TRANSVERSE BEAM STABILITY WITH "ELECTRON LENS"*

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Abstract

Stability analysis is presented for an antiproton beam interacting with an electron beam of an "electron lens" proposed as a beam-beam tune shift compensator. Coherent antiproton-electron interaction causes coupling of the antiproton synchrobetatron modes which may lead to a transverse mode coupling instability (TMCI). Analytical studies and numerical simulations of this effect are presented.

1 INTRODUCTION

An "electron lens" was proposed to compensate beam-beam tune shift in the Tevatron collider [2]. A tune shift of antiprotons on electron beam with total current J_e, radius a_e, length L_e, is equal to

$$\xi^e_{x,y} \approx -\frac{\beta_{x,y}}{4\pi}\frac{2(1+\beta_e)J_e L_e r_{\bar p}}{e v_e a_e^2 \gamma_{\bar p}}, \qquad (1)$$

here $r_{\bar p} = e^2/(M_{\bar p}c^2) \approx 1.53\cdot 10^{-18}$m is the (anti)proton classical radius, $\gamma_{\bar p}$ is relativistic antiproton factor, $v_e = c\beta_e$ is electron beam velocity, $\beta_{x,y}$ is the beta function at the set-up location.

The electron beam create a transverse impedance that can result in collective instabilities of the antiproton bunch. The electron beam is generated by an electron gun cathode, transported through the interaction region, and absorbed in the collector. Therefore, each portion of electrons passes through the $\bar p$ beam only once, and only short distance transverse wake fields are of interest. When the bunch head collides off the electron beam center, it causes electron motion and, as a result, the electron beam acquires a displacement at the moment when it interacts with the tail of the $\bar p$ bunch. This interaction can lead to the strong head-tail instability. To suppress it, a longitudinal magnetic field in the interaction region is assumed to be applied. The magnetic field couples the electron transverse degrees of freedom, transforming a kick in one direction into an offset in another. In the result, the magnetized electron medium creates both conventional and skew wakes.

2 TWO-MODE MODEL

To find the dipole wake function, let us consider a thin antiproton slice with a charge q and transverse offset Δx traveling through the electron beam. After interaction with the slice, electrons acquire a transverse velocity

$$v_{xe} = \frac{2eq\Delta x}{a^2\gamma_e mc}, \qquad (2)$$

where m is the electron mass. Such a kick causes transverse Larmor oscillations in a longitudinal magnetic field B, and after a time interval t, the resulting electron transverse offsets are:

$$x_e = \frac{v_{xe}}{\omega_L}\sin(\omega_L t); \quad y_e = \frac{v_{xe}}{\omega_L}\Big(1-\cos(\omega_L t)\Big), \qquad (3)$$

where $\omega_L = eB/(\gamma_e mc)$ stands for the Larmor frequency. The originally horizontal displacement Δx resulted in both horizontal and vertical displacements. The antiprotons at the distance s behind the slice will experience momentum changes

$$\begin{aligned}\Delta p_x(s) &= -\tfrac{eq}{c}(W_d(s)\Delta x - W_s(s)\Delta y)\\ \Delta p_y(s) &= -\tfrac{eq}{c}(W_s(s)\Delta x + W_d(s)\Delta y)\end{aligned} \qquad (4)$$

where we introduced direct wake function $W_d(s)$ and skew $W_s(s)$ wake function:

$$\begin{aligned}&W_d(s) = W\sin(ks), \quad W_s(s) = W(1-\cos(ks)), \text{for } s>0\\ &W = 4\pi n_e L_e/(Ba^2), \quad k = \omega_L/((1+\beta_e)c)\end{aligned} \qquad (5)$$

Depending on the parameters, one or the other of the two wake functions (5) can give a dominant influence on the antiproton beam stability. The direct wake effects are suppressed if there are many Larmor oscillations periods over the $\bar p$ bunch length σ_s, while the skew force impact decreases with increasing the $x-y$ detuning.

For the parameters under study, the skew wake is found to be more dangerous. To damp the instability, the longitudinal magnetic field B has to be high enough; a two-mode model gives the threshold condition as

$$B \geq B_{th} \approx 2.0\frac{eN_{\bar p}\sqrt{\xi_x\xi_y}}{a^2\sqrt{\Delta\nu\,\min(\Delta\nu, 2.4\nu_s)}}. \qquad (6)$$

For $\xi_x = \xi_y = 0.01$, $N_{\bar p} = 6\cdot 10^{10}, \nu_s = 0.001, \Delta\nu = 0.01, a = 1$ mm it comes out $B_{th} = 12$ kG.

In addition to these simplified analytical calculations, A multi-mode numerical algorithm of Ref. [3] was applied for the stability study. Typical eigenvalues behavior is presented in Fig. 1.

Fig.2 shows the tune shift threshold ξ_e for the first coupling modes versus the tune split in units of the synchrotron tune $\Delta\nu = (\nu_x - \nu_y)$ while the vertical tune is equal to .555. The threshold grows linearly until $\Delta\nu \approx (2-2.5)\nu_s$ and then is approximately proportional to $\sqrt{\Delta\nu}$ - in a good agreement with the two mode model formula (6).

Transverse widening of the electron beam was found to suppress the instability, decreasing the threshold field as $B_{th} \propto a_e^{-2}$.

3 TRACKING SIMULATIONS

Three dimensional numerical simulations of the effects have been done with ECWAKE code written in FORTRAN.

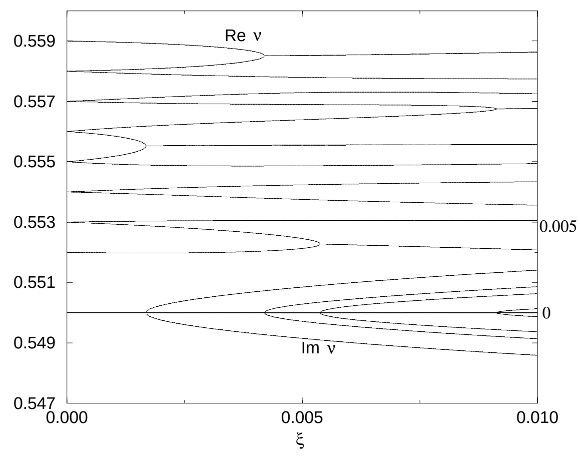

Figure 1: Eigenfrequencies (tunes) of the antiproton bunch oscillation modes versus the antiproton betatron tune shift due to electron beam ξ_e (horizontal axis). Vertical scale on the left is for fractional part of the tunes $Re\nu$ (upper series of lines), the right side scale is for imaginary part of the tunes $Im\nu$ (lower series of lines).

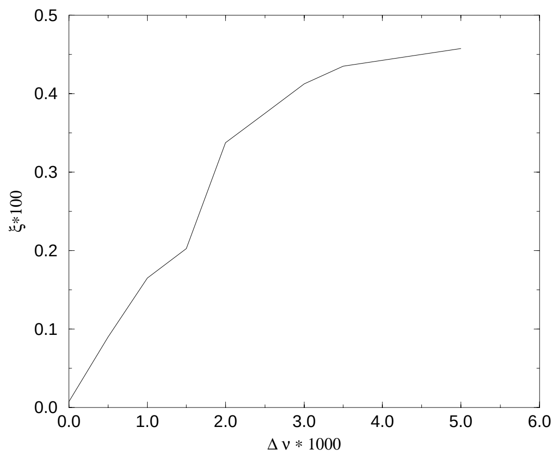

Figure 2: Threshold antiprotons tune shift ξ_e (vertical axis) due to the electron beam versus the difference of antiproton horizontal and vertical tunes $\Delta\nu = \nu_x - \nu_y$. $B = 10$ kG, $\nu_s = 0.001$, $N_{\bar{p}} = 6 \cdot 10^{10}$.

Fig.3 shows the threshold strength of solenoidal magnetic field B_{th} vs. electron beam intensity parameter ξ_e for antiproton bunch population equal to $N_{\bar{p}} = (1, 6, 10) \cdot 10^{10}$ - lower, middle and upper curves, respectively. We define the threshold as the value of B which results in more than 10-fold increase of the initial centroid betatron amplitude over the first 10,000 turns. One can see, that the field is approximately proportional to both ξ_e and $N_{\bar{p}}$ in accordance with Eq.(6).

Dependence of the threshold on the synchrotron tune ν_s

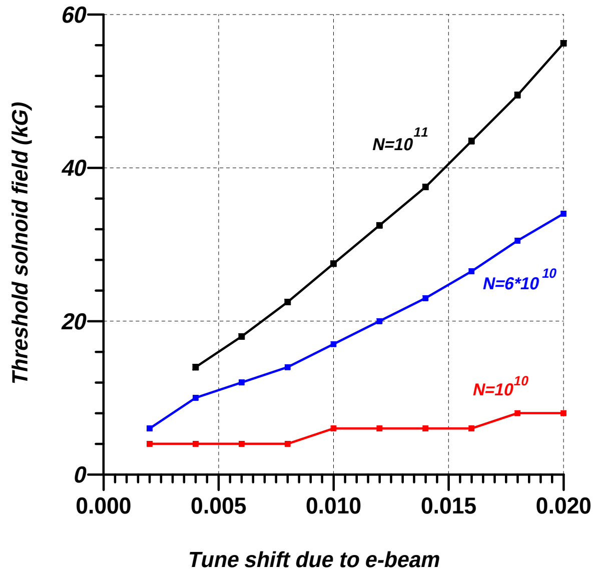

Figure 3: Threshold solenoid field B_{thr} vs tune shift due to electrons $|\xi_e|$ at different bunch populations $N_{\bar{p}} = 1, 6, 10 \cdot 10^{10}$. Focusing lattice tunes $\nu_x = 0.585$, $\nu_y = 0.575$, synchrotron tune $\nu_s = 0.0012$, maximum tune spread $\delta\nu = 0$, the rms size of $\bar{p}$ beam $\sigma_{\bar{p}} = 0.7$ mm.

is depicted in Fig.4. Dots are simulation results with $\nu_x = 0.585$, $\nu_y = 0.575$, $\xi_e = -0.01$, $\delta\nu = 0.002$, $N_{\bar{p}} = 6 \cdot 10^{10}$, $\sigma_{\bar{p}} = 0.7$ mm. The solid line represents a fit $B_{thr} = 17.5[kG]/\sqrt{\nu_s/0.001}$ in line with the two-mode prediction Eq.(6).

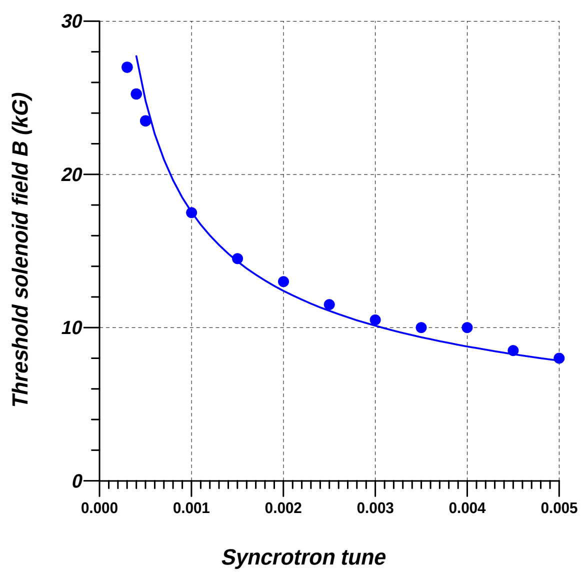

Figure 4: Threshold magnetic field vs synchrotron tune ν_s. Solid line is for $B_{thr} = 12.4[kG]/\sqrt{\nu_s}$. $\nu_x = 0.585$, $\nu_y = 0.575$, $\xi_e = -0.01$, $\delta\nu = 0$, $N_{\bar{p}} = 6 \cdot 10^{10}$, $\sigma_{\bar{p}} = 0.7$ mm.

In order to evaluate importance of the oscillation part of the wakes Eq.5, we performed similar scan without constant part of the skew wake, i.e. with $W_d(s) = W\sin(ks)$ and $W_s(s) = -W\cos(ks)$ and found that about 5 times smaller solenoid field is required for stability. It confirms decisive role of the the constant part of skew wake that is a basic assumption of the two-mode model in Section II.

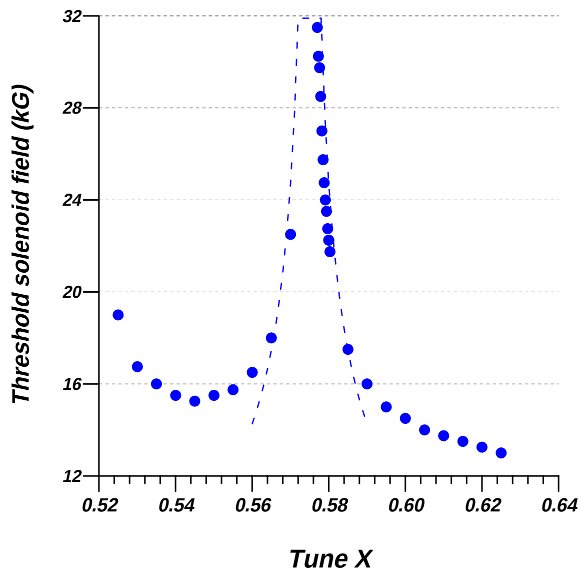

Figure 5: Threshold magnetic field vs horizontal tune ν_x. Dashed line corresponds to $B_{thr} \propto 1/\sqrt{|\nu_x - \nu_y|}$; $\nu_y = 0.575$, $\nu_s = 0.001$, $\xi_e = -0.01$, $\delta\nu = 0.0$, $N_{\bar{p}} = 6 \cdot 10^{10}$, $\sigma_{\bar{p}} = 0.7$ mm.

It is found that the TMCI threshold greatly depends on operation point ν_x, ν_y. Fig.5 presents results of scanning of the horizontal tune ν_x from 0.52 to 0.63 while the vertical tune is $\nu_y = 0.575$. In close vicinity of the coupling resonance $\Delta\nu = |\nu_x - \nu_y| \leq 15\nu_s$ the threshold magnetic field depends on ν_s approximately as $\propto 1/|\Delta\nu|^{\kappa}$, where $2/5 < \kappa < 1/2$. The threshold also goes up near half-integer resonance $\nu_x \rightarrow 0.5$.

In order to compare with the two mode model, one can fit B_{thr} in the form similar to Eq.(6):

$$B_{th} \approx \frac{0.95 e N_{\bar{p}} \xi_e}{\sigma_{\bar{p}}^2 \sqrt{|\nu_x - \nu_y|\nu_s}} = \frac{17.5[kG]\frac{N_{\bar{p}}}{6\cdot 10^{10}}|\frac{\xi_e}{0.01}|}{(\frac{\sigma_{\bar{p}}[mm]}{0.7})^2\sqrt{\frac{\nu_s}{0.001}\frac{|\Delta\nu|}{0.01}}}, \quad (7)$$

- see also dashed line in Fig.5.

These results are in a reasonable agreement with the two-mode analysis and the coupled-mode calculations. The difference ($\simeq 40\%$) in numerical factors between Eq.(7) and Eq.(6) lies within the accuracy limits of the wake calculations and two-mode model.

4 CONCLUSIONS.

We have considered strong head-tail instability of the Tevatron antiproton bunch due to the beam-beam compensation set-up. The head-tail interaction takes place because of the fact that the electron beam is not rigid enough and can be displaced transversely by the bunch head particles. The resulting direct and skew wake forces act on the tail particles and, thus, can lead to the instability. We pursue three approaches to study the instability: a two-mode model with analytical calculations, more sophisticated multi-mode analysis which requires numerical solution of eigenmode equations, and straightforward macroparticle tracking. The results coincide qualitatively and rather well quantitatively agree with each other. For the parameters of the planned Tevatron beam-beam compensation experiment the $\bar{p}$ bunch intensity $eN_{\bar{p}} = 6 \cdot 10^{10}$ and its rms size $\sigma_{\bar{p}} = 0.7$ mm, the tune shift due to electron beam $\xi_e = -0.01$, the distance to the coupling resonance $\Delta\nu = |\nu_x - \nu_y| = 0.01$, and the synchrotron tune $\nu_s = 0.001$, the instability takes place if the longitudinal magnetic field in the set-up is below threshold of about $B_{thr} = 17.5$kG. Essential features of the instability are:

- the constant skew wake plays a major role in the mode coupling;
- the threshold solenoid field B_{thr} is proportional to the transverse charge density of the electron beam, to the transverse charge density of the antiproton beam, and inversely proportional to the product $\sqrt{\nu_s|\nu_x - \nu_y|}$ in vicinity of the coupling resonance $\nu_x - \nu_y = integer$;

Having the electron beam wider than the antiproton beam results in lower threshold magnetic field $B_{thr} \propto (\sigma_{\bar{p}}/a_e)^2$.

We plan to continue investigations of the instability in order to clear some inadequacies of the present studies. In particular, the following effects have to be taken into consideration:

1. non-linear forces with general current distributions in the electron and antiproton beams;
2. instability suppression due to betatron and synchrotron tune spreads;
3. higher order transverse mode coupling.

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5 REFERENCES

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