Available at: http://www.ictp.trieste.it/~pub_off

IC/99/155

United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

ON THE ESSENTIAL SPECTRUM OF UNBOUNDED NONSELF-ADJOINT FRIEDRICHS OPERATOR

M.I. Muminov[1]
Uzbekistan Academy of Sciences (Samarkand Division),
Temur Malik Street no. 3, 703000 Samarkand, Uzbekistan
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

Abstract

In this paper, we consider the unbounded generalized Friedrichs operator H, i.e. the operator of multiplication by the rational function u with the perturbation of integral operator with kernel K. We prove that if the kernel K satisfies some analyticity condition, then the essential spectrum of H coincides with the spectrum of the multiplication operator.

MIRAMARE-TRIESTE
October 1999

[1]E-mail: info@samuni.silk.org

1 Introduction

In $L_2(R)$ we consider the non-self-adjoint unbounded operator H defined by the formula

$$Hf(x) = u(x)f(x) + \int_R K(x,y)f(y)dy, f \in D_u$$

i.e. $H = H_0 + V$, where H_0 is the operator of multiplication by the rational function u and V is integral operator with analytical kernel $K(x,y)$ in $W_\alpha^2 = W_\alpha \times W_\alpha$ satisfying the following condition

$$|K(s+iy, t+iy)| \leq \frac{M}{|s|^\gamma + |t|^\gamma + 1}, \gamma > 1, \tag{K}$$

for all $y \in (-\alpha, \alpha)$, where $W_\alpha = \{z \in C : |Imz| < \alpha\}$, α is a fixed number, C the complex plane and $D_u = \{f \in L_2(R) : \int_R |u(x)f(x)|^2 dx < \infty\}$.

The bounded generalized self-adjoint Friedrichs model is considered for the case of when u and K are analytic functions in [1]. In Lakaev S. N. [1], it was proved that the absolutely continuous spectrum of the operator H coincides with the spectrum of H_0. Moreover, the singular spectrum of H is a finite set. In [2] the structure of essential spectrum for non-self-adjoint bounded Friedrichs operator is described.

First, we remark that $D_u = D(H^*)$, where $D(H^*)-$ is a domain of the adjoint operator H^*. Thus, the operator H is a closed operator (see theorem VIII.1 in [3]). Let $\sigma(H)$ is a spectrum of H.

Definition 1. A point $\lambda \in \sigma(H)$ is called discrete if λ is isolated and the operator

$$P_\lambda = \frac{1}{2\pi i} \oint_{|\mu - \lambda| = \rho} (H - \mu)^{-1} d\mu$$

is finite dimensional, where $(H - \mu)^{-1}$ is a resolvent of the operator H and the number $\rho > 0$ such that $\{\mu : |\mu - \lambda| \leq \rho\} = \{\lambda\}$. We denote by $\sigma_{disc}(H)$ a discrete spectrum of the operator H.

Definition 2. The essential spectrum of operator H is the set

$$\sigma_{ess}(H) = \sigma(H) \setminus \sigma_{disc}(H).$$

Denote by Γ the range of the function u. Let Γ be a set satisfying the following condition

$$C \setminus \Gamma = D_1 \cup ... \cup D_s,$$

where D_i is an unbounded connected open set in C for $i = \overline{1, s}$. We observe that $\Gamma = \sigma(H_0)$, where $\sigma(H_0)$ is the spectrum of the operator H_0.

Theorem. The essential spectrum of operator H concides with the spectrum of the operator H_0, i.e.

$$\sigma_{ess}(H) = \sigma(H_0).$$

2 Auxiliary lemma and the proof of the main result

Let $\Delta(z)$ be the Fredholms determinant of the operator $I+V(H_0-z)^{-1}$, where $z\in C\setminus\sigma(H_0)$, I is a unit operator, $(H_0-z)^{-1}$ is a resolvent of the operator H_0. The function $\Delta(z)$ is represented by the formula

$$\Delta_\mu(z)=1+\sum_{n=1}^{\infty}\frac{d_n(z)}{n!}, \tag{1}$$

$$d_n(z)=\int_R\cdots\int_R D_n(x_1,...,x_n;z)dx_1\dots dx_n, \tag{2}$$

$$D_n(x_1,...,x_n;z)=\begin{vmatrix} K(x_1,x_1) & K(x_1,x_2) & \cdots & K(x_1,x_n)\\ K(x_2,x_1) & K(x_2,x_2) & \cdots & K(x_2,x_n)\\ \vdots & \vdots & \cdots & \vdots\\ K(x_n,x_1) & K(x_n,x_2) & \cdots & K(x_n,x_n)\end{vmatrix}\times$$
$$\times\frac{1}{u(x_1)-z}\cdots\frac{1}{u(x_n)-z}.$$

Proposition 1. The series (1) converges for all $z\in C\setminus\sigma(H_0)$.

Proof. We note that the function $D_n(x_1,...,x_n;z)$ is integrable, due to condition (K). Let N be a positive number. The function $D_n(x_1,...,x_n;z)$ is symmetric. Therefore, the integral (2) is represented by the following expression:

$$d_n(z)=\int_R\cdots\int_R K_n(x_1,...,x_n)\frac{dx_1}{u(x_1)-z}\cdots\frac{dx_n}{u(x_n)-z}=$$
$$=\int_{|x_1|>N}\cdots\int_{|x_n|>N}K_n(x_1,...,x_n)\frac{dx_1}{u(x_1)-z}\cdots\frac{dx_n}{u(x_n)-z}+$$
$$+\dots+C_n^k\int_{|x_1|<N}\cdots\int_{|x_k|<N}\int_{|x_{n-(k+1)}|>N}\cdots\int_{|x_n|>N}K_n(x_1,...,x_n)\times$$
$$\times\frac{dx_1}{u(x_1)-z}\cdots\frac{dx_n}{u(x_n)-z}+$$
$$+\int_{|x_1|<N}\cdots\int_{|x_n|<N}K_n(x_1,...,x_n)\frac{dx_1}{u(x_1)-z}\cdots\frac{dx_n}{u(x_n)-z}$$

where

$$K_n(x_1,...,x_n)=\begin{vmatrix} K(x_1,x_1) & K(x_1,x_2) & \cdots & K(x_1,x_n)\\ K(x_2,x_1) & K(x_2,x_2) & \cdots & K(x_2,x_n)\\ \vdots & \vdots & \cdots & \vdots\\ K(x_n,x_1) & K(x_n,x_2) & \cdots & K(x_n,x_n)\end{vmatrix}.$$

Consequently, we obtain:

$$|d_n(z)|\le\frac{1}{(m_z)^n}\int_{|x_1|>N}\cdots\int_{|x_n|>N}|K_n(x_1,...,x_n)|dx_1\dots dx_n+\dots+$$
$$+C_n^k\frac{1}{(m_z)^n}\int_{|x_1|<N}\cdots\int_{|x_k|<N}\int_{|x_{n-(k+1)}|>N}\cdots\int_{|x_n|>N}|K_n(x_1,...,x_n)|dx_1\dots dx_n+$$
$$+\frac{1}{(m_z)^n}\int_{|x_1|<N}\cdots\int_{|x_n|<N}|K_n(x_1,...,x_n)|dx_1\dots dx_n$$

where $m_z = \inf_{x\in R} |u(x) - z| > 0$ for $z \in C \setminus \sigma(H_0)$.

We can choose a number N such that be following inequlity

$$\int_{|x_1|>N} \cdots \int_{|x_n|>N} |K_n(x_1, ..., x_n)| dx_1 \dots dx_n \leq$$

$$\leq \int_{|x_1|<N} \cdots \int_{|x_k|<N} \int_{|x_{n-(k+1)}|>N} \cdots \int_{|x_n|>N} |K_n(x_1, ..., x_n)| dx_1 \dots dx_n \leq$$

$$\leq \int_{|x_1|<N} \cdots \int_{|x_n|<N} |K_n(x_1, ..., x_n)| dx_1 \dots dx_n,$$

holds.

Then we have

$$|d_n(z)| \leq \frac{2^n}{(m_z)^n} \int_{|x_1|<N} \cdots \int_{|x_n|<N} |K_n(x_1, ..., x_n)| dx_1 \dots dx_n.$$

Thus, by using the condition (K) and applying Hadamard's theorem we obtain the following inequality

$$|d_n(z)| \leq \frac{2^n}{(m_z)^n} M^n (2N)^n \sqrt{n^n}.$$

This proves proposition 1.

Lemma 1. Let i be a fixed positive integer. There exists an unbounded subset $D_i' \subset D_i$ such that $\Delta(z) \to 1$ as $z \to \infty$ and $z \in D_i'$.

Proof. We denote by $a_1, ..., a_k$ the real poles of rational function u, $m_z = \inf_{x\in R} |u(x) - z|$.

a) Let $z \in D_i$ and $m_z \to \infty$ as $z \to \infty$. Then we have

$$|\sum_{n=1}^{\infty} \frac{d_n(z)}{n!}| \leq \sum_{n=1}^{\infty} \frac{1}{n!} \frac{1}{(m_z)^n} \int_R \cdots \int_R |K_n(x_1, ..., x_n)| dx_1 \dots dx_n =$$

$$= \frac{1}{m_z} \sum_{n=1}^{\infty} \frac{1}{(m_z)^{n-1}} \frac{1}{n!} \int_R \cdots \int_R |K_n(x_1, ..., x_n)| dx_1 \dots dx_n. \quad (3)$$

By using (3) we obtain $\lim_{m_z\to\infty} \Delta(z) = 1$ in $z \in D_i$, since

$$\sum_{n=0}^{\infty} \frac{1}{n!} \int_R \cdots \int_R |K_n(x_1, ..., x_n)| dx_1 \dots dx_n < \infty.$$

b) Now let $m_z \to c = const$ as $z \to \infty$ for $z \in D_i$. Then there exists an integer N, a positive number $\delta < \alpha$ and an unbounded connected set $D_i' = D_i'(N, \delta) \subset D_i$ such that

$$u^{-1}(D_i') \subset \Omega = \cup_{i=1}^{k} \{z : |z - a_i| < \delta, Imz > 0\} \cup \{z = x + iy : 0 < y < \delta, |x| > N\},$$

where $N > |a_i|$.

Let $z \in D_i'$ and $\psi_1(z), ..., \psi_l(z) \in \Omega$ be solutions of the equation $u(x) - z = 0$ and $\Gamma_{N,\delta}$ a contour defined by

$$\Gamma_{N,\delta} = \cup_{i=1} k \{z : |z - a_i| = \delta, Imz \geq 0\} \cup \{z = -N + iy : 0 \leq y\delta\} \cup$$

$$\cup\{z = N + iy : 0 \le y \le \delta, |x| \ge N\} \cup \{z = x + i\delta : |x| \ge N\}.$$

Let K(t,s) be an analytic continuation in W_α^2 of the function $K(x, y), x, y \in R$.

The function $d_1(z)$ is represented as

$$d_1(z) = \int_R \frac{K(x,x)dx}{u(x) - z} = \int_{R\setminus\Gamma_{N,\delta}} \frac{K(t,t)dt}{u(t) - z} + \int_{\Gamma_{N,\delta}} \frac{K(t,t)dx}{u(t) - z} =$$

$$= 2\pi i \sum_{j=1}^{l} \frac{K(\psi_j(z), \psi_j(z))}{u'(\psi_j(z))} + \int_{\Gamma_{N,\delta}} \frac{K(x,x)dx}{u(x) - z}.$$

Since $|u(t) - z| \ge \rho > 0$, for all $z \in D_i'$, there exists a number $N > 0$ such that the following inequality

$$\int_{\Gamma_{N,\delta}} \dots \int_{\Gamma_{N,\delta}} |D_n(t_1, ..., t_n; z)| dt_1 \dots dt_n \le$$

$$\le 2 \int_{R_{N,\delta}} \int_{\Gamma_{N,\delta}} \dots \int_{\Gamma_{N,\delta}} |D_n(t_1, ..., t_n; z)| dt_1 \dots dt_n \le$$

$$\le 2^n \int_{R_{N,\delta}} \dots \int_{R_{N,\delta}} |D_n(t_1, ..., t_n)| dt_1 \dots dt_n$$

holds, where

$$R_{N,\delta} = (-N, a_1 - \delta) \cup (a_1 + \delta, a_2 - \delta) \cup \dots \cup (a_{l-1} + \delta, a_l - \delta) \cup (a_l + \delta, N).$$

We denote by K_z a number defined by

$$K_z = max\{\sup |\frac{K(\psi_j(z), \psi_j(z))}{u'(\psi_j(z))}|, \sup_{t\in\Gamma_{N,\delta}} |\frac{K(t, \psi_j(z))}{u'(\psi_j(z))}|, \sup_{t\in\Gamma_{N,\delta}} |\frac{K(t, \psi_j(z))}{u'(\psi_j(z))}|,$$

$$\sup_{t\in\Gamma_{N,\delta}} |\frac{K(t, \psi_j(z))}{u(t) - z}|, \sup_{t\in\Gamma_{N,\delta}} |\frac{K(\psi_j(z)), t}{u(t) - z}|, \sup_{s,t\in R_{N,\delta}} |\frac{K(t,s)}{u(t) - z}|\}. \quad (4)$$

Hence, we get

$$|d_1(z)| \le 2\pi l K_z + 2 \int_{R_{N,\delta}} \frac{|K(t,t)|dt}{|u(t) - z|} \le 2\pi l K_z + 2\rho_{N,\delta} K_z.$$

where $\rho_{N,\delta}$ is a length of the contour $R_{N,\delta}$.

Proposition 2. $\lim_{z\to\infty} K_z = 0$ for $z \in D_i'$.

Proof. Let $u'(x) = const$. Then

$$\lim_{z\to\infty} \psi_j(z) = \infty, \text{for all} j \le l.$$

If $u'(x) \ne const$. Then

$$\lim_{z\to\infty} \psi_j(z) = \infty, j \le l.$$

Thus, we have

$$\sup \lim_{z\to\infty} |\frac{K(\psi_j(z), \psi_j(z))}{u'(\psi_j(z))}| = 0.$$

Note that the contour $R_{N,\delta}$ is bounded. So, we obtain

$$\lim_{z\to\infty} |u(t)-z| = \infty, \quad t \in R_{N,\delta}.$$

Consequently,

$$\lim_{z\to\infty} | \sup_{s,t\in\Gamma_{N,\delta}} \frac{K(t,s)}{u(t)-z}| = 0.$$

The relation $\lim_{z\to\infty} K_z = 0$ is proved analogously for all cases.

Now, we consider

$$d_n(z) = \int_R \cdots \int_R \begin{vmatrix} K(x_1,x_1) & K(x_1,x_2) & \cdots & K(x_1,x_n) \\ K(x_2,x_1) & K(x_2,x_2) & \cdots & K(x_2,x_n) \\ \vdots & \vdots & \cdots & \vdots \\ K(x_n,x_1) & K(x_n,x_2) & \cdots & K(x_n,x_n) \end{vmatrix} \times$$

$$\times \frac{dx_1}{u(x_1)-z} \cdots \frac{dx_n}{u(x_n)-z} =$$

$$\int_{R\setminus\Gamma_{N,\delta}} \cdots \int_{R\setminus\Gamma_{N,\delta}} \begin{vmatrix} K(t_1,t_1) & K(t_1,t_2) & \cdots & K(t_1,t_n) \\ K(t_2,t_1) & K(t_2,t_2) & \cdots & K(t_2,t_n) \\ \vdots & \vdots & \cdots & \vdots \\ K(t_n,t_1) & K(t_n,t_2) & \cdots & K(t_n,t_n) \end{vmatrix} \times$$

$$\times \frac{dt_1}{u(t_1)-z} \cdots \frac{dt_n}{u(t_n)-z} + \ldots +$$

$$+ C_n^k \int_{R\setminus\Gamma_{N,\delta}} \cdots \int_{R\setminus\Gamma_{N,\delta}} \int_{\Gamma_{N,\delta}} dt_{n-k-1} \cdots \int_{\Gamma_{N,\delta}} dt_n \times$$

$$\times \begin{vmatrix} K(t_1,t_1) & K(t_1,t_2) & \cdots & K(t_1,t_n) \\ K(t_2,t_1) & K(t_2,t_2) & \cdots & K(t_2,t_n) \\ \vdots & \vdots & \cdots & \vdots \\ K(t_n,t_1) & K(t_n,t_2) & \cdots & K(t_n,t_n) \end{vmatrix} \times$$

$$\times \frac{1}{u(t_1)-z} \cdots \frac{1}{u(t_{n-k}-z} \frac{dt_{n-k-1}}{u(t_{n-k-1})-z} \cdots \frac{dt_n}{u(t_n)-z} +$$

$$+ \int_{\Gamma_{N,\delta}} \cdots \int_{\Gamma_{N,\delta}} \begin{vmatrix} K(t_1,t_1) & K(t_1,t_2) & \cdots & K(t_1,t_n) \\ K(t_2,t_1) & K(t_2,t_2) & \cdots & K(t_2,t_n) \\ \vdots & \vdots & \cdots & \vdots \\ K(t_n,t_1) & K(t_n,t_2) & \cdots & K(t_n,t_n) \end{vmatrix} \times$$

$$\times \frac{dt_1}{u(t_1)-z} \cdots \frac{dt_n}{u(t_n)-z} =$$

$$= (2\pi i)^n \sum_{j_1=1}^{l} \cdots \sum_{j_n=1}^{l} \begin{vmatrix} K(\psi_{j_1},\psi_{j_1}) & K(\psi_{j_1},\psi_{j_2}) & \cdots & K(\psi_{j_1},\psi_{j_n}) \\ K(\psi_{j_2},\psi_{j_1}) & K(\psi_{j_2},\psi_{j_2}) & \cdots & K(\psi_{j_2},\psi_{j_n}) \\ \vdots & \vdots & \cdots & \vdots \\ K(\psi_{j_n},\psi_{j_1}) & K(\psi_{j_n},\psi_{j_2}) & \cdots & K(\psi_{j_n},\psi_{j_n}) \end{vmatrix} \times$$

$$\times \frac{1}{u'(\psi_{j_1}(z))} \cdots \frac{1}{u'(\psi_{j_n}(z))} +$$

$$+(2\pi i)^{n-1}\sum_{j_1=1}^{l}\cdots\sum_{j_{n-1}=1}^{l}\frac{1}{u'(\psi_{j_1}(z))}\cdots\frac{1}{u'(\psi_{j_{n-1}}(z))}\int_{\Gamma_{N,\delta}}\times$$

$$\times\begin{vmatrix} K(\psi_{j_1},\psi_{j_1}) & K(\psi_{j_1},\psi_{j_2}) & \cdots & K(\psi_{j_1},x_n) \\ K(\psi_{j_2},\psi_{j_1}) & K(\psi_{j_2},\psi_{j_2}) & \cdots & K(\psi_{j_2},x_n) \\ \vdots & \vdots & \cdots & \vdots \\ K(x_n,\psi_{j_1}) & K(x_n,\psi_{j_2}) & \cdots & K(x_n,x_n) \end{vmatrix}\frac{dx_n}{u(x_n)-z}+$$

$$+\ldots+C_n^k(2\pi i)^{n-k}\sum_{j_1=1}^{l}\cdots\sum_{j_{n-k}=1}^{l}\frac{1}{u'(\psi_{j_1}(z))}\cdots\frac{1}{u'(\psi_{j_{n-k}}(z))}\int_{\Gamma_{N,\delta}}\cdots\int_{\Gamma_{N,\delta}}\times$$

$$\times\begin{vmatrix} K(\psi_{j_1},\psi_{j_1}) & \cdots & K(\psi_{j_1},\psi_{j_{n-k}}) & K(\psi_{j_1},t_{n-k-1}) & \cdots & K(\psi_{j_1},t_n) \\ K(\psi_{j_2},\psi_{j_1}) & \cdots & K(\psi_{j_2},\psi_{j_{n-k}}) & K(\psi_{j_2},t_{n-k-1}) & \cdots & K(\psi_{j_2},t_n) \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ K(t_n,\psi_{j_1}) & \cdots & K(t_n,\psi_{j_{n-k}} & K(t_n,t_{n-k-1}) & \cdots & K(t_n,t_n) \end{vmatrix}\times$$

$$\frac{dt_{n-k-1}}{u(t_{n-k-1})-z}\frac{dt_n}{u(t_n)-z}+\int_{\Gamma_{N,\delta}}\cdots\int_{\Gamma_{N,\delta}}\begin{vmatrix} K(t_1,t_1) & K(t_1,t_2) & \cdots & K(t_1,t_n) \\ K(t_2,t_1) & K(t_2,t_2) & \cdots & K(t_2,t_n) \\ \vdots & \vdots & \cdots & \vdots \\ K(t_n,t_1) & K(t_n,t_2) & \cdots & K(t_n,t_n) \end{vmatrix}\times$$

$$\times\frac{dt_1}{u(t_1)-z}\cdots\frac{dt_n}{u(t_n)-z}.$$

Arguing as above (as for the case n=1) by using (4) and applying Hadamard's theorem we get the following inequality

$$|d_n(z)|\le l^nK_z^n\sqrt{n^n}+nl^{n-1}2\rho_{N'\delta}K_z^n\sqrt{n^n}++C_n^kl^{n-k}(2\rho_{N,\delta})^kK_z^n\sqrt{n^n}+\ldots+$$

$$+(2\rho_{N,\delta})^nK_z^n\sqrt{n^n}<2^n(2\rho_{N,\delta})^nK_z^n\sqrt{n^n},$$

for all $z\in D_i'$ and $n=1,2,\ldots$. As consequence we get

$$|\Delta_\mu(z)-1|\le\sum_{n=1}^{\infty}\frac{|d_n(z)|}{n!}\le\sum_{n=1}^{\infty}\frac{2^n(2\rho_{N,\delta})^nK_z^n\sqrt{n^n}}{n!}=K_z\sum_{n=1}^{\infty}\frac{2^n(2\rho_{N,\delta})^nK_z^{n-1}\sqrt{n^n}}{n!},$$

for all $z\in D_i'=D_i'(N,\delta)$.

Now, by proposition 2, we have the proof of lemma 1.

Proof of theorem. The operator $H-z$ is represented as

$$H-z=(I+V(H_0-z)^{-1})(H_0-z)$$

for $z\in C\setminus\sigma(H)$. Thus, $(H-z)^{-1}$ exists if and only if $(I+V(H_0-z)^{-1}))^{-1}$ exists. By proposition 1, Fredholm's determinant $\Delta(z)$ is defined for all $z\in C\setminus\sigma(H_0)$. V is a compact operator. The function $V(H_0-z)^{-1}$ is compact valued analytic in $C\setminus\sigma(H)$. By using the analytic Fredholm theorem, we conclude that $(I+V(H_0-z)^{-1}))^{-1}$ exists on $C\setminus\sigma(H_0)$ except for discrete set $D\subset C\setminus\sigma(H_0)$, since by lemma 1 $(I+V(H_0-z)^{-1}))^{-1}$ exists for certain $z\in C\setminus\sigma(H_0)$. So $\sigma_{ess}(H)\subset\sigma(H_0)$.

Now let λ_0 be an arbitrary element of $\Gamma(\lambda_0 \neq \infty)$ and $u(x_0) = \lambda_0$, u is a rational function. Then there exists positive $N_0 > 0$ such that function u is continuous in $[x_0.x_0 + \frac{1}{N_0}]$. We set

$$f_n(x) = \begin{cases} \sqrt{n(n+1)}, & \text{as} \quad x \in (x_0 + \frac{1}{n+1}, x_0 + \frac{1}{n}] \\ 0 & \text{as} \quad x \notin (x_0 + \frac{1}{n+1}, x_0 + \frac{1}{n}] \end{cases}$$

for integer $n > N_0$. Evidently, $f_n(x) \in D_u$ and $\{f_n\}$ is an orthonormalized system. It is easy to show that

$$\lim_{n\to\infty} \| (H - z\lambda_0) f_n \| = 0.$$

Thus, $\lambda_0 \in \sigma(H)$. In other words $\Gamma = \sigma(H_0) \subset \sigma(H)$. Consequently, $\sigma(H_0) \subset \sigma_{ess}(H)$.

References

1. Lakaev S. N. Some spectral properties of generalized Friedrichs model. Trudy seminara im. I. G. Petrovskogo, 1986. no 11, s 210-238 (in Russian).

2. Lakaev S. N., Muminov M. I. On essential spectrum nonself-adjoint generalized Friedrichs model. Doklady Uzbek Academy of Sciences, no 4,(1997) c. 8-10 (in Russian).

3. M. Reed, B. Simon. Methods of modern mathematical Physics, vol. 4. Analysis of operators, AMS (MOS) 1970.