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GRAVITOMAGNETIC EFFECTS IN CONDUCTOR IN APPLIED MAGNETIC FIELD

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Abstract

The electromagnetic measurements of general relativistic gravitomagnetic effects which can be performed within a conductor embedded in the space-time of slow rotating gravitational object in the presence of magnetic field are proposed.

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The general relativistic electromagnetic effects arising from the gravitomagnetic field in noncurrent carrying (super-)conductors with no applied magnetic field present have been discussed by several authors (see, for review, [1]). However, the general-relativistic effects can be amplified by the interplay between gravitomagnetic field and either electric current or magnetic field and in this respect, we discuss here a new test of gravitomagnetic field of Earth by using conductors embedded in an external magnetic field while in the previous paper [2] we have already shown that the interaction between the gravitomagnetic field and electric current can lead to the galvanogravitomagnetic effect.

Space-time outside a spherically symmetric mass M with angular momentum a is described by the Kerr metric. This differs from the Schwarzschild solution for a static body by having non-diagonal terms, which imply a local inertial frame to be rotating with respect to the distant stars at infinity with the Lense-Thirring angular velocity [3] $\omega(r) \equiv 2aM/r^3$. Then the metric of the reference frame corotating with the slowly rotating gravitational object with mass M (in the linear angular momentum a approximation) is

$$ds^2 = -N^2c^2dt^2 + N^{-2}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2 + 2\bar{\omega}r^2\sin^2\theta dtd\varphi, \tag{1}$$

where $N \equiv (1-2M/r)^{1/2}$, $\bar{\omega} \equiv \Omega - \omega(r)$, Ω is angular velocity of rotation of gravitational object with respect to the distant stars.

Suppose that the material relations between inductions and fields have linear character

$$\begin{aligned} H_{\alpha\beta} &= \frac{1}{\mu}F_{\alpha\beta} + \frac{1-\epsilon\mu}{\mu}(u_\alpha F_{\sigma\beta} - u_\beta F_{\sigma\alpha})u^\sigma, \\ F_{\alpha\beta} &= \mu H_{\alpha\beta} + \frac{\epsilon\mu - 1}{\epsilon}(u_\alpha H_{\sigma\beta} - u_\beta H_{\sigma\alpha})u^\sigma \end{aligned} \tag{2}$$

and the general relativistic Ohm's law for conduction current j^α is

$$\frac{j_\alpha}{\lambda} = F_{\alpha\beta}u^\beta - R_H(F_{\alpha\sigma} + F_{\rho\sigma}u^\rho u_\alpha)j^\sigma + R_{gg}j^\beta A_{\alpha\beta} + \Lambda^{-1/2} \overset{\perp}{\nabla}_\alpha (\Lambda^{1/2}\mu_e), \tag{3}$$

here $F_{\alpha\beta}$ and $H_{\alpha\beta}$ are the tensors of electromagnetic field and induction, respectively, ϵ and μ are the parameters for the conductor, μ_e is the electrochemical potential per unit charge, $R_{gg} = 2mc/ne^2$ is the parameter for the conductor called as galvano-gravitomagnetic one, n is the concentration of the conduction electrons, obviously λ is the electrical conductivity, R_H is the Hall constant, u_α is the four-velocity of the conductor, $w_\alpha = u_{\alpha;\beta}u^\beta$ is the absolute acceleration, $A_{\beta\alpha} = u_{\alpha,\beta} + u_{[\beta}w_{\alpha]}$ is the relativistic rate of rotation of the conductor, $\overset{\perp}{\nabla}_\alpha$ denotes the spatial part of covariant derivative and [ ] represents antisymmetrization. The gravitational field is assumed to be stationary that is space-time metric $g_{\alpha\beta}$ admits a timelike Killing vector $\xi^\alpha_{(t)}$ that is $L_{\xi_t}g_{\alpha\beta} = 0$ (L_{ξ_t} denotes the Lie derivative with respect to $\xi^\alpha_{(t)}$, $\Lambda = -\xi^\alpha_{(t)}\xi_{(t)\alpha}$).

Then the general relativistic expression for the charge distribution inside conductor [4]

$$\rho_0 = \frac{\epsilon\mu R_H}{c}\jmath^2 + \frac{1}{4\pi}\{(\frac{\epsilon}{\lambda}\jmath^\alpha)_{;\alpha} + [\epsilon^2\mu R_H(\frac{1}{\lambda}\jmath^2 + \Lambda^{-1/2}\jmath^\nu \overset{\perp}{\nabla}_\nu (\Lambda^{1/2}\mu_e))u^\alpha]_{;\alpha}$$
$$-\epsilon R_{gg}A_{\alpha\beta}w^\alpha\jmath^\beta + g^{\alpha\beta}(\epsilon R_{gg}\jmath^\nu A_{\alpha\nu})_{;\beta} - \frac{\epsilon}{\lambda}w^\alpha\jmath_\alpha - \epsilon w^\alpha\Lambda^{-1/2}\overset{\perp}{\nabla}_\alpha (\Lambda^{1/2}\mu_e)$$
$$+g^{\alpha\beta}(\epsilon\Lambda^{-1/2}\overset{\perp}{\nabla}_\alpha (\Lambda^{1/2}\mu_e))_{;\beta} + H^{\alpha\beta}[A_{\beta\alpha} + \epsilon\mu R_H w_\alpha \jmath_\beta + (\epsilon\mu R_H \jmath_\alpha)_{,\beta}]\} \qquad (4)$$

can be derived from the general relativisic Maxwell equations

$$e^{\alpha\beta\mu\nu}F_{\beta\mu,\nu} = 0, \quad H^{\alpha\beta}{}_{;\beta} = \frac{4\pi}{c}J^\alpha, \quad J^\alpha \equiv c\rho_0 u^\alpha + \jmath^\alpha. \qquad (5)$$

by using material relationships (2) and (3).

The charge density ρ_0 inside a conductor which has no conduction current $\jmath = 0$ but embedded in an external magnetic field $\mathbf{B}$ is

$$\rho_0 = \frac{1}{4\pi}\{\epsilon A w^2 - (\epsilon A w^\alpha)_{;\alpha} + F^{\alpha\beta}A_{\beta\alpha}\} \qquad (6)$$

and has two contributions: the first one is due to the absolute acceleration w_α and second one is due to the relativistic rate of rotation of the conductor $A_{\beta\alpha}$ and can be adjusted and amplified by the magnetic field penetrating inside the conductor. Here $F_{\alpha\beta} = 2\tau_{[\alpha}E_{\beta]} + \eta_{\alpha\beta\mu\nu}\tau^\mu B^\nu$ is the electromagnetic field tensor, A is the parameter for the conductor, τ^α is four-velocity of observer, E^α and B^α are the electric and magnetic fields measured by observer.

In our approximation the charge density, inside a conductor at rest in the orbiting station (1), is

$$\rho_0 = -\frac{1}{2\pi}\left\{F^{23}A_{23} + F^{13}A_{13}\right\}. \qquad (7)$$

We do not consider the charge redistribution arising from the absolute acceleration of the conductor since it does not depend on electromagnetic field characteristics.

If the electromagnetic field tensor components are

$$F^{31} = \frac{NB^\theta}{r\sin\theta}, \qquad F^{23} = \frac{B^r}{r^2\sin\theta} \qquad (8)$$

and the nonvanishing components of the relativistic rate of rotation have form

$$A_{13} = \frac{\Omega r + \omega r/2}{cN}\sin^2\theta, \qquad A_{23} = \frac{\bar{\omega}r^2}{cN}\sin\theta\cos\theta, \qquad (9)$$

then in our approximation, the space charge density inside the conductor at rest in the frame of reference (1) is

$$\rho_0 = \frac{\Omega}{2\pi c}\left[B^\theta\sin\theta - \frac{B^r\cos\theta}{N}\right] + \frac{\omega}{4\pi c}\left[\frac{2B^r\cos\theta}{N} + B^\theta\sin\theta\right], \qquad (10)$$

where the magnetic field components are measured by zero angular momentum observers with four-velocity $\tau_\alpha \equiv \{-N, 0, 0, 0\}$.

The first term in the right hand side of equation (10) results from angular velocity Ω and last one is due to the gravitomagnetic field of the Earth and has pure general relativistic nature.

On Earth, the angular velocity of the conductor is given by [1] $\mathbf{\Omega}_{cond} = \mathbf{\Omega} - \mathbf{\Omega}_{Th} - \mathbf{\Omega}_S - \omega$, where $\mathbf{\Omega}_{Th}$ and $\mathbf{\Omega}_S$ are the contributions of the Thomas precession arising from non-gravitational forces and of the de Sitter or geodetic precession. In order to measure ω one should measure $\mathbf{\Omega}_{cond}$ and then substract from it the independently measured value of $\mathbf{\Omega}$ with Very Long Baseline Intererometry [5] and the contributions due to the Thomas and de Sitter precession.

In contrast to (10), for a superconductor embedded in the gravitational field (1) the space charge density $\rho_0(sc) = 0$, that is according to the solutions of the general-relativistic Maxwell equations and London equations, the magnetic field penetrating superconductor is proportional to $\bar{\omega}$ and consequently the charge density is at least of order of $\bar{\omega}^2$. Therefore, if the temperature T is increased then in the point of the phase transition $T = T_c$ the applied magnetic field penetrates inside the sample and induces a nonvanishing charge density with the corresponding flow of charges.

For the Earth with mass $M = 0.44cm$ and radius $R \approx 6.37 \times 10^8 cm$, $\omega(r) = \frac{4M}{5R}\Omega \approx 0.6 \times 10^{-9}\Omega$ near the surface. If the value of applied magnetic field around conductor is $10^3 G$ and the relaxation time $t_{rel} = 10^{-8}s$ then one can find a typical value of charge exchange current arising from gravitomagnetic Lense-Thirring frequency is of order $10^{-14}A$ which is within capacity of modern technical measurements. However, in the present case, there are serious problems arising from environment and the design of proposed experiment is under consideration.

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