CERN-TH/2000-027
hep-th/0001166

A Note on Solitons in Brane Worlds

Donam Youm[1]

Theory Division, CERN, CH-1211, Geneva 23, Switzerland

Abstract

We obtain the zero mode effective action for gravitating objects in the bulk of dilatonic domain walls. Without additional fields included in the bulk action, the zero mode effective action reproduces the action in one lower dimensions obtained through the ordinary Kaluza-Klein (KK) compactification, only when the transverse (to the domain wall) component of the bulk metric does not have non-trivial term depending on the domain wall worldvolume coordinates. With additional fields included in the bulk action, non-trivial dependence of the transverse metric component on the domain wall worldvolume coordinates appears to be essential in reproducing the lower-dimensional action obtained via the ordinary KK compactification. We find, in particular, that the effective action for the charged $(p+1)$-brane in the domain wall bulk reproduces the action for the p-brane in one lower dimensions.

CERN-TH/2000-027
January, 2000

[1]E-mail: Donam.Youm@cern.ch

1 Introduction

Recently, phenomenolgists have actively considered the possibility that the existence of additional compact spatial dimensions may account for the large hierarchy between the electroweak scale and the Planck scale, the so-called hierarchy problem in particle physics. In this scenario, our four-dimensional world is confined within the worldvolume of a three-brane, within which the fields of Standard Model are contained. The earlier proposal [1, 2] relies on the large enough volume of the compact extra space for solving the hierarchy problem. More compelling scenario proposed by Randall and Sundrum (RS) [3, 4, 5] assumes that the spacetime is non-factorizable, in contrast to the conventional view of the Kaluza-Klein (KK) theory that the spacetime is the direct product of the four-dimensional spacetime and the compact extra space. Such a point of view on spacetime was also previously taken [6, 7, 8, 9, 10, 11, 12, 13] as an alternative to the compact compactification, namely as a mechanism to trap matter within the four-dimensional hypersurface without having to assume that the extra space is compact. However, what is new in the RS model is that it is not just matter but also gravity that is effectively trapped within the hypersurface, i.e. the three-brane or the five-dimensional domain wall. The exponential fall-off (as one moves away from the brane) of the warp factor in the metric of the non-factorizable spacetime accounts for the large hierarchy between the electroweak and the Planck scales in our four-dimensional world, which is assumed to be located away from the wall. Gravity in our four-dimensional world is relatively weak also because the wave function for the graviton zero mode, which is localized around the wall, falls off as one moves away from the wall.

Since gravity is shown to be effectively compactified to one lower dimensions (even when the extra spatial dimension is infinite) in the background of the RS type domain walls [4], it is of interest to study various gravitating objects in such background. (Some of the previous works on the related subject are Refs. [14, 15, 16, 17, 18].) In our previous works [19, 20, 21], we attempted to understand charged branes in the bulk background of the RS type domain wall. It turns out that the domain wall bulk background is so restrictive about the possible gravitating objects that non-dilatonic domain wall bulk background in general does not allow charged branes. One of ways to get around this difficulty is to allow the cosmological constant term in the bulk action to have the dilaton factor. Fortunately, it is observed [19] that even the dilatonic domain wall background effectively compactifies gravity, if the dilaton coupling parameter in the cosmological constant term is small enough. The warp factor in the metric of the dilatonic domain wall with small enough dilaton coupling parameter also decreases as one moves away from the wall within the finite allowed coordinate interval around the wall as a power-law, instead of exponentially within the infinite allowed coordinate interval around the wall just like non-dilatonic domain wall of the RS model [3, 4, 5], and becomes zero at the end of the allowed finite coordinate interval. So, one can also use such dilatonic domain walls for tackling the hierarchy problem.

It is realized [20] that charged p-branes, as observed in one lower dimensions, should rather be regarded as charged $(p+1)$-branes in the bulk of domain wall, because charged p-branes in the bulk of domain wall background are not effectively compactified to the charged p-branes in one lower dimensions on the hypersurface of the wall. We studied [21] the dynamics of probes in the background of such charged branes for the purpose of understanding the spacetime properties of charged branes in the bulk background of the domain walls. In this paper, we check whether such charged $(p+1)$-branes in the domain wall bulk effectively describe the corresponding charged p-branes in one lower dimensions or describe different physics in one lower dimensions by obtaining the effective action (in one lower dimensions) for such charged $(p+1)$-branes in the bulk of the domain walls. We find that the effective action has exactly the same form as the action for the p-brane in one lower dimensions that is obtained from the action for the $(p+1)$-brane through the ordinary KK compactification on S^1.

The paper is organized as follows. In section 2, we discuss dilatonic domain wall solution that we studied in our previous works. In sections 3, we obtain the effective action for the charged branes in the bulk of the dilatonic domain wall.

2 Dilatonic Domain Wall Solution

We begin by discussing the D-dimensional extreme dilatonic domain wall solution studied in Ref. [21]. Generally, the total action for the RS type model is the sum of the D-dimensional action $S_{\rm bulk}$ for the fields in the bulk of the domain wall and the $(D-1)$-dimensional action $S_{\rm DW}$ on the domain wall worldvolume:

$$S = S_{\rm bulk} + S_{\rm DW}. \tag{1}$$

The bulk action contains the following action for the dilatonic domain wall solution:

$$S_{\rm bulk} \supset \frac{1}{2\kappa_D^2}\int d^Dx\sqrt{-G}\left[\mathcal{R}_G - \frac{4}{D-2}\partial_M\phi\partial^M\phi + e^{-2a\phi}\Lambda\right], \tag{2}$$

and the $(D-1)$-dimensional action contains the following worldvolume action S_σ for the dilatonic domain wall:

$$S_{\rm DW} \supset S_\sigma = -\sigma_{\rm DW}\int d^{D-1}x\sqrt{-\gamma}e^{-a\phi}. \tag{3}$$

Here, $\sigma_{\rm DW}$ is the tension or the energy density of the domain wall and γ is the determinant of the induced metric $\gamma_{\mu\nu} = \partial_\mu X^M\partial_\nu X^N G_{MN}$ on the domain wall worldvolume, where $M, N = 0, 1, ..., D-1$ and $\mu, \nu = 0, 1, ..., D-2$.

The domain wall solution has the following form:

$$G_{MN}dx^M dx^N = \mathcal{W}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \qquad e^{2\phi} = \mathcal{W}^{\frac{(D-2)^2 a}{4}}, \tag{4}$$

where the warp factor $\mathcal{W}$ is given by

$$\begin{aligned}
\mathcal{W}(y) &= \left(1 + \frac{2(D-1)+(D-2)\Delta}{(D-2)\Delta}Q|y|\right)^{\frac{4}{2(D-1)+(D-2)\Delta}} \\
&= \left(1 + \frac{a^2(D-2)^2}{a^2(D-2)^2-4(D-1)}Q|y|\right)^{\frac{8}{a^2(D-2)^2}}, \\
\Delta &= \frac{(D-2)a^2}{2} - \frac{2(D-1)}{D-2},
\end{aligned} \tag{5}$$

for $a \neq 0$, and

$$\mathcal{W}(y) = \exp\left(-\frac{2Q}{D-1}|y|\right), \tag{6}$$

for $a = 0$. The domain wall solution (4) solves the equations of the motion of the combined actions (2) and (3), provided the bulk cosmological constant Λ and the domain wall tension $\sigma_{\rm DW}$ in the action are related to the parameter Q in the solution as

$$\Lambda = -\frac{2Q^2}{\Delta}, \qquad \sigma_{\rm DW} = \frac{4}{|\Delta|}\frac{Q}{\kappa_D^2}. \tag{7}$$

In this paper, we assume that $Q > 0$, so that the tension $\sigma_{\rm DW}$ of the wall is positive.

One can bring the domain wall metric (4) to the conformally flat form by applying the following coordinate transformation:

$$\begin{aligned}
z &= {\rm sgn}(y)\frac{a^2(D-2)^2-4(D-1)}{a^2(D-2)^2-4}Q^{-1}\left[\left(1 + \frac{a^2(D-2)^2}{a^2(D-2)^2-4(D-1)}Q|y|\right)^{\frac{a^2(D-2)^2-4}{a^2(D-2)^2}} - 1\right] \\
&= {\rm sgn}(y)\frac{\Delta}{\Delta+2}Q^{-1}\left[\left(1 + \frac{2(D-1)+(D-2)\Delta}{(D-2)\Delta}Q|y|\right)^{\frac{(D-2)(\Delta+2)}{2(D-1)+(D-2)\Delta}} - 1\right],
\end{aligned} \tag{8}$$

for $a \neq 0$, and

$$z = {\rm sgn}(y)\frac{D-1}{Q}\left[\exp\left(\frac{Q}{D-1}|y|\right) - 1\right], \tag{9}$$

for $a = 0$. The resulting domain wall metric has the following form:

$$G_{MN}dx^M dx^N = \mathcal{C}\left[\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right], \qquad e^{2\phi} = \mathcal{C}^{\frac{(D-2)^2 a}{4}}, \tag{10}$$

where the conformal factor $\mathcal{C}$ is given by

$$\mathcal{C}(z) = \left(1 + \frac{\Delta+2}{\Delta}Q|z|\right)^{\frac{4}{(D-2)(\Delta+2)}}$$

$$= \left(1 + \frac{(D-2)^2a^2 - 4}{(D-2)^2a^2 - 4(D-1)} Q|z|\right)^{\frac{8}{(D-2)^2a^2-4}} . \tag{11}$$

In particular, the domain wall solution of the RS model [3, 4, 5] corresponds to the $(D, a) = (5, 0)$ case of the above general solution. The parameter Q in the above metric is related to the parameter k in the RS domain wall metric as $Q = 4k$. Note, also the difference in the definition of the cosmological constant from that of Refs. [3, 4, 5], where the gravitational constant does not multiply the cosmological constant term and there is a negative sign. So, in order to relate the cosmological constant Λ in Eq. (7) of the action (1) to the cosmological constant of Refs. [3, 4, 5], one has to multiply Λ in Eq. (7) by $-1/(2\kappa_5^2)$, where $\kappa_5^2 = 1/(4M^3)$ in the notation of Refs. [3, 4, 5].

3 Zero Mode Effective Action

In this section, we study the $(D-1)$-dimensional effective action for the RS type model obtained from the total action $S_{\rm bulk} + S_{\rm DW}$ by integrating over the extra space coordinate. We shall consider only the zero modes of the fields, in part because the harmonic functions for branes in the domain wall bulk under consideration, i.e., those studied in Ref. [21], are independent of the extra space coordinate.

In Ref. [4], it is shown that gravity in one lower dimensions is reproduced by allowing the following form of the small perturbation of the domain wall metric (10):

$$G_{MN}dx^M dx^N = \mathcal{C}\left[(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + dz^2\right], \tag{12}$$

where the metric perturbation $h_{\mu\nu}$ satisfies the transverse traceless gauge condition $h^\mu{}_\mu = 0 = \partial^\mu h_{\mu\nu}$. However, in this paper, we consider the case where there is an additional perturbation, depending on the domain wall worldvolume coordinates x^μ, in the extra space direction, i.e. the y or the z direction. This is motivated by the fact that the extra space component of the metric of charged branes in the bulk of the domain wall generally depends on the worldvolume coordinates of the domain wall. So, we consider the following form of the D-dimensional metric:

$$G_{MN}dx^M dx^N = \mathcal{C}\left[g_{\mu\nu}dx^\mu dx^\nu + h^2 dz^2\right], \tag{13}$$

where $\mathcal{C}$ is given by Eq. (11), and $g_{\mu\nu}$ and h are zero modes, i.e., depend on the $(D-1)$-dimensional coordinates x^μ, only. Note, the term h^2 in the above metric does not have anything to do with the radion [3, 22, 23], which determines the scale of the distance between the visible and the hidden domain walls of the first RS model [3], because we are considering only one domain wall, called the TeV brane, and assuming the extra space dimension to be infinite just like the second RS model [4, 5].

First, we consider the case where the total action is given by the actions for the dilatonic domain wall, only, i.e., the sum of the actions (2) and (3). For the Ansätze for the fields, we use Eq. (13) for the metric G_{MN} and Eq. (10) for the dilaton ϕ, where $\mathcal{C}$ is given by Eq. (11). Then, the total action reduces to the following form:

$$\begin{aligned} S &= \frac{1}{2\kappa_D^2}\int d^Dx\sqrt{-G}\left[\mathcal{R}_G - \frac{4}{D-2}\partial_M\phi\partial^M\phi + e^{-2a\phi}\Lambda\right] - \sigma_{\rm DW}\int d^{D-1}x\sqrt{-\gamma}e^{-a\phi} \\ &= \frac{1}{2\kappa_D^2}\int d^{D-1}xdz\sqrt{-g}\varpi^{\frac{2}{\Delta+2}}\left[h\mathcal{R}_g - 8\frac{D-1}{D-2}\frac{Q}{\Delta}h^{-1}\left\{\delta(z) - \frac{DQ}{4(D-1)}\varpi^{-2}\right\}\right. \\ &\qquad \left. - \frac{2Q^2}{\Delta}\varpi^{-2}h\right] - \frac{4}{|\Delta|}\frac{Q}{\kappa_D^2}\int d^{D-1}x\sqrt{-g} \\ &= \frac{1}{2\kappa_D^2}\int d^{D-1}x\sqrt{-g}\left[-\frac{2\Delta Q^{-1}}{\Delta+4}h\mathcal{R}_g - \frac{4Q}{\Delta}(h^{-1}+h-2)\right], \end{aligned} \qquad (14)$$

where $\varpi \equiv 1 + \frac{\Delta+2}{\Delta}Q|z|$. Since the domain wall is located at $z = 0$, the fields in the domain wall worldvolume action S_σ take the forms $e^{-a\phi} = 1$ and $\gamma_{\mu\nu} = g_{\mu\nu}$ (in the static gauge of the worldvolume action). Note, in the last equality, we integrated over all the possible values of the extra space coordinate z. Namely, $\frac{\Delta}{\Delta+2}Q^{-1} \leq z \leq -\frac{\Delta}{\Delta+2}Q^{-1}$ for the $-2 < \Delta < 0$ case, and $-\infty < z < \infty$ for the $\Delta < -2$ case (Cf. see Eq. (11)). When $\Delta > 0$, integration over all the possible values of z, i.e. $-\infty < z < \infty$, will make the Einstein term in the above action diverge, meaning that gravity is not effectively compactified. So, in the worldvolume action S_σ, we have taken $|\Delta| = -\Delta$.

When the dilatonic domain wall metric (10) does not have a perturbation in the extra space direction, i.e., when the D-dimensional metric is of the form Eq. (13) with $h = 1$, the effective action (14) reduces to the action for the $(D-1)$-dimensional general relativity with the gravitational constant given by

$$\kappa_{D-1}^2 = -\frac{\Delta+4}{2\Delta}Q\kappa_D^2, \qquad (15)$$

as can be seen from the last line of Eq. (14). Note, Δ, defined in Eq. (5), is always greater than -4 for $D > 4$, so $\kappa_{D-1}^2 > 0$ if $\Delta < 0$ and $Q > 0$. Note, it is essential that Δ should be negative in order for the Einstein's gravity in one lower dimensions to be reproduced. Namely, if Δ had been positive, (i) the Einstein term would have diverged, (ii) no cancellation of the extra terms in the action (14) would have occurred because of the contribution from the worldvolume action S_σ with the opposite sign (i.e., $|\Delta| = +\Delta$ for $\Delta > 0$), and (iii) the gravitational constant κ_{D-1}^2 in Eq. (15) would not have been positive with $Q > 0$. This is in accordance with the result of our previous work [19] that gravity cannot be trapped within the domain wall if Δ is positive. So, any D-dimensional gravitating objects in the dilatonic domain wall background with the bulk action given by Eq. (2) and the metric given by Eq. (13) with $h = 1$ effectively describe the corresponding configurations in the general relativity in one lower dimensions, as long as the dilaton coupling parameter a is such that $\Delta < 0$. This confirms that the RS model can be extended to the dilatonic domain wall case.

However, when the dilatonic domain wall metric (10) has a perturbation in the transverse direction, i.e., when h in Eq. (13) is a non-trivial function of the domain wall worldvolume coordinates x^μ, the effective action (14) has an additional undesirable term. Namely, the ordinary KK compactification of the D-dimensional Einstein gravity on S^1 by using the KK metric Ansatz given by Eq. (13) with $\mathcal{C}(z) = 1$ leads to the following $(D-1)$-dimensional action:

$$S_{\rm KK} = \frac{1}{2\kappa^2_{D-1}} \int d^{D-1}x\sqrt{-g}h\mathcal{R}_g, \tag{16}$$

where the scalar h can be identified as a Brans-Dicke (BD) scalar of the BD theory [24, 25] with the BD parameter $\omega = 0$, and the $(D-1)$-dimensional gravitational constant κ^2_{D-1}, in this case, is given in terms of the volume $V(S^1)$ of S^1 as $\kappa^2_{D-1} = \kappa^2_D/V(S^1)$. However, the effective $(D-1)$-dimensional action (14) in the domain wall background has an additional term, which is the potential term for the scalar h. So, a D-dimensional gravitating object with non-trivial h in a domain wall background does not effectively describe the corresponding $(D-1)$-dimensional configuration that would have been obtained through the ordinary KK compactification on S^1.

Recently, some efforts [26, 27, 28] have been made to understand the KK modes of graviton in the domain wall background associated with general perturbations around the non-dilatonic domain wall metric with non-trivial transverse perturbations. It is observed there that the zero mode of transverse metric perturbation has to be zero, meaning that only solutions in the domain wall background with $h = 1$ are allowed in the system with the action given by (2) plus (3). And it is further observed [28] that the RS gauge (i.e., the metric perturbation of the form (12) with $h_{\mu\nu}$ satisfying the transverse traceless gauge condition) is classically stable. In fact, the scalar potential in the effective action (14) has the minimum at $h = 1$, implying that a solution with the metric (13) with $h = 1$ is the classically preferred stable configuration. So, we see that the result of Refs. [26, 27, 28], which is valid only perturbatively, continues to hold non-perturbatively, as well. However, this does not mean that there does not exist solutions to the equations of motion of the action (16) with non-trivial h, but it is just that the domain wall bulk background seems to prefer solutions with $h = 1$. An example of solution with a non-trivial h in the system with the action (16) is the $\omega = 0$ case of the spherically symmetric solution with scalar hair [2] constructed by BD [24, 25]. On the other hand, as we will see in the following, a solution with non-trivial h in the

[2]Note, such a solution is not inconsistent with the Hawking's theorem [29] on black holes in the BD theory that the only stationary black holes in the BD theory are those of Einstein's general relativity, i.e., black hole solutions with the constant BD scalar. The reason is that the BD solution [24, 25] violates one of the conditions of the Hawking's theorem, namely, the weak energy condition.

domain wall background is possible (or rather non-trivial h is required in order for the effective theory in one lower dimensions to have vanishing cosmological constant term or no scalar potential term), if additional fields are added in the bulk action $S_{\rm bulk}$.

It is interesting to note that the domain wall bulk background naturally induces a potential of the KK scalar h (in the effective action (14)), which was previously added to the KK action by hand in Ref. [30] in an attempt to remedy the problem of the KK theory that it corresponds to the BD theory with the BD parameter $\omega = 0$, which is outside of the range of the constraint $\omega > 500$ set by the solar system experiment [31, 32]. Note, the solar system experiment does not constrain the allowed values of the BD parameter ω, if the BD scalar has a potential with the minimum which allows the BD scalar to have a non-zero vacuum expectation value at low temperature. A scalar potential was also added *ad hoc* with some justifications in the extended inflationary model [33], which adopts a metric theory of gravity by introducing a scalar of the BD theory in an attempt to solve the "graceful exit" problem of the old inflation model [34, 35] through slowing down of the inflationary expansion from exponential to power-law in time, in order to make the value of the BD scalar settle down at some large expectation value in the true-vacuum phase, i.e., after the inflationary phase [36, 37]. So, the noncompact compactification through the RS type domain wall appears to be more desirable than the ordinary KK compactification in this respect.

Now, we consider the case when the domain wall bulk spacetime contains a charged $(p+1)$-brane, where one of the longitudinal directions of the brane is along the transverse direction of the domain wall, whose explicit solution is study in our previous work [21]. The total action S for this case is give by the sum of the following bulk action:

$$S_{\rm bulk} = \frac{1}{2\kappa_D^2}\int d^Dx\sqrt{-G}\left[\mathcal{R}_G - \frac{4}{D-2}(\partial\phi)^2 + e^{-2a\phi}\Lambda - \frac{1}{2\cdot(p+3)!}e^{2a_{p+1}\phi}F^2_{p+3}\right], \tag{17}$$

the domain wall worldvolume action:

$$S_{\rm DW} = -\sigma_{\rm DW}\int d^{D-1}x\sqrt{-\gamma}e^{-a\phi}, \tag{18}$$

and the following additional worldvolume action for the charged $(p+1)$-brane:

$$\begin{aligned} S_{p+1} &= -T_{p+1}\int d^{p+2}\xi\left[e^{-a_{p+1}\phi}\sqrt{-\det\,\partial_aX^M\partial_bX^NG_{MN}}\right.\\ &\qquad\left.+\frac{\sqrt{\Delta_{p+1}}}{2}\frac{1}{(p+2)!}\epsilon^{a_1\ldots a_{p+2}}\partial_{a_1}X^{M_1}\ldots\partial_{a_{p+2}}X^{M_{p+2}}A_{M_1\ldots M_{p+2}}\right],\\ \Delta_{p+1} &= \frac{(D-2)a^2_{p+1}}{2} + \frac{2(p+2)(D-p-4)}{(D-2)}. \end{aligned} \tag{19}$$

The consistency of the equations of motion requires that the dilaton coupling parameters a and a_{p+1} satisfy the following constraint:

$$aa_{p+1} = -\frac{4(D-p-4)}{(D-2)^2}. \tag{20}$$

Guided by the explicit solution presented in Ref. [21], we take Eq. (13) as the D-dimensional metric Ansatz and the following as the remaining fields Ansätze:

$$e^{2\phi} = \mathcal{C}^{\frac{(D-2)^2 a}{4}} e^{2\tilde{\phi}}, \qquad A_{p+2} = \mathcal{C}^{\frac{D-2}{2}} \tilde{A}_{p+2}, \tag{21}$$

where the tilded fields depend on the domain wall worldvolume coordinates x^μ, only, and $\mathcal{C}$ is given by Eq. (11). Substituting the above Ansätze for the fields into the total action S and integrating over the extra space coordinate z, we obtain the following effective action:

$$\begin{aligned} S &= \frac{1}{2\kappa_D^2}\int d^{D-1}x dz \sqrt{-g}\varpi^{\frac{2}{\Delta+2}} \left[h\mathcal{R}_g - \frac{4}{D-2}h(\partial\tilde{\phi})^2 - \frac{1}{2\cdot(p+2)!}h^{-1}e^{2a_{p+1}\tilde{\phi}}\tilde{F}_{p+2}^2 \right. \\ &\quad \left. -8\frac{D-1}{D-2}\frac{Q}{\Delta}h^{-1}\left\{\delta(z) - \frac{DQ}{4(D-1)}\varpi^{-2}\right\} - \frac{2Q^2}{\Delta}\varpi^{-2}he^{-2a\tilde{\phi}}\right] \\ &\quad -\frac{4}{|\Delta|}\frac{Q}{\kappa_D^2}\int d^{D-1}x\sqrt{-g}e^{-a\tilde{\phi}} + S_{p+1} \\ &= \frac{1}{2\kappa_{D-1}^2}\int d^{D-1}x\sqrt{-g}\left[h\mathcal{R}_g - \frac{4}{D-2}h(\partial\tilde{\phi})^2 - \frac{1}{2\cdot(p+2)!}h^{-1}e^{2a_{p+1}\tilde{\phi}}\tilde{F}_{p+2}^2\right. \\ &\quad \left. +2\frac{(\Delta+4)Q^2}{\Delta^2}(h^{-1} + he^{-2a\tilde{\phi}} - 2e^{-a\tilde{\phi}})\right] + S_{p+1}, \end{aligned} \tag{22}$$

where $\tilde{F}_{p+2} = d\tilde{A}_{p+1}$ with $(\tilde{A}_{p+1})_{\mu_1\dots\mu_{p+1}} \equiv (\tilde{A}_{p+2})_{\mu_1\dots\mu_{p+1}z}$, κ_{D-1}^2 is given by Eq. (15), and we let $\Delta < 0$ and made use of the constraint (20) in the kinetic term for the form potential. Note, the explicit expressions for h and $e^{\tilde{\phi}}$ are given in terms of the harmonic function H_{p+1} for the D-dimensional $(p+1)$-brane as [21]:

$$h = H_{p+1}^{-\frac{2(D-p-4)}{(D-2)\Delta_{p+1}}}, \qquad e^{\tilde{\phi}} = H_{p+1}^{\frac{(D-2)a_{p+1}}{2\Delta_{p+1}}}. \tag{23}$$

So, the last line of the $(D-1)$-dimensional action in Eq. (22) becomes zero. The $(D-1)$-dimensional effective action in Eq. (22), therefore, becomes of the form of the bulk action for the dilatonic p-brane in $D-1$ dimensions, obtained from the bulk action for the D-dimensional dilatonic $(p+1)$-brane through the ordinary KK compactification on S^1 along one of its longitudinal directions. Next, we show that the effective action for the worldvolume action S_{p+1} for the dilatonic $(p+1)$-brane in the bulk of D-dimensional domain wall has the form of the worldvolume action for the dilatonic p-brane in $D-1$ dimensions. In the static gauge with constant transverse (to the $(p+1)$-brane) target space coordinates, the $(p+1)$-brane worldvolume action (19) takes the following form:

$$S_{p+1} = -T_{p+1}\int d^{p+2}x\left[e^{-a_{p+1}\phi}\sqrt{-\det G_{ab}} + \frac{\sqrt{\Delta_{p+1}}}{2}A_{tx_1\ldots x_p z}\right], \tag{24}$$

where $a,b = t, x_1, ..., x_p, z$. After substituting the Ansätze for the fields in the above and integrating over the extra space coordinate z, we obtain the following effective action:

$$\begin{aligned} S_{p+1} &= -T_{p+1}\int d^{p+1}x dz\, \varpi^{\frac{2}{\Delta+2}}\left[e^{-a_{p+1}\tilde{\phi}}h\sqrt{-\det g_{\tilde{a}\tilde{b}}} + \frac{\sqrt{\Delta_{p+1}}}{2}\tilde{A}_{tx_1\ldots x_p z}\right] \\ &= -T_p\int d^{p+1}x\left[e^{-a_{p+1}\tilde{\phi}}h\sqrt{-\det g_{\tilde{a}\tilde{b}}} + \frac{\sqrt{\Delta_p}}{2}\tilde{A}_{tx_1\ldots x_p}\right], \end{aligned} \tag{25}$$

where $T_p \equiv -\frac{2\Delta}{(\Delta+4)Q}T_{p+1}$ and $\tilde{a},\tilde{b} = t, x_1, ..., x_p$. This is of the form of the worldvolume action for the $(D-1)$-dimensional p-brane obtained from the worldvolume action for the D-dimensional $(p+1)$-brane through the ordinary KK compactification on S^1 along one of its longitudinal directions. Therefore, the effective action for the $(p+1)$-brane in the bulk of D-dimensional dilatonic domain wall has the same form as the action for the p-brane in $D-1$ dimensions, obtained from the action for the D-dimensional $(p+1)$-brane through the ordinary KK compactification on S^1.

This result indicates that in the case where there are additional fields in the bulk action the gravity can be trapped within the domain wall even if the domain wall metric has a non-trivial perturbation along the transverse (to the domain wall) direction. The zero mode of this transverse perturbation is identified as a scalar in one lower dimensions. As we have seen from the dilatonic $(p+1)$-brane solution in the domain wall bulk, such zero mode of transverse metric perturbation conspires with the zero mode of the dilaton perturbation in such a way that the possible scalar potential term or the cosmological term in one lower dimensions is eliminated. Thereby, a configuration in the domain wall bulk effectively describes the corresponding configuration in *an asymptotically flat spacetime* in one lower dimensions [3]. One the other hand, the domain wall bulk spacetime is very restrictive about the possible gravitating configurations. As

[3] In Ref. [21], we argued that the dynamics of a test particle in the background of the charged $(p+1)$-brane in the bulk of the domain wall does not reproduce that of a test particle in the background of the corresponding p-brane in one lower dimensions because of the non-trivial dependence of the transverse (to the wall) component of the metric on the longitudinal coordinates of the wall. This is due to the fact that the simple trick that was used in Refs. [15, 21] to study the test particle dynamics (with non-trivial motion along the extra space direction) is not applicable in such case. If we instead solve the full non-linear coupled geodesic equations $\ddot{x}^\mu + \Gamma^\mu_{\nu\rho}\dot{x}^\nu\dot{x}^\rho = 0$, which is rather a difficult task, it may be possible to reproduce the geodesic motion in one lower dimensions.

pointed out in Refs. [26, 27, 28] and also we have seen in the above, the gravitating configurations with a transverse (to the wall) metric component that depends on the longitudinal coordinates of the wall is not preferred, if there are no additional fields in the bulk action. And only the dilatonic charged brane with the dilaton coupling parameter that satisfies the constraint (20) is allowed in the bulk of a dilatonic domain wall. Because of this restriction, non-dilatonic domain wall bulk background cannot admit charged branes (with asymptotically flat spacetime in one lower dimensions) and the current RS type models (dilatonic and non-dilatonic) cannot admit, for example, the Reissner-Nordstrom black holes in one lower dimensions.

We comment on an alternative realization of charged branes in the RS type models. One may just want to regard charged branes as living within the domain wall. Namely, one may regard the form fields that the charged branes couple to as being contained in the worldvolume action $S_{\rm DW}$ of the domain wall, not in the bulk action $S_{\rm bulk}$. The combined bulk and worldvolume action has the following form:

$$\begin{aligned} S &= \frac{1}{2\kappa_D^2}\int d^{D-1}x dz\sqrt{-G}\left[\mathcal{R}_G - \frac{4}{D-2}\partial_M\phi\partial^M\phi + e^{-2a\phi}\Lambda\right] \\ &\quad + \int d^{D-1}x\sqrt{-\gamma}\left[\mathcal{L} - e^{-a\phi}\sigma_{\rm DW}\right], \end{aligned} \tag{26}$$

where the form fields that charged branes couple to are contained in the Lagrangian $\mathcal{L}$ on the domain wall worldvolume. Then, by using the D-dimensional bulk metric Ansatz given by Eq. (13) with $h = 1$, one can bring the action (26) to the following form of the effective $(D-1)$-dimensional action after integrating over the extra space coordinate z:

$$S = \frac{1}{2\kappa_{D-1}^2}\int d^{D-1}x\sqrt{-g}\left[\mathcal{R}_g - \frac{\Delta+4}{2\Delta Q}\mathcal{L}\right], \tag{27}$$

where the $(D-1)$-dimensional gravitational constant κ_{D-1} is given by Eq. (15). An advantage of this approach is that there is no restriction on the possible gravitating configurations within the domain wall. So, even the non-dilatonic domain wall of the original RS model [3, 4, 5] can admit charged black hole solutions, including the Reissner-Nordstrom solution. However, one of setbacks is that such description is inconsistent with the recent string theory view on charged black holes that enabled microscopic interpretation of the Bekenstein-Hawking entropy [38]. Namely, if charged black holes are to be interpreted as being compactified from intersecting branes in ten or eleven dimensions, then the charged black holes have to be coupled to the fields in the bulk action $S_{\rm bulk}$.

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, "The hierarchy problem and new dimensions at a millimeter," Phys. Lett. **B429** (1998) 263, hep-ph/9803315.

[2] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, "New dimensions at a millimeter to a Fermi and superstrings at a TeV," Phys. Lett. **B436** (1998) 257, hep-ph/9804398.

[3] L. Randall and R. Sundrum, "A large mass hierarchy from a small extra dimension," Phys. Rev. Lett. **83** (1999) 3370, hep-ph/9905221.

[4] L. Randall and R. Sundrum, "An alternative to compactification," Phys. Rev. Lett. **83** (1999) 4690, hep-th/9906064.

[5] J. Lykken and L. Randall, "The shape of gravity," hep-th/9908076.

[6] D.W. Joseph, "Coordinate covariance and the particle spectrum," Phys. Rev. **126** (1962) 319.

[7] K. Akama, "Pregeometry. (Talk)," *In *Nara 1982, Proceedings, Gauge Theory and Gravitation*, 267-271*, hep-th/0001113.

[8] V.A. Rubakov and M.E. Shaposhnikov, "Do we live inside a domain wall?," Phys. Lett. **B125** (1983) 136.

[9] V.A. Rubakov and M.E. Shaposhnikov, "Extra space-time dimensions: towards a solution to the cosmological constant problem," Phys. Lett. **B125** (1983) 139.

[10] M. Visser, "An exotic class of Kaluza-Klein models," Phys. Lett. **B159** (1985) 22, hep-th/9910093.

[11] E.J. Squires, "Dimensional reduction caused by a cosmological constant," Phys. Lett. **B167** (1986) 286.

[12] G.W. Gibbons and D.L. Wiltshire, "Space-Time as a membrane in higher dimensions," Nucl. Phys. **B287** (1987) 717.

[13] C. Wetterich, "Dimensional reduction of fermions in generalized gravity," Nucl. Phys. **B242** (1984) 473.

[14] A. Chamblin and G.W. Gibbons, "Supergravity on the brane," hep-th/9909130.

[15] A. Chamblin, S.W. Hawking and H.S. Reall, "Brane-world black holes," hep-th/9909205.

[16] R. Emparan, G.T. Horowitz and R.C. Myers, "Exact description of black holes on branes," JHEP **0001** (2000) 007, hep-th/9911043.

[17] J. Garriga and M. Sasaki, "Brane-world creation and black holes," hep-th/9912118.

[18] R. Emparan, G.T. Horowitz and R.C. Myers, "Exact description of black holes on branes. II: Comparison with BTZ black holes and black strings," hep-th/9912135.

[19] D. Youm, "Solitons in brane worlds," hep-th/9911218.

[20] D. Youm, "Probing solitons in brane worlds," hep-th/9912175.

[21] D. Youm, "Solitons in brane worlds II," hep-th/0001018.

[22] W.D. Goldberger and M.B. Wise, "Modulus stabilization with bulk fields," Phys. Rev. Lett. **83** (1999) 4922, hep-ph/9907447.

[23] W.D. Goldberger and M.B. Wise, "Phenomenology of a stabilized modulus," hep-ph/9911457.

[24] C. Brans and R.H. Dicke, "Mach's principle and a relativistic theory of gravitation," Phys. Rev. **124** (1961) 925.

[25] R.H. Dicke, "Mach's principle and invariance under transformation of units," Phys. Rev. **125** (1962) 2163.

[26] M.G. Ivanov and I.V. Volovich, "Metric fluctuations in brane worlds," hep-th/9912242.

[27] Y.S. Myung and G. Kang, "Comment on 'Metric fluctuations in brane worlds'," hep-th/0001003.

[28] Y.S. Myung, G. Kang and H.W. Lee, "Randall-Sundrum gauge in the brane world," hep-th/0001107.

[29] S.W. Hawking, "Black holes in the Brans-Dicke theory of gravitation," Comm. Math. Phys. **25** (1972) 167.

[30] H.H. Soleng, "Trapping the Kaluza-Klein scalar," Gen. Rel. Grav. **23** (1991) 313.

[31] R.D. Reasenberg *et al.*, "Viking relativity experiment: Verification of signal retardation by solar gravity," Astrophys. J. **234** (1979) L219.

[32] C.M. Will, "Theory and experiments in gravitational physics," *Cambridge, UK: Univ. Pr. (1981) 342p.*

[33] D. La and P.J. Steinhardt, "Extended inflationary cosmology," Phys. Rev. Lett. **62** (1989) 376.

[34] D. Kazanas, "Dynamics of the universe and spontaneous symmetry breaking," Astrophys. J. **241** (1980) L59.

[35] A.H. Guth, "The inflationary universe: A possible solution to the horizon and flatness problems," Phys. Rev. **D23** (1981) 347.

[36] E.J. Weinberg, "Some problems with extended inflation," Phys. Rev. **D40** (1989) 3950.

[37] D. La, P.J. Steinhardt and E.W. Bertschinger, "Prescription for successful extended inflation," Phys. Lett. **B231** (1989) 231.

[38] A. Strominger and C. Vafa, "Microscopic origin of the Bekenstein-Hawking entropy," Phys. Lett. **B379** (1996) 99, hep-th/9601029.