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Measuring Higgs boson couplings at the LHC

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Abstract

For an intermediate mass Higgs boson with SM-like couplings the LHC allows observation of a variety of decay channels in production by gluon fusion and weak boson fusion. Cross section ratios provide measurements of various ratios of Higgs couplings, with accuracies of order 15% for 100 fb⁻¹ of data in each of the two LHC experiments. For Higgs masses above 120 GeV, minimal assumptions on the Higgs sector allow for an indirect measurement of the total Higgs boson width with an accuracy of 10 to 20%, and of the $H \rightarrow WW$ partial width with an accuracy of about 10%.

I. INTRODUCTION

Investigation of the symmetry breaking mechanism of the electroweak $SU(2) \times U(1)$ gauge symmetry will be one of the prime tasks of the LHC. Correspondingly, major efforts have been concentrated on devising methods for Higgs boson discovery, for the entire mass range allowed within the Standard Model (SM) (100 GeV $\lesssim m_H \lesssim 1$ TeV, after LEP2), and for Higgs boson search in extensions of the SM, like the MSSM [1,2]. While observation of one or more Higgs scalar(s) at the LHC appears assured, discovery will be followed by a more demanding task: the systematic investigation of Higgs boson properties. Beyond observation of the various CP even and CP odd scalars which nature may have in store for us, this means the determination of the couplings of the Higgs boson to the known fermions and gauge bosons, i.e. the measurement of Htt, Hbb, $H\tau\tau$ and HWW, HZZ, $H\gamma\gamma$ couplings, to the extent possible.

Clearly this task very much depends on the expected Higgs boson mass. For $m_H > 200$ GeV and within the SM, only the $H \to ZZ$ and $H \to WW$ channels are expected to be observable, and the two gauge boson modes are related by SU(2). Above $m_H \approx 250$ GeV, where detector effects will no longer dominate the mass resolution of the $H \to ZZ \to 4\ell$ resonance, additional information is expected from a direct measurement of the total Higgs boson width, Γ_H . A much richer spectrum of decay modes is predicted for the intermediate mass range, i.e. if a SM-like Higgs boson has a mass between the reach of LEP2 ($\lesssim 110$ GeV) and the Z-pair threshold. The main reasons for focusing on this range are present indications from electroweak precision data, which favor $m_H < 250$ GeV [3], as well as expectations within the MSSM, which predicts the lightest Higgs boson to have a mass $m_h \lesssim 130$ GeV [4].

Until recently, the prospects of detailed and model independent coupling measurements at the LHC were considered somewhat remote [5], because few promising search channels were known to be accessible, for any given Higgs boson mass. Taking ATLAS search scenarios as an example, these were [1]

$$gg \to H \to \gamma\gamma$$
, for $m_H \lesssim 150 \text{ GeV}$, (1)

$$gg \to H \to ZZ^* \to 4\ell$$
, for $m_H \gtrsim 130 \text{ GeV}$, (2)

and

$$gg \to H \to WW^* \to \ell \bar{\nu} \bar{\ell} \nu$$
, for $m_H \gtrsim 150 \text{ GeV}$, (3)

with the possibility of obtaining some additional information from processes like WHand/or $t\bar{t}H$ associated production with subsequent $H \rightarrow \bar{b}b$ and $H \rightarrow \gamma\gamma$ decay for Higgs boson masses near 100 GeV. Throughout this paper, " $gg \rightarrow H$ " stands for inclusive Higgs production, which is dominated by the gluon fusion process for a SM-like Higgs boson.

This relatively pessimistic outlook is changing considerably now, due to the demonstration that weak boson fusion is a promising Higgs production channel also in the intermediate mass range. Previously, this channel had only been explored for Higgs masses above 300 GeV. Specifically, it was recently shown in parton level analyses that the weak boson fusion channels, with subsequent Higgs decay into photon pairs [6,7],

$$qq \to qqH, \ H \to \gamma\gamma, \qquad \text{for} \quad m_H \lesssim 150 \text{ GeV},$$
(4)

into $\tau^+\tau^-$ pairs [7–9],

$$qq \to qqH, \ H \to \tau\tau , \qquad \text{for} \quad m_H \lesssim 140 \text{ GeV} ,$$
 (5)

or into W pairs [7,10]

$$qq \to qqH, \ H \to WW^{(*)} \to e^{\pm} \mu^{\mp} \not\!\!\!/_T, \qquad \text{for} \quad m_H \gtrsim 120 \text{ GeV},$$
 (6)

can be isolated at the LHC. Preliminary analyses, which try to extend these parton level results to full detector simulations, look promising [11]. The weak boson fusion channels utilize the significant background reductions which are expected from double forward jet tagging [12–14] and central jet vetoing techniques [15,16], and promise low background environments in which Higgs decays can be studied in detail. The parton level results predict highly significant signals with (substantially) less than 100 fb⁻¹.

The prospect of observing several Higgs production and decay channels, over the entire intermediate mass range, suggests a reanalysis of coupling determinations at the LHC [5]. In this paper we attempt a first such analysis, for the case where the branching fractions of an intermediate mass Higgs resonance are fairly similar to the SM case. i.e. we analyze a SM-like Higgs boson only. We make use of the previously published analyses for the inclusive Higgs production channels [1,2] and of the weak boson fusion channels [6-10]. The former were obtained by the experimental collaborations and include detailed detector simulations. The latter are based on parton level results, which employ full QCD tree level matrix elements for all signal and background processes. We will not discuss here differences in the performance expected for the ATLAS and CMS detectors nor details in the theoretical assumptions which lead to different estimates for expected signal and background rates. The reader is referred to the original publications from which numbers are extracted. In Section II we summarize expectations for the various channels, including expected accuracies for cross section measurement of the various signals for an integrated luminosity of 100 $\rm fb^{-1}$. Implications for the determination of coupling ratios and the measurement of Higgs boson (partial) decay widths are then obtained in Section III. A final summary is given in Section IV.

II. SURVEY OF INTERMEDIATE MASS HIGGS CHANNELS

The various Higgs channels listed in Eqs. (1–6) and their observability at the LHC have all been discussed in the literature. Where available, we give values as presently quoted by the experimental collaborations. In order to compare the accuracy with which the cross sections of different Higgs production and decay channels can be measured, we need to unify these results. For example, K-factors of unity are assumed throughout. Our goal in this section is to obtain reasonable estimates for the relative errors, $\Delta \sigma_H / \sigma_H$, which are expected after collecting 100 fb⁻¹ in each the ATLAS and the CMS detector, i.e. we estimate results after a total of 200 fb⁻¹ of data have been collected at the

TABLE I. Number of expected events for the inclusive SM $H \rightarrow \gamma \gamma$ signal and expected backgrounds, assuming an integrated luminosity of 100 fb⁻¹ and high luminosity performance. Numbers correspond to optimal $\gamma \gamma$ invariant mass windows for CMS and ATLAS. The expected relative statistical errors on the signal cross section are given for the individual experiments and are combined in the last line.

	m_H	100	110	120	130	140	150
CMS [17,18]	N_S	865	1038	1046	986	816	557
	N_B	29120	22260	16690	12410	9430	7790
	$\Delta \sigma_H / \sigma_H$	20.0%	14.7%	12.7%	11.7%	12.4%	16.4%
ATLAS [1]	N_S	1045	1207	1283	1186	973	652
	N_B	56450	47300	39400	33700	28250	23350
	$\Delta \sigma_H / \sigma_H$	22.9%	18.2%	15.7%	15.7%	17.6%	23.8%
Combined	$\Delta \sigma_H / \sigma_H$	15.1%	11.4%	9.9%	9.4%	10.1%	13.5%

LHC. Presumably these data will be taken with a mix of both low and high luminosity running.

We find that the measurements are largely dominated by statistical errors. For all channels, event rates with 200 fb⁻¹ of data will be large enough to use the Gaussian approximation for statistical errors. The experiments measure the signal cross section by separately determining the combined signal + background rate, N_{S+B} , and the expected number of background events, $\langle N_B \rangle$. The signal cross section is then given by

$$\sigma_H = \frac{N_{S+B} - \langle N_B \rangle}{\epsilon \int \mathcal{L} dt} = \frac{N_S}{\epsilon \int \mathcal{L} dt} , \qquad (7)$$

where ϵ denotes efficiency factors. Thus the statistical error is given by

$$\frac{\Delta\sigma_H}{\sigma_H} = \frac{\sqrt{N_{S+B}}}{N_S} = \frac{\sqrt{N_S + N_B}}{N_S} , \qquad (8)$$

where in the last step we have dropped the distinction between the expected and the actual number of background events. Systematic errors on the background rate are added in quadrature to the background statistical error, $\sqrt{N_B}$, where appropriate.

Well below the $H \to WW$ threshold, the search for $H \to \gamma\gamma$ events is arguably the cleanest channel for Higgs discovery. LHC detectors have been designed for excellent two-photon invariant mass resolution, with this Higgs signal in mind. We directly take the expected signal and background rates for the inclusive $H \to \gamma\gamma$ search from the detailed studies of the CMS and ATLAS collaborations [17,18,1], which were performed for an integrated luminosity of 100 fb⁻¹ in each detector. Expectations are summarized in Table I. Rates correspond to not including a K-factor for the expected signal and background cross sections in CMS and ATLAS. Cross sections have been determined with MRS (R1) parton distribution functions (pdf's) for CMS, while ATLAS numbers are based on CTEQ2L pdf's.

The inclusive $H \to \gamma \gamma$ signal will be observed as a narrow $\gamma \gamma$ invariant mass peak on top of a smooth background distribution. This means that the background can be directly measured from the very high statistics background distribution in the sidebands. We expect any systematic errors on the extraction of the signal event rate to be negligible compared to the statistical errors which are given in the last row of Table I. With 100 fb⁻¹ of data per experiment $\sigma(gg \to H) \cdot B(H \to \gamma \gamma)$ can be determined with a relative error of 10 to 15% for Higgs masses between 100 and 150 GeV. Here we do not include additional systematic errors, e.g. from the luminosity uncertainty or from higher order QCD corrections, because we will mainly consider cross section ratios in the final analysis in the next Section. These systematic errors largely cancel in the cross section ratios. Systematic errors common to several channels will be considered later, where appropriate.

A Higgs search channel with a much better signal to background ratio, at the price of lower statistics, however, is available via the inclusive search for $H \rightarrow ZZ^* \rightarrow 4\ell$ events. Expected event numbers for 100 fb⁻¹ in both ATLAS [1] and CMS [19] are listed in Table II. These numbers were derived using CTEQ2L pdf's and are corrected to contain

TABLE II. Number of expected events for the inclusive SM $H \to ZZ^* \to \ell^+ \ell^- \ell^+ \ell^-$ signal and expected backgrounds, assuming an integrated luminosity of 100 fb⁻¹ and high luminosity performance. Numbers correspond to optimal four-lepton invariant mass windows for CMS and ATLAS and to the combined total. Rates in parentheses correspond to numbers interpolated, according to $H \to ZZ^*$ branching ratios for the signal. The expected relative statistical errors on the signal cross section are given for each experiment and are combined in the last line.

	m_H	120	130	140	150	160	170	180
CMS [19]	N_S	19.2	55.3	(99)	131.4	(48)	29.4	(76.5)
	N_B	12.9	17.1	(20)	22.5	(26)	27.5	(27)
	$\Delta \sigma_H / \sigma_H$	29.5%	15.4%	11.0%	9.4%	17.9%	25.7%	13.3%
ATLAS [1]	N_S	10.3	28.7	(51)	67.6	(31)	19.1	49.7
	N_B	4.44	7.76	(8)	8.92	(8)	8.87	8.81
	$\Delta \sigma_H / \sigma_H$	37.3%	21.0%	15.1%	12.9%	20.1%	27.7%	15.4%
Combined	$\Delta \sigma_H / \sigma_H$	23.1%	12.4%	8.9%	7.6%	13.4%	18.8%	10.1%

no QCD K-factor. For those Higgs masses where no ATLAS or CMS prediction is available, we interpolate/extrapolate the results for the nearest Higgs mass, taking the expected $H \to ZZ^*$ branching ratios into account for the signal. Similar to the case of $H \to \gamma \gamma$ events, the signal is seen as a narrow peak in the four-lepton invariant mass distribution, i.e. the background can be extracted directly from the signal sidebands. The combined relative error on the measurement of $\sigma(gg \to H) \cdot B(H \to ZZ^*)$ is listed in the last line of Table II. For Higgs masses in the 130–150 GeV range, and above Z-pair threshold, a 10% statistical error on the cross section measurement is possible. In the intermediate range, where $H \to WW$ dominates, and for lower Higgs masses, where the Higgs is expected to dominantly decay into $\bar{b}b$, the error increases substantially.

Above $m_H \approx 135 \text{ GeV}, H \to WW^{(*)}$ becomes the dominant SM Higgs decay channel. The resulting inclusive $WW \to \ell^+ \nu \ell^- \bar{\nu}$ signal is visible above backgrounds, after

TABLE III. Number of expected events for the inclusive SM $H \to WW^* \to \ell^+ \nu \ell^- \bar{\nu}$ signal and expected backgrounds, assuming an integrated luminosity of 30 fb⁻¹. Numbers correspond to optimized cuts, varying with the mass of the Higgs boson being searched for. The expected relative errors on the signal cross section are given for each experiment, separating the statistical error, the effect of a systematic 5% error of the background level, and the two added in quadrature. The combined error for the two experiments assumes 100% correlation of the systematic errors on the background determination.

	m_H	120	130	140	150	160	170	180	190
CMS [20]	N_S	44	106	279	330	468	371	545	
	N_B	272	440	825	732	360	360	1653	
	$\Delta \sigma_H / \sigma_H (\text{stat.})$	40.4%	22.0%	11.9%	9.9%	6.1%	7.3%	8.6%	
	$\Delta \sigma_H / \sigma_H (\text{syst.})$	30.9%	20.8%	14.8%	11.1%	3.8%	4.9%	15.2%	
	$\Delta \sigma_H / \sigma_H (\text{comb.})$	50.9%	30.3%	19.0%	14.9%	7.3%	8.8%	17.4%	20.6%
ATLAS $[1]$	N_S				240	400	337	276	124
	N_B				844	656	484	529	301
	$\Delta \sigma_H / \sigma_H$ (stat.)				13.7%	8.1%	8.5%	10.3%	16.6%
	$\Delta \sigma_H / \sigma_H$ (syst.)				17.6%	8.2%	7.2%	9.6%	12.1%
	$\Delta \sigma_H / \sigma_H \text{ (comb.)}$	50.9%	30.3%	19.0%	22.3%	11.5%	11.1%	14.1%	20.6%
Combined	$\Delta \sigma_H / \sigma_H \ (\text{comb.})$	42.1%	26.0%	17.0%	14.8%	7.0%	8.0%	13.6%	16.9%

exploiting the characteristic lepton angular correlations for spin zero decay into W pairs near threshold [20]. The inclusive channel, which is dominated by $gg \to H \to WW$, has been analyzed by ATLAS for $m_H \ge 150$ GeV and for integrated luminosities of 30 and 100 fb⁻¹ [1] and by CMS for $m_H \ge 120$ GeV and 30 fb⁻¹ [20]. The expected event numbers for 30 fb⁻¹ are listed in Table III. The numbers are derived without QCD K-factors and use CTEQ2L for ATLAS and MRS(A) pdf's for CMS results.

Unlike the two previous modes, the two missing neutrinos in the $H \to WW$ events

do not allow for a reconstruction of the narrow Higgs mass peak. Since the Higgs signal is only seen as a broad enhancement of the expected background rate in lepton-neutrino transverse mass distributions, with similar shapes of signal and background after application of all cuts, a precise determination of the background rate from the data is not possible. Rather one has to rely on background measurements in phase space regions where the signal is weak, and extrapolation to the search region using NLO QCD predictions. The precise error on this extrapolation is unknown at present, the assumption of a 5% systematic background uncertainty appears optimistic but attainable. It turns out that with 30 fb^{-1} already, the systematic error starts to dominate, because the background exceeds the signal rate by factors of up to 5, depending on the Higgs mass. Running at high luminosity makes matters worse, because the less efficient reduction of $\bar{t}t$ backgrounds, due to less stringent b-jet veto criteria, increases the background rate further. Because of this problem we only present results for 30 fb^{-1} of low luminosity running in Table III. Since neither of the LHC collaborations has presented predictions for the entire Higgs mass range, we take CMS simulations below 150 GeV and ATLAS results at 190 GeV, but divide the resultant statistical errors by a factor $\sqrt{2}$, to take account of the presence of two experiments. Between 150 and 180 GeV we combine both experiments, assuming 100% correlation in the systematic 5% normalization error of the background.

The previous analyses are geared towards measurement of the inclusive Higgs production cross section, which is is dominated by the gluon fusion process. 15 to 20% of the signal sample, however, is expected to arise from weak boson fusion, $qq \rightarrow qqH$ or corresponding antiquark initiated processes. The weak boson fusion component can be isolated by making use of the two forward tagging jets which are present in these events and by vetoing additional central jets, which are unlikely to arise in the color singlet signal process [15]. A more detailed discussion of these processes can be found in Ref. [7] from which most of the following numbers are taken.

TABLE IV. Number of expected $\gamma\gamma jj$ events from the $qq \rightarrow qqH$, $H \rightarrow \gamma\gamma$ weak boson fusion signal and expected backgrounds, assuming an integrated luminosity of 100 fb⁻¹. Numbers correspond to optimal $\gamma\gamma$ invariant mass windows for CMS and ATLAS and to the combined total, as projected from the parton level analysis of Refs. [6,7]. The expected relative statistical errors on the signal cross section are given for each experiment and are combined in the last line.

	m_H	100	110	120	130	140	150
projected CMS	N_S	37	48	56	56	48	33
performance	N_B	33	32	31	30	28	25
	$\Delta \sigma_H / \sigma_H$	22.6%	18.6%	16.7%	16.6%	18.2%	23.1%
projected ATLAS	N_S	42	54	63	63	54	37
performance	N_B	61	60	56	54	51	46
	$\Delta \sigma_H / \sigma_H$	24.2%	19.8%	17.3%	17.2%	19.0%	24.6%
combined	$\Delta \sigma_H / \sigma_H$	16.5%	13.6%	12.0%	11.9%	13.1%	16.8%

The $qq \rightarrow qqH$, $H \rightarrow \gamma\gamma$ process was first analyzed in Ref. [6], where cross sections for signal and background were obtained with full QCD tree level matrix elements. The parton level Monte Carlo determines all geometrical acceptance corrections. Additional detector effects were included by smearing parton and photon 4-momenta with expected detector resolutions and by assuming trigger, identification and reconstruction efficiencies of 0.86 for each of the two tagging jets and 0.8 for each photon. Resulting cross sections were presented in Ref. [7] for a fixed $\gamma\gamma$ invariant mass window of total width $\Delta m_{\gamma\gamma} = 2$ GeV. We correct these numbers for m_H dependent mass resolutions in the experiments. We take 1.4σ mass windows, as given in Ref. [1] for high luminosity running, which are expected to contain 79% of the signal events for ATLAS. The 2 GeV window for $m_H = 100$ GeV at CMS [17,18] is assumed to scale up like the ATLAS

TABLE V. Number of expected signal and background events for the $qq \rightarrow qqH \rightarrow \tau\tau jj$ channel, for 100 fb⁻¹ and two detectors. Cross sections are added for $\tau\tau \rightarrow \ell^{\pm}h^{\mp}\not{p}_{T}$ and $\tau\tau \rightarrow e^{\pm}\mu^{\mp}\not{p}_{T}$ events as given in Refs. [7,9]. The last line gives the expected statistical relative error on the $qq \rightarrow qqH$, $H \rightarrow \tau\tau$ cross section.

m_H	100	110	120	130	140	150
N_S	211	197	169	128	79	38
N_B	305	127	51	32	27	24
$\Delta \sigma_H / \sigma_H$	10.8%	9.1%	8.8%	9.9%	13.0%	20.7%

resolution and assumed to contain 70% of the Higgs signal. The expected total signal and background rates for 100 fb⁻¹ and resulting relative errors for the extraction of the signal cross section are given in Table IV. Statistical errors only are considered for the background subtraction, since the background level can be measured independently by considering the sidebands to the Higgs boson peak.

The next weak boson fusion channel to be considered is $qq \rightarrow qqH$, $H \rightarrow \tau\tau$. Again, this channel has been analyzed at the parton level, including some estimates of detector effects, as discussed for the $H \rightarrow \gamma\gamma$ case. Here, a lepton identification efficiency of 0.95 is assumed for each lepton $\ell = e, \mu$. Two τ -decay modes have been considered so far: $H \rightarrow \tau\tau \rightarrow \ell^{\pm}h^{\mp}p_{T}$ [8] and $H \rightarrow \tau\tau \rightarrow e^{\pm}\mu^{\mp}p_{T}$ [9]. These analyses were performed for low luminosity running. Some deterioration at high luminosity is expected, as in the analogous $H/A \rightarrow \tau\tau$ channel in the MSSM search [1]. At high luminosity, pileup effects degrade the p_{T} resolution significantly, which results in a worse $\tau\tau$ invariant mass resolution. At a less significant level, a higher p_{T} threshold for the minijet veto technique will increase the QCD and $t\bar{t}$ backgrounds. The τ -identification efficiency is similar at high and low luminosity. We expect that the reduced performance at high luminosity can be compensated for by considering the additional channels $H \rightarrow \tau\tau \rightarrow$

TABLE VI. Number of events expected for $qq \rightarrow qqH$, $H \rightarrow WW^{(*)} \rightarrow \mu^{\pm}e^{\mp}\not{p}_{T}$ in 200 fb⁻¹ of data, and corresponding backgrounds [10]. The expected relative statistical error on the signal cross section is given in the last line.

m_H	120	130	140	150	160	170	180	190
N_S	136	332	592	908	1460	1436	1172	832
N_B	136	160	188	216	240	288	300	324
$\Delta \sigma_H / \sigma_H$	12.1%	6.7%	4.7%	3.7%	2.8%	2.9%	3.3%	4.1%

 $e^+e^-\not\!p_T$, $\mu^+\mu^-\not\!p_T$. Z+jets and ZZ+jets backgrounds (with $ZZ \to \ell^+\ell^-\nu\bar{\nu})$ are strongly suppressed by rejecting same flavor lepton pairs which are compatible with Z decays $(m_{\ell\ell} = m_Z \pm 6 \text{ GeV})$. Drell-Yan plus jets backgrounds are further reduced by requiring significant $\not\!p_T$. Since these analyses have not yet been performed, we use the predicted cross sections for only those two channels which have already been discussed in the literature and scale event rates to a combined 200 fb⁻¹ of data. Results are given in Table V.

The previous two weak boson channels allow reconstruction of the Higgs resonance as an invariant mass peak. This is not the case for $H \to WW \to \ell^+ \nu \ell^- \bar{\nu}$ as discussed previously for the inclusive search. The weak boson fusion channel can be isolated separately by employing forward jet tagging and color singlet exchange isolation techniques in addition to tools like charged lepton angular correlations which are used for the inclusive channel. The corresponding parton level analysis for $qq \to qqH$, $H \to WW^{(*)} \to \mu^{\pm}e^{\mp}\not{p}_{T}$ has been performed in Ref. [10] and we here scale the results to a total integrated luminosity of 200 fb⁻¹, which takes into account the availability of two detectors. As for the tau case, the analysis was done for low luminosity running conditions and somewhat higher backgrounds are expected at high luminosity. On the other hand the $WW^{(*)} \to \mu^+\mu^-\not{p}_T$ and $WW^{(*)} \to e^+e^-\not{p}_T$ modes should roughly double the available statistics since very few signal events have lepton pair invariant masses compatible with $Z \to \ell \ell$ decays. Therefore our estimates are actually conservative. Note that the expected background for this weak boson fusion process is much smaller than for the corresponding inclusive measurement. As a result modest systematic uncertainties will not degrade the accuracy with which $\sigma(qq \to qqH) \cdot B(H \to WW^{(*)})$ can be measured. A 10% systematic error on the background, double the error assumed in the inclusive case, would degrade the statistical accuracy by, typically, a factor 1.2 or less. As a result, we expect that a very precise measurement of $\sigma(qq \to qqH) \cdot B(H \to WW^{(*)})$ can be performed at the LHC, with a statistical accuracy of order 5% or even better in the mass range $m_H \ge 140$ GeV. Even for m_H as low as 120 GeV a 12% measurement is expected.

III. MEASUREMENT OF HIGGS PROPERTIES

One would like to translate the cross section measurements of the various Higgs production and decay channels into measurements of Higgs boson properties, in particular into measurements of the various Higgs boson couplings to gauge fields and fermions. This translation requires knowledge of NLO QCD corrections to production cross sections, information on the total Higgs decay width and a combination of the measurements discussed previously. The task here is to find a strategy for combining the anticipated LHC data without undue loss of precision due to theoretical uncertainties and systematic errors.

For our further discussion it is convenient to rewrite all Higgs boson couplings in terms of partial widths of various Higgs boson decay channels. The Higgs-fermion couplings g_{Hff} , for example, which in the SM are given by the fermion masses, $g_{Hff} = m_f(m_H)/v$, can be traded for the $H \to \bar{f}f$ partial widths,

$$\Gamma_f = \Gamma(H \to \bar{f}f) = c_f \frac{g_{Hff}^2}{8\pi} \left(1 - \frac{4m_f^2}{m_H^2}\right)^{\frac{3}{2}} m_H .$$
(9)

Here c_f is the color factor (1 for leptons, 3 for quarks). Similarly the square of the HWW coupling $(g_{HWW} = gm_W$ in the SM) or the HZZ coupling is proportional to

the partial widths $\Gamma_W = \Gamma(H \to WW^*)$ or $\Gamma_Z = \Gamma(H \to ZZ^*)$ [21]. Analogously we trade the squares of the effective $H\gamma\gamma$ and Hgg couplings for $\Gamma_{\gamma} = \Gamma(H \to \gamma\gamma)$ and $\Gamma_g = \Gamma(H \to gg)$. Note that the Hgg coupling is essentially proportional to g_{Htt} , the Higgs' coupling to the top quark.

The Higgs production cross sections are governed by the same squares of couplings. This allows to write e.g. the $gg \to H$ production cross section as [22]

$$\sigma(gg \to H) = \Gamma(H \to gg) \frac{\pi^2}{8m_H^3} \tau \int_{\tau}^1 \frac{dx}{x} g(x, m_H^2) g(\frac{\tau}{x}, m_H^2) , \qquad (10)$$

where $\tau = m_H^2/s$. Similarly the $qq \rightarrow qqH$ cross sections via WW and ZZ fusion are proportional to $\Gamma(H \rightarrow WW^*)$ and $\Gamma(H \rightarrow ZZ^*)$, respectively. In the narrow width approximation, which is appropriate for the intermediate Higgs mass range considered here, these production cross sections need to be multiplied by the branching fractions for final state j, $B(H \rightarrow j) = \Gamma_j/\Gamma$, where Γ denotes the total Higgs width. This means that the various cross section measurements discussed in the previous Section provide measurements of various combinations $\Gamma_i \Gamma_j/\Gamma$.

The production cross sections are subject to QCD corrections, which introduces theoretical uncertainties. While the K-factor for the gluon fusion process is large [23], which suggests a sizable theoretical uncertainty on the production cross section, the NLO corrections to the weak boson fusion cross section are essentially identical to the ones encountered in deep inelastic scattering and are quite small [24]. Thus we can assign a small theoretical uncertainty to the latter, of order 5%, while we shall use a larger theoretical error for the gluon fusion process, of order 20% [23]. The problem for weak boson fusion is that it consists of a mixture of $ZZ \rightarrow H$ and $WW \rightarrow H$ events, and we cannot distinguish between the two experimentally. In a large class of models the ratio of HWW and HZZ couplings is identical to the one in the SM, however, and this includes the MSSM. We therefore make the following W, Z-universality assumption:

The H → ZZ* and H → WW* partial widths are related by SU(2) as in the SM,
 i.e. their ratio, z, is given by the SM value,

$$\Gamma_Z = z \ \Gamma_W = z_{SM} \ \Gamma_W \ . \tag{11}$$

Note that this assumption can be tested, at the 15-20% level for $m_H > 130$ GeV, by forming the ratio $B\sigma(gg \to H \to ZZ^*)/B\sigma(gg \to H \to WW^*)$, in which QCD uncertainties cancel (see Table VII).

With W, Z-universality, the three weak boson fusion cross sections give us direct measurements of three combinations of (partial) widths,

$$X_{\gamma} = \frac{\Gamma_W \Gamma_{\gamma}}{\Gamma} \qquad \text{from } qq \to qqH, \ H \to \gamma\gamma , \qquad (12)$$

$$X_{\tau} = \frac{\Gamma_W \Gamma_{\tau}}{\Gamma} \qquad \text{from } qq \to qqH, \ H \to \tau\tau \ , \tag{13}$$

$$X_W = \frac{\Gamma_W^2}{\Gamma} \qquad \text{from } qq \to qqH, \ H \to WW^{(*)} , \qquad (14)$$

with common theoretical systematic errors of 5%. In addition the three gluon fusion channels provide measurements of

$$Y_{\gamma} = \frac{\Gamma_g \Gamma_{\gamma}}{\Gamma} \quad \text{from } gg \to H \to \gamma\gamma ,$$
 (15)

$$Y_Z = \frac{\Gamma_g \Gamma_Z}{\Gamma} \qquad \text{from } gg \to H \to ZZ^{(*)} , \qquad (16)$$

$$Y_W = \frac{\Gamma_g \Gamma_W}{\Gamma} \qquad \text{from } gg \to H \to WW^{(*)} , \qquad (17)$$

with common theoretical systematic errors of 20%.

The first precision test of the Higgs sector is provided by taking ratios of the X_i 's and ratios of the Y_i 's. In these ratios the QCD uncertainties, and all other uncertainties related to the initial state, like luminosity and pdf errors, cancel. Beyond testing W, Zuniversality, these ratios provide useful information for Higgs masses between 100 and 150 GeV and 120 to 150 GeV, respectively, where more than one channel can be observed in the weak boson fusion and gluon fusion groups. Typical errors on these cross section ratios are expected to be in the 15 to 20% range (see Table VII). Accepting an additional systematic error of about 20%, a measurement of the ratio Γ_g/Γ_W , which determines the *Htt* to *HWW* coupling ratio, can be performed, by measuring the cross section ratios

TABLE VII. Summary of the accuracy with which various ratios of partial widths can be determined with 200 fb⁻¹ of data. The first two columns give the ratio considered and indicate the method by which it is measured. Y_Z/Y_W , for example, indicates a measurement of $\sigma B(H \to ZZ^*)/\sigma B(H \to WW^*)$ in gluon fusion, while X_i ratios correspond to weak boson fusion (see text for details). The statistical combination of several channels for a given width ratio is indicated by \oplus . 5% and 20% theoretical uncertainties for weak boson and gluon fusion cross sections affect the mixed gluon/weak boson fusion ratios only, which are needed for a measurement of Γ_g/Γ_W . The effect of this systematic error is indicated in the last line.

m_H		100	110	120	130	140	150	160	170	180
$z = \Gamma_Z / \Gamma_W$	Y_Z/Y_W			48%	29%	19%	17%	15%	20%	17%
	$\frac{Y_Z}{Y_\gamma} \frac{X_\gamma}{X_W}$			30%	21%	19%	23%			
	$rac{Y_Z}{Y_W} \oplus rac{Y_Z}{Y_\gamma} rac{X_\gamma}{X_W}$			29%	19%	15%	14%	15%	20%	17%
Γ_{γ}/Γ_W	$rac{Y_{\gamma}}{Y_W} \oplus rac{X_{\gamma}}{X_W}$			16%	12%	11%	13%			
Γ_{τ}/Γ_W	$\frac{X_{\tau}}{X_W}$			15%	12%	14%	21%			
$\Gamma_{ au}/\Gamma_{\gamma}$	$\frac{X_{\tau}}{X_{\gamma}}$	20%	16%	15%	16%	18%	27%			
Γ_g/Γ_W	$rac{Y_\gamma}{X_\gamma} \oplus rac{Y_W}{X_W}$	22%	18%	15%	13%	12%	13%	8%	9%	14%
	$rac{Y_{\gamma}}{X_{\gamma}} \oplus rac{Y_W}{X_W} \oplus 21\%$	30%	27%	25%	24%	24%	24%	22%	22%	25%

 $B\sigma(gg \to H \to \gamma\gamma)/\sigma(qq \to qqH)B(H \to \gamma\gamma)$ and $B\sigma(gg \to H \to WW^*)/\sigma(qq \to qqH)B(H \to WW^*)$. Expected accuracies are listed in Table VII. In these estimates the systematics coming from understanding detector acceptance is not included.

Beyond the measurement of coupling ratios, minimal additional assumptions allow an indirect measurement of the total Higgs width. First of all, the τ partial width, properly normalized, is measurable with an accuracy of order 10%. The τ is a third generation fermion with isospin $-\frac{1}{2}$, just like the *b*-quark. In all extensions of the SM with a common source of lepton and quark masses, even if generational symmetry is broken, the ratio of *b* to τ Yukawa couplings is given by the fermion mass ratio. We thus assume, in addition to W, Z-universality, that

• The ratio of b to τ couplings of the Higgs is given by their mass ratio, i.e.

$$y = \frac{\Gamma_b}{\Gamma_\tau} = 3c_{QCD} \frac{g_{Hbb}^2}{g_{H\tau\tau}^2} = 3c_{QCD} \frac{m_b^2(m_H)}{m_\tau^2} , \qquad (18)$$

where c_{QCD} is the known QCD and phase space correction factor.

• The total Higgs width is dominated by decays to $\bar{b}b$, $\tau\tau$, WW, ZZ, gg and $\gamma\gamma$, i.e. the branching ratio for unexpected channels is small:

$$\epsilon = 1 - \left(B(H \to b\bar{b}) + B(H \to \tau\tau) + B(H \to WW^{(*)}) + B(H \to ZZ^{(*)}) + B(H \to gg) + B(H \to \gamma\gamma) \right) \ll 1.$$
(19)

Note that, in the Higgs mass range of interest, these two assumptions are satisfied for both CP even Higgs bosons in most of the MSSM parameter space. The first assumption holds in the MSSM at tree level, but can be violated by large squark loop contributions, in particular for small m_A and large $\tan \beta$ [25,26]. The second assumption might be violated, for example, if the $H \rightarrow \bar{c}c$ partial width is exceptionally large. However, a large up-type Yukawa coupling would be noticeable in the Γ_g/Γ_W coupling ratio, which measures the *Htt* coupling.

With these assumptions consider the observable

$$\tilde{\Gamma}_W = X_\tau (1+y) + X_W (1+z) + X_\gamma + \tilde{X}_g$$
$$= \left(\Gamma_\tau + \Gamma_b + \Gamma_W + \Gamma_Z + \Gamma_\gamma + \Gamma_g\right) \frac{\Gamma_W}{\Gamma} = (1-\epsilon)\Gamma_W , \qquad (20)$$

where $\tilde{X}_g = \Gamma_g \Gamma_W / \Gamma$ is determined by combining Y_W and the product $Y_\gamma X_W / X_\gamma$. $\tilde{\Gamma}_W$ provides a lower bound on $\Gamma(H \to WW^{(*)}) = \Gamma_W$. Provided ϵ is small ($\epsilon < 0.1$ suffices for practical purposes), the determination of $\tilde{\Gamma}_W$ provides a direct measurement of the $H \to WW^{(*)}$ partial width. Once Γ_W has been determined, the total width of the Higgs boson is given by

$$\Gamma = \frac{\Gamma_W^2}{X_W} = \frac{1}{X_W} \left(X_\tau (1+y) + X_W (1+z) + X_\gamma + \tilde{X}_g \right)^2 \frac{1}{(1-\epsilon)^2} \,. \tag{21}$$

For a SM-like Higgs boson the Higgs width is dominated by the $H \to \bar{b}b$ and $H \to WW^{(*)}$ channels. Thus, the error on $\tilde{\Gamma}_W$ is dominated by the uncertainties of the X_W and X_τ measurements and by the theoretical uncertainty on the *b*-quark mass, which enters the determination of *y* quadratically. According to the Particle Data Group, the present uncertainty on the *b* quark mass is about $\pm 3.5\%$ [27]. Assuming a luminosity error of $\pm 5\%$ in addition to the theoretical uncertainty of the weak boson fusion cross section of $\pm 5\%$, the statistical errors of the $qq \to qqH$, $H \to \tau\tau$ and $qq \to qqH$, $H \to WW$ cross sections of Tables V and VI lead to an expected accuracy of the $\tilde{\Gamma}_W$ determination of rig. 1.

The extraction of the total Higgs width, via Eq. (21), requires a measurement of the $qq \rightarrow qqH, H \rightarrow WW^{(*)}$ cross section, which is expected to be available for $m_H \gtrsim$ 115 GeV [10]. Consequently, errors are large for Higgs masses close to this lower limit (we expect a relative error of $\approx 20\%$ for $m_H = 120$ GeV and $\epsilon < 0.05$). But for Higgs boson masses around the WW threshold, $\Gamma(1 - \epsilon)^2$ can be determined with an error of about 10%. Results are shown in Fig. 1 and look highly promising.

IV. SUMMARY

In the last section we have found that various ratios of Higgs partial widths can be measured with accuracies of order 10 to 20%, with an integrated luminosity of 100 fb⁻¹ per experiment. This translates into 5 to 10% measurements of various ratios of coupling constants. The ratio Γ_{τ}/Γ_{W} measures the coupling of down-type fermions relative to the Higgs couplings to gauge bosons. To the extent that the $H\gamma\gamma$ triangle diagrams are dominated by the W loop, the width ratio $\Gamma_{\tau}/\Gamma_{\gamma}$ measures the same relationship. The fermion triangles leading to an effective Hgg coupling are expected to



FIG. 1. Expected accuracy with which the Higgs boson width can be measured at the LHC, with 100 fb⁻¹ of data in each experiment. Results are shown for the extraction of the the $H \to WW$ partial with, Γ_W , and and the total Higgs boson width, Γ . ϵ is the sum of the residual (small) branching ratios of unobserved channels, mainly $H \to c\bar{c}$ (see text for detail).

be dominated by the top-quark, thus, Γ_g/Γ_W probes the coupling of up-type fermions relative to the *HWW* coupling. Finally, for Higgs boson masses above ≈ 120 GeV, the absolute normalization of the *HWW* coupling is accessible via the extraction of the $H \rightarrow WW^{(*)}$ partial width in weak boson fusion.

Note that these measurements test the crucial aspects of the Higgs sector. The HWW coupling, being linear in the Higgs field, identifies the observed Higgs boson as the scalar responsible for the spontaneous breaking of $SU(2) \times U(1)$: a scalar without a vacuum expectation value couples to gauge bosons only via HHWW or HHW vertices at tree level, i.e. the interaction is quadratic in scalar fields. The absolute value of the HWW coupling, as compared to the SM expectation, reveals whether H may be the only mediator of spontaneous symmetry breaking or whether additional Higgs bosons await

discovery. Within the framework of the MSSM this is a measurement of $|\sin(\beta - \alpha)|$, at the ±0.05 level. The measurement of the ratios of g_{Htt}/g_{HWW} and $g_{H\tau\tau}/g_{HWW}$ then probes the mass generation of both up and down type fermions.

The results presented here constitute a first look only at the issue of coupling extractions for the Higgs. This is the case for the weak boson fusion processes in particular, which prove to be extremely valuable if not essential. Our analysis is mostly an estimate of statistical errors, with some rough estimates of the systematic errors which are to be expected for the various measurements of (partial) widths and their ratios. A number of issues need to be addressed in further studies, in particular with regard to the weak boson fusion channels.

- (a) The weak boson fusion channels and their backgrounds have only been studied at the parton level, to date. Full detector level simulations, and optimization of strategies with more complete detector information is crucial for further progress.
- (b) A central jet veto has been suggested as a powerful tool to suppress QCD backgrounds to the color singlet exchange processes which we call weak boson fusion. The feasibility of this tool and its reach need to be investigated in full detector studies, at both low and high luminosity.
- (c) In the weak boson fusion studies of $H \to WW$ and $H \to \tau\tau$ decays, double leptonic $e^+e^-\not p_T$ and $\mu^+\mu^-\not p_T$ signatures have not yet been considered. Their inclusion promises to almost double the statistics available for the Higgs coupling measurements, at the price of additional ZZ+jets and Drell-Yan plus jets backgrounds which are expected to be manageable.
- (d) Other channels, like WH or $t\bar{t}H$ associated production with subsequent decay $H \rightarrow \bar{b}b$ or $H \rightarrow \gamma\gamma$, provide additional information on Higgs coupling ratios, which complement our analysis at small Higgs mass values, $m_H \lesssim 120$ GeV [2,5]. These channels need to be included in the analysis.

- (e) Much additional work is needed on more reliable background determinations. For the $H \to WW^{(*)} \to \ell^+ \ell'^- \not p_T$ channel in particular, where no narrow Higgs resonance peak can be reconstructed, a precise background estimate is crucial for the measurement of Higgs couplings. Needed improvements include NLO QCD corrections, single top quark production backgrounds, the combination of shower Monte Carlo programs with higher order QCD matrix element calculations and more.
- (f) Both in the inclusive and WBF analyses any given channel contains a mixture of events from $gg \to H$ and $qq \to qqH$ production processes. The determination of this mixture adds another source of systematic uncertainty, which was not included in the present study. In ratios of X observables (or of different Y_i) these uncertainties largely cancel, except for the effects of acceptance variations due to different signal selections. Since an admixture from the wrong production channel is expected at the 10 to 20% level only, these systematic errors are not expected to be serious.
- (g) We have only analyzed the case of a single neutral, CP even Higgs resonance with couplings which are close to the ones predicted in the SM. While this case has many applications, e.g. for the large m_A region of the MSSM, more general analyses, in particular of the MSSM case, are warranted and highly promising.

While much additional work is needed, our study clearly shows that the LHC has excellent potential to provide detailed and accurate information on Higgs boson interactions. The observability of the Higgs boson at the LHC has been clearly established, within the SM and extensions like the MSSM. The task now is to sharpen the tools for accurate measurements of Higgs boson properties at the LHC.

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