FERMI-BOSE SUPERSYMMETRY

(SUPERGAUGE SYMMETRY IN FOUR DIMENSIONS)

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1. INTRODUCTION

Fermi-Bose supersymmetry was introduced by Wess and the author[1]. It connects Bosons with Fermions. Its existence was suggested by dual models[2] (when formulated as two-dimensional field theories) and the name supergauge symmetry in four dimensions seemed a natural choice. The supergauge algebra having only a finite number of generators in four dimensions, it seems now reasonable to avoid the word gauge and adopt the expression Fermi-Bose supersymmetry, or simply supersymmetry, suggested recently by Salam and Strathdee.

The supersymmetry algebra is very simple. Let Q_i be a constant Majorana spinor (we use the Majorana representation where the γ matrices are real; then Q_i is a hermitian spinor). The algebra is

$$\{Q_i, \bar{Q}_j\} = -2(\gamma^\mu)_{ij} P_\mu, \quad \bar{Q} = \tilde{Q}\gamma^o \qquad (1)$$

$$[P_\mu, Q_i] = [P_\mu, P_\lambda] = 0$$

where $P^\mu = (H, \vec{P})$ is the energy-momentum. In a supersymmetric theory the spinor charges are

$$Q_i = \int J_i^o \, d^3x \qquad (2)$$

where the vector-spinor current is conserved

$$\partial\mu \, J_i^\mu = 0 \qquad (3)$$

Lorentz transformations and parity transform Q_i as a spinor and P_μ as a vector. The algebra(1) is not a Lie algebra since it contains both commutators and anticommutators.

If one introduces Majorana spinors α_1, α_2 etc (they commute with tensors and anticommute with spinors and among themselves), the relation(1)can be written as a commutation relation

$$[\bar{\alpha}_1 Q, \bar{\alpha}_2 Q] = -2\bar{\alpha}_1\gamma^\mu\alpha_2 P_\mu. \qquad (4)$$

These objects have been studied in mathematics[3]. From(1)one finds an expression for the total Hamiltonian in terms of the spinor charges

$$H = \frac{1}{4}\sum_{i=1}^{4} Q_i^2 \qquad (5)$$

valid, in presence of interaction for any supersymmetric theory, and similar expressions for the momentum.

Supersymmetry is not an ordinary Lie algebra and avoids no-go theorems[4] of relativistic SU(6): there exist non trivial (and renormalizable) Lagrangian theories which are exactly invariant under supersymmetry.

Our motivation was to show the feasibility of supermultiplets containing interacting particles with both integral and half integral spin. From a rather different point of view the same algebra (1) was considered independently by Volkov and Akulov[5]. They gave a non-linear realization in terms of a single spinor field (see Section 4) and suggested that it may be relevant as a description of the properties of the neutrino. Their Lagrangian is non-renormalizable. In reference[1] an algebra larger than (1) was described, which contains also Lorentz transformations, dilatations, conformal[6] and chiral transformations.

That larger algebra was later abandoned[7], to avoid the problems of scale and conformal anomalies.

2. SUPERMULTIPLETS AND LAGRANGIANS

The simplest supermultiplet consists of a scalar field A, a pseudoscalar field B, a Majorana spinor, and two auxiliar fields F and G. Writing

$$\delta A = [\bar{\alpha}Q, A] \text{ etc.} \tag{6}$$

for an infinitesimal supertransformation, one has

$$\begin{cases} \delta A = i\bar{\alpha}\psi \\ \delta B = i\bar{\alpha}\gamma_5\psi \\ \delta\psi = \partial_\mu(A - \gamma_5 B)\gamma^\mu\alpha + (F + \gamma_5 G)\alpha \\ \delta F = i\bar{\alpha}\gamma^\mu\partial_\mu\psi \\ \delta G = i\bar{\alpha}\gamma_5\gamma^\mu\partial_\mu\psi \end{cases} \tag{7}$$

The commutator of two supertransformations is a translation. For instance

$$[\delta_2, \delta_1] = 2i\bar{\alpha}_1\gamma^\mu\alpha_2\partial_\mu A \tag{8}$$

With this supermultiplet Wess and the author[7] constructed the first non trivial supersymmetric model, with Lagrangian

$$\begin{aligned} L = & -\tfrac{1}{2}\left[(\partial_\mu A)^2 + (\partial_\mu B)^2 + i\bar{\psi}\gamma^\mu\partial_\mu\psi - F^2 - G^2\right] \\ & + m(FA + GB - \frac{i}{2}\bar{\psi}\psi) \\ & + g(FA^2 - FB^2 + 2GAB - i\bar{\psi}\psi A + i\bar{\psi}\gamma_5\psi B). \end{aligned} \tag{9}$$

The various terms of the Lagrangian each change by a divergence under(7): the action integral is invariant. The auxiliary fields F and G can be eliminated by using their own equations of motion and the Lagrangian takes the more familiar form

$$\begin{aligned} L = & -\tfrac{1}{2}\left[(\partial_\mu A)^2 + (\partial_\mu B)^2 + i\bar{\psi}^\mu\partial_\mu\psi + m^2A^2 + \right. \\ & \left. m^2B^2 + im\bar{\psi}\psi\right] - gmA(A^2 + B^2) - \frac{g^2}{2}(A^2 + B^2)^2 \\ & - ig\bar{\psi}(A - \gamma_5 B)\psi \end{aligned} \tag{10}$$

As a consequence of supersymmetry all masses are equal and all couplings are expressed in terms of the single coupling constant g. One verifies that the supercurrent

$$J^\mu = \gamma^\lambda\partial_\lambda(A - \gamma_5 B)\gamma^\mu\psi - (F + \gamma_5 G)\gamma^\mu\psi \tag{11}$$

is conserved as a consequence of the equations of motion.

This model has been studied in great detail by Ferrara, Iliopoulos and the author[8]. It can be regularized in a supersymmetric way by introducing higher order derivatives in the kinetic term of the Lagrangian. The Ward identities corresponding to the conservation of the supercurrent (11) can be written and used to prove that renormalization does not spoil the relations among masses and coupling constants due to supersymmetry. The model is less divergent than the generic theory with the same kind of couplings. Only one single wave function renormalization constant Z common to all fields is required. The renormalized mass and coupling constant are

$$m_r = Zm_o, \; g_r = Z^{\frac{3}{2}}g_o \tag{12}$$

The Callan-Symanzik functions β and γ are proportional to each other. As a consequence one can argue that β cannot vanish except at the origin and that the effective coupling increases indefinitely with energy.

Supersymmetry can be broken softly, by adding to the Lagrangian a term proportional to A. Just as in the analogous case of the σ-model, the renormalization program can still be carried out. The masses of the various fields of the multiplet are no longer equal. Instead in the tree approximation

$$m_A^2 + m_B^2 = 2m_\psi^2 \tag{13}$$

In higher orders this is corrected by finite terms.

The fact that the supersymmetric model is less divergent than the generic theory of its kind, leads one to ask whether a supersymmetric theory might not be renormalizable even if it does not appear to be so by simple power counting. To answer this question, Laing and Wess[9] have replaced, in the Lagrangian(9), the renormalizable interaction proportional to g with the interaction

$$\begin{aligned} & f\{FA^3 - GB^3 + 3GA^2B - 3FAB^2 - \tfrac{3}{2}i(A^2 - B^2)\bar{\psi}\psi \\ & + 3iAB\bar{\psi}\gamma_5\psi\} \end{aligned} \tag{14}$$

This interaction is supersymmetric but non-renormalizable by power counting.
While this question is being investigated, it appears that, in this particular model, supersymmetry, in spite of the compensation of some divergences, is not sufficient to render the theory renormalizable.

3. GAUGE INVARIANCE AND SUPERSYMMETRY

The existence of Lagrangian theories which are both gauge invariant and supersymmetric was first shown by Wess and the author[10]. They use a supermultiplet consisting of a vector field v_μ, a Majorana spinor λ and an auxiliary field D, transforming as

$$\begin{aligned} \delta v_\mu &= i\bar{\alpha}\gamma_\mu\lambda \\ \delta\lambda &= -\tfrac{1}{4}v_{\mu\nu}\gamma^\mu\gamma^\nu\alpha + D\gamma_5\alpha \qquad (15) \\ \delta D &= i\bar{\alpha}\gamma_5\gamma^\mu\partial_\mu\lambda \qquad v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu \end{aligned}$$

under a supersymmetry transformation. Using real fields the Lagrangian can be written as

$$\begin{aligned} L = &-\tfrac{1}{4}v_{\mu\nu}{}^2 - \tfrac{i}{2}\bar{\lambda}\gamma^\mu\partial_\mu\lambda + \tfrac{1}{2}D^2 - \tfrac{1}{2}\sum_{i=1}^{2}\left[(\partial_\mu A_i)^2 + (\partial_\mu B_i)^2 + i\bar{\psi}_i\gamma^\mu\partial_\mu\psi_i + m^2(A_i^2 + B_i^2) + im\bar{\psi}_i\psi_i\right] \\ &+ g\left[D(A_1B_2 - A_2B_1) - v^\mu(A_1\partial_\mu A_2 - A_2\partial_\mu A_1 + B_1\partial_\mu B_2 - B_2\partial_\mu B_1 - i\bar{\psi}_1\gamma_\mu\psi_2)\right. \\ &\left.- i\bar{\lambda}\{(A_1 + \gamma_5B_1)\psi_2 - (A_2 + \gamma_5B_2)\psi_1\}\right] - \frac{g^2}{2}v_\mu^2\,(A_1^2 + A_2^2 + B_1^2 + B_2^2) \qquad (16) \end{aligned}$$

where the fields F_i and G_i $(i = 1, 2)$ have already been eliminated. The gauge transformation rotates the fields with the subscripts 1 and 2 into each other and changes v_μ by a four-gradient. The fields λ and D are gauge invariant. The Lagrangian (16) is a sort of supersymmetric extension of quantum electrodynamics. All couplings are expressed in terms of the single coupling constant g, as a consequence of supersymmetry of the Lagrangian which has been shown to be renormalizable in the one-loop approximation in a manner consistent with gauge invariance and supersymmetry. A preliminary investigation of higher orders supports this conclusion.

4. SPONTANEOUS SYMMETRY BREAKING

The model described in the previous section can give an example of spontaneous breaking of supersymmetry, with the corresponding emergence of a "Goldstone" spinor. Fayet and Iliopoulos[11] have added to the Lagrangian(16) a parity violating, gauge and supersymmetry invariant term $\xi D/g$. Upon elimination of D this gives a term

$$-\xi(A_1B_2 - A_2B_1). \qquad (17)$$

Introducing the new fields

$$\begin{aligned} a_1 &= \frac{1}{\sqrt{2}}(A_1 - B_2) \qquad & a_2 &= \frac{1}{\sqrt{2}}(B_1 + A_2) \\ b_1 &= \frac{1}{\sqrt{2}}(A_1 + B_2) \qquad & b_2 &= \frac{1}{\sqrt{2}}(-B_1 + A_2) \end{aligned} \qquad (18)$$

this results in the potential (tree approximation)

$$\tfrac{1}{2}(m^2 - \xi)(a_1^2 + a_2^2) + \tfrac{1}{2}(m^2 + \xi)(b_1^2 + b_2^2) + \frac{g^2}{8}(a_1^2 + a_2^2 - b_1^2 - b_2^2)^2. \qquad (19)$$

Supersymmetry is spontaneously broken, since the masses of the fields are not longer equal. For $|\xi| < m^2$ the fields a_i and b_i have vanishing vacuum expectation value, while $\langle D\rangle = -\xi/g$ and, from (15)

$$\delta\lambda = -\frac{\xi}{g}\gamma_5\alpha + \ldots \qquad (20)$$

where the dots denote terms containing other fields. The field λ is a Goldstone Fermion (germion).
For $|\xi| > m^2$ one of the quadratic terms in (19) has a negative coefficient. The gauge invariance is also spontaneously broken and the vector field acquires a mass $m_v^2 = 2(\xi - m^2)$ (Higgs mechanism).

The Goldstone Fermion which is now a linear combination λ^1 of the fields $\lambda, \gamma_5\psi_1$ and ψ_2, transforms as

$$\delta\lambda^1 = -\frac{m}{g}\sqrt{2\zeta - m^2}\gamma_5\alpha + \ldots \qquad (21)$$

The Goldstone Fermions are massless spinors arising from the spontaneous breaking of supersymmetry, just as Goldstone bosons arise when chiral symmetry is spontaneously broken. In analogy with that case one may ask whether non-linear realizations of supersymmetry exist. The first non-linear realization was given by Volkov and Akulov[5];

$$\delta\chi = \frac{1}{a}\alpha + ia(\bar{\alpha}\gamma^\mu\chi)\partial_\mu\chi \qquad (22)$$

A different realization was found by the author:

$$\delta\chi = \frac{1}{a}\alpha + ia(\bar{\alpha}\gamma^\mu\chi)\partial_\mu\chi + ia(\bar{\alpha}\gamma_5\gamma^\mu\chi)\gamma_5\partial_\mu\chi. \qquad (23)$$

One may inquire about the correction between linear and non-linear realizations. There exist functions of the field χ transforming as in (23) which transform linearly as in (7).

5. SUPERSPACE AND SUPERFIELDS

Supersymmetry representations have been studied with various techniques [12,13]. Salam and Strathdee[14] have introduced the very interesting concept of superfield and have described supersymmetry transformations as operations on superfields. It has been extended by Ferrara, Wess and the author[15]. A general review is given by Salam and Strathdee[16]. Superspace was considered by Volkov and Akulov[5] in their non-linear realizations.

Consider a space (superspace) whose points are determined by coordinates x_μ, θ and $\bar{\theta}$, where x_μ are the usual space-time coordinates, θ is a (totally anti-commuting) two-component spinor and $\bar{\theta}$ its complex conjugate.

Supersymmetry transformations are geometrical transformations in superspace

$$\begin{cases} \delta x_\mu = i\theta\sigma_\mu\bar{\xi} - i\xi\sigma_\mu\bar{\theta} & \sigma_o = 1 \\ \delta\theta = \xi & \\ \delta\bar{\theta} = \bar{\xi} & \end{cases} \qquad (24)$$

The commutator of two such transformations is a translation - furthermore (24) and translations leave invariant the differential form

$$\omega_\mu = dx_\mu + i\bar{\theta}\sigma_\mu d\theta - i\theta\sigma_\mu d\bar{\theta} \qquad (25)$$

Adjoining Lorentz transformations the "line element"

$$\omega_\mu\omega^\mu \qquad (26)$$

is invariant. A superfield is a field in superspace, $V(x,\theta,\bar{\theta})$. Its power series in θ and $\bar{\theta}$, terminates after a finite number of terms:

$$V(x,\theta,\bar{\theta}) = C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + \ldots + \theta\theta\bar{\theta}\bar{\theta}\tfrac{1}{2}D(x) \qquad (27)$$

Therefore a superfield corresponds to a finite supermultiplet of ordinary fields. A superfield is taken to transform as a scalar in superspace under (24)

$$V'(x,\theta,\bar{\theta}) = V(x',\theta',\bar{\theta}') \qquad (28)$$

and can have spinor or vector indices which determine how it transforms under Lorentz transformations. From (28) and (27) one derives the transformation of the fields C, χ etc of the supermultiplet. Observe that the new coordinates

$$Z_\mu = x_\mu + i\theta\sigma_\mu\bar{\theta} \qquad (29)$$

gives

$$\delta z_\mu = 2i\theta\sigma_\mu\bar{\xi}, \qquad (30)$$

which does not contain $\bar{\theta}$. Therefore, it is consistent to require that a superfield be a function only of z_μ and θ, $S(z,\theta)$, or that it satisfy

$$\bar{D}S = 0 \qquad (31)$$

where

$$\bar{D} = -\frac{\partial}{\partial\bar{\theta}}\Big|_z = -\frac{\partial}{\partial\bar{\theta}} - i\theta\sigma^\mu\partial_\mu \qquad (32)$$

$\bar{D}$ is a covariant derivative(under (24) and so is

$$D = \frac{\partial}{\partial\theta}\Big|_x + i\sigma^\mu\bar{\theta}\partial_\mu \qquad (33)$$

A superfield satisfying (31) is called left-handed, one satisfying the covariant constraint

$$DS = 0 \qquad (34)$$

is called right-handed. A left-handed superfield (together with its right handed complex conjugate) corresponds to the multiplet described in section 2.

Volkov and Soroka[18] have developed a description of curved superspace which combines gravitational theory with the interaction of particles of spin 3/2, 1 and 1/2. Can a theory of this kind, because of the compensation of divergences due to supersymmetry, provide a renormalizable description of gravitational interactions?

6. YANG-MILLS GAUGES AND SUPERSYMMETRY

Using superfields Salam and Strathdee[19] and Ferrara and the author[20] have constructed theories which are both supersymmetric and invariant under non-abelian gauge transformations. We give only the results. The simplest supersymmetric and gauge invariant theory is the ordinary Yang Mills theory of vectors in interaction with a multiplet of Majorana spinors belonging to the regular (adjoint) representation of the internal symmetry group. For instance, for SU(N), using NxN matrix notation

$$L = \mathrm{Tr}(-\tfrac{1}{4}v_{\mu\nu}^{2} - \tfrac{1}{2}\bar{\lambda}\gamma^{\mu}D_{\mu}\lambda + \tfrac{1}{2}D^{2})$$

$$v_{\mu\nu} = \partial_{\mu}v_{\nu} - \partial_{\nu}v_{\mu} + ig[v_{\mu}, v_{\nu}] \tag{35}$$

$$D_{\mu}\lambda = \partial_{\mu}\lambda + ig[v_{\mu},\lambda]$$

One checks that the supercurrent

$$J^{\mu} = -\tfrac{1}{4}\mathrm{Tr}(v_{\nu\rho}[\gamma^{\nu},\gamma^{\rho}]\ \gamma^{\mu}\lambda) \tag{36}$$

is conserved as a consequence of the equations of motion.

The multiplet v_{μ},λ,D can be coupled to "matter multiplets" A, B, ψ, F, G belonging to any representation of the internal group. For the case of a matter multiplet in the regular representation the Lagrangian is given by (35) plus

$$\mathrm{Tr}\Big\{-\tfrac{1}{2}\big[(D_{\mu}A)^{2} + (D_{\mu}B)^{2} + i\bar{\psi}\gamma^{\mu}D_{\mu}\psi - F^{2} - G^{2}\big] + m(FA + GB - \tfrac{i}{2}\bar{\psi}\psi) + igD[A, B] + g\bar{\lambda}[A + \gamma_{5}B,\psi]\Big\} \tag{37}$$

Theories of this kind contain scalar and pseudoscalar fields - since all coupling constants are expressed in terms of the single Yang-Mills coupling g, they will be asymptotically free provided the Callan-Symanrik function β is negative (and provided the theories are renormalizable in accordance with supersymmetry!). For (35) plus n matter multiplets like (37) it is

$$\beta = -\frac{g^{3}}{16\pi^{2}}(3-n)N \tag{38}$$

where N refers to SU(N).

If one adds (35) plus (37) for m = 0, eliminates the field D and combines the two Majorana fields λ and ψ into a complex spinor

$$\phi = \frac{1}{\sqrt{2}}(\lambda + i\psi) \tag{39}$$

one obtains the Lagrangian

$$\mathrm{Tr}\{-\tfrac{1}{4}v_{\mu\nu}^{2} - \tfrac{1}{2}(D_{\mu}A)^{2} - \tfrac{1}{2}(D_{\mu}B)^{2} - i\bar{\phi}\gamma^{\mu}D_{\mu}\phi - ig\bar{\phi}[A + \gamma_{5}B,\phi] - \tfrac{g}{2}(i[A, B])^{2}\} \tag{40}$$

which is invariant under the transformation

$$\phi \to e^{i\omega}\phi \quad \phi^{*} \to e^{-i\omega}\phi^{*} \tag{41}$$

The conserved supercurrent for (40) is[21]

$$J = \mathrm{Tr}\{-\tfrac{1}{4}v_{\nu\rho}[\gamma^{\nu},\gamma^{\rho}]\gamma^{\mu}\phi + ig[A, B]\gamma_{5}\gamma^{\mu}\phi - i\gamma^{\lambda}D_{\lambda}(A-\gamma_{5}B)\gamma^{\mu}\phi\} \tag{42}$$

One can also obtain a V±A scheme with Fermion number[22]. One need only combine two multiplets $v_{\mu}^{(1)}$, $\lambda^{(1)}$ and $v_{\mu}^{(2)}$, $\lambda^{(2)}$ described by Lagrangians like (35). The total Lagrangian can be rewritten in terms of the complex spinor

$$\phi = \tfrac{1}{2}(1-i\gamma_{5})\lambda^{(1)} + \tfrac{1}{2}(1 + i\gamma_{5})\lambda^{(2)} \tag{43}$$

and of the vector and axial vector fields

$$v_{\mu} = \frac{1}{\sqrt{2}}(v_{\mu}^{(1)} + v_{\mu}^{(2)}),\quad a_{\mu} = \frac{1}{\sqrt{2}}(v_{\mu}^{(1)} - v_{\mu}^{(2)}), \tag{44}$$

and is again invariant under (41).

The examples of this section show that one can now attempt to construct realistic supersymmetric models. The main difficulty at present is that one does not have a device for generating masses without spoiling the renormalizability of the theory. The Fayet-Iliopoulos trick described in Section 4, can be applied only to an invariant abelian subgroup.

Perhaps consideration of the effective potential in higher orders will provide a solution to this problem.

7. NON-TRIVIAL MIXING OF INTERNAL SYMMETRY AND SUPERSYMMETRY

The combination of internal symmetry and supersymmetry described in the previous section attributes all fields of a supermultiplet to the same representation of the internal symmetry group. A more interesting possibility was suggested by Salam and Strathdee[13] and by Wess and the author[23]. It consists in an actual combination of the two symmetries to a new algebra. Imagine the algebra (1) written in two component notation, by using a representation of the γ matrices in which γ_5 is diagonal. The supercharges will consist of a two component spinor and its conjugate. One can give these two component spinors an internal symmetry index i or j and write the algebra as

$$\{Q_{\alpha i}, Q_{\beta j}\} = \{\bar{Q}_{\dot{\alpha}}{}^{i}, Q_{\dot{\beta}}{}^{j}\} = 0$$

$$\{Q_{\alpha i}, \bar{Q}_{\dot{\beta}}{}^{j}\} = 2\delta_{i}{}^{j}(\sigma_{\mu})_{\alpha\dot{\beta}}P^{\mu} \qquad (45)$$

$$[Q_{\alpha i}, P_{\mu}] = [P_{\lambda}, P_{\mu}] = 0$$

For instance, the indices i and j could refer to the N-dimensional representation of SU(N) (upper indices to the complex conjugate). The representations of this algebra can be studied by the method of superfields[24], in a superspace labelled by x_{μ}, $\theta_{\alpha i}$, $\theta_{\dot{\alpha}}{}^{j}$. A simpler technique has been suggested by Salam and Strathdee [13]. One can go to the rest frame $P^{\mu} \rightarrow (m, 0, 0, 0)$.

The algebra (45) becomes then an algebra of creation and destruction operators. As an example for SU(2) the quantum numbers (I, J) = (isospin, spin) for the fields of the multiplet corresponding to a left-handed superfield, are (0, 0), ($\frac{1}{2}$, $\frac{1}{2}$), (1, 0), (0, 1), ($\frac{1}{2}$, $\frac{1}{2}$), (0, 0).

The symmetry described by (45), or other similar generalizations, seems well worth exploring, by any available techniques. Work along these lines is presently being done by several groups.

8. CONCLUSION

Three lines of future development come to mind. The first would use supersymmetry as a way of classifying hadronic states and their interactions, a kind of relativistic SU(6), connecting Bosons and Fermions and free of contradictions. The Lagrandian model described in Section 2 gives an extremely simplified version of this kind of theory. The inclusion of internal symmetries could be effected either by introducing them as an additional group commuting with the supersymmetry or by enlarging the algebra as indicated in Section 7. For instance, the indices i, j occurring in (45) can be extended to include, besides the physical SU(N), a colour SU(N).

The second line of development is suggested by the gauge invariant model of Section 3. It is very tempting to interpret the supermultiplet (15) as containing the photon and the (electron) neutrino. The non-abelian generalizations of the model of Section 3, given in Section 6, are the first step towards a description of weak, electromagnetic and possibly strong interactions.

The third line of development could be a generalized (possibly renormalizable) theory of gravitation, as indicated at the end of Section 5.

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RECENT WORK ON STRING MODELS

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My main subject will be the interacting-string vertex, showing directly that it possesses the required Lorentz-transformation properties. The conformal invariance of the amplitude will follow. Some of the arguments of ref. 1 can then be simplified considerably.

The Lorentz generators of Goddard, Goldstone, Rebbi and Thorn are as follows: