CERN-TH/2000-031
hep-ph/0001292

VORTEX PHASES IN CONDENSED MATTER AND COSMOLOGY

M. LAINE
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Placing a high-T_c superconductor in an increasing external magnetic field, the flux first penetrates the sample through an Abrikosov vortex lattice, and then a first order transition is observed by which the system goes to the normal phase. We discuss the cosmological motivation for considering the electroweak phase transition in the presence of an external magnetic field, the analogies this system might have with the superconductor behaviour described above, and in particular whether at large physical Higgs masses, corresponding to the high-T_c regime, an analogue of the vortex phase and an associated first order phase transition could be generated.

1 Introduction

One possible explanation for the baryon asymmetry of the Universe is that it was generated in a 1st order electroweak phase transition[1]. This possibility cannot be realized in the Standard Model (SM), though, since there is no 1st order transition for $m_H \gtrsim 72$ GeV [2]. In the MSSM there is still room for a strong 1st order transition, if the right-handed stop is lighter than the top[3]: a consequence of such a circumstance would be a Higgs lighter than ~ 110 GeV, not yet experimentally excluded for the MSSM.

In this paper, we will consider another possibility for increasing the strength of the electroweak phase transition. Indeed, remaining in the SM but imposing an external magnetic field H_{ext} on the system, has a strengthening effect[4]. It turns out that for baryon number non-conservation there is an opposing effect due to the sphaleron dipole moment[5], but we nevertheless consider it interesting to map out the phase diagram in some detail for $H_{\text{ext}} \neq 0$, as an analogy with superconductors (Sec. 3) suggests that the system might have quite unexpected properties. The case of small Higgs masses in $H_{\text{ext}} \neq 0$, as well as the first results on large Higgs masses, were already reviewed in[6], and we concentrate here on the physical case $m_H \gtrsim m_Z$.

2 Cosmological motivation for $H_{\text{ext}} \neq 0$

The physical relevance of considering $H_{\text{ext}} \neq 0$ comes from the observation that the existence of galactic magnetic fields today may well imply the existence of primordial seed fields in the Early Universe. In order to get large enough length scales, it seems conceivable that even in the most favourable case of strongly "helical" fields, the seed fields should have a correlation length at least of the order of the horizon radius at the electroweak (EW) epoch[7]. Such large length scales could possibly be produced during the inflationary period of Universe expansion (see, e.g., ref.[8] and references therein).

If a primordial spectrum of magnetic fields is generated during inflation, it is on the other hand also true that after a while the fields are essentially homogeneous at small length scales. Indeed, magnetohydrodynamics,

$$\frac{\partial \vec{H}_Y}{\partial t} = \frac{1}{\sigma}\nabla^2 \vec{H}_Y + \nabla \times (\vec{v} \times \vec{H}_Y), \tag{1}$$

tells that magnetic fields diffuse away at scales $l \lesssim (t/\sigma)^{1/2} \sim (M_{\rm Pl}/T)^{1/2} T^{-1}$, where σ is conductivity. At the EW epoch $T \sim 100$ GeV, this gives $l_{\rm EW} \sim 10^7/T$, a scale much larger than the typical correlation lengths $\sim$ a few $\times T^{-1}$.

A further question is the magnitude of magnetic fields. An equipartition argument would say that only a small fraction of the total (free) energy density can be in magnetic fields. This leads to $H_Y/T^2 \lesssim 2$. In conclusion, there could well be essentially homogeneous and macroscopic (hypercharge) magnetic fields around at $T \sim 100$ GeV, with a magnitude $H_Y/T^2 \sim 1$.

3 Superconductors in $H_{\rm ext} \neq 0$

As a further motivation for studying in detail the electroweak case, let us recall the very rich structure found in quite an analogous system, superconductors under an external magnetic field. Denoting the inverses of the spatial scalar and vector correlation lengths by m_H, m_W (and $x \equiv m_H^2/(2m_W^2)$), the usual starting point for superconductor studies, the 3d continuum scalar electrodynamics (or the Ginzburg-Landau, GL, theory), predicts at the tree-level two qualitatively different responses of the system to an external magnetic field:

In the type I case, $m_H < m_W$, a flux cannot penetrate the superconducting phase. However, superconductivity is destroyed by $H_{\rm ext}$. The way in which this transition has to take place is that the superconducting and the normal phases coexist at $H_{\rm ext}^c$. This implies a *1st order* transition.

In the type II case, $m_H > m_W$, on the other hand, the flux can penetrate the system via an Abrikosov vortex lattice. At a large enough $H_{\rm ext}$ the system then *continuously* changes to the normal phase.

It is now a very interesting observation that fluctuations change the nature of the tree-level type II transition described above in an essential way. Indeed, much of the vortex lattice phase is observed to be removed, but it is also found in high-T_c superconductors (which are strongly of type II) that the continuous

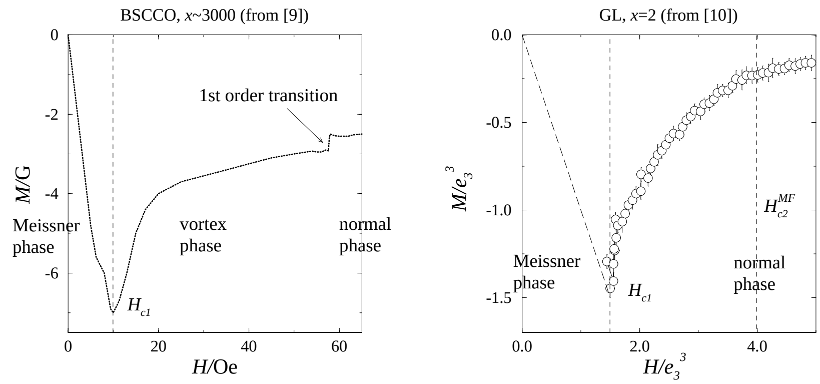

Figure 1. Left: magnetization M as a function of the field strength H as observed for the high-T_c material BSCCO ($x \sim 3000$); a qualitative reproduction of Fig. 2 in[9]. The tree-level ("mean field") upper critical field $H_{c2}^{\rm MF}$ is huge due to the large value of x, $H_{c2}^{\rm MF} \sim 10^3 H_{c1}$, and the transition observed takes place much below $H_{c2}^{\rm MF}$. The system is macroscopic in the sense that there are several hundred flux quanta at the point of the 1st order transition. Right: magnetization M as a function of H in the GL model at $x = 2$; data from[10]. Measurements were only carried out for $H \geq H_{c1}$. At $H_{c2}^{\rm MF}$, there are ~ 40 flux quanta in the system. No 1st order transition is observed within this resolution.

transition changes to a 1st order one: for a particularly clear signal, see Fig. 2 in[9], reproduced in Fig. 1(left).

At the same time, high-T_c superconductors are of course a complicated layered and highly anisotropic material, so it is not immediately clear whether the 1st order transition observed is also a property of, say, the simple continuum GL theory. Let us list arguments in favour of and against this possibility:

- There is an analytic prediction of a 1st order transition[11], starting just from the GL theory. However, it is based on an ϵ-expansion around $d = 6$, and relies on $\epsilon = 3$ being small. Other analytic arguments (see, e.g., [12]) also lack a small expansion parameter. A set of lattice simulations have been carried out which favour the possibility of a phase transition directly in the GL theory[13]. However, the theory actually simulated is not GL but some approximation thereof, and moreover, the effects of discretization artifacts in the simulations have not been systematically investigated.

- There are, on the other hand, other simulation results which argue that a layered structure *is* essential for the 1st order transition[14]. However, these simulations use again an approximate form of the theory, whose validity for the full GL model is not clear. Finally, direct lattice simulations in the full GL model[10] have so far failed to see a transition, see Fig. 1(right). However, one can argue that due to the high computational cost, these simulations do not necessarily yet represent the thermodynamical limit with respect to the number of vortices.

To summarize, we consider it at the moment an open problem what is the "minimal" continuum model which may display a 1st order transition between the vortex phase and the normal phase. Understanding this issue would be very important for, e.g., the considerations to which we now turn.

4 The Electroweak Theory in $H_{\mathbf{ext}} \neq 0$ at Tree-Level

To analyse the behaviour of the electroweak theory in an external magnetic field, we can directly consider the dimensionally reduced 3d action[15]

$$\mathcal{L}_{3\mathrm{d}} = \frac{1}{4}G^a_{ij}G^a_{ij} + \frac{1}{4}F_{ij}F_{ij} + (D_i\phi)^\dagger D_i\phi + y\phi^\dagger\phi + x(\phi^\dagger\phi)^2. \qquad (2)$$

Here G^a_{ij}, F_{ij} are the SU(2) and $\mathrm{U}_Y(1)$ field strengths, and ϕ is the Higgs doublet. In terms of the physical 4d parameters, x and y are expressed as

$$x \sim 0.12\frac{m_H^2}{m_W^2}, \quad y \sim 4.5\frac{T-T_0}{T_0}, \qquad (3)$$

where T_0 equals the critical temperature up to radiative corrections. By a magnetic field we now mean, in the symmetric phase of the theory, an Abelian $\mathrm{U}_Y(1)$ magnetic field H_Y. In the broken phase, this goes dynamically to the electromagnetic field H_{EM}.

The tree-level phase diagram of the theory in Eq. (2) is shown in Fig. 2. This is quite similar to that of the GL model. For $m_H > m_Z$ the ground state solution of the equations of motion is inhomogeneous in a certain range of H_Y. This Ambjørn-Olesen (AO) phase[16] is the analogue of the Abrikosov vortex lattice of superconductors. There are some differences, as well: for instance, the Higgs phase is not really a Meissner phase, as at low temperatures and fields, the magnetic field can pass through the system in a homogeneous configuration. Another notable difference is that the "vortices" (see Fig. 2(right)) are not topological objects in the same sense as in superconductors, as the Higgs vev does not vanish at the core of the profile.

For future reference, let us recall one way of understanding the appearance of the "instability" leading to the AO-phase. The point is that at tree-level, there are charged excitations in both phases of the system which can be arbitrarily light close enough to the phase transition. In the presence of

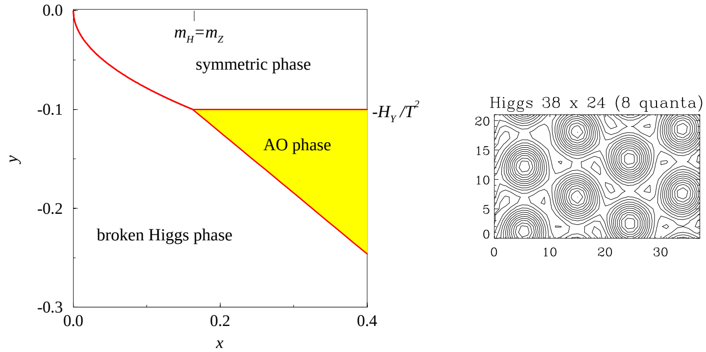

Figure 2. Left: The tree-level phase diagram of the electroweak theory in a magnetic field. In the superconductor analogy, symmetric phase ↔ normal phase, AO phase ↔ Abrikosov lattice, Higgs phase ↔ Meissner phase. Right: The Higgs profile in the AO phase[17]. At the point of the minima, the Higgs vev is only ∼ 10...20% smaller than elsewhere.

a magnetic field, the corresponding energies behave as Landau levels. It can then happen that some excitations become essentially "tachyonic", leading to an instability: e.g. in the broken phase for large $H_{\rm EM}$,

$$m_{W,\rm eff}^2 = m_W^2 - eH_{\rm EM} < 0. \tag{4}$$

5 The Electroweak Theory in $H_{\rm ext} \neq 0$ with Fluctuations

In order to include systematically the effects of fluctuations, we have studied the system in Eq. (2) with lattice simulations[15]. We refer there for the details of the simulations, as well as for the justification of the following main result: for the values of $H_{\rm ext}$ studied, we have *not observed* the AO phase, nor any phase transition at all for $m_H > m_Z$! Let us discuss here to what extent we can now understand such a contrast with the high-T_c behaviour.

For small values of H_Y, the discrepancy can be understood as being due to SU(2) confinement. For instance, the W is always massive in contrast to perturbation theory, so that Eq. (4) cannot be satisfied for arbitrarily small $H_{\rm EM}$. It is however difficult to turn this argument into a quantitative one.

Another way to express the issue is that the only gauge-invariant degrees of freedom which can become massless are a neutral scalar (the Higgs), and the photon[18]. Close to the endpoint (see Fig. 3), the system can thus be non-perturbatively described by an effective theory of the form[2] ($\phi \in \mathbb{R}$)

$$\mathcal{L} = \frac{1}{4}F_{ij}F_{ij} + \frac{1}{2}(\partial_i\phi)^2 + h\phi + \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \gamma_1\phi F_{ij}F_{ij} + \ldots . \tag{5}$$

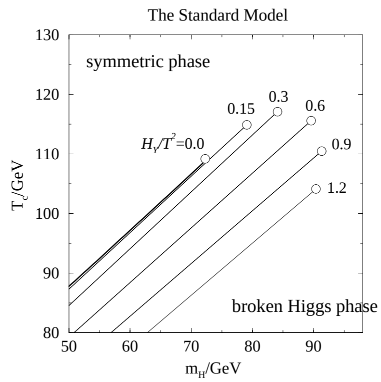

Figure 3. The non-perturbative phase diagram of the electroweak theory in a magnetic field (no errorbars shown). A solid line indicates a 1st order transition, and an open circle a 2nd order endpoint. Based on ref.[15] and the preliminary results of ref.[17].

However, in this theory there are no charged excitations, hence no Landau levels and instabilities, unlike at tree-level!

On the other hand, the effective theory in Eq. (5) can in principle break down for very large fields, and also far away from the endpoint, and one may ask what happens then? It is here that the case of superconductors again becomes relevant. As discussed at the end of Sec. 3, it might be that even in superconductors some extra structure such as layers is needed in order to have a vortex phase and the associated 1st order transition. If so, then it is unlikely that there would be any remnant of the AO phase in the fluctuating electroweak system even at large H_Y. If no layers are needed, on the contrary, there just might be one.

6 Conclusions

It appears that even if there is an external magnetic field present, the SM electroweak transition terminates at $m_H \lesssim 90$ GeV, and above that there is no structure at all, see Fig. 3. In particular, the Ambjørn-Olesen phase seems not to be realized at realistic magnetic fields. Thus an electroweak phase transition within the SM does not leave a cosmological remnant. An interesting theoretical open issue is still what happens at very large magnetic field strengths — a question which involves quite intriguing analogies also with the behaviour of experimentally accessible high-T_c superconductors.

Acknowledgments

I thank K. Kajantie, T. Neuhaus, P. Pennanen, A. Rajantie, K. Rummukainen, M. Shaposhnikov and M. Tsypin for collaboration and discussions on various topics mentioned in this talk. This work was partly supported by the TMR network *Finite Temperature Phase Transitions in Particle Physics*, EU contract no. FMRX-CT97-0122.

References

1. V.A. Kuzmin *et al*, *Phys. Lett.* B **155**, 36 (1985).
2. K. Rummukainen *et al*, *Nucl. Phys.* B **532**, 283 (1998) [hep-lat/9805013].
3. D. Bödeker *et al*, *Nucl. Phys.* B **497**, 387 (1997) [hep-ph/9612364]; M. Carena *et al*, *Nucl. Phys.* B **524**, 3 (1998) [hep-ph/9710401]; M. Laine and K. Rummukainen, *Phys. Rev. Lett.* **80**, 5259 (1998) [hep-ph/9804255]; J.M. Cline and G.D. Moore, *Phys. Rev. Lett.* **81**, 3315 (1998) [hep-ph/9806354]; M. Losada, *Nucl. Phys.* B **537**, 3 (1999) [hep-ph/9806519].
4. M. Giovannini and M.E. Shaposhnikov, *Phys. Rev.* D **57**, 2186 (1998).
5. D. Comelli *et al*, *Phys. Lett.* B **458**, 304 (1999) [hep-ph/9903227].
6. M. Laine, Proceedings of *SEWM '98*, eds. J. Ambjørn *et al*, p. 319 (World Scientific, Singapore, 1999) [hep-ph/9902282].
7. D.T. Son, *Phys. Rev.* D **59**, 063008 (1999) [hep-ph/9803412]; G.B. Field and S.M. Carroll, astro-ph/9811206.
8. M.S. Turner and L.M. Widrow, *Phys. Rev.* D **37**, 2743 (1988); B. Ratra, *Astrophys. J. Lett.* **391**, L1 (1992); A. Dolgov and J. Silk, *Phys. Rev.* D **47**, 3144 (1993); M. Gasperini, M. Giovannini and G. Veneziano, *Phys. Rev. Lett.* **75**, 3796 (1995) [hep-th/9504083].
9. E. Zeldov *et al*, *Nature* **375**, 373 (1995).
10. K. Kajantie *et al*, *Nucl. Phys.* B **559**, 395 (1999) [hep-lat/9906028].
11. E. Brézin, D.R. Nelson and A. Thiaville, *Phys. Rev.* B **31**, 7124 (1985).
12. Z. Tešanović, *Phys. Rev.* B **59**, 6449 (1999) [cond-mat/9801306].
13. A.K. Nguyen and A. Sudbø, *Phys. Rev.* B **60**, 15307 (1999) [cond-mat/9907385]; and references therein.
14. A.K. Kienappel and M.A. Moore, *Phys. Rev.* B **60**, 6795 (1999) [cond-mat/9809317]; and references therein.
15. K. Kajantie *et al*, *Nucl. Phys.* B **544**, 357 (1999) [hep-lat/9809004].
16. J. Ambjørn and P. Olesen, *Nucl. Phys.* B **315**, 606 (1989); *Phys. Lett.* B **218**, 67 (1989); *Nucl. Phys.* B **330**, 193 (1990).
17. K. Kajantie *et al*, in preparation.
18. K. Kajantie *et al*, *Nucl. Phys.* B **493**, 413 (1997) [hep-lat/9612006].