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United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

PHOTOPRODUCTION OF DILATONS IN AN EXTERNAL MAGNETIC FIELD

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Abstract

An attempt is made to present some experimental predictions of the five dimension Kaluza–Klein theory. The conversion of photons into dilatons in the static magnetic fields are considered in detail. The differential cross sections are presented for the conversions in a magnetic field of the flat condensor and a magnetic field of the solenoid. A numerical evaluation shows that in the present technical scenario, the creation of dilatons at high energies may have the observable value.

MIRAMARE – TRIESTE
October 1999

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1. Introduction

In the 1920's, Kaluza and Klein suggested a method to unify gravity and electromagnetism in a unique theory which was, in fact, an Eisntein theory in five dimensions where a scalar particle called dilaton, arose as a result of a dimensional reduction to four dimensions [1,2]. Besides that, a low–energy effective string field theory shows that the dilaton fields naturally appear, coupled to the Einstein–Maxwell fields [3]. These theories and the inflationary cosmology have revived the interest in the dilaton coupled with matter since they are important for the understanding of more general theories (as the Einstein - Maxwell - dilatons, configurations representing charged black holes [4] and other kinds of stationary dilatons with arbitrary electromagnetic (EM) field [5]) which have been extensively studied recently. Much work has been done with the dilatons based on the unified theories in order to establish the main properties of them [6].

The study on the interaction between EM fields and gravitational fields is a significant work of research on gravitational radiations [7,8]. In our earlier work [9], we presented conversions of photons into gravitons in external EM fields by Feynman techniques. It is shown that in the present technical scenario the creation of high frequency gravitational waves may be observed. To complete the experiment, all possible effects have to be looked for. The purpose of this paper is to consider the conversion of photons into dilatons in external magnetic fields.

2. Cross sections

From the Einstein - Hilbert action in the five dimension Kaluza-Klein theory, after some manipulation and performing the fifth coordinate integration, we have [1,2]

$$L_{EH} = \int \sqrt{-g}\left[\frac{R^{(4)}}{16\pi G} + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\sqrt{3}K}{4}\sigma F_{\mu\nu}F^{\mu\nu} + O(K^2)\right]dx^4. \quad (1)$$

Let us consider the process in which the initial state has the photon γ with momentum q and the external EM field and the final state has the same external EM field and the dilaton σ with the momentum p. For the above mentioned process, the relevant coupling is the fourth term in (1). In the linear approximation [7], the Lagrangian L_{EH} yields the following vertex

$$L_{\sigma\gamma\gamma} = -\frac{\sqrt{3}K}{2}\eta_{\mu\alpha}\eta_{\nu\beta}\sigma F^{\mu\nu}F^{\alpha\beta}_{class}, \tag{2}$$

where $K = \sqrt{16\pi G}$, G is the Newton constant, and $\eta_{\mu\nu} = diag(+,-,-,-)$. Using the Feynman rules we get the following expression for the matrix element

$$\langle p|M|q\rangle = -\frac{\sqrt{3}K}{2(2\pi)^2 q_0}\varepsilon^{\lambda}(\vec{q},\sigma)\eta_{\lambda\beta}q_{\alpha}\int_V e^{i\vec{k}\vec{r}}F^{class}_{\alpha\beta}d\vec{r} \tag{3}$$

where $\vec{k} \equiv \vec{q} - \vec{p}$ the momentum transfer to the EM field, and $\varepsilon^{\mu}(\vec{q},\sigma)$ represents the polarization vector of the photon. Expression (3) is valid for an arbitrary external EM field. In the following we shall use it for two cases, namely conversions of the photon into the dilaton in a magnetic field of the flat condensor and in a static magnetic field of the solenoid. Here we use the following notations: $q \equiv |\vec{q}|, p \equiv |\vec{p}|$ and θ is the angle between $\vec{p}$ and $\vec{q}$.

Photoproduction in the magnetic field of the flat condensor. Now we take the EM field is a homogeneous magnetic field of the flat condensor of size $a \times b \times c$. We shall use the coordinate system with the z axis is parallel to the direction of the field, i.e., $F^{12} = -F^{21} = B$ then the matrix element is given by

$$\langle p|M|q\rangle = \frac{\sqrt{3}K}{2(2\pi)^2 q_0}\varepsilon^{i}(\vec{q},\sigma)(q_2\eta_{i1} - q_1\eta_{i2})F_m(\vec{k}) \tag{4}$$

where a form factor for the magnetic region [9]

$$F_m(\vec{k}) = \int_V e^{i\vec{k}\vec{r}}B(\vec{r})d\vec{r} = 8B\sin(\frac{1}{2}ak_x)\sin(\frac{1}{2}bk_y)\sin(\frac{1}{2}ck_z)(k_xk_yk_z)^{-1}. \tag{5}$$

Squaring the matrix element (4) we get the following expression for the differential cross section (DCS)

$$\frac{d\sigma^m(\gamma\to\sigma)}{d\Omega'} = \frac{6K^2B^2}{\pi^2}\left[\frac{\sin(\frac{1}{2}ak_x)\sin(\frac{1}{2}bk_y)\sin(\frac{1}{2}ck_z)}{k_xk_yk_z}\right]^2(q_x^2+q_y^2). \tag{6}$$

From (6) we see that if the photon moves in the direction of the magnetic field i.e., $q^{\mu} = (q,0,0,q)$ then DCS vanishes (since in this case $q_x = q_y = 0$). If the momentum of photon is parallel to the x axis, i.e., q^{μ} =(q, q, 0, 0) then Eq.(6) becomes

$$\begin{aligned}\frac{d\sigma^m(\gamma\to\sigma)}{d\Omega'} &= \frac{6K^2B^2}{\pi^2q^4}\left[\sin\left(\frac{aq}{2}(1-\cos\theta)\right)\sin\left(\frac{bq\sin\theta\sin\varphi'}{2}\right)\right.\\ &\quad\left.\times\sin\left(\frac{cq\sin\theta\cos\varphi'}{2}\right)\right]^2\left(\sin^2\theta\sin\varphi'\cos\varphi'(1-\cos\theta)\right)^{-2}\end{aligned} \tag{7}$$

where φ' is the angle between the z axis and the projection of $\vec{p}$ on the xz plane [9]. From (7) we have

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} = \frac{3K^2B^2V^2q^2}{32\pi^2}; \qquad V = a \times b \times c \tag{8}$$

for $\theta \approx 0$, and

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} = \frac{3K^2B^2b^2}{2\pi^2q^2} \sin^2\left(\frac{aq}{2}\right) \sin^2\left(\frac{cq}{2}\right), \tag{9}$$

for $\theta = \frac{\pi}{2}, \varphi' = 0$, and then

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} = \frac{3K^2B^2c^2}{2\pi^2q^2} \sin^2\left(\frac{aq}{2}\right) \sin^2\left(\frac{bq}{2}\right), \tag{10}$$

for $\theta = \frac{\pi}{2}, \varphi' = \frac{\pi}{2}$. In the C.G.S units, Eqs. (8), (9) and (10) become, respectively

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} \approx 3.9 \times 10^{-49} \frac{B^2V^2}{\lambda^2}, \tag{11}$$

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} \approx 1.6 \times 10^{-50} B^2b^2\lambda^2 \sin^2\left(\frac{a\pi}{\lambda}\right) \sin^2\left(\frac{c\pi}{\lambda}\right), \tag{12}$$

and

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} \approx 1.6 \times 10^{-50} B^2c^2\lambda^2 \sin^2\left(\frac{a\pi}{\lambda}\right) \sin^2\left(\frac{b\pi}{\lambda}\right), \tag{13}$$

where λ is the wavelength of photons. From Eqs. (9) and (10) it implies that when $b = c$ then the probabilities of the creation of dilatons in the y and z directions are the same. From (8) we see that DCS in the direction of the dilaton motion depends quadratically on the *intensity* B, the *volume* V of condensor, and the *photon momentum* q. *This is the best case for conversions.* In the case of the electric field of size $a \times b \times c$, it is easy to get the same results.

Photoproduction in the magnetic field of the solenoid. – Now we move on the conversion of photons into dilatons in a homogeneous magnetic field of the solenoid with a radius R and a lenght h, and without loss of generality suppose that the direction of the magnetic field is parallel to the z axis. The form factor for the magnetic region of the solenoid is given by [9]:

$$F_m(\vec{k}) = \frac{4\pi BR}{k_z\sqrt{k_x^2 + k_y^2}} J_1(R\sqrt{k_x^2 + k_y^2}) \sin\left(\frac{hk_z}{2}\right), \tag{14}$$

where J_1 is the one-order spherical Bessel function. From Eqs. (4) and (14) we obtain the DCS for the conversion of photons into dilatons as

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} = \frac{3K^2R^2B^2}{2k_z^2(k_x^2 + k_y^2)} J_1^2\left(Rq\sqrt{k_x^2 + k_y^2}\right) \sin^2\left(\frac{k_zh}{2}\right) \left(q_x^2 + q_y^2\right) \tag{15}$$

From (15) it follows that when the momentum of the photon is parallel to the z axis (the direction of the magnetic field), DCS vanishes. This result is the same as the previous item. It implies that *if the momentum of the photon is parallel to the EM field then there is no conversion.* If the momentum of the photon is parallel to the x axis, i.e., $q^\mu = (q, q, 0, 0)$ then Eq.(15) gets the final form

$$\begin{aligned}
\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} &= \frac{3K^2R^2B^2J_1^2\left(Rq\sqrt{(1-\cos\theta)^2+\sin^2\theta\cos^2\varphi'}\right)}{2q^2\sin^2\theta\sin^2\varphi'[(1-\cos\theta)^2+\sin^2\theta\cos^2\varphi']} \\
&\times \sin^2\left(\frac{hq}{2}\sin\theta\sin\varphi'\right), \qquad (16)
\end{aligned}$$

where φ' is the angle between the y axis and the projection of $\vec{p}$ on the yz plane [9]. From (16) with the notice that

$$\lim_{p\to q}\frac{J_1\left(R(q-p)\right)}{q-p} = \frac{R}{2},$$

then it is easy to see that

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} = \frac{3K^2V^2B^2q^2}{32\pi^2}; \qquad V \equiv \pi R^2 h \qquad (17)$$

for $\theta \approx 0$, and

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} = \frac{3K^2}{16}R^2h^2B^2J_1^2\left(Rq\sqrt{2}\right), \qquad (18)$$

for $\theta = \frac{\pi}{2}, \varphi' = 0$, and

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} = \frac{3K^2}{2q^2}R^2B^2J_1^2\left(Rq\right)\sin^2(\frac{hq}{2}), \qquad (19)$$

for $\theta = \frac{\pi}{2}, \varphi' = \frac{\pi}{2}$. From (18) it follows that DCS vanishes when $q_n = \frac{\mu_n}{R\sqrt{2}}$ with $n = 0, \pm1 \pm 2...$, and has its largest value

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} = \frac{3}{16}K^2R^2h^2B^2J_1^2(\mu'_n) \qquad (20)$$

for $q_n = \frac{\mu'_n}{R\sqrt{2}}$, where μ_n and μ'_n are the roots of $J_1(\mu_n) = 0$ and $J'_1(\mu'_n) = 0$. In the C.G.S units, Eqs. (17), (18), and (19) become, respectively

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} \approx 1.6 \times 10^{-48}\frac{B^2V^2}{\lambda^2}, \qquad (21)$$

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} \approx 8 \times 10^{-49}R^2B^2h^2J_1^2\left(\frac{2\sqrt{2}R\pi}{\lambda}\right), \qquad (22)$$

and

$$\frac{d\sigma^m(\gamma \to \sigma)}{d\Omega'} \approx 1.6 \times 10^{-49} R^2 B^2 \lambda^2 J_1^2 \left(\frac{2R\pi}{\lambda}\right) \sin^2 \left(\frac{h\pi}{\lambda}\right). \tag{23}$$

From Eqs. (18) and (19) it implies that in the limit $q \to 0$, creations of dilatons in the two directions are the same. To complete the experiment, note that the cross section for the reverse process coincides exactly with the above results, so that for the conversion photon-dilaton-photon, the cross section is the square of the previous evaluations.

Discussion

From Eqs.(8) and (17) it implies that when the momentum of photons is perpendicular to the EM field then in this case, *the conversion cross - sections are largest in direction of the photon motion.* In the C.G.S units, suppose that the volume of the external fields is $V = 1m \times 1m \times 1m$, the intensity of the magnetic field $B = 10^6 cm^{-1/2} g^{1/2} s^{-1}$, the photon length $\lambda = 10^{-5}$ cm, then the cross section given by (11) is $\frac{d\sigma(\gamma \to a)}{d\Omega} \simeq 4 \times 10^{-15} cm^2$, while (21) is $\frac{d\sigma(\gamma \to a)}{d\Omega} \simeq 1.6 \times 10^{-14} cm^2$. *We see that considered process may be have the observable value at the high energies.*

Finally, note that the scattering of photons in an external EM field is the most important effect because it has the following advantages [8]: the first, this is the unique effect giving non-zero cross-section in the first order of the pertubation theory, the second, since the EM field is classical we can increase the scattering probability as much as possible by increasing the intensity of the field or the volume containing the field. *This is an important point in order to apply it in experiments*

Acknowledgements

One of the authors (DVS) would like to thank Professor Antonio Masiero for continuous help at ICTP, Trieste, Italy. This work was supported in part by the Vietnam National Basic Research Programme in Natural Siences and also Hanoi Institute for Physics, NCST.

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