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A q-DEFORMATION OF VIRASORO AND U(1) KAC-MOODY ALGEBRAS WITH HOPF STRUCTURE

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Abstract

Using a general formalism of a q-deformation of an arbitrary Lie algebra, new kinds of q-deformed centerless Virasoro and U(1) Kac-Moody algebras are found. This q-deformation is associated to an R-matrix of unit square satisfying the quantum Yang-Baxter equation and allows a nontrivial Hopf structure. The central extension is also incorporated in this formalism.

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1   Introduction

In recent years, a lot of attention has been paid to the study of quantum deformation of Lie algebras and groups [1]-[4]. This is due to its close connection to the Yang-Baxter equation, which is central in various physically interesting models and theories, such as two dimensional conformal field theories, integrable models and so on [5]-[7]. These quantum deformations are mathematically characterized by the existence of the R-matrix satisfying the Yang-Baxter equation, and nontrivial Hopf algebra structure (nonco-commutative or noncommutative or both) and the dependence upon some parameter "q" (one or more). The latter is crucial in the deformation theory and permits the connection with the classical counterpart, which is recovered in the limit $q = 1$. Although this is the general mathematical framework of the quantum deformation, there are other attempts based only on the deformation parameters. The latter deformations are nothing more than analogies. This is the case of many q-deformation versions of the Virasoro algebra and some other infinite algebras. The first version of q-deformed centerless Virasoro algebra was introduced by Curtright and Zachos [8], its central extension was later furnished in [9]. However the existence of a Hopf structure for this algebra is still an open question. Another type of q-deformation is proposed in [10] but with a trivial Hopf structure. The Hopf structure has been investigated in [11] by introducing an additional set of generators indices.

In this paper we will follow a general way of deformation of Lie algebras to give a new type of q-deformation of the Virasoro and U(1) Kac-Moody algebras to which we associate an R-matrix satisfying the quantum Yang-Baxter equation and unit square condition. This q-deformation allows a nontrivial Hopf structure. In sect. 2, we describe the general formalism for an arbitrary Lie algebra. We generalize the antisymmetry and Jacobi identity principles in the sense of q-deformation by introducing an R-matrix of unit square and satisfying the quantum Yang-Baxter equation. Then, we construct the associated q-deformed universal enveloping algebra. To provide the latter with a nontrivial Hopf structure, we introduce additional generators which are degenerate with the unit in the undeformed case. Finally, we incorporate the central extension in this general formalism. In sects. 3 and 4 we apply the above formalism to give a new q-deformation of Virasoro and U(1) Kac-Moody algebras, respectively. Finally, sect. 5 contains some concluding remarks.

2   Quantum Lie algebra

Let us first recall the definition of the classical Lie algebra. Let $\mathcal{A}$ be a vector space over the field K ($K = R$ or C) with an additional linear operation

$$[,] : \mathcal{A} \otimes \mathcal{A} \to \mathcal{A}$$

$\mathcal{A}$ is called a Lie algebra if the following properties hold

$$[,] = -[,] \circ \sigma \qquad (antisymmetry) \tag{1}$$

$$[,](id \otimes [,])(id + \sigma^{12}\sigma^{23} + \sigma^{23}\sigma^{12}) = 0 \qquad (Jacobi\ identity) \tag{2}$$

where $\sigma(a \otimes b) = b \otimes a$, $a, b \in A$, $\sigma^{12} = \sigma \otimes 1$ and $\sigma^{23} = 1 \otimes \sigma$. When $\mathcal{A}$ is a finite dimensional Lie algebra with the basis X_i, $i = 1, \ldots, n$, the linear operation defining this Lie structure is determined by

$$[X_i, X_j] = C_{ij}{}^k X_k \qquad i, j, k = 1, \ldots. n \tag{3}$$

(with the summation over repeated indices) where $n = dim\mathcal{A}$, and the coefficients C_{ij}^k are the structure constants of $\mathcal{A}$ (with respect to the basis X_i). The relations (3) may also hold for the infinite dimensional algebras but with $n = n(X_i, X_j) \in N$. In terms of these structure constants, the conditions (1) and (2) become

$$C_{ij}{}^k = -C_{ji}{}^k \tag{4}$$

$$C_{jk}{}^m C_{im}{}^l + C_{ij}{}^m C_{km}{}^l + C_{ki}{}^m C_{jm}{}^l = 0. \tag{5}$$

Now, it is natural to think about more general operators S than the permutation operator σ. In this case, it is easy to check that the generalized analogues of the conditions (1) and (2)

$$[,]_q = -[,]_q \circ S \qquad (q-antisymmetry) \tag{6}$$

$$[,]_q(id \otimes [,]_q)(id + S^{12}S^{23} + S^{23}S^{12}) = 0 \qquad (q-Jacobi\ identity) \tag{7}$$

are satisfied if the operator S satisfies the quantum Yang-Baxter equation:

$$S^{12}S^{23}S^{12} = S^{23}S^{12}S^{23}, \tag{8}$$

and

$$S^2 = 1. \tag{9}$$

We suppose that the operator S depends on some arbitrary complex parameter q such that in the limit $q = 1$ we recover the classical Lie structure given by (1) and (2) (this is the meaning of the index q appearing in (6) and (7)). In this sense this generalization may be called a quantum deformation or a q-deformation. The corresponding q-deformed Lie algebra is denoted by $\mathcal{A}_q$. In what follows we will simply refer to (6) and (7) as q-A and q-JI, respectively. Introducing a basis (X_i) of $\mathcal{A}_q$ on which we suppose that the operator S acts linearly, namely

$$S(X_i \otimes X_j) = S_{ij}{}^{kl} X_k \otimes X_l, \tag{10}$$

the q-deformed Lie structure may be of the following form:

$$[X_i, X_j]_q = C_{ij}{}^k(q) X_k. \tag{11}$$

Then, using (10), the conditions q-A, q-JI, (8), and (9) become

$$C_{ij}{}^m(q) = -S_{ij}{}^{kl} C_{kl}{}^m(q), \tag{12}$$

$$C_{jk}{}^l(q)C_{il}{}^m(q) \; + \; S_{jk}{}^{ln}S_{il}{}^{pr}C_{rn}{}^s(q)C_{ps}{}^m(q) \; + \; S_{ij}{}^{ln}S_{nk}{}^{pr}C_{pr}{}^s(q)C_{ls}{}^m(q) \; = \; 0, \tag{13}$$

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$$S_{ij}{}^{kl} S_{lm}{}^{rs} S_{kr}{}^{uv} = S_{jm}{}^{kl} S_{ik}{}^{ur} S_{rl}{}^{vs}, \tag{14}$$

$$S_{ij}{}^{kl} S_{kl}{}^{mn} = \delta_i^m \delta_j^n. \tag{15}$$

To the q-deformed Lie algebra $\mathcal{A}_q$ we associate the q-deformed universal enveloping algebra (QUEA) $\mathcal{U}_q(\mathcal{A}_q)$ defined by

$$[X_i, X_j]_q = X_i \otimes X_j - S_{ij}{}^{kl} X_k \otimes X_l = C_{ij}^k(q) X_k, \tag{16}$$

where the first equality defines the so-called q-deformed commutator or q-deformed commutation relation. Note that an analogue deformation to (16) was derived in [12], but from a different formalism and with other conditions than (12)-(15). Using the following q-deformed commutation relations

$$[X_i, X_j \otimes X_k]_q = X_i \otimes X_j \otimes X_k - S^{23} S^{12}(X_i \otimes X_j \otimes X_k), \tag{17}$$

it follows that the q-deformed commutator (16) satisfies the q-A and q-JI properties when (8) and (9) hold. In what follows we will omit the symbol "$\otimes$" in (16).

The Hopf structure of $\mathcal{U}_q(\mathcal{A}_q)$ is defined by :

$$\Delta(X_i) = X_i \otimes 1 + M_i^j \otimes X_j, \tag{18}$$

$$\varepsilon(X_i) = 0, \tag{19}$$

$$\gamma(X_i) = -\gamma(M_i^j) X_j, \tag{20}$$

where M_i^j are additional elements of $\mathcal{U}_q(\mathcal{A}_q)$ which reduce to δ_i^j in the limit $q = 1$, and from the consistency with the basic axioms of the Hopf algebra [13], their costructure should be

$$\Delta(M_i^j) = M_i^k \otimes M_k^j, \tag{21}$$

$$\varepsilon(M_i^j) = \delta_i^j, \tag{22}$$

$$\gamma(M_i^j) = (M^{-1})_i^j. \tag{23}$$

Further, the consistency of this Hopf structure with (16) implies the following q-deformed commutation relations:

$$\begin{aligned}
S_{mn}{}^{kl} M_k^r M_l^s &= M_m^k M_n^l S_{kl}{}^{rs}, \\
M_m^k M_n^l C_{kl}^r(q) &= C_{mn}^k(q) M_k^r, \\
M_m^r X_n &= S_{mn}{}^{kl} X_k M_l^r.
\end{aligned} \tag{24}$$

Up to now we have neglected the central extension of $\mathcal{A}_q$. However, it is straightforward to incorporate it into our theory. The extended version of (16) may be written as

$$[X_i, X_j]_q = X_i X_j - S_{ij}{}^{kl} X_k X_l = C_{ij}{}^k(q) X_k + \phi_q(i,j)\tilde{c}, \tag{25}$$

$$[X_i, \tilde{c}]_q = X_i\,\tilde{c} - A_i^j\,\tilde{c}\,X_j = 0 \tag{26}$$

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where $A_i{}^j$ are arbitrary functions depending on the parameter q and reduce to δ_i^j in the limit $q = 1$. Using q-A and q-JI, it follows that the q-deformed two-cocycle $\phi_q(i, j)$ in (25) satisfies :

$$\phi_q(i, j) = -S_{ij}{}^{kl}\phi_q(k, l) \tag{27}$$

$$C_{jk}{}^l(q)\phi_q(i, l) + S_{jk}{}^{lm}S_{il}{}^{rs}C_{sm}{}^l(q)\phi_q(r, l) + S_{ij}{}^{lm}S_{mk}{}^{rs}C_{rs}{}^p(q)\phi_q(l, p) = 0. \tag{28}$$

The Hopf structure on $\mathcal{U}_q(\mathcal{A}_q)$ defined by (25), and (26) may be deduced in the same way as for the centerless case. The costructure of the generators X_i and "$M_i{}^j$" is analogous to that given in (18) - (23). For the central charge we set

$$\begin{aligned}\Delta(\tilde{c}) &= \tilde{c} \otimes 1 \; + \; H \otimes \tilde{c}, \\ \varepsilon(\tilde{c}) &= 0, \qquad \gamma(\tilde{c}) = -\gamma(H)\tilde{c}\end{aligned} \tag{29}$$

where H is an additional element which reduces to 1 in the limit $q = 1$. From the consistency with the basic axioms of the Hopf algebra, its costructure should be

$$\begin{aligned}\Delta(H) &= H \otimes H, \\ \varepsilon(H) &= 1, \qquad \gamma(H) = H^{-1}.\end{aligned} \tag{30}$$

From the consistency of this Hopf structure with (25) and (26) we find, in addition to q-deformed commutation relations analogous to (24), the following ones

$$\begin{aligned}X_i H &= A_i{}^j H X_j, \\ M_i{}^j H A_j{}^k &= A_i{}^j H M_j{}^k, \\ A_i{}^j \tilde{c} M_j{}^k &= M_i{}^k \tilde{c}, \\ M_i{}^k M_j{}^l \phi_q(k, l) &= \phi_q(i, j) H.\end{aligned} \tag{31}$$

Further, if we multiply both sides of (25) from the right by H and pull it to the left, using the first equation of (31) and supposing that $\tilde{c}H = H\tilde{c}$, we find the following conditions on the functions $A_i{}^j$:

$$\begin{aligned}A_i{}^m A_j{}^n S_{mn}{}^{rs} &= S_{ij}{}^{mn}A_m{}^r A_n{}^s, \\ A_i{}^m A_j{}^n C_{mn}^r(q) &= C_{ij}^k(q)A_k{}^r, \\ A_i{}^m A_j{}^n \phi_q(m, n) &= \phi_q(i, j).\end{aligned} \tag{32}$$

3 Quantum deformed Virasoro algebra

We will now apply the above formalism to derive a new quantum deformation of the classical Virasoro algebra with nontrivial Hopf structure. We will also investigate its q-deformed central extension. Before doing this, let us recall the classical version of this algebra for convenience and comparison. The classical Virasoro algebra without central extension is generated by the set $\{L_m, m \in \mathbb{Z}\}$ with the following Lie operation :

$$[L_m, L_n] = (m - n)L_{m+n}. \tag{33}$$

Note that if we restrict the generators to L_0, L_1, L_{-1}, we get the $SU(2)$ algebra :

$$[L_0, L_1] = -L_1, \qquad [L_0, L_{-1}] = L_{-1}, \qquad [L_1, L_{-1}] = 2L_0, \tag{34}$$

which appears as a subalgebra of the Virasoro algebra. The Hopf structure on this algebra is trivial. Its central extension is usually given by

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{(m^3 - m)}{12}c\delta_{m+n,0}, \tag{35}$$

$$[L_m, c] = 0. \tag{36}$$

Now, let us take a solution of the operator S satisfying (14) and (15) as

$$S_{mn}{}^{kl} = q^{2(n-m)}\delta_m^l \delta_n^k. \tag{37}$$

Consequently, we associate the following q-deformation of the Virasoro algebra, and its corresponding QUEA :

$$[L_m, L_n]_q = L_m L_n - q^{2(n-m)}L_n L_m = C_{mn}(q)L_{m+n} \tag{38}$$

where the q-deformed commutator is the same as the one introduced by Curtright and Zachos [8], and the q-deformed structure constants $C_{mn}(q)$ are to be determined from the following explicit q-A and the q-JI :

$$C_{mn}(q) = -q^{2(n-m)}C_{nm}(q) \tag{39}$$

$$C_{rs}(q)C_{m,r+s}(q) + q^{2(s-r)}q^{2(s-m)}C_{mr}(q)C_{s,m+r}(q) + q^{2(s-m)}q^{2(r-m)}C_{sm}(q)C_{r,m+s}(q) = 0 \tag{40}$$

from which we get

$$C_{mn}(q) = q^{2n}(m - n). \tag{41}$$

Then the q-deformed Virasoro algebra and its QUEA are given by

$$[L_m, L_n]_q = L_m L_n - q^{2(n-m)}L_n L_m = q^{2n}(m - n)L_{m+n} \tag{42}$$

where the q-deformed structure constants are different from the ones given in [8]. The Hopf structure may be deduced using (18)-(23). In this case the q-deformed commutation relations between $M_m{}^n$ and L_m are given by

$$\begin{aligned} M_m{}^r L_n &= q^{2(n-m)}L_n M_m{}^r, \\ M_n{}^r M_m{}^s &= q^{2(m-n+r-s)} M_m{}^s M_n{}^r, \\ \sum_l M_m{}^{r-l} M_n{}^l q^{2l}(r - 2l) &= q^{2n}(m - n)M_{m+n}{}^r. \end{aligned} \tag{43}$$

We suppose that for each row m there are finite nonzero elements of the infinite matrix $M_m{}^n$, so we may simply choose

$$M_m{}^n = K_m \delta_m^n. \tag{44}$$

Then, the q-deformed commutation relations (43) reduce to a simple form as follows

$$\begin{aligned} K_m L_n &= q^{2(n-m)}L_n K_m, \\ K_m K_n &= K_n K_m = K_{m+n}, \end{aligned} \tag{45}$$

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and the Hopf structure is given by

$$
\begin{aligned}
\Delta(L_m) &= L_m \otimes 1 + K_m \otimes L_m, & \Delta(K_m) &= K_m \otimes K_m \\
\varepsilon(L_m) &= 0, & \varepsilon(K_m) &= 1, \\
\gamma(L_m) &= -K_m^{-1} L_m, & \gamma(K_m) &= K_m^{-1}.
\end{aligned}
\tag{46}
$$

If we restrict the generators to the L_0, L_{-1}, and L_1 we find a new type of quantum deformation $\mathcal{U}_q(SU_q(2))$, namely

$$
\begin{aligned}
[L_0, L_1]_q &= L_0 L_1 - q^2 L_1 L_0 = -q^2 L_1, \\
[L_0, L_{-1}]_q &= L_0 L_{-1} - q^{-2} L_{-1} L_0 = q^{-2} L_{-1}, \\
[L_1, L_{-1}]_q &= L_1 L_{-1} - q^{-4} L_{-1} L_1 = 2q^{-2} L_0.
\end{aligned}
\tag{47}
$$

Note that, to construct the Hopf structure in [11] the author also introduced additional generators of the same costructure and the same q-deformed commutation relations as we find here though our q-deformation is different.

We will now introduce the central extension of (42). It may be written as :

$$
[L_m, L_n]_q = L_m L_n - q^{2(n-m)} L_n L_m = q^{2n}(m-n) L_{m+n} + \varphi_q(m,n) \hat{c},
\tag{48}
$$

and

$$
[L_m, \hat{c}]_q = L_m \hat{c} - f_m \hat{c} L_m = 0,
\tag{49}
$$

where the $A_m{}^n$ introduced in (26) is replaced by $f_m \delta_m^n$, which is compatible with (32). The q-deformed two-cocycle $\varphi_q(m,n)$ in (48) should satisfy the following conditions:

$$
\varphi_q(m,n) = -q^{2(n-m)} \varphi_q(n,m),
\tag{50}
$$

$$
q^{2s}(n-s)\varphi_q(m,n+s) + q^{2(2s-m)}(m-n)\varphi_q(s,n+m) + q^{2(n+s-m)}(s-m)\varphi_q(n,s+m) = 0, \tag{51}
$$

deduced from the q-A and q-JI, respectively. A solution of the equations (50) and (51) should be consistent with the classical limit given by $q = 1$. At this limit the classical counterpart may be the central extended Virasoro algebra (35) and (36) as well as the centerless one (33). In the first case we have

$$
\varphi_q(m,n) = q^{n-m} \frac{(\mathrm{m}^3 - \mathrm{m})}{12} \delta_{m+n,0}.
\tag{52}
$$

In the second case, we have

$$
\varphi_q(m,n) = \lambda q^{n-m} \frac{(\mathrm{m}^3 - \mathrm{m})}{12} \delta_{m+n,0}
\tag{53}
$$

where $\lambda = q - q^{-1}$, which vanishes in the classical limit. This central extension is due only to the quantum deformation fluctuations.

A solution of f_m compatible with (32), (53) and (52) is given by

$$
f_m = q^{-2m}.
\tag{54}
$$

Then, the q-deformed commutation relations (31) are now

$$
\begin{aligned}
L_m H &= q^{-2m} H L_m, \\
K_m H &= H K_m, \\
\tilde{c} K_m &= q^{2m} K_m \tilde{c}, \\
K_m K_{-m} &= H.
\end{aligned}
\tag{55}
$$

Comparing the last relation in (55) with the second one in (45) we deduce that $H = K_0$, which is consistent with all other q-deformed commutation relations. The coproduct, the counit, and the antipode actions on the generators L_m and the corresponding additional generators K_m are similar to that in (46). Their q-deformed commutation relations are analogues to (45). Finally, the costructure on the central charge is given by

$$
\begin{aligned}
\Delta(\hat{c}) &= \hat{c} \otimes 1 \; + \; K_0 \otimes \hat{c}, \\
\varepsilon(\hat{c}) &= 0, \qquad \gamma(\hat{c}) = -K_0^{-1} \hat{c}.
\end{aligned}
\tag{56}
$$

4 Quantum deformed U(1)-Kac-Moody algebra

This algebra is an important one in physics, it may be related, for example, to the oscillator algebra. Its classical form without central extension is generated by the basis $\{J_m, m \in \mathbb{Z}\}$ with the following commutation relations

$$
[J_m, J_n] = 0.
\tag{57}
$$

The case with central charge is given by :

$$
[J_m, J_n] = m \delta_{m+n,0} c,
\tag{58}
$$

$$
[J_m, c] = 0.
\tag{59}
$$

To deform it we will follow the same procedure as we have done for the Virasoro algebra. We take the same R-matrix as in (37) to derive the q-deformed analogues of (57), (58) and (59), respectively. The q-deformed centerless $U(1)$ Kac-Moody algebra and its QUEA are given by:

$$
[J_m, J_n]_q \;=\; J_m J_n \;-\; q^{2(n-m)} J_n J_m \;=\; 0,
\tag{60}
$$

where the two constraints q-A and q-JI are trivially satisfied. The Hopf structure may be deduced using (18)-(23). In this case, the q-deformed commutation relations between $M_m{}^n$ and J_m are as follows

$$
\begin{aligned}
M_m{}^r J_n &= q^{2(n-m)} J_n M_m{}^r, \\
M_n{}^r M_m{}^s &= q^{2(m-n+r-s)} M_m{}^s M_n{}^r.
\end{aligned}
\tag{61}
$$

Here, we also suppose that for each row m there are finite nonzero elements of the infinite matrix $M_m{}^n$, and we take the following simple diagonal matrix

$$
M_m{}^n \;=\; T_m \delta_m^n.
\tag{62}
$$

So, the q-deformed commutation relations (61) reduce to the following

$$
\begin{aligned}
T_m J_n &= q^{2(n-m)} J_n T_m, \\
T_m T_n &= T_n T_m,
\end{aligned}
\tag{63}
$$

and the Hopf structure is given by

$$
\begin{aligned}
\Delta(J_m) &= J_m \otimes 1 + T_m \otimes J_m, & \Delta(T_m) &= T_m \otimes T_m \\
\varepsilon(T_m) &= 0, & \varepsilon(T_m) &= 1, \\
\gamma(J_m) &= -T_m^{-1} J_m, & \gamma(T_m) &= T_m^{-1}.
\end{aligned}
\tag{64}
$$

Now let us look at the q-deformed central extension. We set

$$
[J_m, J_n]_q = J_m J_n - q^{2(n-m)} J_n J_m = \psi_q(m,n)\tilde{c},
\tag{65}
$$

and

$$
[J_m, \tilde{c}]_q = J_m \tilde{c} - q^{-2m}\tilde{c}J_m = 0,
\tag{66}
$$

where $A_m{}^n = q^{-2n}\delta_m^n$ for the same reasons as in the Virasoro algebra case. The q-JI is trivially satisfied, however the q-A gives

$$
\psi_q(m,n) = -q^{2(n-m)}\psi_q(n,m)
\tag{67}
$$

from which we get two solutions. The first one is

$$
\psi_q(m,n) = \lambda q^{(n-m)}[m-n]_q \delta_{m+n,0}
\tag{68}
$$

($[x]_q = [q^x - q^{-x}]/(q - q^{-1})$) which vanishes in the limit $q = 1$. This central extension is allowed by the quantum deformation. The second solution is given by

$$
\psi_q(m,n) = q^{n-m} m \delta_{m+n,0},
\tag{69}
$$

which reduces to the classical central term in (58) in the limit $q = 1$. Note that the solution of $A_m{}^n$ in (66) is compatible with (68) and (69). These central extensions may be endowed with the Hopf structure by applying the formalism of the first section. The coproduct, the counit, and the antipode actions on the generators J_m and the corresponding additional generators T_m are similar to that in (64) and we have analogous q-deformed commutation relations to (63). For the central charge we have the following costructure

$$
\begin{aligned}
\Delta(\tilde{c}) &= \tilde{c} \otimes 1 + H \otimes \tilde{c}, \\
\varepsilon(\tilde{c}) &= 0, & \gamma(\tilde{c}) &= -H^{-1}\tilde{c}.
\end{aligned}
\tag{70}
$$

The q-deformed commutation relations (31) are now

$$
\begin{aligned}
J_m H &= q^{-2m} H J_m, \\
T_m H &= H T_m, \\
\tilde{c} T_m &= q^{-2m} T_m \tilde{c}, \\
T_m T_{-m} &= H.
\end{aligned}
\tag{71}
$$

5 Concluding remarks

We have presented a new type of q-deformed centerless Virasoro and U(1) Kac-Moody algebras based on a general q-deformation formalism of arbitrary Lie algebra. We have associated to these

q-deformed algebras an R-matrix, which is of unit square and satisfies the quantum Yang-Baxter equation. We have also investigated the possibility of extending these q-deformed algebras to have a q-deformed central charge. It appeared that one can find two solutions, one of them is purely a quantum deformation fluctuation in the sense that it disappears in the classical limit $q = 1$. The other is the q-deformed analogue of the usual classical one. The main advantage of this construction is that it allows a nontrivial Hopf structure on the corresponding QUEA's by introducing additional elements, which degenerate with the unit element in the undeformed case. Of course, this is an essential ingredient for application to integrable systems. The super-extension of this work will be discussed elsewhere [14]. It will also be interesting to look for the realization of these q-deformed algebras.

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