# The QCD pomeron at TESLA - motivation and exclusive $J / \psi$ production ${ }^{\text {¢ }}$ 

J. Kwieciński ${ }^{a}$, L. Motyka ${ }^{b}$ and A. De Roeck ${ }^{c}$<br>${ }^{a}$ Department of Theoretical Physics, H. Niewodniczański Institute of Nuclear Physics, Cracow, Poland<br>${ }^{b}$ Institute of Physics, Jagellonian University, Cracow, Poland<br>${ }^{c}$ CERN, Geneva, Switzerland


#### Abstract

We briefly present the motivation for studying the processes mediated by the QCD pomeron at high energy $e^{+} e^{-}$colliders. We describe the behaviour of the cross-section for the reaction $\gamma \gamma \rightarrow J / \psi J / \psi$ obtained from the BFKL equation with dominant non-leading corrections. We give the predictions for the rates of anti-tagged $e^{+} e^{-} \rightarrow e^{+} e^{-} J / \psi J / \psi$ events in TESLA and conclude that such reactions may be excellent probes of the hard pomeron.


[^0]The pomeron exchange is one of the most intriguing phenomena in quantum chromodynamics. It governs the leading behaviour of the scattering amplitude for two objects which may interact by exchanging colour degrees of freedom. The characteristic feature of these amplitudes is an approximate power-like dependence on the collision energy, which supports the picture of the pomeron as an isolated Regge pole in the complex angular momentum plane. The exponent characterizing the pomeron exchange is universal for soft processes and thus it is convenient to use phenomenological Regge motivated models [1] for the pomeron. For hard processes at high energies it is possible to go beyond the level of models and describe the pomeron in the framework of perturbative QCD in the high energy limit [2, 3]. However, at every order of the perturbative expansion there appear large logarithms of the collision energy which accompany the coupling constant and one cannot rely on the fixed order perturbation theory. The necessary resummation of ladder diagrams with reggeized gluons along the chain may be performed with the use of the Balitzkij, Fadin, Kuraev, Lipatov (BFKL) equation (H) Formally, this equation gives the exact answer in the leading logarithmic approximation, but the recently obtained NLO corrections (5) are very large, that could make the BFKL approach not useful for phenomenology. One is also forced to face the problem of contributions to the amplitudes coming from the infra-red domain. In fact one can control both effects in an approximate way and construct reasonable phenomenology on the basis of the BFKL equation. The important role of the experiments is therefore to verify the quality of the approximations and provide the guideline for further development of the pomeron theory.

Unfortunately, for most observables, it is difficult to disentangle the genuine hard pomeron effects from other contributions. The ideal high energy process should have the following properties:
(i) The virtualities of the gluons along the ladder should be large enough to assure the relevance of the perturbative expansion. The necessary hard scale may be provided either by the coupling of the ladder to scattering particles, that contain a hard scale themselves, or by large momentum transfer carried by the gluons.
(ii) In order to distinguish the genuine BFKL from DGLAP evolution effects it is convenient to focus on processes in which the scales on the both ends of the ladder are of comparable size. Then, the amplitude is free from the DGLAP corrections due to zero length of the DGLAP evolution.
(iii) Finally, one requires the non-perturbative effects to factor out.

Since these requirements are rather stringent, the list of possible measurements in hadron-hadron and lepton-hadron colliders is limited. It would be therefore desirable to study pomeron physics also for the two (virtual) photon processes at TESLA. The quality of such measurements is expected to be excellent due to very large luminosity for high energy photon-photon collisions and the clean experimental signatures. In


Figure 1: The QCD pomeron exchange mechanism of the processes $\gamma \gamma \rightarrow J / \psi J / \psi$.
what follows we shall present the estimates of the cross-section of the double exclusive $J / \psi$ production at TESLA which have been performed using the BFKL equation with non-leading corrections [6]. A typical diagram illustrating this process is given in Fig. 1.

The doubly exclusive $J / \psi$ productions is an unique process since it allows to test the differential cross-section for arbitrary momentum transfer. Besides that, the pomeron exchange amplitude enters the cross-section in the second power making the dependence on the energy very robust. The hard scale is given by the relatively large mass of the $c$-quark and thus the perturbative calculation is valid even for real photons or photons with small virtuality. The flux of such photons in an electron contains a large enhancement proportional to the logarithm of the beam energy which moves this rather exotic process into the measurable domain. Since the $e^{+} e^{-}$cross-section for $J / \psi$ production is dominated by small virtualities of the mediating photons it is convenient to focus on the anti-tagged $e^{+} e^{-}$events. The cross-section for the process $e^{+} e^{-} \rightarrow e^{+} e^{-}+Y$ for anti-tagged $e^{ \pm}$corresponds to the production of the hadronic state $Y$ in $\gamma \gamma$ collision and is given by a standard convolution integral (7].

The imaginary part $\operatorname{Im} A\left(W^{2}, t=-Q_{P}^{2}\right)$ of the amplitude for the considered process which corresponds to the diagram in Fig. 1 can be written in the following form:

$$
\begin{equation*}
\operatorname{Im} A\left(W^{2}, t=-Q_{P}^{2}\right)=\int \frac{d^{2} \boldsymbol{k}}{\pi} \frac{\Phi_{0}\left(k^{2}, Q_{P}^{2}\right) \Phi\left(x, \boldsymbol{k}, \boldsymbol{Q}_{P}\right)}{\left[\left(\boldsymbol{k}+\boldsymbol{Q}_{P} / 2\right)^{2}+s_{0}\right]\left[\left(\boldsymbol{k}-\boldsymbol{Q}_{P} / 2\right)^{2}+s_{0}\right]} \tag{1}
\end{equation*}
$$

In this equation $x=m_{J / \psi}^{2} / W^{2}$ where $W$ denotes the total CM energy of the $\gamma \gamma$ system, $m_{J / \psi}$ is the mass of the $J / \psi$ meson, $\boldsymbol{Q}_{P} / 2 \pm \boldsymbol{k}$ denote the transverse momenta of the exchanged gluons and $\boldsymbol{Q}_{P}$ is the transverse part of the momentum transfer. The role of the parameter $s_{0}$ will be explained later. The impact factor $\Phi_{0}\left(k^{2}, Q_{P}^{2}\right)$ describes the $\gamma J / \psi$ transition induced by two gluons and may be calculated in QCD [8]. The function $\Phi\left(x, \boldsymbol{k}, \boldsymbol{Q}_{P}\right)$ satisfies the non-forward BFKL equation which may be
represented symbolically as follows:

$$
\begin{equation*}
\Phi\left(\boldsymbol{Q}_{P}\right)=\Phi_{0}\left(\boldsymbol{Q}_{P}\right)+\frac{3 \alpha_{s}\left(\mu^{2}\right)}{2 \pi^{2}} \mathcal{K}\left(\boldsymbol{Q}_{P}\right) \times \Phi\left(\boldsymbol{Q}_{P}\right) \tag{2}
\end{equation*}
$$

where the dependence of the impact factors $\Phi$ 's and the BFKL kernel $\mathcal{K}$ on the transverse momenta and on the longitudinal momentum fraction(s) is not indicated. The scale of the QCD coupling $\alpha_{s}$ which appears in the impact factors and in Eq. (22) will be set to $\mu^{2}=k^{2}+Q_{P}^{2} / 4+m_{c}^{2}$ where $m_{c}$ denotes the mass of the charmed quark. The differential cross-section is related in the following way to the amplitude $A$ :

$$
\begin{equation*}
\frac{d \sigma}{d t}=\frac{1}{16 \pi}\left|A\left(W^{2}, t\right)\right|^{2} \tag{3}
\end{equation*}
$$

It follows from the solution to the leading order BFKL equation (2) (with the fixed coupling constant) that the differential cross-section (3) rises for asymptotically large energies $W$ as $\left(W^{2}\right)^{2 \lambda_{P}}$ with $\lambda_{P}$ given at the leading order by the famous BFKL formula $\lambda_{P}=12 \alpha_{s} \log (2) / \pi$. However, this exponent is subject to very large NLO corrections (5]:

$$
\begin{equation*}
\lambda_{P}^{(N L O)} \simeq \lambda_{P}\left(1-6.2 \alpha_{s}\right) . \tag{4}
\end{equation*}
$$

For all values of $\alpha_{s}$, relevant for experiments, this expansion is ill defined. It seems that the source of this problem can be traced back to non-conservation of energy and momentum in the gluon configurations resummed by the BFKL ladder. Although these effects are formally subleading, they appear to be numerically large as it can be seen e.g. in Eq. (4). At present, it is not known how to impose the exact phase space conditions on the multiple gluon emissions which build up the BFKL ladder. One of the possible approximate solutions to this problem was proposed in [g] and developed in [10]. The proper kinematics is partially restored when one requires that the virtualities of the gluons propagating along the ladder are dominated by the transverse momenta squared. This modification, which we call kinematical or consistency constraint does not introduce any effect at the leading order, but when taken into account at the NLO accuracy, it exhausts about $70 \%$ of the exact NLO result. Moreover this constraint eliminates double transverse logarithms in the DGLAP limits of the NLO BFKL kernel as required by the renormalization group equations [11]. Thus, besides equation (2) we shall consider a similar equation, with the kinematical constraint imposed on the BFKL kernel. The modified BFKL equation is more reliable and the cross-sections arising from BFKL at LO are kept only as a reference.

Let as also mention how we treat the infra-red domain. We introduce a parameter $s_{0}$ in the propagators of exchanged gluons. This parameter can be viewed as the effective representation of the inverse of the colour confinement radius squared. The parameter $s_{0}$ will be varied within the range $(0.2 \mathrm{GeV})^{2}<s_{0}<(0.4 \mathrm{GeV})^{2}$. Sensitivity of the cross-section to its magnitude can serve as an estimate of the sensitivity of the results
to the contribution coming from the infrared region. It should be noted that formula ( $\mathbb{1})$ gives finite result in the limit $s_{0}=0$. The results with finite $s_{0}$ are however more realistic.


Figure 2: Energy dependence of the cross-section for the process $\gamma \gamma \rightarrow J / \psi J / \psi$. The two lower curves correspond to the calculations based on the BFKL equation with kinematical constraint and the values of $s_{0}$ equal to $0.04 \mathrm{GeV}^{2}$ (the continuous line) and to $0.16 \mathrm{GeV}^{2}$ (dashed line). The two upper curves correspond to the BFKL equation in the leading logarithmic approximation with $s_{0}=0.04 \mathrm{GeV}^{2}$ (dash-dotted line) and $s_{0}=0.16 \mathrm{GeV}^{2}$ (short-dashed line).

In Fig. 2 we show the obtained cross-section for the process $\gamma \gamma \rightarrow J / \psi J / \psi$ plotted as a function of the total CM energy $W$. We show results based on the BFKL equation in the leading logarithmic approximation as well as those which include the dominant non-leading effects. The calculations were performed for two values of the parameter $s_{0}$ i.e. $s_{0}=0.04 \mathrm{GeV}^{2}$ and $s_{0}=0.16 \mathrm{GeV}^{2}$.

Let us discuss the main features of the obtained results. We see from Fig. 2 that the effect of the non-leading contributions is very important and that they significantly reduce the magnitude of the cross-section and slow down its increase with increasing CM energy $W$. The cross-section obtained from the BFKL equation with non-leading corrections exhibits an approximate $\left(W^{2}\right)^{2 \lambda_{P}}$ dependence. The parameter $\lambda_{P}$, which slowly varies with the energy $W$, takes the values $\lambda_{P} \sim 0.23-0.28$ within the energy range $20 \mathrm{GeV}<W<500 \mathrm{GeV}$ relevant for LEP2 and for possible TESLA measurements. The cross-section calculated with the BFKL equation in the leading logarith-
mic approximation gives a much stronger energy dependence of the cross-section. The enhancement of the cross-section is still appreciable after including the dominant nonleading contribution which follows from the kinematical constraint. Thus while in the Born approximation (i.e. for the elementary two gluon exchange which gives energy independent cross-section) we get $\sigma_{\text {tot }} \sim 1.9-2.6 \mathrm{pb}$ the cross-section calculated from the solution of the BFKL equation with the non-leading effects taken into account can reach the value 4 pb at $W=20 \mathrm{GeV}$ and 26 pb for $W=100 \mathrm{GeV}$ i.e. for energies which should be easily accessible at TESLA. The magnitude of the cross-section decreases with increasing magnitude of the parameter $s_{0}$ which controls the contribution coming from the infrared region. The energy dependence of the cross-section is practically unaffected by the parameter $s_{0}$.

In our calculations we have assumed dominance of the imaginary part of the production amplitude. The effect of the real part can be taken into account by multiplying the cross-section by the correction factor $1+t g^{2}\left(\pi \lambda_{P} / 2\right)$ which for $\lambda_{P} \sim 0.25$ can introduce an additional enhancement of about $20 \%$.

In order to link our results to the $e^{+} e^{-}$observables we consider the most inclusive quantity which is the total cross-section $\sigma_{\text {tot }}\left(e^{+} e^{-} \rightarrow e^{+} e^{-} J / \psi J / \psi\right)$. In fact, it is convenient to impose additionally the anti-tagging condition. Taking $\theta_{\max }=30 \mathrm{mrad}$ and the minimal $\gamma \gamma$ energy to be 15 GeV we get for $\sigma_{\text {tot }}\left(e^{+} e^{-} \rightarrow e^{+} e^{-} J / \psi J / \psi\right)$ the values of about 0.74 pb at $\sqrt{s}=500 \mathrm{GeV}$ (i.e. for typical energies at TESLA). Thus, at TESLA for an $e^{+} e^{-}$energy of 500 GeV and integrated luminosity of $100 \mathrm{fb}^{-1}$ we expect about 74000 events, which is a large number, even if the acceptance for the $J / \psi$ detection is low.

The measurability of the $J / \psi$ mesons has been studied with the Monte Carlo generator PYTHIA, adapted for this purpose [12]. Here the reaction $\gamma \gamma \rightarrow J / \psi J / \psi$ is generated, with zero transverse momentum transfer. The decays products of the $J / \psi$ will generally have small angles with respect to the beam particle direction, i.e. stay close to the beampipe, rendering experimental detection difficult. The most promising detectable decay channels are the leptonic decays $J / \psi \rightarrow e^{+} e^{-}$and $J / \psi \rightarrow \mu^{+} \mu^{-}$. Fig. 3 shows the $W$ distribution of the generated events and the distribution of the angle of the decay muons with respect to the beam direction.

A generic detector at the LC [13] has an acceptance for muons and electrons starting from polar angles $\theta>100 \mathrm{mrad}$. If the background from the interaction region would be unexpectedly too large, the minimum reachable $\theta$ angle will have a larger value, e.g. 150 mrad. Reaching smaller angles is not impossible: the inner mask, which occupies the detector region below 100 mrad , can be instrumented for detecting particles in the region $\theta=20-25$ to 100 mrad (14. Since this region will suffer from background
electrons and photons, in case the $J / \psi$ decays into electrons, these electrons will have to have at least 50 GeV to be detectable above this relatively uniform background. In table 1 we show the acceptance (not taking into account the magnetic field) for muons as function of the minimum angle $\theta_{\min }$ and momentum $p_{t o t}$ for detecting the particles, for TESLA at $\sqrt{s}=500 \mathrm{GeV}$. The detection efficiencies for the electrons are similar apart from the low angle region, where a high momentum of 50 GeV is always required. The corresponding values for the electrons with that additional requirement are given in brackets in the table.

| $p_{\text {tot }}$ | $\theta_{\text {min }}$ | Efficiency |
| :---: | :---: | :---: |
| 1 GeV | 20 mrad | $73 \%(48 \%)$ |
| 1 GeV | 100 mrad | $46 \%$ |
| 1 GeV | 150 mrad | $38 \%$ |
| 2.5 GeV | 20 mrad | $40 \%(18 \%)$ |
| 2.5 GeV | 100 mrad | $17 \%$ |
| 2.5 GeV | 150 mrad | $10 \%$ |
| 5 GeV | 20 mrad | $25 \%(8 \%)$ |
| 5 GeV | 100 mrad | $6 \%$ |
| 5 GeV | 150 mrad | $2 \%$ |

Table 1: Detection efficiency for $J / \psi$ 's decaying into muons or electrons (values between brackets take into account the 50 GeV momentum requirement for the electrons in the region of the mask). A $J / \psi$ is considered lost if one of its decay particles is not within the acceptance region.

A realistic choice is $p_{t o t}>2.5 \mathrm{GeV}$, for which the efficiency is $40 \%$ for the channel $J / \psi \rightarrow \mu^{+} \mu^{-}$, for the full angular region, and $17 \%$ for the region $\theta_{\text {min }}>100 \mathrm{mrad}$. Taking into account the branching ratio for $J / \psi \rightarrow \mu^{+} \mu^{-}(\sim 6 \%)$, the total number of detectable events in the 4 -muon channel is 43 events for $\theta_{\text {min }}>20 \mathrm{mrad}$ and 100 $\mathrm{fb}^{-1}$. When also the electron channels are included, this increases to 88 events. In case $\theta_{\min }>100 \mathrm{mrad}$, one expects 8 events (4-muons) and 32 events (muon and electron channels). These numbers show that this measurement will need high luminosity and an as large as possible angular acceptance.

These efficiencies were estimated for events with four-momentum transfer $t=0$. The detectability increases with increasing $t$ : at $|t|=4 \mathrm{GeV}^{2}$, the efficiency is a factor four better. The efficiency decreases with increasing $W$ due to the kinematics: for $p_{\text {tot }}>2.5 \mathrm{GeV}, \theta_{\text {min }}>100 \mathrm{mrad}$, and $W>100 \mathrm{GeV}$, the $J / \psi$ detection efficiency is reduced to $5 \%$.


Figure 3: The $W$ distribution of the generated events and the distribution of the angle of the decay muons with respect to the beam direction.

In conclusion, the doubly exclusive $J / \psi$ production may be successfully studied at TESLA providing important tests of our understanding of the QCD pomeron dynamics. A high integrated luminosity is however indispensable ( $>100 \mathrm{fb}^{-1}$ ) and a good angular acceptance for the muons and/or electrons down to small angles ( $\theta_{\text {min }} \sim 20 \mathrm{mrad}$ ) mandatory.

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