Standard Model Vacua in Heterotic M–Theory

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Abstract

We present a class of $N = 1$ supersymmetric "standard" models of particle physics, derived directly from heterotic M–theory, that contain three families of chiral quarks and leptons coupled to the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. These models are a fundamental form of "brane world" theories, with an observable and hidden sector each confined, after compactification on a Calabi–Yau threefold, to a BPS three-brane separated by a higher dimensional bulk space with size of the order of the intermediate scale. The requirement of three families, coupled to the fundamental conditions of anomaly freedom and supersymmetry, constrains these models to contain additional five-branes located in the bulk space and wrapped around holomorphic curves in the Calabi–Yau threefold.

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1 Introduction

In fundamental work, it was shown by Hořava and Witten [1, 2] that if M–theory is compactified on the orbifold S^1/Z_2, a chiral $N = 1$, E_8 gauge supermultiplet must exist in the twisted sector of each of the two ten-dimensional orbifold fixed planes. It is important to note that, in this theory, the chiral gauge matter is confined solely to the orbifold planes, while pure supergravity inhabits the bulk space between these planes. Thus, Hořava-Witten theory is a concrete and fundamental representation of the idea of a "brane world".

Witten then showed [3] that, if further compactified to four dimensions on a Calabi–Yau three-fold, the $N = 1$ supersymmetric low–energy theory exhibits realistic gauge unification and gravitational coupling strength provided the Calabi–Yau radius, R, is of the order of 10^{16}GeV and that the orbifold radius, ρ, is larger than R. Thus, Hořava–Witten theory has a "large" internal bulk dimension, although it is of order the intermediate scale and not the TeV size bulk dimensions, or larger, discussed recently [4].

When compactifying the Hořava–Witten theory, it is possible that all or, more likely, a subset of the E_8 gauge fields do not vanish classically in the Calabi–Yau threefold directions. Since these gauge fields "live" on the Calabi–Yau manifold, $3 + 1$-dimensional Lorentz invariance is left unbroken. Furthermore, by demanding that the associated field strengths satisfy the constraints $F_{ab} = F_{\bar{a}\bar{b}} = g^{a\bar{b}}F_{a\bar{b}} = 0$, $N = 1$ supersymmetry is preserved. However, these gauge field vacua do spontaneously break the E_8 gauge group as follows. Suppose that the non-vanishing gauge fields are associated with the generators of a subgroup G, where $G \times H \subseteq E_8$. Then the E_8 gauge group is spontaneously broken to H, which is the commutant subgroup of G in E_8. This mechanism of gauge group breaking allows one, in principle, to reduce the E_8 gauge group to smaller and phenomenologically more interesting gauge groups such as unification groups E_6, $SO(10)$ and $SU(5)$ as well as the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The spontaneous breaking of E_8 to E_6 by taking $G = SU(3)$ and identifying it with the spin connection of the Calabi–Yau threefold, the so-called "standard embedding", was discussed in [1, 2]. A general discussion of non-standard embeddings in this context and their low energy implications was presented in [5, 6]. We will refer to Hořava–Witten theory compactified to lower dimensions with arbitrary gauge vacua as heterotic M–theory.

It is, therefore, of fundamental interest to know, given a Calabi–Yau threefold X, what non-Abelian gauge field vacuum configurations associated with a subgroup $G \subseteq E_8$ can be defined on it. One approach to this problem is to simply attempt to solve the six-dimensional Yang–Mills equations with the appropriate boundary conditions subject to the above constraints on the field strengths. However, given the complexity of Calabi–Yau threefolds, this approach becomes very

difficult at best and is probably untenable. One, therefore, must look for an alternative construction of these Yang-Mills connections. Such an alternative was presented by Donaldson [7] and Uhlenbeck and Yau [8], who recast the problem in terms of holomorphic vector bundles. These authors prove that for every semi-stable holomorphic vector bundle with structure group G over X, there exists a solution to the six-dimensional Yang–Mills equations satisfying the above constraints on the field strengths, and conversely. Thus, the problem of determining the allowed gauge vacua on a Calabi–Yau threefold is replaced by the problem of constructing semi–stable holomorphic vector bundles over the same threefold.

It was shown in recent publications [9, 10, 11, 12, 13], relying heavily on work on holomorphic vector bundles by several authors [14, 15, 16], that a wide class of semi-stable holomorphic vector bundles with structure groups $SU(n) \subset E_8$ can be explicitly constructed over elliptically fibered Calabi–Yau threefolds. The restriction to $SU(n)$ subgroups was for simplicity, other structure subgroups being possible as well. Thus, using holomorphic vector bundles and the Donaldson, Uhlenbeck, Yau theorem, it has been possible to classify and give the properties of a large class of $SU(n)$ gauge vacua even though the associated solutions of the Yang–Mills equations are unknown.

As presented in [9, 10], three–family vacua with phenomenologically interesting unification groups such as E_6, $SO(10)$ and $SU(5)$ could be obtained, corresponding to vector bundle structure groups $SU(3)$, $SU(4)$ and $SU(5)$ respectively. However, it was not possible to break E_8 directly to the standard gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ in this manner. A natural solution to this problem is to use non-trivial Wilson lines to break the GUT group down to the standard gauge group [17, 18]. This requires that the fundamental group of the Calabi–Yau threefold be non-trivial. Unfortunately, one can show that all elliptically fibered Calabi–Yau threefolds are simply connected, with the exception of such threefolds over an Enriques base which, however [10], is not consistent with the requirement of three families of quarks and leptons.

With this in mind, recall that an elliptic fibration is simply a torus fibration that admits a zero section. We were able to show that it is the requirement of a zero section that severely restricts the fundamental group of the threefold to be, modulo the one exception mentioned above, trivial. Hence, if one lifts the zero section requirment, and considers holomorphic vector bundles over torus-fibered Calabi–Yau threefolds without section, then one expects to find non-trivial first homotopy groups and Wilson lines in vacua that are consistent with the three-family requirement. In [19] we gave the relevant mathematical properties of a specific class of torus-fibered Calabi–Yau threefolds without section and constructed holomorphic vector bundles over such threefolds. We then used these results to explicitly construct a number of three-family vacua with unification group $SU(5)$ which is spontaneously broken to the standard gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ by Wilson lines.

The results of [19] represent $N = 1$ "standard" models of particle physics derived directly from M–theory. Each of these vacua has three families of chiral quarks and leptons coupled to the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. As discussed above, this "observable sector" lives on a $3 + 1$ dimensional "brane world". It was shown in [20, 21] that this $3 + 1$ dimensional space is the worldvolume of a BPS three–brane. It is separated from a "hidden sector" three–brane by a bulk space with an intermediate scale "large" extra dimension. The requirement of three families, coupled to the fundamental condition of anomaly freedom and supersymmetry, constrains the theory to admit an effective class describing the wrapping of additional five-branes around holomorphic curves in the Calabi–Yau threefold. These five-branes "live" in the bulk space and represent new, non-perturbative aspects of particle physics vacua.

In this talk, we present the rules for building phenomenological particle physics "standard" models in heterotic M-theory on torus-fibered Calabi–Yau threefolds without section realized as quotient manifolds $Z = X/\tau_X$. These quotient threefolds have a non-trivial first homotopy group $\pi_1(Z) = \mathbb{Z}_2$. Specifically, we construct three-family particle physics vacua with GUT group $SU(5)$. Since $\pi_1(Z) = \mathbb{Z}_2$, these vacua have Wilson lines that break $SU(5)$ to the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. We then present several explicit examples of these "standard" model vacua for the base surface $B = F_2$ of the torus fibration. We refer the reader to [19] for the mathematical details and a wider set of examples, including the base $B = dP_3$.

2 Rules for Realistic Particle Physics Vacua

In this section, we give the rules required to construct realistic particle physics vacua, restricting our results to vector bundles with structure group $SU(n)$ for n odd. The rules presented here lead to $N = 1$ supersymmetric theories with three families of quarks and leptons with the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$.

The first set of rules deals with the selection of the elliptically fibered Calabi–Yau threefold X with two sections, the choice of the involution and constraints on the vector bundles, such that the bundles descend to vector bundles on $Z = X/\tau_X$. If one was using this construction to construct vector bundles for each of the two E_8 groups in Hořava-Witten theory, then this first set of constraints is applicable to each bundle individually. The rules are

- Two Section Condition: Choose an elliptically fibered Calabi–Yau threefold X which admits two sections σ and ξ. This is done by selecting the base manifold B of X to be a 1) del Pezzo, 2) Hirzebruch, 3) blown-up Hirzebruch or 4) an Enriques surface. The threefold X with two

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sections is then specified by its Weierstrass model with an explicit choice of

$$g_2 = 4(a^2 - b), \qquad g_3 = 4ab. \tag{2.1}$$

The discriminant is then given by

$$\Delta = \Delta_1 \Delta_2^2, \tag{2.2}$$

where

$$\Delta_1 = a^2 - 4b, \qquad \Delta_2 = 4(2a^2 + b). \tag{2.3}$$

- Choice of Involution: Using the properties of the base, explicitly specify an involution τ_B on B. Now choose sections a and b to be invariant under τ_B. This allows one to construct an involution τ_X on X. Find the set of fixed points $\mathcal{F}_{\tau_B}$ under τ_B and show that

$$\mathcal{F}_{\tau_B} \cap \{\Delta = 0\} = \emptyset. \tag{2.4}$$

- Bundle Constraint: Consider semi-stable holomorphic vector bundles V over X. To construct any such vector bundle one must specify a divisor class η in the base B as well as coefficients λ and κ_i. These coefficients satisfy

$$\lambda - \tfrac{1}{2} \in \mathbb{Z}, \qquad \kappa_i - \tfrac{1}{2}m \in \mathbb{Z}, \tag{2.5}$$

with m an integer. Furthermore, we must have that

$$\eta \text{ is effective} \tag{2.6}$$

as a class on B.

- Bundle Involution Condition: In order for V to descend to a vector bundle V_Z over Z, the class η in B and the coefficients κ_i must satisfy the constraints

$$\begin{aligned} \tau_B(\eta) &= \eta, \\ \sum_i \kappa_i &= \eta \cdot c_1 \end{aligned} \tag{2.7}$$

The second set of rules is directly particle physics related. The first of these is the requirement that the theory have three families of quarks and leptons. The number of generations associated with the vector bundle V_Z over Z is given by

$$N_{\text{gen}} = \tfrac{1}{2}c_3(V_Z). \tag{2.8}$$

Requiring $N_{\text{gen}} = 3$ leads to the following rule for the associated vector bundle V over X.

- Three-Family Condition: To have three families we must require

$$6 = \lambda\eta(\eta - nc_1). \tag{2.9}$$

The second such rule is associated with the anomaly cancellation requirement that

$$[W_Z] = c_2(TZ) - c_2(V_{Z1}) - c_2(V_{Z2}), \tag{2.10}$$

where $[W_Z]$ is the class associated with non-perturbative five-branes in the bulk space of the Hořava-Witten theory. Vector bundles V_{Z1} and V_{Z2} are located on the "observable" and "hidden" orbifold planes respectively. In this talk, for simplicity, we will always take V_{Z2} to be the trivial bundle. Hence, gauge group E_8 remains unbroken on the "hidden" sector, $c_2(V_{Z2})$ vanishes and condition (2.10) simplifies accordingly. Using the definition

$$[W_Z] = \frac{1}{2}q_*[W], \tag{2.11}$$

condition (2.10) can be pulled-back onto X to give

$$[W] = c_2(TX) - c_2(V). \tag{2.12}$$

It follows that

$$[W] = \sigma_* W_B + c(F - N) + dN \tag{2.13}$$

where

$$W_B = 12c_1 - \eta \tag{2.14}$$

and

$$c = c_2 + \left(\frac{1}{24}(n^3 - n) + 11\right)c_1^2 - \frac{1}{2}\left(\lambda^2 - \frac{1}{4}\right)n\eta\,(\eta - nc_1) - \sum_i \kappa_i^2, \tag{2.15}$$

$$d = c_2 + \left(\frac{1}{24}(n^3 - n) - 1\right)c_1^2 - \frac{1}{2}\left(\lambda^2 - \frac{1}{4}\right)n\eta\,(\eta - nc_1) - \sum_i \kappa_i^2 + \sum_i \kappa_i. \tag{2.16}$$

The class $[W_Z]$ must represent an actual physical holomorphic curve in the Calabi–Yau threefold Z since physical five-branes are required to wrap around it. Hence, $[W_Z]$ must be an effective class and, hence, its pull-back $[W]$ is an effective class in the covering threefold X. This leads to the following rule.

- Effectiveness Condition: For $[W]$ to be an effective class, we require

$$W_B \text{ is effective in } B, \quad c \geq 0, \quad d \geq 0. \tag{2.17}$$

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Finally, consider subgroups of E_8 of the form

$$G \times H \subset E_8. \tag{2.18}$$

If G is chosen to be the structure group of the vector bundle, then, naively, one would expect the commutant subgroup H to be the subgroup preserved by the bundle. However, Rajesh, Berglund and Mayr [22] have shown that this will be the case if and only if the vector bundle satisfies a further constraint. If this constraint is not satisfied, then the actual preserved subgroup of E_8 will be larger than H. Although not strictly necessary, we find it convenient in model building to demand that this constraint hold.

- Stability Constraint: Let $G \times H \subset E_8$ and G be the structure group of the vector bundle. Then H will be the largest subgroup preserved by the bundle if and only if

$$\eta > nc_1. \tag{2.19}$$

If one follows the above rules, then the vacua will correspond to a grand unified theory with unification group H and three families of quarks and leptons. In this talk, we will only consider the maximal subgroup $SU(5) \times SU(5) \subset E_8$. We then choose

$$G = SU(5). \tag{2.20}$$

Therefore, the unification group will be

$$H = SU(5). \tag{2.21}$$

However, these vacua correspond to vector bundles over the quotient torus-fibered Calabi–Yau threefold Z which has non-trivial homotopy group

$$\pi_1(Z) = \mathbb{Z}_2. \tag{2.22}$$

It follows that the GUT group will be spontaneously broken to the standard model gauge group

$$SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y, \tag{2.23}$$

if we adopt the following rule.

- Standard Gauge Group Condition: Assume that the bundle contains a non-vanishing Wilson line with generator

$$\mathcal{G} = \begin{pmatrix} \mathbf{1}_3 & \\ & -\mathbf{1}_2 \end{pmatrix}. \tag{2.24}$$

Armed with the above rules, we now turn to the explicit construction of phenomenologically relevant non-perturbative vacua.

3   Three Family Models

We begin by choosing the base of the Calabi–Yau threefold to be the Hirzebruch surface

$$B = F_2. \tag{3.1}$$

As discussed in the Appendix of [10], the Hirzebruch surfaces are $\mathbb{CP}^1$ fibrations over $\mathbb{CP}^1$. There are two independent classes on F_2, the class of the base $\mathcal{S}$ and of the fiber $\mathcal{E}$. Their intersection numbers are

$$\mathcal{S} \cdot \mathcal{S} = -2, \qquad \mathcal{S} \cdot \mathcal{E} = 1, \qquad \mathcal{E} \cdot \mathcal{E} = 0. \tag{3.2}$$

The first and second Chern classes of F_2 are given by

$$c_1(F_2) = 2\mathcal{S} + 4\mathcal{E}, \tag{3.3}$$

and

$$c_2(F_2) = 4. \tag{3.4}$$

We now need to specify the involution τ_B on the base and how it acts on the classes on B. We recall that there is a single type of involution on $\mathbb{CP}^1$. If (u, v) are homogenous coordinates on $\mathbb{CP}^1$, the involution can be written as $(u, v) \to (-u, v)$. This clearly has two fixed points, namely the origin $(0, 1)$ and the point at infinity $(1, 0)$ in the u-plane. To construct the involution τ_B, we combine an involution on the base $\mathbb{CP}^1$ with one on the fiber $\mathbb{CP}^1$. Thus $\mathcal{F}_{\tau_B}$ contains four fixed points.

To ensure that we can construct a freely acting involution τ_X from τ_B, we need to show that the discriminant curve can be chosen so as not to intersect these fixed points. We recall that the two components of the discriminant curve are given by

$$\Delta_1 = a^2 - 4b, \qquad \Delta_2 = 4\left(2a^2 + b\right), \tag{3.5}$$

and that parameters a and b are sections of K_B^{-2} and K_B^{-4} respectively, where K_B is the canonical bundle of the base. In order to lift τ_B to an involution of X, we required that

$$\tau_B(a) = a, \qquad \tau_B(b) = b. \tag{3.6}$$

This restricts the allowed sections a and b and, consequently, the form of Δ_1 and Δ_2. One can show that, for a generic choice of a and b satisfying conditions (3.6), there is enough freedom so that the discriminant curves do not intersect any of the fixed points.

We now want to consider curves η in F_2 that are invariant under the involution τ_B. This can be done by first determining how this involution acts on the effective classes. We find that the involution preserves both $\mathcal{S}$ and $\mathcal{E}$ separately, so that

$$\tau_B(\mathcal{S}) = \mathcal{S}, \qquad \tau_B(\mathcal{E}) = \mathcal{E}. \tag{3.7}$$

Since any class η is a linear combination of $\mathcal{S}$ and $\mathcal{E}$, we see that an arbitrary η satisfies $\tau_B(\eta) = \eta$.

We can now search for η, λ and κ_i satisfying the three family, effectiveness and stability conditions given above. We find that there are two classes of solutions

$$
\begin{aligned}
\text{solution 1:} \quad & \eta = 14\mathcal{S} + 22\mathcal{E}, \quad \lambda = \tfrac{3}{2}, \\
& \sum_i \kappa_i = \eta \cdot c_1 = 44, \quad \sum_i \kappa_i^2 \leq 60, \\
\text{solution 2:} \quad & \eta = 24\mathcal{S} + 30\mathcal{E}, \quad \lambda = -\tfrac{1}{2}, \\
& \sum_i \kappa_i = \eta \cdot c_1 = 60, \quad \sum_i \kappa_i^2 \leq 76.
\end{aligned}
\tag{3.8}
$$

First note that the coefficients λ satisfy the bundle constraint (2.5). Furthermore, one can find many examples of κ_i with $i = 1, \ldots, 4\eta \cdot c_1$, satisfying the bundle constraint (2.5), the given conditions on $\sum_i \kappa_i^2$ and the invariance condition $\sum_i \kappa_i = \eta \cdot c_1$.

Using $n = 5$, (3.3), (3.8) and the intersection relations (3.2), one can easily verify that both solutions satisfy the three-family condition (2.9).

Next, from (2.13), (2.14), (2.15) and (2.16), as well as $n = 5$, (3.3), (3.4), (3.8) and the intersection relations (3.2), we can calculate the five-brane curves W associated with each of the solutions. We find that

$$
\begin{aligned}
\text{solution 1:} \quad & [W] = \sigma_* \left(10\mathcal{S} + 26\mathcal{E}\right) + \left(112 - k\right)\left(F - N\right) + \left(60 - k\right)N, \\
\text{solution 2:} \quad & [W] = \sigma_* \left(18\mathcal{E}\right) + \left(132 - k\right)\left(F - N\right) + \left(76 - k\right)N,
\end{aligned}
\tag{3.9}
$$

where

$$k = \sum_i \kappa_i^2 \tag{3.10}$$

It follows that the base components for $[W]$ are given by

$$
\begin{aligned}
\text{solution 1:} \quad & W_B = 10\mathcal{S} + 26\mathcal{E}, \\
\text{solution 2:} \quad & W_B = 18\mathcal{E},
\end{aligned}
\tag{3.11}
$$

which are both effective. Furthermore, we note that for each five-brane curve the c and d coefficients of classes $F - N$ and N respectively are non-negative integers (given the constraints on k). Hence, effectiveness condition (2.17) is satisfied.

Finally, note that for $n = 5$ the stability condition becomes $\eta > 5c_1$. In both of the above solutions

$$\eta > 5c_1 = 10\mathcal{S} + 20\mathcal{E} \tag{3.12}$$

so that the stability condition is satisfied. Note that this condition is consistent with the somewhat stronger condition used in [19] since η and c_1 have integer coefficients.

We conclude that, over a Hirzebruch base $B = F_2$, one can construct torus-fibered Calabi–Yau threefolds, Z, without section with non-trivial first homotopy group $\pi_1(Z) = \mathbb{Z}_2$. Assuming a trivial gauge vacuum on the hidden brane, we have shown that we expect these threefolds to admit precisely two classes of semi-stable holomorphic vector bundles V_Z, (3.8), associated with an $N = 1$ supersymmetric theory with three families of chiral quarks and leptons and GUT group $H = SU(5)$ on the observable brane world. Since $\pi_1(Z) = \mathbb{Z}_2$, Wilson lines break this GUT group as

$$SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y, \tag{3.13}$$

to the standard model gauge group. Anomaly cancellation and supersymmetry require the existence of non-perturbative five-branes in the extra dimension of the bulk space. These five-branes are wrapped on holomorphic curves in Z whose homology classes, (3.9), are exactly calculable.

Acknowledgments

References

[1] P. Hořava and E. Witten, *Nucl. Phys.* **B460** (1996) 506.

[2] P. Hořava and E. Witten, *Nucl. Phys.* **B475** (1996) 94.

[3] E. Witten, *Nucl. Phys.* **B471** (1996) 135.

[4] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B429** (1998) 263; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B436** (1998) 257; L. Randall and R. Sundrum, *An Alternative to Compactification*, hep-th/9906064; K. R. Dienes, E. Dudas and T. Gherghetta, *Phys. Lett.* **B436** (1998) 55.

[5] A. Lukas, B. A. Ovrut and D. Waldram, *Phys.Rev.* **D59** (1999) 106005.

[6] Z. Lalak, S. Pokorski and S. Thomas, *Nucl. Phys.* **B549**, (1999) 63.

[7] S. Donaldson, *Proc. London Math. Soc.* **3** 1 (1985).

[8] K. Uhlenbeck and S.-T. Yau, Comm. Pure App. Math.**39** 257 (1986), **42** 703 (1986).

[9] R. Donagi, A. Lukas, B. A. Ovrut and D. Waldram, *JHEP* **05** (1999) 018.

[10] R. Donagi, A. Lukas, B.A. Ovrut, D. Waldram, *JHEP* **06** (1999) 034.

[11] R. Donagi, B.A. Ovrut and D. Waldram, *Moduli Spaces of Fivebranes on Elliptic Calabi-Yau Threefolds*, hep-th/9904054.

[12] G. Curio, *Phys. Lett.* **B435** 39 (1998).

[13] B. Andreas, *JHEP* **01** (1999) 011.

[14] R. Friedman, J. Morgan and E. Witten, *Commun. Math. Phys.* **187**, 679 (1997).

[15] R. Donagi, *Asian. J. Math.* **1**, 214 (1997).

[16] M. Bershadsky, A. Johansen, T. Pantev and V. Sadov, *Nuc. Phys.* **B505** 165 (1997).

[17] J.D. Breit, B.A. Ovrut, G.C. Segre, *Phys. Lett.* **158B** (1985) 33.

[18] E. Witten, *Nucl. Phys.* **B258** (1985) 75.

[19] R. Donagi, B.A. Ovrut, T. Pantev and D. Waldram, hep-th/9912208.

[20] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, *Phys. Rev.* **D59** (1999) 086001.

[21] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, *Nucl. Phys* **B552** (1999) 246.

[22] G. Rajesh, *JHEP* **12** (1998) 018; P. Berglund and P. Mayr, *Adv. Theor. Math. Phys.* **2** (1999) 1307-1372.