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A NONLOCAL MODEL OF THE YUKAWA AND DIPOLE INTERACTIONS

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Abstract

By analogy of the nonlocal electromagnetic interaction of the ring charge, nonlocal Yukawa and dipole interactions are constructed. In this scheme the Yukawa and the dipole potentials and corresponding propagators of massive scalar and massless spinor particles are changed in accordance with the concept of extended particle-strings with a finite size characterized by some length ℓ . It is shown that a formal Euclidean extension of the potential theory is useful for the construction of nonlocal fields and their nonlocal interaction in the Yukawa and dipole theories. In the dipole theory a neutrino like massless spinor particle (nonlocal photon field) carrying a medium range interaction appears.

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1 Introduction

In the Standard Model (Glashow, 1961; Weinberg, 1967; and Salam, 1968) the Higgs boson plays an important role in the mechanism of breaking the $SU(2) \times U(1)$ symmetry and generating the W and Z boson masses. By means of the Yukawa interaction the Higgs couples to quarks and leptons, of mass m_f , with a strength of $gm_f/2M_W$ (g is the coupling constant of the $SU(2)$ gauge theory). Higgs boson properties and its mass are not predicted by the model. We know only that its coupling to stable matter is very small, and production and detection are very difficult. Consequently the nature of the Higgs boson becomes mystery for physicists. The Yukawa interaction of Higgs boson with fundamental fermions should shed light on understanding the mass scales of fermions and their origin through the Higgs mechanism (Namsrai, 1996)

In this paper we propose a simple model of the Yukawa and the dipole interactions where force transmitting quanta-Higgs bosons and massless fermions are spread-out and possess string-like innermost structure. Such a model leads to a finite theory of the nonlocal Yukawa and the dipole interactions. In a previous paper (Namsrai, 1997) we constructed the nonlocal electromagnetic interaction of the ring charge. Here our purpose is to generalize the mathematical method of this approach for the massive and the dipole cases. In Section 2 we briefly discuss the corresponding rule between electromagnetic and the Yukawa interactions in the ring theory. Section 3 deals with the Poisson like equation for the Yukawa potential and its solutions. A formal four-dimensional Euclidean extension of the potential theory in both massless and massive cases is studied in Section 4. Within the framework of the nonlocal model in Section 5 we discuss some possibilities of an existence of a new medium range interaction (carrier of which is a massless spinor particle) and the confinement of a particle which does not exist in free states, i.e., poles in those propagators are absent in momentum space. Section 6 is devoted to the construction of spread-out fields corresponding to the ring theory and their nonlocal Yukawa and dipole interaction Lagrangians. Consequences of the theory will be considered elsewhere.

2 The Corresponding Rule Between the Long- and Short-Range Interactions

Earlier (Namsrai, 1997) we have considered an unoriented stochastic ring charge and a modified Coulomb's potential

$$U_c^\ell = \frac{e}{2\pi^2} \frac{1}{r} \arcsin \frac{r}{\sqrt{r^2 + \ell^2}} \quad (1)$$

and the corresponding propagator of the nonlocal photon field is

$$\tilde{D}_{\mu\nu}^\ell(p) = -g_{\mu\nu} \frac{e^{-\sqrt{-p^2}\ell^2}}{-p^2 - i\varepsilon} \quad (2)$$

where $p^2 = p_o^2 - \mathbf{p}^2$. In accordance with the corresponding rule the propagator of the massive particle carrying the short range Yukawa interaction takes the form

$$\tilde{\mathbf{D}}_m^\ell(p) = \frac{1}{m^2 - p^2 - i\varepsilon} \exp \left[-\sqrt{(m^2 - p^2)\ell^2} \right] \quad (3)$$

in the ring theory. In the static limit equation (3) gives the modified Yukawa potential

$$U_Y^\ell(r) = \frac{g}{(2\pi)^3} \int d^3p e^{i\mathbf{p}\mathbf{r}} \tilde{\mathbf{D}}_m^\ell(\mathbf{p}) \quad (4)$$

and an explicit form of which is

$$U_Y^\ell(r) = \frac{g}{4\pi} \frac{e^{-mr}}{r} - \frac{gm}{2\pi^2} \int_0^\ell d\lambda \frac{1}{\sqrt{r^2 + \lambda^2}} K_1(m\sqrt{r^2 + \lambda^2}) \quad (5)$$

where $K_1(z)$ is the modified Bessel function and g is a coupling constant. It is natural that the inverse Fourier transform of (4) yields

$$\begin{aligned} \tilde{\mathbf{D}}_m^\ell(\mathbf{p}) &= \frac{1}{g} \int d^3r e^{-i\mathbf{p}\mathbf{r}} U_Y^\ell(r) = \\ &= \frac{1}{m^2 + \mathbf{p}^2} + \frac{e^{-\ell\sqrt{m^2 + \mathbf{p}^2}}}{m^2 + \mathbf{p}^2} - \frac{1}{m^2 + \mathbf{p}^2} = \frac{e^{-\ell\sqrt{m^2 + \mathbf{p}^2}}}{m^2 + \mathbf{p}^2} \end{aligned} \quad (6)$$

as should be. Further, making use of changing $g \rightarrow e$ and taking the limit $m \rightarrow 0$ in expressions (5) and (6) one gets (1) and (2) exactly. This fact tells us that the corresponding rule is valid in the given case.

3 The Poisson-like Equation for the Yukawa Potential Theory

First, let us consider the local Yukawa theory, potential of which satisfies the Poisson equation

$$\Delta U_Y(r) = g\rho_Y(r) \quad (7)$$

where $\rho_Y(r) = \frac{m^2}{4\pi r} e^{-mr}$ with the condition

$$\int d^3r \rho_Y(r) = 1 \quad (8)$$

The solution of equation (7) has the integral form

$$U_Y(r) = \frac{g}{4\pi} \int d^3r' \frac{\delta(\mathbf{r}') - \rho_Y(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \quad (9)$$

The Fourier transform (3) in the static limit and in the local theory is given by

$$\tilde{\mathbf{D}}_m^o(\mathbf{p}^2) = \frac{1}{g} \int d^3r e^{-i\mathbf{p}\mathbf{r}} \frac{g}{4\pi} \int d^3r' \frac{\Omega_Y^o(\mathbf{r} - \mathbf{r}')}{|r'|} \quad (10)$$

in accordance with (6) and (9). Here

$$\Omega_Y^o(\mathbf{r} - \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') - \rho_Y(\mathbf{r} - \mathbf{r}') = \frac{1}{(2\pi)^3} \int d^3 q e^{-iq(\mathbf{r}-\mathbf{r}')} \tilde{\Omega}_Y^o(q) \quad (11)$$

and

$$\frac{1}{|\mathbf{r}'|} = \frac{1}{2\pi^2} \int d^3 p e^{i\mathbf{p}\mathbf{r}'} \frac{1}{\mathbf{p}^2} \quad (12)$$

where $\tilde{\Omega}_Y^o(q)$ is the Fourier transform of the density $\Omega_Y^o(\mathbf{r})$ in (10). Therefore we see immediately that

$$\tilde{D}_m^o(\mathbf{p}^2) = \frac{1}{\mathbf{p}^2} \tilde{\Omega}_Y^o(\mathbf{p}) \quad (13)$$

where

$$\tilde{\Omega}_Y^o(\mathbf{p}) = \frac{\mathbf{p}^2}{m^2 + \mathbf{p}^2},$$

and

$$\left(\widetilde{\frac{1}{4\pi r}} \right)(p) = \frac{1}{\mathbf{p}^2}$$

It is easy to generalize the above formulas in the ring theory. For example, equation (7) acquires the form

$$\Delta U_Y^\ell(r) = g \rho_Y^\ell(r) \quad (14)$$

$$\begin{aligned} \rho_Y^\ell(r) &= \frac{m^2}{4\pi r} e^{-mr} - \frac{m}{2\pi^2} \int_0^\ell d\lambda \times \\ &\times \left[-\frac{3m}{r^2 + \lambda^2} K_2(m\sqrt{r^2 + \lambda^2}) + \frac{m^2 r^2}{(r^2 + \lambda^2)^{3/2}} K_3(m\sqrt{r^2 + \lambda^2}) \right] \end{aligned} \quad (15)$$

and the normalization condition is

$$\int d^3 r \rho_Y^\ell(r) = 1 \quad (16)$$

In this case, solution (9) is modified by

$$U_Y^\ell(r) = \frac{g}{4\pi} \int d^3 r' \frac{\Omega_Y^\ell(\mathbf{r} - \mathbf{r}')}{|\mathbf{r}'|} \quad (17)$$

Here the Fourier transform of $\Omega_Y^\ell(\mathbf{r}) = \delta^{(3)}(r) - \rho_Y^\ell(r)$ reads

$$\begin{aligned} \tilde{\Omega}_Y^\ell(\mathbf{p}) &= \int d^3 r e^{-i\mathbf{p}\mathbf{r}} \Omega_Y^\ell(\mathbf{r}) = 1 - \frac{m^2}{m^2 + \mathbf{p}^2} + \frac{\mathbf{p}^2}{m^2 + \mathbf{p}^2} \times \\ &\times e^{\ell\sqrt{m^2 + \mathbf{p}^2}} - \frac{\mathbf{p}^2}{m^2 + \mathbf{p}^2} = \frac{\mathbf{p}^2}{m^2 + \mathbf{p}^2} e^{-\ell\sqrt{m^2 + \mathbf{p}^2}} \end{aligned}$$

So that the similar relation like (13)

$$\tilde{D}_m^\ell(\mathbf{p}^2) = \frac{1}{\mathbf{p}^2} \tilde{\Omega}_Y^\ell(\mathbf{p}) = \frac{e^{-\ell\sqrt{m^2 + \mathbf{p}^2}}}{m^2 + \mathbf{p}^2} \quad (18)$$

holds in this general case. We see that propagator (3) for the Yukawa particle and its potential (17) are related by the formula (18) in the unified way for the ring theory.

4 The Potential Theory in the Four-Dimensional Euclidean Space

In order to construct relativistic covariant Yukawa interaction theory of nonlocal quantized fields in the ring model it is useful to extend formally the potential theory in the four-dimensional Euclidean space. Now we turn to this problem.

4.1 The massless case

The ring charge potential in the four-dimensional Euclidean space obeys the d'Alembertian equation

$$\square_E U_E^o(x) = -e\rho_E^o(x) \quad (19)$$

where

$$\square_E = \frac{\partial^2}{\partial x_i^2}, x_i = \{x_1, x_2, x_3, x_4\}$$

and

$$\rho_E^o(x, \ell) = \frac{3\ell}{4\pi^2} (r_E^2 + \ell^2)^{-5/2}, (r_E^2 = x_i x_i) \quad (20)$$

As before,

$$\int d^4 x \rho_E^o(x) = 1$$

The solution of (19) has the similar form of (10) and (17):

$$U_E^o(x) = \frac{e}{4\pi^2} \int d^4 y \frac{\rho_E^o(x-y)}{|y|^2} \quad (21)$$

The Fourier transforms of $\rho_E^o(x)$ and $\frac{1}{4\pi^2}|y|^2 = f(y)$ are given by

$$\tilde{\rho}_E^o(p_E) = \int d^4 x e^{-ip_E x} \rho_E^o(x) = e^{-\ell\sqrt{p_E^2}} \quad (22)$$

and

$$\tilde{f}(p) = \left(\frac{1}{4\pi^2|y|^2} \right) = \frac{1}{(2\pi)^2} \int d^4 x e^{-ipx} \frac{1}{x^2} = \frac{1}{p_E^2}, p_E^2 = p_1^2 + \dots + p_4^2 \quad (23)$$

From expressions (22) and (23) we see that in the four-dimensional Euclidean space a similar relation like (13) and (18) is valid. So that

$$D_E^o(p) = \frac{1}{p_E^2} e^{-\ell\sqrt{p_E^2}} \quad (24)$$

An analytic continuation of this formula in the pseudo-Euclidean space is exactly equal to the propagator of the nonlocal photon field for the ring charge. An explicit form of the potential (21) is given by

$$U_E^o(x) = \frac{1}{4\pi^2 r_E^2} \left[1 - \frac{\ell}{\sqrt{r_E^2 + \ell^2}} \right] \quad (25)$$

and the Fourier transform reads

$$\begin{aligned}\widetilde{U_E^o(p)} &= \int d^4y e^{-ipy} U_E^o(y) = \frac{1}{p_E^2} - \frac{\ell}{p} \int_0^\infty d\lambda \frac{J_1(p_E \lambda)}{\sqrt{\ell^2 + \lambda^2}} = \\ &= \frac{1}{p_E^2} - \frac{\ell}{p_E} \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{\ell p_E}} [I_{1/2}(\ell p_E) - L_{1/2}(\ell p_E)]\end{aligned}\quad (26)$$

Here modified Bessel and Struve functions $I_{1/2}(x)$ and $L_{1/2}(x)$ are given by

$$I_{1/2}(\ell p_E) = \frac{1}{\sqrt{2\pi\ell p_E}} (e^{\ell p_E} - e^{-\ell p_E})$$

and

$$L_{1/2}(\ell p_E) = -ie^{-i\pi/4} H_{1/2}(i\ell p_E) = -\sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{\ell p_E}} \left(1 - \frac{1}{2}e^{-\ell p_E} - \frac{1}{2}e^{\ell p_E}\right)$$

Therefore, expression (26) coincides identically with (24).

4.2 The massive case

In the four-dimensional Euclidean space the usual Yukawa local potential theory is extended by the similar method as expounded above. In this case, equation (19) is changed as follows

$$\square_E U_{YE}^o(x) = g\rho_{YE}^o(x) \quad (27)$$

where

$$\rho_{YE}^o(x) = \frac{m^3}{4\pi^2 r_E} K_1(mr_E)$$

with the normalization condition

$$\int d^4x \rho_{YE}^o(x) = 1$$

The solution of (27) acquires the standard form

$$U_{YE}^o(x) = \frac{g}{4\pi^2} \int d^4y \frac{\Omega_{YE}(x-y)}{y^2} \quad (28)$$

Here the Fourier transforms of quantities: $\Omega_{YE}^o(x) = \delta^{(4)}(x) - \rho_{YE}^o(x)$ and $f_Y(x) = \frac{1}{4\pi^2} \frac{1}{x_E^2}$ are given by

$$\widetilde{\Omega}_{YE}(p) = 1 - \frac{m^2}{m^2 + p_E^2}$$

and

$$\widetilde{f}_Y(p) = \left(\widetilde{\frac{1}{4\pi x_E^2}}\right)(p) = \frac{1}{p_E^2}$$

respectively. Therefore, the Euclidean propagator of the massive particle can be written in the usual form

$$\widetilde{D}_{YE}^o(p_E) = \frac{1}{g} \int d^4x e^{-ip_E x} U_{YE}^o(x) = \frac{\widetilde{\Omega}_{YE}^o(p_E)}{p_E^2} = \frac{1}{m^2 + p_E^2} \quad (29)$$

by virtue of (28). An explicit form of the four-dimensional Yukawa potential is

$$U_{YE}^o(x) = \frac{g}{4\pi^2} \frac{m}{r_E} K_1(mr_E) \quad (30)$$

In the ring theory the four-dimensional modified Yukawa potential obeys the equation

$$\square_E U_{YE}^\ell(x) = g\rho_{YE}^\ell(x) \quad (31)$$

where

$$\begin{aligned} \rho_{YE}^\ell(x) = & \frac{m}{4\pi^2} \left\{ \frac{m^2}{r_E} K_1(mr_E) - \sqrt{\frac{2m}{\pi}} \int_0^\ell d\lambda \times \right. \\ & \left. \left[-\frac{4m}{(r^2 + \lambda^2)^{5/4}} K_{5/2}(m\sqrt{r_E^2 + \lambda^2}) + \frac{m^2 r_E^2}{[r_E^2 + \lambda^2]^{7/4}} K_{7/2}(m\sqrt{r_E^2 + \lambda^2}) \right] \right\} \end{aligned} \quad (32)$$

satisfying the condition

$$\int d^4x \rho_{YE}^\ell(x) = 1$$

In order to calculate the Fourier transform of (32) we use the following integrals:

$$\begin{aligned} I_1 &= \int_0^\infty d\rho \rho K_1(m\rho) J_1(p_E \rho) = \frac{p_E}{m} \frac{1}{m^2 + p_E^2} \\ I_2 &= \int_0^\infty d\rho \frac{\rho^2}{(\rho^2 + \lambda^2)^{5/4}} K_{5/2}(m\sqrt{\rho^2 + \lambda^2}) J_1(\rho p_E) = \\ &= \sqrt{\frac{\pi}{2}} \frac{p_E}{\lambda} m^{-5/2} \exp \left[-\lambda \sqrt{m^2 + p_E^2} \right] \\ I_3 &= \int_0^\infty d\rho \frac{\rho^4}{[\rho^2 + \lambda^2]^{7/4}} K_{7/2}(m\sqrt{\rho^2 + \lambda^2}) J_1(\rho p_E) = \\ &= 4p_E m^{-7/2} \sqrt{\frac{\pi}{2}} \frac{1}{\lambda} e^{-\lambda \sqrt{m^2 + p_E^2}} - p_E^3 m^{-7/2} \sqrt{\frac{\pi}{2}} (m^2 + p_E^2)^{-1/2} \times \\ &\quad \times e^{-\lambda \sqrt{m^2 + p_E^2}} \end{aligned} \quad (33)$$

As usual, the solution of (31) can be represented in the integral form

$$U_{YE}^\ell(x) = \frac{g}{4\pi^2} \int d^4y \frac{\Omega_{YE}^\ell(x-y)}{y^2} \quad (34)$$

Making use of (33) one gets

$$\tilde{D}_{YE}^\ell(p_E) = \frac{1}{g} \int d^4x e^{-ip_E x} U_{YE}^\ell(x) = \frac{\tilde{\Omega}_{YE}^\ell(p_E)}{p_E^2} = \frac{e^{-\ell \sqrt{m^2 + p_E^2}}}{m^2 + p_E^2} \quad (35)$$

as should be. Here

$$\Omega_{YE}^\ell(x-y) = \delta^{(4)}(x-y) - \rho_{YE}^\ell(x-y)$$

and

$$\tilde{\Omega}_{YE}^\ell(p_E) = 1 - \left[1 - \frac{p_E^2}{m^2 + p_E^2} e^{-\ell\sqrt{m^2 + p_E^2}} \right]$$

in accordance with formulas (32) and (33). In the ring theory, an explicit form of the potential $U_{YE}^\ell(x)$ in (31) and (34) is given by

$$U_{YE}^\ell(x) = \frac{gm}{4\pi^2 r_E} \left\{ K_1(mr_E) - \sqrt{\frac{2}{\pi}} mr_E \int_0^\ell d\lambda \frac{K_{3/2}(m\sqrt{r_E^2 + \lambda^2})}{(r_E^2 + \lambda^2)^{3/4}} \right\} \quad (36)$$

It should be noted that if we use an another integral representation

$$\rho_E^m(x) = \frac{1}{4\pi^2} \frac{1}{r_E} \int_0^\infty d\rho \frac{\rho^4}{\rho^2 + m^2} e^{-\ell\sqrt{m^2 + \rho^2}} J_1(\rho r_E)$$

for the density of the "charge" g . Then equation (31) leads to the form

$$\square_E U_{YE}^\ell(x) = -g\rho_E^m(x) \quad (37)$$

with the solution given by the integral equation:

$$U_{YE}^\ell(x) = \frac{g}{4\pi^2} \int d^4 y \frac{\rho_E^m(x-y)}{y^2} \quad (38)$$

Here the Fourier transform of $\rho_E^m(x)$ defines as

$$\begin{aligned} \tilde{\rho}_E^m(p) &= \frac{1}{4\pi^2} \int_0^\infty \frac{d\rho \cdot \rho^4}{\rho^2 + m^2} e^{-\ell\sqrt{m^2 + \rho^2}} \frac{4\pi^2}{p_E} \int_0^\infty d\lambda \cdot \lambda \cdot J_1(p_E \lambda) \times \\ &\times J_1(\lambda \rho) = \frac{p_E^2}{p_E^2 + m^2} e^{-\ell\sqrt{m^2 + p_E^2}} \end{aligned} \quad (39)$$

where we have used the well-known relation

$$\int_0^\infty d\lambda \cdot \lambda J_1(p_E \lambda) J_1(\rho \lambda) = \frac{1}{p_E} \delta(\rho - p_E) \quad (40)$$

In this case, we get again:

$$\tilde{D}_{YE}^\ell(p_E) = \frac{\tilde{\rho}_E^m(p_E)}{p_E^2} \quad (41)$$

in virtue of (38). Below, we are interested in a specific density $\rho_{SE}^\ell(x)$ which will be used to construct spread-out fields of massive particles. Let us consider the following formal equation

$$\square_E U_{SE}^\ell(x) = -\rho_{SE}^\ell(x) \quad (42)$$

where

$$\rho_{SE}^\ell(x) = \frac{1}{4\pi^2 r_E} \int_0^\infty d\rho \cdot \rho^2 J_1(\rho r_E) \exp \left[-\frac{\ell}{2} \sqrt{m^2 + \rho^2} \right] =$$

$$= \frac{\ell}{8\pi^2} \sqrt{\frac{2}{\pi}} m^{5/2} \frac{K_{5/2} \left(m \sqrt{r_E^2 + \frac{1}{4}\ell^2} \right)}{\left(r_E^2 + \frac{1}{4}\ell^2 \right)^{5/4}} \quad (43)$$

The Fourier transform of (43) is given by

$$\begin{aligned} \widetilde{\rho_{SE}^\ell}(p) &= \int d^4x e^{-ip_E x_E} \rho_{SE}^\ell(x) = \\ &= \frac{\ell}{2p_E} \sqrt{\frac{2}{m}} m^{5/2} \int_0^\infty d\rho \frac{\rho^2}{\left(\rho^2 + \frac{1}{4}\ell^2 \right)^{5/4}} K_{5/2} \left(m \sqrt{\rho^2 + \frac{1}{4}\ell^2} \right) J_1(p_E \rho) = \\ &= \exp \left[-\frac{\ell}{2} \sqrt{m^2 + p_E^2} \right] \end{aligned} \quad (44)$$

The integral representation for $U_{SE}^\ell(x)$ in (42) reads

$$U_{SE}^\ell(x) = \frac{1}{4\pi^2 r_E} \int_0^\infty d\rho J_1(\rho r_E) e^{-\frac{\ell}{2} \sqrt{m^2 + \rho^2}} \quad (45)$$

which is not a real potential field because of

$$\int d^4x \rho_{SE}^\ell(x) = e^{-\frac{1}{2}m\ell} \quad (46)$$

In this specific case the limit $m \rightarrow 0$ reproduces the electromagnetic ring charge theory while the limit $\ell \rightarrow 0$ ($m \rightarrow 0$) corresponds to the local electromagnetic interaction of the point charge. For the latter case, the four-dimensional Euclidean extension of the theory does not work, since

$$\int d^4x \rho_{SE}^o(x, m=0) = \infty$$

5 Does a Medium Range Interaction Exist?

Above, we have expounded the long-range electromagnetic and the short-range Yukawa interactions from a point of view of the potential theory. In this connection the following question arises. Does a medium-range interaction between these two kinds of interactions exist in nature? Roughly speaking, a potential of such a type interaction, if it exists, should behave as $1/r^\alpha$, $1 < \alpha < \infty$ at the limit $r \rightarrow \infty$. It is an interesting fact that Fourier transforms of these singular potentials exist for two cases $\alpha = 2$ and $\alpha = 1$ only. The latter corresponds to the electromagnetic potential for which there exists force transmitting quanta, i.e., photon and its propagator is just the Fourier transform of the Coulomb potential in the static limit. The case $\alpha = 2$ leads to the potential of the dipole

$$U_d(r) = \frac{1}{4\pi\epsilon_o} \frac{\lambda \cos \theta}{r^2} \quad (47)$$

satisfying an inhomogeneous Poisson equation

$$\left(\Delta - \frac{2}{r^2} \right) U_d(r) = 0 \quad (48)$$

In expression (47) $\lambda = ed$ (d is a length of the dipole) is the electric dipole moment, r is a distance from the midpoint of a dipole, this line makes an angle θ with the dipole axis. Below we will assume $\varepsilon_o = 1$. The inverse Fourier transform for (47)

$$D_d(\mathbf{p}^2) = \frac{1}{e \cos \theta} \int d^3 r e^{-i\mathbf{p}\mathbf{r}} U_d(r) = \frac{L}{\sqrt{\mathbf{p}^2}}, (L = \pi d/2) \quad (49)$$

defines a propagator of a particle in the static limit, which may be absorbed or emitted by the dipole and its properties are clear from further consideration. As before for the cases of point and ring charges the direct Fourier transform reads

$$U_d(r) = \frac{e \cos \theta}{(2\pi)^3} \int d^3 p e^{i\mathbf{p}\mathbf{r}} D_d(\mathbf{p}^2) \quad (50)$$

From equation (49) we can see that the dipole is some kind of ring, i.e., the rigid one. It is more clear from the following exact definition

$$U_d^{st}(r) = \frac{1}{4\pi} \frac{ed \cos \theta}{r^2 + \ell^2} \quad (51)$$

Here ℓ is a parameter dimension of length ($\ell = d/2$) characterizing nonlocality of the dipole. If we want to see a stochastic unoriented dipole then we assume that θ is a random quantity with some distribution $W(\theta)$ ($-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$) satisfying conditions (Namsrai, 1997):

$$\int_{-\pi/2}^{\pi/2} d\theta W(\theta) = 1, \quad \int_{-\pi/2}^{\pi/2} d\theta \cdot \theta W(\theta) = 0, \quad \int_{-\pi/2}^{\pi/2} d\theta \cdot \theta^2 W(\theta) = const \quad (52)$$

The choice (Namsrai, 1997) $W(\theta) = \frac{1}{2} \cos \theta$ leads to the averaged dipole potential

$$U_d^\ell(r) = \langle U_d^{st}(r) \rangle_\theta = \frac{ed}{16} \frac{1}{r^2 + \ell^2} \quad (53)$$

This definition is more acceptable from a physical point of view. In this case an inhomogeneous Poisson equation is given by

$$\Delta U_d^\ell(r) = e \rho_d^\ell(r) \quad (54)$$

with the solution

$$U_d^\ell(r) = \frac{e}{4\pi} \int d^3 r' \frac{\rho_d^\ell(\mathbf{r} - \mathbf{r}')}{|\mathbf{r}'|} \quad (55)$$

Here

$$\rho_d^\ell(r) = \frac{d}{16} \left\{ \frac{6}{(r^2 + \ell^2)^2} - \frac{8r^2}{(r^2 + \ell^2)^3} \right\}$$

satisfying the condition

$$\int d^3 r \rho_d^\ell(r) = 0 \quad (56)$$

as should be for the dipole consisting from opposite electric charges. A nonlocal propagator for a "spread-out particle" (like nonlocal photon field) which may be absorbed or emitted by the nonlocal dipole-string acquires the form

$$\tilde{D}_d^\ell(\mathbf{p}^2) = \frac{1}{e\pi/4} \int d^3 r e^{-i\mathbf{p}\mathbf{r}} U_d^\ell(r) = \frac{L}{\sqrt{\mathbf{p}^2}} e^{-\ell\sqrt{\mathbf{p}^2}} \quad (57)$$

or its analytic continuation expression in four-dimensional Minkowski space is

$$\tilde{D}_d^\ell(p^2) = \frac{L}{\sqrt{-p^2}} e^{-\ell\sqrt{-p^2}}, \quad p^2 = p_0^2 - \mathbf{p}^2 \quad (58)$$

This formula is very important for us. As we expected, the factor $\exp[-\ell\sqrt{-p^2}]$ corresponding to the string property of a particle also appears in this case. It means that our approach has a universal character for the different potential theories. What kind of a particle does exist if its propagator is defined by formulas (49) and (57) or (58) for the local and nonlocal theories, respectively? From the Minkowski extension of (49) one gets

$$\tilde{D}_d(p^2) = \frac{L}{\sqrt{-p^2}} = \frac{L}{i} \frac{1}{\sqrt{\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}}} = \frac{L}{i} \frac{\hat{\mathbf{p}}}{p^2} \quad (59)$$

or analogously (58) yields

$$\tilde{D}_d^\ell(p^2) = \frac{L}{i} \frac{\hat{\mathbf{p}}}{p^2} e^{-\ell\sqrt{-p^2}} \quad (60)$$

Here $\hat{\mathbf{p}} = \gamma_i p_i$, γ_i are the Dirac γ -matrices. From definitions (59) and (60) we can easily see that these formulas correspond to the propagators of the massless fermion particles like neutrino. Do the dipoles emit or absorb neutrino-like particles? This question is now open. Here the dipole may be rotated with very high speeds. If such particles are detected then we would like to call these ones nonlocal photons or photinos - the fermion partners of the gauge bosons (photons) in the supersymmetric theory (Bailin and Love, 1994).

It should be noted that for the dipole-string case, the four-dimensional Euclidean extension of our formalism is the same as before. For this purpose, let us write down another representation for the propagator (57):

$$\tilde{D}_d^\ell(\mathbf{p}^2) = \frac{1}{(e\pi/4)} \int d^3 r e^{-i\mathbf{p}\mathbf{r}} U_d^\ell(r) = \frac{4}{\pi} \tilde{\rho}_d^\ell(p) \left[\frac{\widetilde{1}}{4\pi} \frac{1}{r} \right] (p) \quad (61)$$

which follows from expression (55), where

$$\left[\frac{\widetilde{1}}{4\pi r} \right] (p) = \int d^3 r e^{-i\mathbf{p}\mathbf{r}} \frac{1}{4\pi r} = \frac{1}{\mathbf{p}^2}$$

and

$$\begin{aligned} \tilde{\rho}_d^\ell(p) &= \frac{d}{16} \frac{4\pi}{p} \int_0^\infty dx \cdot x \cdot \sin px \left[\frac{6}{(x^2 + \ell^2)^2} - \frac{8x^2}{(x^2 + \ell^2)^3} \right] = \\ &= \frac{\pi^2}{8} d \cdot p e^{-\ell p}, \quad (p = \sqrt{\mathbf{p}^2}) \end{aligned} \quad (62)$$

The Euclidean potential corresponding to the propagator (58) acquires the form

$$U_{dE}^\ell(x) = \frac{e\pi}{4} \frac{1}{(2\pi)^4} \int d^4 p e^{ipx} \tilde{D}_{dE}^\ell(p^2) = \frac{e}{16\pi} L \frac{1}{(r_E^2 + \ell^2)^{3/2}} \quad (63)$$

In this case, the Poisson equation (54) is changed as follows

$$\square_E U_{dE}^\ell(x) = -e \rho_{dE}^\ell(x) \quad (64)$$

where

$$\rho_{dE}^\ell(x) = \frac{3L}{16\pi} \left[\frac{4}{(r_E^2 + \ell^2)^{5/2}} - \frac{5r_E^2}{(r_E^2 + \ell^2)^{7/2}} \right] \quad (65)$$

or

$$\rho_{dE}^\ell(x) = \frac{L}{16\pi r_E} \int_0^\infty d\rho \rho^3 e^{-\ell\rho} J_1(\rho r_E) \quad (66)$$

with the condition

$$\int d^4x \rho_{dE}^\ell(x) = 0 \quad (67)$$

The solution of equation (64) is given by the integral representation:

$$U_{dE}^\ell(x) = \frac{1}{4\pi^2} \int d^4y \frac{\rho_{dE}^\ell(x-y)}{y^2} \quad (68)$$

Here a similar relation of (61) holds

$$\tilde{D}_{dE}^\ell(x) = \frac{4}{\pi} \widetilde{\rho_{dE}^\ell}(p) \left[\frac{1}{4\pi^2} \frac{1}{y_E^2} \right] (p) \quad (69)$$

Thus, we observe that for the very different potential theories there exist the same mathematical methods.

Now we briefly discuss some possibilities of the confinement of the particle, propagator of which does not have poles in momentum space (Efimov and Ivanov, 1993). Let us consider a more singular simple potential

$$U_c^\ell(r) = \frac{g}{\pi^2} L_c^2 \frac{\ell}{(r^2 + \ell^2)^2} \quad (70)$$

with respect to (51). Here g is some coupling constant, L is a parameter characterizing a size of a system ($L_c \sim 1/m$, ℓ means nonlocality of the theory. The corresponding propagator for (70) defines as

$$\tilde{D}_c^\ell(p) = \frac{1}{g} \int d^3r e^{-i\mathbf{p}\mathbf{r}} U_c^\ell(r) = L_c^2 e^{-\ell\sqrt{\mathbf{p}^2}} \quad (71)$$

In the case of (70), a modified Poisson equation takes the form

$$\Delta U_c^\ell(r) = -g\rho_c^\ell(r) \quad (72)$$

where

$$\rho_c^\ell(r) = 12 \frac{\ell L_c^2}{\pi^2} \frac{\ell^2 - r^2}{(r^2 + \ell^2)^4} \quad (73)$$

with the remarkable property

$$\int d^3r \rho_c^\ell(r) = 0 \quad (74)$$

Equalities (74) and (71) tell us that potential (70) corresponds to a dipole-like string consisting of opposite charges g and this system does not exist in free states but only in bounded or virtual ones-like quarks, gluons and even Higgs bosons (Namsrai, 1996). For the massive case, expressions (70) and (71) lead to formulas:

$$U_{cm}^\ell(r) = \frac{g}{2\pi^2} \ell \frac{K_2(m\sqrt{r^2 + \ell^2})}{r^2 + \ell^2} \quad (75)$$

and

$$\tilde{D}_{cm}^\ell(\mathbf{p}) = \frac{1}{g} \int d^3r e^{-i\mathbf{p}\mathbf{r}} U_{cm}^\ell(r) = m^{-2} e^{-\ell\sqrt{m^2+\mathbf{p}^2}} \quad (76)$$

Analogously, equation (72) is changed as

$$\Delta U_{cm}^\ell(r) = -g\rho_{cm}^\ell(r) \quad (77)$$

Here, the charge density is given by

$$\rho_{cm}^\ell(r) = \frac{\ell}{2\pi^2} \left[3m \frac{K_3(m\sqrt{r^2+\ell^2})}{(r^2+\ell^2)^{3/2}} - m^2 r^2 \frac{K_4(m\sqrt{r^2+\ell^2})}{(r^2+\ell^2)^2} \right] \quad (78)$$

The direct calculation gives

$$\int d^3r \rho_{cm}^\ell(r) = 0 \quad (79)$$

as should be in accordance with the universal rule. As before, one can represent the solution of (77) in the standard integral form

$$U_{cm}^\ell(r) = \frac{g}{4\pi} \int d^3r' \frac{\rho_{cm}^\ell(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}'|} \quad (80)$$

So that there exists another formula for (76). That is

$$\tilde{D}_{cm}^\ell(\mathbf{p}) = \tilde{\rho}_{cm}^\ell(p) \left[\frac{1}{4\pi} \frac{1}{|x|} \right] (p) \quad (81)$$

where after some calculations we get

$$\tilde{\rho}_{cm}^\ell(p) = \int d^3r e^{i\mathbf{p}\mathbf{r}} \rho_{cm}^\ell(r) = \frac{\mathbf{p}^2}{m^2} e^{-\ell\sqrt{m^2+\mathbf{p}^2}} \quad (82)$$

Similar relations hold for equation (72). For the given cases, the Euclidean extension is also trivial.

It should be noted again that the propagators

$$\tilde{D}_c^\ell(p^2) = L_c^2 e^{-\ell\sqrt{-p^2}} \quad (83)$$

and

$$\tilde{D}_{cm}^\ell(p^2) = m^{-2} e^{-\ell\sqrt{m^2-p^2}}, \quad (p^2 = p_0^2 - \mathbf{p}^2) \quad (84)$$

describe the behavior of virtual particles in momentum space for the ring theory. From (70) and (75) we see that for these potentials there does not exist their local counterparts in the limit $\ell \rightarrow 0$ and therefore they are only responsible for pure nonlocal effects. Indeed the limit $\ell \rightarrow 0$ reduces a size of the system to the point where opposite charges are annihilated mutually and as a result the given system disappears from our consideration.

6 The Construction of Spread-out Fields in the Nonlocal Yukawa and the Dipole Interactions

Now let us study spread-out or nonlocal fields propagators of which are given by formulas (3) and (60) in momentum space. In accordance with the Efimov nonlocal theory (Efimov, 1977 and Namsrai, 1986) we construct nonlocal fields by using the nonlocal distributions

$$N_Y^\ell(x) = N_Y^\ell(\ell^2 \square) \delta^{(4)}(x) \quad (85)$$

$$N_d^\ell(x) = N_d^\ell(\ell^2 \square) \delta^{(4)}(x) \quad (86)$$

for the Yukawa and dipole interactions, respectively. Here

$$N_Y^\ell(\ell^2 \square) = \exp \left[-\frac{1}{2} \sqrt{m^2 - \square} \ell \right] \quad (87)$$

$$N_d^\ell(\ell^2 \square) = \exp \left[-\frac{1}{2} \sqrt{-\square} \ell \right] \quad (88)$$

It is easy to see that the Fourier transforms of generalized functions (85) and (86) can be written in the forms

$$\tilde{N}_Y^\ell(k^2 \ell^2) = \int d^4 x e^{ikx} N_Y^\ell(x) = \exp \left[-\frac{1}{2} \sqrt{m^2 - k^2} \ell \right] \quad (89)$$

$$\tilde{N}_d^\ell(k^2 \ell^2) = \int d^4 x e^{ikx} N_d^\ell(x) = \exp \left[-\frac{1}{2} \sqrt{-k^2} \ell^2 \right] \quad (90)$$

In the Euclidean metric the Fourier transforms of the generalized functions $N_d^\ell(x)$ and $N_Y^\ell(x)$ in (86) and (85) coincide with the Fourier transforms of the formal Euclidean extensions of the charge density $\rho_E^0(x, \frac{\ell}{2})$ in (20) and $\rho_{SE}^\ell(x,)$ in (43), respectively. It is obvious that functions (20) [or (86)] and (43) [or(85)] satisfy δ -function properties at the limit $\ell \rightarrow 0$, i.e.,

$$\lim_{\ell \rightarrow 0} N_i^\ell(x) = \delta^{(4)}(x)$$

$$\lim_{\ell \rightarrow 0} \int d^4 x f(x) N_i^\ell(x) = f(0), (i = Y, d) \quad (91)$$

Here

$$N_Y^\ell(x) = \frac{\ell}{8\pi^2} \sqrt{\frac{2}{\pi}} m^{5/2} \frac{K_{5/2} \left(m \sqrt{\frac{1}{4} \ell^2 - x^2} \right)}{\left(\frac{1}{4} \ell^2 - x^2 \right)^{5/4}} \quad (92)$$

and

$$N_d^\ell(x) = \frac{3\ell}{8\pi^2} \left(\frac{1}{4} \ell^2 - x^2 \right)^{-5/2}, \quad (x^2 = x_0^2 - \mathbf{x}^2) \quad (93)$$

Thus, we arrive at the square-root Klein-Gordon type, $\sqrt{m^2 - \square}$, differential operators for the string model of the Yukawa and dipole interactions. Let us construct the spread-out Yukawa and the nonlocal photon (arising from the dipole interaction) fields by using these operators:

$$\phi_Y^\ell(x) = \int d^4 y N_Y^\ell(x-y) \phi_Y^0(y) \quad (94)$$

and

$$P^\ell(x) = \int d^4y N_d^\ell(x-y) P^o(y) \quad (95)$$

where $\phi_Y^o(x)$ and $P^o(x)$ are the local Yukawa and the local neutrino-like nonlocal photon fields, respectively. The usual propagators of these fields are defined as

$$\begin{aligned} \Delta_Y^o(x-y) &= \langle 0|T \{ \phi_Y^o(x) \phi_Y^o(y) \} |0 \rangle = \\ &= \frac{1}{(2\pi)^4 i} \int \frac{d^4p e^{-ip(x-y)}}{m^2 - p^2 - i\varepsilon} \end{aligned} \quad (96)$$

$$\begin{aligned} \Delta_{pt}^o(x-y) &= \langle 0|T \{ P^\ell(x) \bar{P}^\ell(y) \} |0 \rangle = \\ &= \frac{1}{(2\pi)^4 i} \int \frac{d^4p \hat{p} \cdot e^{-ip(x-y)}}{-p^2 - i\varepsilon} \end{aligned} \quad (97)$$

where $\Delta_{pt}^o(x)$ means the local propagator of the neutrino-like field. Making use of formulas (96) and (97) one gets

$$\begin{aligned} D_Y^\ell(x-y) &= \overbrace{\phi_Y^\ell(x) \phi_Y^\ell(y)} = \int d^4y_1 \int d^4y_2 N_Y^\ell(x-y_1) \times \\ &N_Y^\ell(y_1-y_2) \langle 0|T \{ \phi_Y^o(y_1) \phi_Y^o(y_2) \} |0 \rangle = \\ &= \frac{1}{(2\pi)^4 i} \int \frac{d^4k \left[\tilde{N}_Y^\ell(k^2 \ell^2) \right]^2}{m^2 - k^2 - i\varepsilon} e^{-ik(x-y)} \end{aligned} \quad (98)$$

and

$$\begin{aligned} D_{pt}^\ell(x-y) &= \overbrace{P^\ell(x) P^\ell(y)} = \int d^4y_1 \int d^4y_2 N_d^\ell(x-y_1) \times \\ &N_Y^\ell(y_1-y_2) \langle 0|T \left[P^\ell(y_1) P^o(y_2) \right] |0 \rangle = \\ &= \frac{L}{(2\pi)^4} \int d^4k \frac{\hat{k}}{-k^2 - i\varepsilon} \left[\tilde{N}_d^\ell(k^2 \ell^2) \right]^2 e^{-ik(x-y)} \end{aligned} \quad (99)$$

In expressions (98) and (99) the Fourier transforms \tilde{N}_Y^ℓ and \tilde{N}_d^ℓ are given by (89) and (90) in accordance with the string theory discussed above. Now we turn to discuss the problem of how to formulate interaction Lagrangians for different fields in the Yukawa and the dipole theories. Let us consider a simple scheme where fermions $\psi_i (i = e, \mu, \dots, u, d, \dots, t)$ interact with the Higgs boson $\phi_H^o(x)$ through some nonlocal (Efimov, 1977; Namsrai, 1986) or averaged (Reuter and Wetterich, 1993) interaction of the Yukawa scalar form

$$\begin{aligned} L_{in}^Y(x) &= \frac{1}{2} \delta_i \int d^4y_1 \int d^4y_2 N_Y^\ell(y_1) N_Y^\ell(y_2) \bar{\psi}_i(x-y_1-y_2) \times \\ &\times \psi_i(x-y_1-y_2) [\phi_H^o(x-y_1) + \phi_H^o(x-y_2)] \end{aligned} \quad (100)$$

or in differential form

$$L_{in}^Y(x) = \delta_i \bar{\psi}_i(x) \psi_i(x) \phi_H^\ell(x) \quad (101)$$

where δ_i are some coupling constants. The latter corresponds to the assumption that the Higgs boson carries nonlocality only:

$$\phi_H^0(x) \Rightarrow \phi_H^\ell(x) = \int d^4y N_Y^\ell(x-y) \phi_H^o(y) = N_Y^\ell(\ell^2 \square) \phi_H^o(x) \quad (102)$$

Here $N_Y^\ell(x) = N_Y^\ell(\ell^2 \square) \delta^{(4)}(x)$ is the generalized function (85) arising from the nonlocal (string) theory. For the cases (100) and (102), the propagator of the Higgs boson is given by the formula (98). For the interaction Lagrangian (101) symbolic scheme of the construction of the finite S -matrix in the perturbation theory can be represented in the form

$$S = \lim_{\Lambda \rightarrow \infty} T_\Lambda \exp \left\{ i \int d^4x L_{in}(x, \Lambda) \right\} \quad (103)$$

where T_Λ means chronological ordering operators plus some regularization procedure leading to finite matrix elements in series of the perturbation theory and Λ is a parameter of the regularization. The limit $\Lambda \rightarrow \infty$ means the removal of the regularization (Efimov, 1977). The exact form of the interaction Lagrangian for the dipole case will be given elsewhere.

In order to construct the perturbation series for the S -matrix (103) with Lagrangians (101) by prescription of the usual local theory, it is necessary to change (in the Feynman diagrams) $\Delta_Y^o(x-y) \Rightarrow D_Y^\ell(x-y)$ and $\Delta_{pt}^o(x-y) \Rightarrow D_{pt}^\ell l(x-y)$ in virtue of (98) and (99) and to keep the usual local fermion propagator for the field ψ_i in corresponding interaction Lagrangians (101). Here the following Mellin representations for propogators of Yukawa and nonlocal photon fields in momentum space:

$$\tilde{D}_Y^\ell(k^2) = \frac{\ell^2}{2i} \int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{v(\xi)}{\sin \pi \xi} [\ell^2(m^2 - k^2 - i\varepsilon)]^{\xi-1} \quad (104)$$

and

$$\tilde{D}_d^\ell(k^2) \equiv \tilde{D}_{pt}^\ell(k^2) = \frac{\ell^2}{2i} \int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{v(\xi)}{\sin \pi \xi} [\ell^2(-k^2 - i\varepsilon)]^{\xi-1} \hat{k} \quad (105)$$

are useful for calculation purposes, where

$$v(\xi) = \frac{1}{\cos \pi \xi, (1 + 2\xi)}, \quad -\frac{1}{2} < \beta < 0 \quad (106)$$

In (105) the parameter $L = \pi d/2$ is omitted. Finally, it should be noted that calculation of matrix elements for the S -matrix in the perturbation series for the Yukawa and dipole (string) theories will be presented elsewhere. Definition of the square-root differential operators and their actions on field functions are given in papers (Smith, 1993, Lammerzahi, 1993 and Nam-srai, 1997).

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