pi^\pm \pi^0) < 12 \cdot 10^{-6} \quad (61)$$

These measurements are still quoted as an average over charge conjugated modes (e.g. $B^+ \to \pi^0 K^+$ and $B^- \to \pi^0 K^-$). Measured separately, a difference in branching ratio for these conjugated processes would reveal CP violation.

To give an illustration of how the above results can, in principle, be used to constrain CKM parameters, let us first consider the ratio

$$R = \frac{B(B \to \pi^\pm K^\mp)}{B(B^\pm \to \pi^\pm K)} \quad (62)$$

The following method was first suggested by Fleischer and Mannel [138] and triggered a lively debate in the subsequent literature. The decay mode $B^0 \to \pi^- K^+$ proceeds at the quark level as $\bar{b}(d) \to \bar{s}u\bar{u}(d)$, which can be generated by tree-level transitions (simple W-exchange $\bar{b} \to \bar{s}u\bar{u}$ at zeroth order in strong interactions) or by penguin contributions. The amplitude has therefore two components, 'tree' t and 'penguin' p, as can also be read off from (48):

$$A(B^0 \to \pi^- K^+) = \lambda^2 e^{i\gamma} t + p \quad (63)$$

The weak phase of the tree contribution is $\arg(V^*_{ub} V_{us}) = \gamma$, the one of the penguin $\arg(V^*_{ub} V_{us}) \simeq 0$, in usual phase conventions. By contrast the process $B^+ \to \pi^+ K^0$ comes from $\bar{b}(u) \to \bar{s}d\bar{d}(u)$ and has only a penguin contribution, related to the previous one by isospin symmetry. Thus one expects

$$A(B^+ \to \pi^+ K^0) \approx p \quad (64)$$

neglecting electroweak penguins and rescattering effects for the moment. Assuming (63) and (64), and the corresponding relations for the CP conjugated modes (where $\gamma \to -\gamma$), it is straightforward to show that $\cos\gamma$ has to satisfy the bound [138]

$$|\cos\gamma| \geq \sqrt{1-R} \quad (65)$$

independently of the hadronic quantity p/t. The Fleischer-Mannel bound in its original form (65) is useful if $R < 1$. In this case the bound excludes in the (ϱ, η) plane an angular region centered around the positive η-axis. While earlier results from CLEO seemed to indicate a low value of R, the most recent numbers (57), (58) are well compatible with $R \approx 1$. Nevertheless the present simple example is useful to illustrate the basic idea behind similar, more general approaches that could still be used to extract information from R [139, 140]. A further comment is in order concerning the bound (65). As mentioned above, the simple form of (64), leading to (65), neglects rescattering effects. At the quark level those proceed through $(u)\bar{b} \to (u)\bar{u}u\bar{s} \to (u)\bar{d}d\bar{s}$, where $u\bar{u}$ rescatters into $d\bar{d}$, similarly to the penguin contraction shown in fig. 6. From the $\bar{b} \to \bar{u}$ transition a component with weak phase γ is thus induced also in (64), modifying the above argument. In the partonic picture this effect is expected to be very small. The validity of this description has subsequently been the subject of a controversial discussion in the literature [141]. The recent developments in the theory of hadronic B-decays [126] lend support to a quark-level description and the original assumption of a small rescattering contribution to $B^+ \to \pi^+ K^0$. Correspondingly, the $B^+ \to \pi^+ K^0$ amplitude would be dominated by a single weak phase and direct CP violation in the channels $B^\pm \to \pi^\pm K$ would be negligible. This can be tested experimentally.

Another strategy to constrain γ was discussed by Neubert and Rosner [142, 143], based on earlier work in [144, 145]. Here one starts from the ratio

$$R_*^{-1} = \frac{2B(B^\pm \to \pi^0 K^\pm)}{B(B^\pm \to \pi^\pm K)} \quad (66)$$

Still neglecting rescattering effects, but incorporating dominant electroweak penguin contributions using SU(3) flavour symmetry, [142] obtain a bound of the form ($\delta_{EWP} \approx 0.7$)

$$|\cos\gamma - \delta_{EWP}| > \frac{\sqrt{B(B^\pm \to \pi^\pm K)}}{0.38\sqrt{B(B \to \pi^\pm \pi^0)}} \left(1 - \sqrt{R_*}\right) \quad (67)$$

that can be translated into a bound on γ.

These ideas can be generalized to systematic analyses of $B \to \pi K$ decays and the determination of γ. Theoretical issues that can limit the practical usefulness of a given approach, and that need to be addressed in this context, are the role of electroweak penguins, rescattering effects (final state interactions (FSI)) and corrections to SU(3) flavour symmetry. Additional information can be gained by combining $B \to \pi K$ with other modes, such as $B^+ \to \pi^+\pi^0$, $B \to K^+K^-$ and using results on direct CP violation, which should become available in the future. General analyses of this type have been presented in [139, 143, 146, 147]. Further theoretical insight into the dynamics of exclusive nonleptonic decays in the heavy-quark limit, discussed above, could be useful for a successful implementation of this program.

5.4. Other Strategies

To illustrate the variety of opportunities for probing CP violation with B decays we briefly present a selection of further strategies that have been proposed in the literature. We concentrate on approaches with good control of theoretical uncertainties.

A clean extraction of γ can in principle be obtained from $B^\pm \to D_{CP}K^\pm$, $B^\pm \to D^0K^\pm$ and $B^\pm \to \bar{D}^0K^\pm$ decays (D_{CP} denotes a neutral D meson seen in a final state that is an eigenstate of CP). The method was proposed in [148] and refined in [149].

Another interesting approach makes use of $B_s \to D_s^\pm K^\mp$. From a time-dependent analysis one can determine $\sin^2\gamma$ [150]. As in the previous example, the underlying quark-level transitions are $b \to u\bar{c}s$, $b \to c\bar{u}s$. Hence, no penguin contributions are possible. Another advantage of this strategy are the sizable branching ratios ($\sim 10^{-4}$), while a disadvantage is the need to resolve the rapid time oscillations in the B_s–$\bar{B}_s$ system.

B_s mesons can also be used to study the mixing induced CP asymmetry $\mathcal{A}_{CP}(B_s \to \Psi\phi)$, which is a B_s analog of $\mathcal{A}_{CP}(B_d \to \Psi K_S)$. Instead of $\sin 2\beta$, $\mathcal{A}_{CP}(B_s \to \Psi\phi)$ measures $2\lambda^2\eta \approx 3\%$. The asymmetry is thus very small in the Standard Model. It is therefore also interesting as a probe of New Physics in B_s–$\bar{B}_s$ mixing, which could give much larger effects.

Additional ways of exploiting the rich information from B meson decays have been discussed by R. Fleischer at this conference. One of these possibilities is the combined use of $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ [135, 136], already briefly mentioned above. These processes are transformed into each other by simply exchanging d and s quarks. The hadronic dynamics governing the two decays is therefore related by U-spin symmetry, the subgroup of flavour SU(3) operating on d and s. The time-dependent CP asymmetries of $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ offer enough information to determine, in the U-spin symmetry limit, the CKM angles β and γ independently of further hadronic input as emphasized in [136]. There it is also shown how the impact of potential U-spin breaking might be controled, for instance by using β (which will be well measured in the near future) as an input to the analysis. The approach looks promising for the LHC era, where $B_s \to K^+K^-$ should be well accessible.

U-spin symmetry may also be exploited for probing γ with combined information from $B_s \to J/\Psi K_S$ and $B_d \to J/\Psi K_S$ [135, 151]. Further options for extracting β and γ are provided by investigating time-dependent angular distributions in $B_d \to J/\Psi\varrho$ and $B_s \to J/\Psi\phi$ [152].

Recent discussions on New Physics effects in B decays and CP violation are given in [153, 154, 155, 156].

Besides the standard use of purely exclusive modes, the possibility of studying CP violation in semi-inclusive decays has been proposed in the literature. In particular the processes $B \to X_s\phi$ [157], $B \to K^{(*)}X$ [158] and $b \to \pi^-u$, $b \to K^-u$ [159] have been discussed. It is also possible to search for CP violation in fully inclusive B decays, to which we turn in the following section.

5.5. Inclusive CP Asymmetries

CP violation in the B_d–$\bar{B}_d$ mixing matrix, measured by $a \equiv \mathrm{Im}(\Gamma_{12}/M_{12})$, is one example of an inclusive CP asymmetry. It arises from a relative phase between the mass-matrix element M_{12} in B–$\bar{B}$ mixing and the off-diagonal element in the B–$\bar{B}$ decay rate matrix $\Gamma_{12} = \sum_f \langle B|f\rangle\langle f|\bar{B}\rangle$, an inclusive quantity. Γ_{12} can be computed using the heavy-quark expansion (HQE) [160, 161], exploiting the fact that $m_b \gg \Lambda_{QCD}$ and assuming local quark-hadron duality. The resulting Standard Model expectation is $a \lesssim 10^{-3}$ for B_d mesons (and smaller still for B_s). The effect could be enhanced by New Physics to typically $a \sim 10^{-2}$ [162].

The quantity a can be measured using a lepton charge asymmetry in time-dependent B decay

$$\mathcal{A}_l = \frac{\Gamma(\bar{B}(t) \to l^+) - \Gamma(B(t) \to l^-)}{\Gamma(\bar{B}(t) \to l^+) + \Gamma(B(t) \to l^-)} = a \qquad (68)$$

Using a similar method ALEPH finds (B. Petersen, these proceedings)

$$a = -0.05 \pm 0.03 \pm 0.01 \qquad (69)$$

which is compatible with zero at a level of sensitivity of a few percent. The same quantity a can also be determined from the tagged and time-dependent totally inclusive B decay asymmetry [163]

$$\begin{aligned} \mathcal{A}_{all}(t) &= \frac{\Gamma(B(t) \to \mathrm{all}) - \Gamma(\bar{B}(t) \to \mathrm{all})}{\Gamma(B(t) \to \mathrm{all}) + \Gamma(\bar{B}(t) \to \mathrm{all})} \\ &= a\left[\frac{x}{2}\sin\Delta Mt - \sin^2\frac{\Delta Mt}{2}\right] \qquad (70) \end{aligned}$$

where ΔM is the mass difference between eigenstates in the B–$\bar{B}$ system, $x = \Delta M/\Gamma$ (with Γ the total decay rate) and 'all' denotes all possible final states. Using this method a has been measured to be ([164, 165, 166, 167] and B. Petersen, R. Hawkings, these proceedings)

$$
\begin{aligned}
a &= -0.006 \pm 0.043^{+0.011}_{-0.009} \qquad \text{(ALEPH)} \\
a &= 0.005 \pm 0.055 \pm 0.013 \qquad \text{(OPAL)} \\
a &= -0.04 \pm 0.12 \pm 0.05 \qquad \text{(SLD)} \\
a &= -0.022 \pm 0.030 \pm 0.011 \qquad \text{(DELPHI)}
\end{aligned}
$$

in agreement with (69). Also direct CP violation may be looked for inclusively. The most up-to-date analysis of such asymmetries in B decays to charmless final states X

$$\mathcal{A}_{dir,incl} = \frac{\Gamma(B^+ \to X) - \Gamma(B^- \to X)}{\Gamma(B^+ \to X) + \Gamma(B^- \to X)} \qquad (71)$$

is given in [168]. There it is estimated that

$$
\begin{aligned}
\mathcal{A}_{dir,incl}(\Delta S = 0) &= \left(2.0^{+1.2}_{-1.0}\right)\% \qquad (72) \\
\mathcal{A}_{dir,incl}(\Delta S = 1) &= (-1.0 \pm 0.5)\% \qquad (73)
\end{aligned}
$$

for transitions with $\Delta S = 0$ and $\Delta S = 1$, respectively.

Finally, flavour-specific inclusive decays (such as $b \to u\bar{u}d$) may also be used to construct mixing-induced CP asymmetries [163].

6. Summary

At present it is still true that CP violation has been firmly established only in a few decay channels of the long-lived neutral kaon, namely $K_L \to \pi\pi$, $K_L \to \pi l\nu$, $K_L \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-e^+e^-$. The main effect is described by a single complex parameter ε, which is well measured and whose size is in good agreement with the Standard Model. Despite some hadronic uncertainties, this agreement is non-trivial and naturally accomodates what is known from independent sources for m_t, V_{cb} or V_{ub}. Simultaneously, what is thus learned from ε in the kaon system implies that large mixing-induced CP violation should be seen in the B sector, $\sin 2\beta \approx 0.7 \pm 0.1$.

Although continuous progress has occurred over the years, leading to important refinements, the basic empirical knowledge on CP nonconservation has nevertheless remained rather limited. This situation is expected to change considerably in the near future. It is probably no exaggeration to say that we are at the beginning of a new era in flavour physics and CP violation. In fact this new phase is already marked by several highlights that were recently reported and presented at this conference:

- Direct CP violation is now, for the first time, firmly established in $K \to \pi\pi$ decays (KTeV, NA48).
- T violation has been demonstrated by the CPLEAR collaboration in $K \leftrightarrow \bar{K}$ transitions.
- The rare decay $K_L \to \pi^+\pi^-e^+e^-$ has been observed and found to exhibit a large, T-odd, CP violating asymmetry in the decay angular distribution (KTeV, NA48), in nice agreement with theoretical predictions.
- Brookhaven experiment E871 has measured $B(K_L \to e^+e^-) = (8.7^{+5.7}_{-4.1}) \cdot 10^{-12}$, the smallest decay branching fraction yet observed.
- Brookhaven experiment E787 has seen one event of the 'golden' decay $K^+ \to \pi^+\nu\bar{\nu}$, corresponding to a branching ratio of $(1.5^{+3.5}_{-1.3}) \cdot 10^{-10}$. The continuing search is approaching Standard Model sensitivity, a development that opens the way to high precision flavour physics.
- First evidence for large CP violation in the B sector is reported from CDF, providing a 2σ signal for the asymmetry in $B/\bar{B} \to \Psi K_S$ and $\sin 2\beta = 0.79^{+0.41}_{-0.44}$.
- The CLEO experiment at Cornell has performed the first measurements of the branching fractions of several rare ($\sim 10^{-5}$), nonleptonic B decays, among them $B \to K^+\pi^-$, $K^+\pi^0$, $K^0\pi^+$, $K^0\pi^0$ and $B \to \pi^+\pi^-$. This class of processes will undoubtedly play an important role for the phenomenology of CP violation in the coming years.

These highlights give us a first glimpse of the rich prospects offered by the world-wide program of precision flavour physics. It is obvious that for a comprehensive view both dedicated rare kaon experiments as well as detailed studies of B decays are necessary. Together they will provide the complementary information that is crucial for a decisive test of the CKM paradigm and for probing what may lie beyond. Other sources of information, as D meson decays, electric dipole moments, etc., could well harbour some surprizes and should also be pursued.

The B factory experiments BaBar at SLAC and Belle at KEK, and the KLOE experiment at the Frascati ϕ factory have already started. In the near future they will be joined by HERA-B, CLEO-III, and the upgraded CDF and D0 detectors at Fermilab for Tevatron Run-II, while several kaon experiments are still ongoing at CERN (NA48), Fermilab (KTeV), Brookhaven and KEK. For the future, second generation B precision experiments (LHC-b at CERN, BTeV at Fermilab) are taking shape, supplemented by ambitious rare K decay projects at Brookhaven

(E926 and E949), Fermilab (KAMI and CKM) and KEK (JHF). Theoretical efforts are continuing to accompany the experimental and technological progress. The weak decay Hamiltonians are now routinely treated at the next-to-leading order in QCD, theoretically clean observables have been identified, phenomenological strategies are being proposed, various field theoretical methods (lattice QCD, QCD sum rules, $1/N_c$ expansion, chiral perturbation theory, factorization) improve our understanding of weak hadronic processes.

The years to come therefore clearly provide exciting opportunities for the phenomenology of weak decays and CP violation, resulting from a close interplay of experimental discovery and theoretical interpretation.

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References

[1] Grossman Y, Nir Y, Rattazzi R hep-ph/9701231
[2] Giudice G F hep-ph/9912279
[3] Buras A J, Lautenbacher M E, Ostermaier G 1994 *Phys. Rev.* D **50** 3433
[4] Herrlich S, Nierste U 1995 *Phys. Rev.* D **52** 6505
[5] Buras A J, Jamin M, Weisz P H 1990 *Nucl. Phys.* B **347** 491
[6] Buras A J hep-ph/9905437
[7] Parodi F et al. hep-ex/9903063
[8] Ali A, London D 1999 *Eur. Phys. J.* C **9** 687
[9] Mele S 1999 *Phys. Rev.* D **59** 113011
[10] Checchia P et al. hep-ph/9907300
[11] Wolfenstein L 1964 *Phys. Rev. Lett.* **13** 562
[12] Alavi-Harati A et al. (KTeV) 1999 *Phys. Rev. Lett.* **83** 22
[13] Fanti V et al. (NA48) 1999 *Phys. Lett.* B **465** 335
[14] Buchalla G, Buras A J, Lautenbacher M E 1996 *Rev. Mod. Phys.* **68** 1125
[15] Flynn J M, Randall L 1989 *Phys. Lett.* B **224** 221; 1990 erratum ibid. **235** 412
[16] Buchalla G, Buras A J, Harlander M K 1990 *Nucl. Phys.* B **337** 313
[17] Buras A J hep-ph/9908395
[18] Buras A J, Jamin M, Lautenbacher M E, Weisz P H 1992 *Nucl. Phys.* B **370** 69; Addendum-ibid. B **375** 501; 1993 *Nucl. Phys.* B **400** 37; Buras A J, Jamin M, Lautenbacher M E 1993 *Nucl. Phys.* B **400** 75
[19] Ciuchini M, Franco E, Martinelli G, Reina L 1994 *Nucl. Phys.* B **415** 403
[20] Bardeen W A, Buras A J, Gérard J-M, 1986 *Phys. Lett.* B **180** 133; 1987 *Nucl. Phys.* B **293** 787; 1987 *Phys. Lett.* B **192** 138
[21] Gupta R hep-ph/9801412; Kenway R D hep-ph/9810054; Lellouch L hep-ph/9906497
[22] Jamin M, Münz M 1995 *Z. Phys.* C **66** 633; Narison S 1995 *Phys. Lett.* B **358** 113; Chetyrkin K G, Pirjol D, Schilcher K 1997 *Phys. Lett.* B **404** 337; Colangelo P et al. 1997 *Phys. Lett.* B **408** 340; Jamin M 1998 *Nucl. Phys. Proc. Suppl.* B **64** 250; Narison S hep-ph/9905264
[23] Bosch S et al. hep-ph/9904408
[24] Becirevic D et al. 1998 *Phys. Lett.* B **444** 401
[25] Aoki S et al. (JLQCD) 1999 *Phys. Rev. Lett.* **82** 4392
[26] Garden J et al. (ALPHA/UKQCD) hep-lat/9906013
[27] Göckeler M et al. (QCDSF) hep-lat/9908005
[28] Ryan S hep-ph/9908386
[29] Bhattacharya T, Gupta R 1997 *Phys. Rev.* D **55** 7203
[30] Gough B et al. 1997 *Phys. Rev. Lett.* **79** 1622
[31] Lellouch L, de Rafael E, Taron J 1997 *Phys. Lett.* B **414** 195; Ynduráin F J 1998 *Nucl. Phys.* B **517** 324; Dosch H G, Narison S 1998 *Phys. Lett.* B **417** 173
[32] Pich A, Prades J 1999 *JHEP* **10** 004 (hep-ph/9909244)
[33] Barate R et al. (ALEPH) hep-ex/9903015
[34] Chetyrkin K G, Kühn J H, Pivovarov A A 1998 *Nucl. Phys.* B **533** 473
[35] Ciuchini M et al. hep-ph/9910236
[36] Hambye T et al. hep-ph/9906434
[37] Bertolini S, Fabbrichesi M, Eeg J O hep-ph/9802405; Bertolini S hep-ph/9908268
[38] Belkov A A et al. hep-ph/9907335
[39] Hambye T et al. 1998 *Phys. Rev.* D **58** 014017; Hambye T, Köhler G O, Soldan P H 1999 *Eur. Phys. J.* C **10** 271
[40] Soldan P H hep-ph/9608281; Hambye T, Soldan P H hep-ph/9908232
[41] Pekurovsky D, Kilcup G hep-lat/9812019
[42] Martinelli G et al. 1995 *Nucl. Phys.* B **445** 81
[43] Kilcup G, Gupta R, Sharpe S R 1998 *Phys. Rev.* D **57** 1654
[44] Conti L et al. 1998 *Phys. Lett.* B **421** 273

[45] Gupta R, Bhattacharya T, Sharpe S R 1997 *Phys. Rev.* D **55** 4036

[46] Bijnens J, Prades J 1999 *JHEP* **01** 023 (hep-ph/9811472); Bijnens J hep-ph/9907307; hep-ph/9907514

[47] Donoghue J F, Golowich E hep-ph/9911309

[48] Knecht M, Peris S, de Rafael E 1999 *Phys. Lett.* B **457** 227

[49] Blum T et al. (Riken-BNL-Columbia) hep-lat/9908025

[50] Gardner S, Valencia G hep-ph/9909202

[51] Ecker G et al. hep-ph/9912264

[52] Keum Y, Nierste U, Sanda A I 1999 *Phys. Lett.* B **457** 157

[53] Pallante E, Pich A hep-ph/9911233

[54] Nierste U hep-ph/9910257

[55] Buras A J et al. hep-ph/9908371

[56] Kagan A L, Neubert M 1999 *Phys. Rev. Lett.* **83** 4429

[57] Jamin M hep-ph/9911390

[58] Littenberg L 1989 *Phys. Rev.* D **39** 3322

[59] Novikov V A et al. 1977 *Phys. Rev.* D **16** 223; Ellis J, Hagelin J S 1983 *Nucl. Phys.* B **217** 189; Dib C, Dunietz I, Gilman F J 1991 *Mod. Phys. Lett.* A **6** 3573

[60] Buchalla G, Buras A J 1993 *Nucl. Phys.* B **398** 285; 1993 *Nucl. Phys.* B **400** 225; 1994 *Nucl. Phys.* B **412** 106

[61] Misiak M, Urban J 1999 *Phys. Lett.* B **451** 161

[62] Buchalla G, Buras A J 1999 *Nucl. Phys* B **548** 309

[63] Marciano W, Parsa Z 1996 *Phys. Rev.* D **53** R1

[64] Rein D, Sehgal L M 1989 *Phys. Rev.* D **39** 3325

[65] Hagelin J S, Littenberg L S, 1989 *Prog. Part. Nucl. Phys.* **23** 1; Lu M, Wise M 1994 *Phys. Lett.* B **324** 461

[66] Buchalla G, Isidori G 1998 *Phys. Lett.* B **440** 170

[67] Buchalla G, Buras A J 1998 *Phys. Rev.* D **57** 216

[68] Buchalla G, Buras A J 1996 *Phys. Rev.* D **54** 6782

[69] Grossman Y, Nir Y 1997 *Phys. Lett.* B **398** 163

[70] Nir Y, Worah M P 1998 *Phys. Lett.* B **423** 326

[71] Hattori T, Hasuike T, Wakaizumi S hep-ph/9804412

[72] Buras A J, Romanino A, Silvestrini L 1998 *Nucl. Phys.* B **520** 3

[73] Buras A J, Silvestrini L 1999 *Nucl. Phys.* B **546** 299

[74] Colangelo G, Isidori G 1998 *JHEP* **09** 009 (hep-ph/9808487)

[75] Adler S et al. (BNL E787) 1997 *Phys. Rev. Lett.* **79** 2204

[76] Komatsubara T K hep-ex/9905014

[77] Sato T, talk presented at Workshop on Physics and Detectors for Daphne, Frascati, Nov. 1999

[78] BNL E949 collaboration, http://www.phy.bnl.gov/e949/

[79] Coleman R et al. (CKM) 1998 FERMILAB-P-0905

[80] Shinkawa T, in JHF98 Proceedings, KEK, Tsukuba

[81] Alavi-Harati A et al. (E799-II/KTeV) hep-ex/9907014

[82] BNL E926 collaboration, http://sitka.triumf.ca/e926/

[83] Cheu E et al. (KAMI) hep-ex/9709026

[84] Inagaki T, in JHF98 Proceedings, KEK, Tsukuba

[85] Bossi F, Colangelo G, Isidori G 1999 *Eur. Phys. J.* C **6** 109

[86] Angelopoulos A et al. (CPLEAR) 1998 *Phys. Lett.* B **444** 43; Apostolakis A et al. (CPLEAR) 1999 *Phys. Lett.* B **456** 297

[87] Sehgal L M, Wanninger M 1992 *Phys. Rev.* D **46** 1035; erratum ibid. 5209; Heiliger P, Sehgal L M 1993 *Phys. Rev.* D **48** 4146

[88] Elwood J K, Wise M B, Savage M J 1995 *Phys. Rev.* D **52** 5095; 1996 erratum ibid. **53** 2855; Elwood J K et al. 1996 *Phys. Rev.* D **53** 4078

[89] Alavi-Harati A et al. (KTeV) hep-ex/9908020

[90] Wolfenstein L 1999 *Phys. Rev. Lett.* **83** 911

[91] Ellis J, Mavromatos N E hep-ph/9903386

[92] Alvarez-Gaumé L, Kounnas C, Lola S, Pavlopoulos P 1999 *Phys. Lett.* B **458** 347

[93] Bigi I I, Sanda A I hep-ph/9904484

[94] Sehgal L M, van Leusen J hep-ph/9908426

[95] Garisto R, Kane G 1991 *Phys. Rev.* D **44** 2038

[96] Abe M et al. (KEK E246) 1999 *Phys. Rev. Lett.* **83** 4253

[97] Diwan M (BNL E923) hep-ex/9801023

[98] Zhitnitsky A R 1980 *Sov. J. Nucl. Phys.* **31** 529

[99] Hiller G, Isidori G 1999 *Phys. Lett.* B **459** 295

[100] Littenberg L, Valencia G 1993 *Ann. Rev. Nucl. Part. Sci.* **43** 729

[101] Winstein B, Wolfenstein L 1993 *Rev. Mod. Phys.* **65** 1113

[102] Ritchie J L, Wojcicki S G 1993 *Rev. Mod. Phys.* **65** 1149

[103] Buchholz P, Renk B 1997 *Prog. Part. Nucl. Phys.* **39** 253

[104] D'Ambrosio G, Isidori G 1998 *Int. J. Mod. Phys.* A **13** 1

[105] Ambrose D et al. (BNL E871) 1998 *Phys. Rev. Lett.* **81** 4309

[106] Valencia G 1998 *Nucl. Phys.* B **517** 339

[107] Gómez Dumm D, Pich A 1998 *Phys. Rev. Lett.* **80** 4633

[108] Ciuchini M, Franco E, Martinelli G, Silvestrini L 1997 *Nucl. Phys.* B **501** 271; 1998 erratum ibid. **531** 656

[109] Buras A J, Silvestrini L hep-ph/9812392

[110] Bauer M, Stech B, Wirbel M 1987 *Z. Phys.* C **34** 103

[111] Neubert M, Stech B hep-ph/9705292

[112] Ali A, Greub C 1998 *Phys. Rev.* D **57** 2996; Cheng H-Y, Tseng B 1998 *Phys. Rev.* D **58** 094005; Ali A, Kramer G, Lü C-D 1998 *Phys. Rev.* D **58** 094009

[113] Buras A J, Silvestrini L 1999 *Nucl. Phys.* B **548** 293

[114] Buras A J, Gérard J-M, Rückl R 1986 *Nucl. Phys.* B **268** 16

[115] Bjorken J D 1989 *Nucl. Phys. Proc. Suppl.* B **11** 325

[116] Dugan M J, Grinstein B 1991 *Phys. Lett.* B **255** 583

[117] Szczepaniak A, Henley E M, Brodsky S J 1990 *Phys. Lett.* B **243** 287

[118] Simma H, Wyler D 1991 *Phys. Lett.* B **272** 395

[119] Dahm M, Jakob R, Kroll P 1995 *Z. Phys.* C **68** 595

[120] Melić B 1999 *Phys. Rev.* D **59** 074005

[121] Yeh T-W, Li H-n 1997 *Phys. Rev.* D **56** 1615

[122] Blok B, Shifman M 1993 *Nucl. Phys.* B **389** 534

[123] Halperin I, 1995 *Phys. Lett.* B **349** 548

[124] Khodjamirian A, Rückl R hep-ph/9801443

[125] Donoghue J F, Petrov A A 1997 *Phys. Lett.* B **393** 149

[126] Beneke M, Buchalla G, Neubert M, Sachrajda C T 1999 *Phys. Rev. Lett.* **83** 1914

[127] Ball P 1998 *JHEP* **09** 005 (hep-ph/9802394)

[128] Farrar G R, Jackson D R 1979 *Phys. Rev. Lett.* **43** 246

[129] Efremov A V, Radyushkin A V 1980 *Phys. Lett.* B **94** 245

[130] Lepage G P, Brodsky S J 1980 *Phys. Rev.* D **22** 2157

[131] Politzer H D, Wise M B 1991 *Phys. Lett.* B **257** 399

[132] Affolder T et al. (CDF) hep-ex/9909003

[133] Ackerstaff D et al. (OPAL) 1998 *Eur. Phys. J.* C **5** 379; Abe F et al. (CDF) 1998 *Phys. Rev. Lett.* **81** 5513

[134] Gronau M, London D 1990 *Phys. Rev. Lett.* **65** 3381

[135] Dunietz I 1993 in Proceedings of the Workshop on B Physics at Hadron Accelerators, Snowmass, CO, eds. McBride P, Mishra C S, p. 83

[136] Fleischer R 1999 *Phys. Lett.* B **459** 306

[137] Kwon Y et al. (CLEO) hep-ex/9908039

[138] Fleischer R, Mannel T 1998 *Phys. Rev.* D **57** 2752

[139] Buras A J, Fleischer R 1999 *Eur. Phys. J.* C **11** 93

[140] Fleischer R 1998 *Phys. Lett.* B **435** 221; 1999 *Eur. Phys. J.* C **6** 451

[141] Gronau M, Rosner J L 1998 *Phys. Rev.* D **57** 6843; Gérard J-M, Weyers J 1999 *Eur. Phys. J.* C **7** 1; Neubert M 1998 *Phys. Lett.* B **424** 152; Falk A F et al. 1998 *Phys. Rev.* D **57** 4290; Atwood D, Soni A 1998 *Phys. Rev.* D **58** 036005; Buras A J, Fleischer R, Mannel T 1998 *Nucl. Phys.* B **533** 3

[142] Neubert M, Rosner J L 1998 *Phys. Lett.* B **441** 403; 1998 *Phys. Rev. Lett.* **81** 5076

[143] Neubert M 1999 *JHEP* **02** 014 (hep-ph/9812396)

[144] Gronau M, Rosner J L, London D 1994 *Phys. Rev. Lett.* **73** 21

[145] Deshpande N G, He X-G 1995 *Phys. Rev. Lett.* **74** 26

[146] Gronau M, Pirjol D 1999 *Phys. Lett.* B **449** 321; 2000 *Phys. Rev.* D **61** 013005; Gronau M, Pirjol D, Yan T-M 1999 *Phys. Rev.* D **60** 034021

[147] Agashe K, Deshpande N G 1999 *Phys. Lett.* B **451** 215; ibid. **454** 359

[148] Gronau M, Wyler D 1991 *Phys. Lett.* B **265** 172

[149] Atwood D, Dunietz I, Soni A 1997 *Phys. Rev. Lett.* **78** 3257

[150] Aleksan R, Dunietz I, Kayser B 1992 *Z. Phys.* C **54** 653

[151] Fleischer R 1999 *Eur. Phys. J.* C **10** 299

[152] Fleischer R 1999 *Phys. Rev.* D **60** 073008

[153] Fleischer R, Matias J hep-ph/9906274

[154] Grossman Y, Neubert M, Kagan A L hep-ph/9909297

[155] Ball P, Frère J-M, Matias J hep-ph/9910211

[156] Ball P, Fleischer R hep-ph/9912319

[157] Deshpande N G et al. 1996 *Phys. Lett.* B **366** 300

[158] Browder T E et al. 1998 *Phys. Rev.* D **57** 6829

[159] Atwood D, Soni A 1998 *Phys. Rev. Lett.* **81** 3324
[160] Chay J, Georgi H, Grinstein B 1990 *Phys. Lett.* B **247** 399
[161] Bigi I I, Uraltsev N G, Vainshtein 1992 *Phys. Lett.* B **293** 430; 1993 erratum ibid. **297** 477; Bigi I I et al. hep-ph/9401298
[162] Cahn R N, Worah M P 1999 *Phys. Rev.* D **60** 076006
[163] Beneke M, Buchalla G, Dunietz I 1997 *Phys. Lett.* B **393** 132
[164] Hüttmann K (ALEPH) ALEPH 98-061 CONF 98-032
[165] Abbiendi G et al. (OPAL) hep-ex/9901017
[166] Daoudi M et al. (SLD) hep-ex/9712031
[167] Feindt M et al. (DELPHI) DELPHI 97-98 CONF 80
[168] Lenz A, Nierste U, Ostermaier G 1999 *Phys. Rev.* D **59** 034008