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A Cosmological Mechanism for Stabilizing Moduli

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Abstract

In this paper, we show how the generic coupling of moduli to the kinetic energy of ordinary matter fields results in a cosmological mechanism that influences the evolution and stability of moduli. As an example, we reconsider the problem of stabilizing the dilaton in a non-perturbative potential induced by gaugino condensates. A well-known difficulty is that the potential is so steep that the dilaton field tends to overrun the correct minimum and to evolve to an observationally unacceptable vacuum. We show that the dilaton coupling to the kinetic or thermal energy of matter fields produces a natural mechanism for gently relaxing the dilaton field into the correct minimum of the potential without fine-tuning of initial conditions. The same mechanism is potentially relevant for stabilizing other moduli fields.

A fundamental problem in supergravity and superstring theories is the stabilization of moduli fields, particularly the dilaton. Perturbatively, $\Phi \equiv \exp(\lambda\phi)$ (the dilaton) has no potential, although it does not behave as a free field because it has non-linear couplings to the kinetic energy of the axion field. (Throughout this paper, we use Φ and ϕ interchangeably to represent the dilaton according to convenience; the constant $\lambda = \sqrt{16\pi}/m_{pl}$, where $m_{pl} \equiv 1.2 \times 10^{19}$ GeV is the Planck scale, is chosen so that ϕ has a canonical kinetic energy density, $\frac{1}{2}\dot{\phi}^2$.) A non-perturbative potential can be induced by gaugino condensates [1–3]. With several gaugino condensates, parameters can be tuned so that there is a locally stable minimum with zero cosmological constant [4]. See the solid curve in Fig. 1. However, the potential is exponentially steep ($V \sim \exp(-\exp(\phi))$) and the desired minimum, Φ_{min}, is separated by an exponentially small barrier (compared to the Planck scale) from an observationally unacceptable anti-de Sitter vacuum [5]. It appears that, unless the initial conditions of the dilaton field are finely-tuned to lie very near the correct minimum, the field will overrun or miss altogether the desired minimum.

In this paper, we present a possible robust solution to this problem based on generic properties of the dilaton and natural cosmological effects. The solution relies on the coupling of the dilaton to the kinetic energy density of ordinary matter fields which has important consequences in the early universe when the thermal (kinetic) energy density is high. In the radiation-dominated epoch, at least three effects come into play, two of which have been considered previously.

First, the energy density in the thermal component increases the Hubble damping, as emphasized by Barreiro *et al* [6]. If the thermal energy density is very large compared to the dilaton energy density, the Hubble damping factor is significantly enhanced and the evolution of the dilaton is slowed. As a result, Φ can be allowed somewhat smaller initial values (corresponding to climbing further up the steep part of the potential in Fig. 1) and still be trapped at Φ_{min}. This is a modest expansion in allowed initial conditions. In the scheme presented here, we find that the range of allowed initial conditions is enormously expanded.

Second, as pointed out by Horne and Moore [7], the dilaton couples non-linearly to the axion field and, if both fields have large initial kinetic energy densities compared to their potential, the non-linear coupling causes Φ to undergo chaotic motion back and forth in its potential over a finite range in Φ that includes the desired minimum. If the chaotic behavior could be sustained, then this would enhance the probability that Φ is trapped in the correct minimum. However, as pointed out by Banks *et al* [8], the axion kinetic energy decays too quickly and spatial inhomogeneities grow too rapidly during the chaotic phase.

This paper points out a third feature of the dilaton in a cosmological setting that can provide a robust mechanism for dilaton stabilization. Namely, although the dilaton couples non-perturbatively to itself, it couples *perturbatively* to the kinetic energy and potential energy of all matter and gauge fields. In studying vacuum solutions, these fields and their kinetic energies are usually set to zero. However, in a cosmological setting, they produce a non-negligible, temperature-dependent contribution to the dilaton effective potential that can allow the dilaton field to be gently lowered into the desired minimum as the universe expands and cools. Whether this mechanism works depends on the functional form of the dilaton coupling to the matter and radiation energy densities. If we take forms suggested by superstring theory, the scenario works. (When the first two effects above, Hubble damping and coupling to the axion, are also included, they help to extend the range of dilaton couplings which work.)

We write the lowest component of dilaton superfield as $S = \Phi + iA/m_{pl}$, where Φ describes the dilaton and A the axion. The non-perturbative dilaton potential, V_{np}, is due to multiple gaugino condensates, arranged to yield a stable minimum with zero cosmological constant ($\Phi = \Phi_{min}$): the racetrack model [4] as shown in Fig. 1. The energy scale has been blown up by more than 60 orders of magnitude compared to the Planck scale in order to make visible the features near Φ_{min}. The minimum is locally stable. There is a barrier at $\Phi > \Phi_{min}$ peaking at $\Phi = \Phi_p$ which separates the desired minimum from an anti-de Sitter vacuum. The height of the barrier is tiny, typically 50 or more orders of magnitude below the Planck density. At $\Phi < \Phi_{min}$ the potential rises exponentially steeply to values $V_{np}[\Phi] \gg V_{np}[\Phi_p]$.

Based on this description and Fig. 1, it is simple to see why it is hard to be trapped at $\Phi = \Phi_{min}$. If Φ begins at $\Phi_0 > \Phi_p$, on the right side of the barrier from Φ_{min}, it is unlikely to be trapped at Φ_{min}. For $\Phi_0 < \Phi_{min}$, there is a very limited range of initial conditions for which Φ is trapped at Φ_{min}. In particular, if $V_{np}[\Phi_0] \gg V_{np}[\Phi_p]$, (e.g. if the initial potential energy density is near the Planck scale or compactification scale, which is much greater than the barrier height) the field tends to roll rapidly down the exponential potential, overshooting Φ_{min} and the barrier ($\Phi = \Phi_p$), ending up in the wrong vacuum.

At high temperatures the relevant terms of a typical Lagrangian have the form:

$$\sqrt{|g|}L = \sqrt{|g|}\left\{\frac{1}{2}(\partial\phi)^2 + \frac{f_A(\Phi)}{2}(\partial A)^2 + \frac{f(\Phi)}{2}|\partial C|^2 - g(\Phi)V_C(C) - V_{np}(\Phi, A)\right\} \tag{1}$$

where C is the complex scalar field in a chiral supermultiplet (a matter field) with potential $V_C(C)$, $f_A(\Phi) \equiv 1/2\Phi^2$ is the dilaton-axion coupling, and $f(\Phi)$ and $g(\Phi)$ are, respectively, the coupling of the dilaton to the kinetic energy and potential energy of C. The exact form of $f(\Phi)$ and $g(\Phi)$ depends on the theory one is considering (see below). $V_{np}(\Phi, A)$ is the racetrack potential, constructed from the superpotential

$$W \propto m_{pl}^3 Z(Z+1)^2 \quad ; \quad Z \equiv e^{-\alpha S} \tag{2}$$

and Kähler potential

$$K = -m_{pl}^2 \ln\left(S + \overline{S}\right) - \ldots . \tag{3}$$

Here α is a constant whose value depends on the gauge group. The result for the potential is

$$V_{np} = e^{K/m_{pl}^2}\left[K^{S\overline{S}} D_S W \overline{D_S W} - \frac{3}{m_{pl}^2} W\overline{W}\right] = \frac{1}{\Phi}\sum_{j=1}^{5} h_j(\Phi, A) e^{-(j+1)\alpha\Phi} \tag{4}$$

here $D_S W \equiv \partial_S W - K_S W/m_{pl}^2$ and the $h_j(\Phi, A)$ are polynomials of degree 2 in Φ. The functional form of W is chosen such that the cosmological constant is zero at the minimum. From Eq. (4) we can see that Φ decreases exponentially fast for $\Phi < \Phi_{min}$; and, as proven in [5], using the holomorphic property of W, V_{np} is forced to have a barrier at some $\Phi = \Phi_p > \Phi_{min}$ separating Φ_{min} from an anti-de Sitter minimum at $\Phi > \Phi_p$. See Fig. 1.

FIGURES

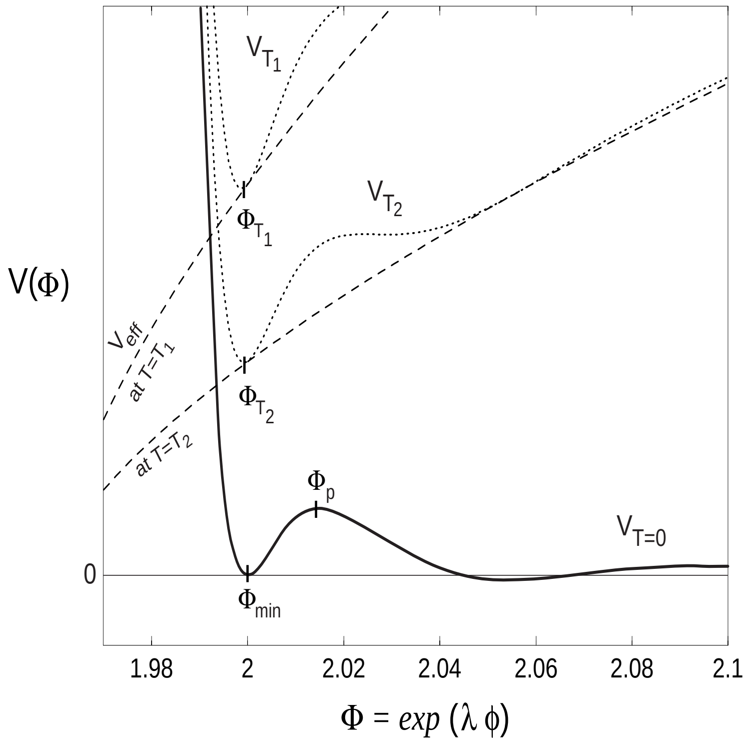

FIG. 1. A schematic of the racetrack potential for the dilaton $\Phi = \exp(\lambda\phi)$, generated by gaugino condensates (λ is a constant). This is represented by the solid curve. The desired minimum at $\Phi = \Phi_{min}$ is separated by a small barrier, peaked at $\Phi = \Phi_p$. Beyond $\Phi = \Phi_p$ (around $\Phi = 2.05$ in this example), there is an unacceptable anti-de Sitter vacuum. (The energy scale has been blown up by more than 60 orders of magnitude to make the barrier visible.) The dashed line represents V_{eff}, the effective potential for Φ stemming from the dilaton coupling $f(\Phi) = g(\Phi) = 1/\Phi$ at temperature $T = T_i$. As T decreases from T_1 to T_2 to zero, this contribution adiabatically decreases. The dotted line represents the total finite temperature potential for Φ, V_{T_i}, which has a minimum at $\Phi = \Phi_{T_i}$.

Note that Eq. (1) includes a perturbative coupling of ϕ to the kinetic energy of the C field. In previous treatments of dilaton stabilization at the minimum of racetrack potentials, this coupling was ignored because the kinetic energy was treated as negligible. While this is justified at zero temperature, the kinetic energy is non-negligible at high temperature and, then, this dilaton coupling is extremely important and should not be ignored.

Stabilization can result under two conditions: (a) coherent oscillation of a homogeneous scalar (matter) field; and (b) thermal excitation of matter fields. Both are plausible sources in the early universe. Let us first consider Case (a), the coherent oscillations of a scalar field C. If the potential energy is $V_C \propto |C|^n$ for integer $n \geq 2$, then the oscillatory C-field energy density ρ_C decays as $a^{-6n/(n+2)}$. For simplicity, we will restrict ourselves to $n = 4$ for which $\rho_C \propto a^{-4}$, similar to radiation. Furthermore, we take $f(\Phi) = g(\Phi)$. Because the field is assumed to be homogeneous, $\nabla C = 0$. Then, the action in Eq. (1) contains the interaction $f(\Phi)\left[\frac{1}{2}|\dot{C}|^2 - V_C(C)\right] \equiv f(\Phi)p_C$, where p_C is the pressure of the oscillatory scalar field. Assuming a Friedmann-Robertson-Walker metric, the equation of motion for $\Phi = \exp(\lambda\phi)$ becomes

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3\dot{\phi}\right) - f' p_C + V_{np,\phi} = 0 \tag{5}$$

where $a(t)$ is the Robertson-Walker scale factor and $f' = df/d\phi$. According to Eq. (5), the pressure due to C exerts a force on ϕ equal to $-f'p_C$. From the equation-of-motion for C, we see

$$\ddot{C} + \left(3H + \frac{\dot{f}}{f}\right)\dot{C} = -V_C'(C) \tag{6}$$

where $V_C'(C) = dV/dC$. Using $p_C \equiv \frac{1}{2}|\dot{C}|^2 - V_C$ and defining $\rho_C \equiv \frac{1}{2}|\dot{C}|^2 + V_C$, Eq. (6) can be recast as

$$\dot{\rho}_C = -\left(3H + \frac{\dot{f}}{f}\right)(\rho_C + p_C). \tag{7}$$

For oscillations in a $V_C \propto C^4$ potential, $p_C = \rho_C/3$, so $p_C = p_C^{(0)}(a^3 f)^{-4/3}$, where $p_C^{(0)}$ is the initial value of the pressure. The force in Eq. (5) then becomes $-p_C^{(0)} f'(a^3 f)^{-4/3}$.

As a specific example, consider the case $f(\Phi) = g(\Phi) = 1/\Phi = \exp(-\lambda\phi)$. This example assumes a single moduli field (the dilaton). Later, we will discuss the case of two or more moduli fields, which is pertinent to perturbative string theory or non-perturbative M-theory [9–12]. For $f(\Phi) = g(\Phi) = 1/\Phi$, an exponentially strong force is induced by p_C that adds an effective potential to $V_{np}(\phi)$ equal to

$$V_{eff}(\phi) = \frac{3p_C^{(0)}}{a^4}\exp(\lambda\phi/3). \tag{8}$$

Note that $1/a^4 \propto T^4$, where the T is the temperature of the radiation background. $V_{eff}(\phi)$ is an exponentially increasing function that provides a force pushing ϕ towards smaller values and opposes V_{np}, which pushes ϕ toward higher values. Note that, expressed in terms of Φ, the effective potential is $V_{eff} \propto T^4 \Phi^{1/3}$.

Case (b), where C is in thermal equilibrium, proceeds similarly. Now the fluctuations in C are non-negligible ($\nabla C \neq 0$) and contribute to the interaction term $(f(\Phi)/2)|\partial C|^2 - g(\Phi)V_C$, which does not obey the same simple relationship to the pressure p_C as above. A different approach must be used to compute V_{eff}. As above, we take a quartic potential $V_C = \epsilon C^4$ Under the assumption that Φ varies slowly compared to thermal interactions, we can transform $C \to \sqrt{f} C$ and $g(\Phi)V_C = \epsilon g C^4 \to (\epsilon g/f^2)C^4 \equiv \epsilon_{eff} C^4$. In thermal equilibrium, the effective potential for a scalar field with quartic interactions is [13] $V_{eff} = -(\pi^2 T^4/30)[1 - (15/8)\epsilon_{eff} + \ldots]$, which includes a Φ-dependent piece proportional to $(\pi^2 T^4/48)(g/f^2)$. Whether this acts as an effective potential term that causes Φ to decrease (stabilizes) or increase (destabilizes) depends critically on the dilaton coupling to the kinetic energy. For example, consider the case $f(\Phi) = g(\Phi) = 1/\Phi$. Naively, based on the potential energy term alone, $g(\Phi)(\epsilon C^4)$, one might suppose that the effective potential is proportional to $g(\Phi) = 1/\Phi$, which is destabilizing. However, when the kinetic energy contribution is properly included,

$$V_{eff} = \frac{\pi^2}{48} \frac{T^4}{f(\Phi)} \propto T^4 \Phi = T^4 \exp(\lambda\phi). \tag{9}$$

As in the case of coherent oscillations, V_{eff} increases as Φ increases, which is the stabilizing condition we need. In the remainder of the paper, we will consider this case with thermal excitations, although the same considerations apply to the coherent oscillation case.

As shown in Fig. 1, the net effect is that $V_{eff} + V_{np}$ at fixed temperature (dotted curves V_{T_i}) has a temperature-dependent minimum, Φ_{T_i}, about which the dilaton Φ oscillates. The minimum lies at $\Phi_{T_i} < \Phi_{min}$. As the universe expands and cools, the temperature decreases and V_{eff} decreases, as well. The energy density at Φ_{T_i} decreases and the value of Φ at the minimum moves gradually towards Φ_{min}.

For this mechanism to work, an issue is that oscillations in Φ about Φ_T must decay sufficiently quickly that Φ does not jump over the barrier at low temperatures. That is, even if Φ_T gently decreases towards Φ_{min}, it is conceivable that Φ is oscillating so wildly about Φ_T that it is carried past the peak Φ_p at low temperatures when $V_{eff}(\Phi_p) \leq V_{np}(\Phi_p)$. The large initial oscillations must be damped rapidly. The greater is the damping rate, the larger can be the initial oscillations, and, hence, the larger is the initial value of Φ that can be stabilized.

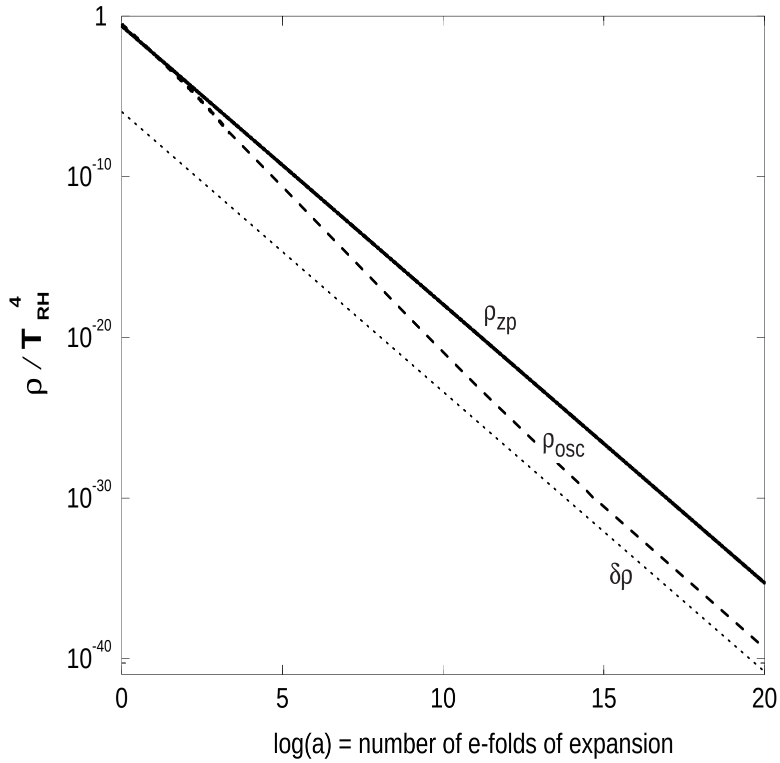

FIG. 2. The evolution of the various energy densities for the case of dilaton coupling $f = g = 1/\Phi$. T_{RH} is the reheat temperature after inflation. The initial value of Φ was chosen to be $\Phi = 10 \gg \Phi_{min}$. The figure shows how the zero point (ρ_{zp}), oscillation (ρ_{osc}), and perturbation ($\delta\rho$) energy densities evolve. In particular, note that, although the system begins with $\rho_{osc} \sim \rho_{zp}$, the oscillations are heavily damped after a few e-folds, leading to $\rho_{osc} \ll \rho_{zp}$. Furthermore, note that $\delta\rho$ (the contribution of inhomogeneity in all fields to the energy density) decays at the same rate as ρ_{zp}, so inhomogeneity in the universe does not come to dominate.

The total dilaton energy (ρ_ϕ) at fixed temperature can be split into the zero-point energy

($\rho_{zp} \equiv V_{np}(\Phi_T) + V_{eff}(\Phi_T)$, where Φ_T is the minimum of the finite temperature effective potential) and oscillation energy ($\rho_{osc} \equiv \rho_\phi - \rho_{zp}$). Thus for stabilization of the dilaton to be robust, we need ρ_{osc} to decay faster than ρ_{zp}. Figure 2 shows the results of a numerical simulation for a typical case starting at a temperature of approximately T_{RH} with $\Phi = 10 \gg \Phi_{min}$ and all of the components of the energy density comparable. Note that initially $\rho_{osc} \sim \rho_{zp}$, but after 10 e-folds of expansion it is about 4 orders of magnitude smaller. The relative damping of oscillation energy can be understood as follows: the effective potential energy for C decreases as T^4, like radiation. As Φ is rolling along V_{eff}, the oscillation energy decays due to the red shifting of its kinetic energy *and* due to the fact that V_{eff} decreases as the temperature decreases. If Φ were frozen ($\dot{\Phi} = 0$) at some value away from the minimum and all that happened is that V_{eff} decreases, the energy in the dilaton would decay at the same rate as V_{eff}. With Φ oscillating ($\dot{\Phi} \neq 0$), one has additionally the red shift of the dilaton kinetic energy; hence, ρ_{osc} decreases more rapidly than V_{eff}. However, the rate of decay of the zero-point energy ρ_{zp} is approximately the same as V_{eff}. Thus, ρ_{osc} decays faster than ρ_{zp} and becomes negligible. That is, the dilaton settles down near the minimum ϕ_T as the temperature decreases.

A more rigorous argument shows that ρ_{osc} decays faster than ρ_{zp} until ρ_{osc}/ρ_ϕ reaches a negligibly small value and then the ratio remains roughly constant (10^{-4} in Fig. 2). The remaining oscillations are not important for our purposes since they are too small to drive Φ past Φ_p. The decay rate of ρ_{osc}/ρ_ϕ is so rapid once oscillations begin that it poses no significant constraint on our scenario. What does limit the range of initial conditions is that, for sufficiently large Φ, there is insufficient time for oscillations to commence. We will return to this point below when we determine how robust the stabilization mechanism is.

Based on what has been learned from this example, it is straightforward to consider couplings different from $f(\Phi) = g(\Phi) = 1/\Phi$. A necessary (but insufficient) condition for the coupling to produce a stabilizing V_{eff} is that $(g/f^2)' = d(g/f^2)/d\Phi > 0$ for the case of thermally excited C-fields. Hence, $f = g \propto 1/\Phi^n$ where $n > 0$ is a satisfactory form. (Since V_{eff} grows exponentially with ϕ for all $n > 0$, the stabilization mechanism is not very sensitive to the power n.)

We have focused on the dilaton coupling $f(\Phi)$ to the kinetic energy of the matter fields because they produce a net, stabilizing, effective potential. We note that S also couples to the gauge fields via an interaction $h(\Phi)F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu}F^{\mu\nu} \approx B^2 - E^2$ in the case of $U(1)$ gauge fields. At high temperature, $< B^2 >=< E^2 >$, and so the gauge interaction adds zero effective potential for Φ. Hence, in the case of abelian gauge fields, $h(\Phi)F_{\mu\nu}F^{\mu\nu}$ can be ignored for our purposes.

The dilaton coupling to the axion is yet another interesting example. The kinetic energy of the axion couples to the dilaton with $f_A(\Phi) = 1/2\Phi^2$, a stabilizing form by the criterion outlined above. However, the axion field is weakly coupled to matter, and so it cannot be expected to be in thermal equilibrium with the matter-fields. Instead, one can imagine that the axion has large coherent time-variation, as discussed by Horne and Moore [7]. This produces a steep, stabilizing, effective potential $\propto \Phi^2 = \exp(2\lambda\phi)$ which forces ϕ towards small values where it eventually gets trapped in the minimum of the combined potential due to the thermally excited C-field and the non-perturbative potential V_{np}. The axion-induced force is not sustained for a very long time because the strength is proportional to its pressure, $p_A \propto 1/a^6$, which decays faster than the thermal energy. However, the brief contribution of the axion-induced force to dilaton capture expands the range of $f(\Phi)$ and initial conditions for the dilaton that are ultimately trapped.

How robust are the various stabilization mechanisms? That is, beginning from initial conditions, what is the probability that Φ is trapped at Φ_{min}? A precise answer is not possible because there is no rigorous understanding of the initial conditions. We use plausible estimates similar to Horne and Moore [7] and others (*e.g.*, we only consider energy densities less than the Planck scale and rough equipartition of kinetic and potential energies). Originally, when the couplings between the dilaton and all other fields were ignored, it appeared that a very narrow range of initial conditions result in Φ being trapped at Φ_{min}. Formally, this is a set of measure zero if one imagines all possible initial values of Φ and $\dot{\Phi}$ as being equally likely. Barreiro *et al.* propose a high-temperature thermal background of particles in order to increase the Hubble damping during the phase when Φ evolves along the potential. By increasing the damping of $\dot{\Phi}$, this effect enhances the range of initial conditions by allowing Φ to lie somewhat further up the steep part of the potential at $\Phi < \Phi_{min}$ and still not overshoot the peak at Φ_p. While this is an improvement, the range of allowed initial Φ remains finite and narrow; formally, this is also a set of measure zero.

Horne and Moore [7] argue that all possible values of Φ are not equally likely, if couplings to the axion are properly included. The nonlinear coupling between axion and dilaton causes the dilaton to follow a chaotic path of back and forth motion in the potential in which large values of $\Phi >> \Phi_{min}$ are exponentially unlikely. They argue that the effect can be taken into account by weighting the probability of Φ according to the Kähler metric, which leads to a finite phase volume. Fig. 3 shows two representations of the phase space of Φ and A. The horizontal bounding curves represent $A = 0$ and $A = 2\pi m_{pl}/\alpha$. The probability of a given Φ' is proportional to the length of the vertical segment joining the upper and lower curves at $\Phi = \Phi'$. Fig. 3a represents the naive expectation that all combinations of initial $1 \leq \Phi \leq \infty$ and $0 \leq A \leq 2\pi m_{pl}/\alpha$ are equally probable (all vertical segments joining the boundary have the same length). In this case, the total volume is infinite. However, the non-linear coupling between Φ and A leads to chaotic dynamics at early times which causes the probability distribution as a function of Φ to fall off as $1/\Phi^2$ [7]. Fig. 3b illustrates this distortion of the phase space volume, which is now finite. Horne and Moore conclude that, within the total volume, the sub-volume of initial conditions that are ultimately trapped at Φ_{min} is $\sim 14\%$ of the total volume, corresponding to Φ near Φ_{min}. However, as later pointed out by Banks *et al.* [8], the chaotic motion also causes the evolution of unacceptably large inhomogeneities in the axion field. In particular, the homogeneous component of the axion energy responsible for the chaotic motion decreases as $1/a^6$, whereas the density inhomogeneities grow as $1/a^4$. So, while the universe may become trapped at $\Phi = \Phi_{min}$, the density distribution is too inhomogeneous.

In judging the stabilization mechanism proposed in this paper, we assume the axion field is excited initially as well as the matter (C) fields. Hence, we adopt the Kähler-weighted finite measure of the phase space for initial ϕ as argued by Horne and Moore. To estimate what initial conditions are trapped, we impose the conservative constraint that our mechanism will rapidly stabilize the dilaton at $\Phi = \Phi_T$ beginning from some high initial temperature, *e.g.*, the reheat temperature after inflation, T_{RH}. We determine the maximum Φ for which the dilaton completes one oscillation about Φ_T before the temperature decreases to $10^{-3}T_{RH}$, say. After this oscillation, ρ_{osc} is already less than ρ_{zp} and Φ is essentially caught near Φ_T. We find that $\Phi \leq 50$ satisfies this conservative condition, which encompasses 98% of the initial phase space volume. If we loosen our constraint by decreasing the bound below $10^{-3}T_{RH}$, the fraction of allowed initial moduli space can be made even closer to unity.

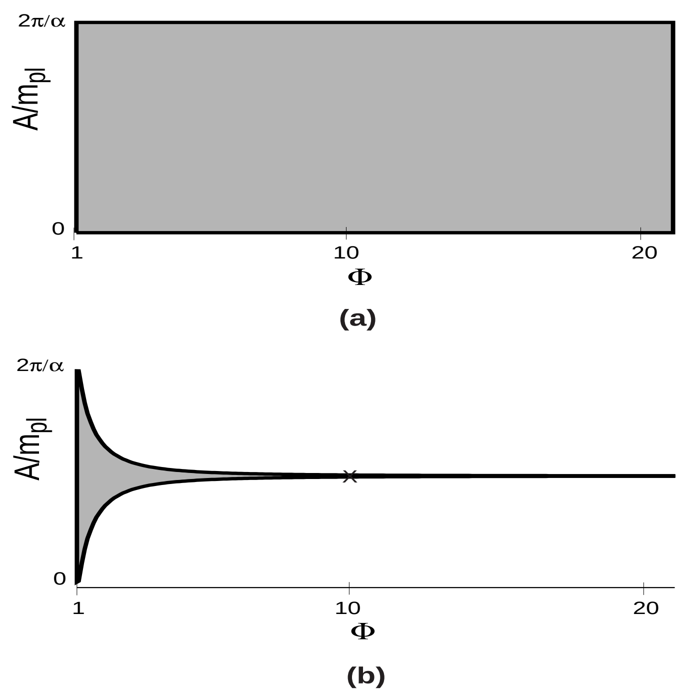

FIG. 3. A schematic illustration of initial phase space volume. The relative likelihood of an initial Φ is represented by the vertical distance between the curves bounding the shaded region. Naively, as shown in (a), all combinations of initial $1 \leq \Phi \leq \infty$ and $0 \leq A \leq 2\pi m_{pl}/\alpha$ might appear equally probable, and the allowed volume of the shaded region is infinite. However, based on the arguments of Horne and Moore, the effective volume of moduli space is defined by the Kähler metric and is finite, as illustrated in (b). The initial conditions used in Fig. 2 are marked by "X."

As an example, consider the case of an initial value $\Phi = 10$, the case depicted in Fig. 2 and marked by an "X" in Fig. 3. This value lies outside the trapped region of Barriero *et al.*, which considers the Hubble damping effect, and the trapped region of Horne and Moore, which considers only the dilaton-axion coupling. But this value lies well within the trapped region in our scenario, which includes the coupling between dilaton and C-field as well. Trapping all initial conditions with $\Phi \leq 10$ would be arguable progress if Fig. 3a were correct, since this range would represent formally a set of measure zero. But, in Fig. 3b, this same range of initial conditions corresponds to 90% of the total phase volume.

Figures 1 and 2 apply for case of dilaton coupling $f(\Phi) = g(\Phi) = 1/\Phi$. For a general $f(\Phi)$, we can ask what fraction of the Kähler-weighted volume of phase space for Φ is trapped at Φ_{min}. Let us assume roughly equipartition initial conditions in which the kinetic plus potential energy density in ϕ is comparable to the matter-field energy density. For $f(\Phi) = g(\Phi) = 1/\Phi^n$, this implies an effective potential $V_{eff} \sim \Phi^{n/3} \sim \exp(n\lambda\phi/3)$, which is exponentially steep, sufficient to trap nearly 100% of all initial conditions.

Unlike the case of Horne and Moore, our scenario does not suffer from the problem of axion energy density inhomogeneities ($\delta\rho$). In their scenario, energy density due to inhomogeneities $\delta\rho$, which decays as $1/a^4$, always overtakes the homogeneous energy component, the axion kinetic energy, which decays as $1/a^6$. In our scenario, the homogeneous energy density is dominated by the thermal energy of the matter and gauge fields, which decays as $1/a^4$. (Here $\delta\rho$ is defined as the deviation in the $0-0$ component of the stress-energy tensor due to perturbations in the dilaton, axion and C fields as well as the metric [14].) Hence, as shown in Fig. 2, $\delta\rho$ decays at the same rate as the total energy density (ρ_{tot}). Assuming that the inhomogeneities are initially negligible, they remain negligible.

When two or more moduli fields exist, the situation becomes more complicated. Both f and g take different forms. An example relevant to perturbative string theory or non-perturbative M-theory [9–12] is $f[S,T] = (3/Re[T]) + (\beta/Re[S])$ and $g[S,T] = 1/(ST^3 f[S,T])$. In models of the Hořava-Witten type, the dilaton S is replaced in the non-perturbative superpotential W by $S - \beta T$, where T is the orbifold modulus. Hence, one can consider trapping in the $S - \beta T$ direction; typically, an independent method is needed to stabilize the $S + \beta T$ direction. If one supposes a mechanism that fixes $Re[S + \beta T] = \kappa$, where $\beta > 0$ and $\kappa = \mathcal{O}(100) > 0$ (as in the standard embedding), then the effective potential along the the $Re[S - \beta T]$ direction is similar to the examples considered above. A technical difference is that, since the physical regime is $S > 0$ and $T > 0$, the constraint, $Re[S + \beta T] = \kappa$, prevents $\Phi = Re[S]$ from exceeding κ; so trapping is only required for $S \leq \kappa = \mathcal{O}(100)$. The non-perturbative potential tends to push $\Phi = Re[S]$ to increase, but the thermal contribution due to the matter fields pulls Φ back to smaller values. As in our toy model (see discussion of Eq. (9)), the critical feature is that the coupling to the kinetic energy produces a a stabilizing contribution to the thermal effective potential. The trapping force becomes small at large Φ. However, an initial axion kinetic energy produces a steep, stabilizing potential at early times (until the axion kinetic energy density becomes negligible compared to the dilaton energy). When all effects are included, the percentage of initial conditions that become trapped rises to nearly 100%, as before.

The lesson to be learned from this study goes beyond finding a long-sought mechanism for stabilizing the dilaton. What we have seen is that the cosmological background can play an important role in the evolution and stabilization of moduli fields and the determination of the present vacuum state. This is especially important for nearly-flat, non-perturbative potentials with multiple vacua, as is common in supergravity and superstring theories, where there is little guidance as to why one vacuum is observed and the others are irrelevant (at least within our Hubble volume). A characteristic feature of these models is non-linear sigma-model type couplings of the moduli fields to the kinetic energy of the matter of the type considered here. Whereas these couplings have been ignored in past considerations of the moduli problem, here we have seen that they can have a strong influence in the early universe. Hence, just as we have demonstrated for the dilaton, we expect the cosmological background to have significant effect on other moduli fields.

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