REMARKS ON e^+e^- ANNIHILATION

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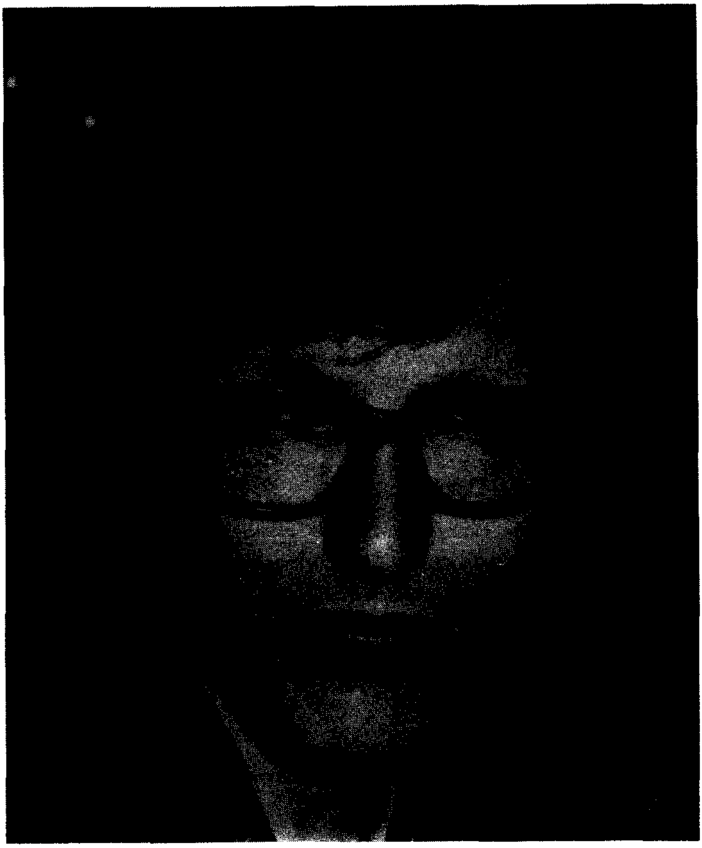

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1. INTRODUCTION

Recent results on e^+e^- annihilation[1-5] have upset many theoretical prejudices. No theoretical consensus has emerged and I for one have not yet had time to study all the interesting questions raised by the data. Furthermore, there are many questions which are likely to be answered experimentally in the very near future. Thus the following remarks are certain to prove ephemeral but it is hoped that they may serve to highlight some of the issues and perhaps provide a starting point for those who wish to study the subject in more depth. For a rather complete coverage of the field up to six months ago (when only the behaviour of $\sigma_{e^+e^- \to \text{Had.}}$ and the charged multiplicity were known in the 3 to 5 GeV range) we refer to the excellent reviews by Bjorken[6] and by Cahn and Ellis[7]; in preparing these remarks I have drawn heavily on these reviews in some places.

2. MAIN FEATURES OF THE DATA

The main features of the data which I shall discuss are[8]:

a) $R = \sigma(e^+e^- \to \text{Had})/\sigma(e^+e^- \to \mu^+\mu^-)$ rises[2-4] from about 2 or 3 at $E_{cm} = 3$ GeV to between 4 and 7 at 5 GeV. Correspondingly, $\sigma_{e^+e^- \to \text{Had.}}$ is about 22 ± 5 nb throughout this range.

b) The charged multiplicity is approximately constant from 3 to 5 GeV with a value $\langle n_{ch} \rangle \sim 4$ [2-4]. The mean momentum per charged particle rises from about 400 MeV at E = 3 GeV to about 500 MeV at 4.8 GeV [3,4]. Taken at face value, the data imply that the ratio of energy in neutral particles to energy in charged particles rises rapidly from about 0.6 at 2.8 GeV to about 1.1 at 5 GeV [9] (the reader must imagine errors, in this case $\sim \pm 20\%$, on all statements in this summary).

c) The data for $e^+e^- \to H(p) + \ldots$ is consistent with the hypothesis that $s(d\sigma/dx)$ scales (i.e. is a function of x only) for $x > \frac{1}{2}$, where $x = 2p/\sqrt{s}$. For $x < \frac{1}{2}$ there is a strong s-dependence [the data are nowhere compatible with the hypothesis that $(1/\sigma)$ $(d\sigma/dx)$ scales].

d) The quantity $E_p(d^3\sigma/d^3p)$ is s-independent within the errors.

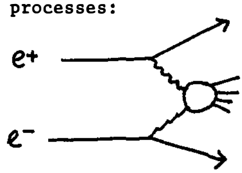
e) The $\pi/K/p$ ratio is very roughly 100/10/1.

3. EXPLANATIONS?

There are three possibilities:

a) One photon annihilation directly into hadrons accounts for most of the cross-section.

b) Two photon processes:



play an important role (this could solve the "missing energy" problem since the leptons take off a lot of energy down the beam pipe). There are good experimental reasons to doubt this[2-4]. However, it is possible to maintain that these processes account for 20 or 30% of the cross-section; I refer to Greco for a discussion of this view[10].

c) Something else occurs.

The rest of this talk is devoted to possibilities a) and c).

4. ONE PHOTON ANNIHILATION DIRECTLY TO HADRONS

4.1 General remarks

4.1.1 Rising R, QED, unitarity, etc.

Unitarity requires that[11]

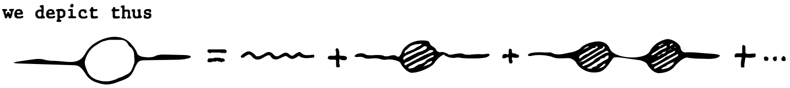
$$\sigma_{J=1}(e^+e^- \to all) \leqslant \frac{3\pi}{s} .$$

Separating the one-photon contribution we can write

$$\sigma_{J=1} = \sigma_{1\gamma} + \sigma_{int.} + \sigma_{rest} \cdot$$

Suppose $\sigma_{1\gamma}$ = 20 nb, then this term on its own would exceed the unitarity limit at a beam energy of about 230 GeV [when R $\sim$ $(137)^2$]. This is allowed but in order to keep $\sigma_{J=1}$ within the unitarity limit, the power series expansion in α would have to break down.

Actually we can argue that perturbation theory would break down at the much lower energy at which R $\sim$ 137. Consider the photon propagator which we depict thus



(the shaded blobs being one particle irreducible) and write as

$$\mathcal{D}(q^2) = \frac{1}{q^2} + \frac{1}{q^2}\Pi(q^2) + \frac{1}{q^2}\Pi^2(q^2) + \dots$$
$$= \frac{1}{q^2(1-\Pi(q^2))}$$

where, to lowest order

$$\Pi(q^2) = \frac{q^2}{4\pi\alpha}\int \frac{ds\,\sigma(e^+e^-\to\gamma\to Had.)}{q^2 - s + i\epsilon}\,.$$

Suppose $\sigma \sim s^p$, $R = \text{const.}\ (s)^{p+1}$. Note first that if $p \geq 0$, the dispersion relation for π will need a subtraction, thus spoiling the renormalizability of QED [12]. With $0 > p > -1$ and q^2 space-like, the integral gives[6]

$$\mathcal{D} = \frac{1}{q^2}\left[1 + \frac{\alpha}{3\sin\pi(p+1)}\,\mathcal{R}(q^2) + \dots\right.$$

Clearly perturbation theory will fail when $R \gtrsim 137$ [with $\sigma(e^+e^- \to \gamma \to \text{Had.}) =$ $= 20$ nb, $R = 137$ at the comparatively modest energy of 12 GeV per beam] and a large rising R implies relatively large corrections to QED predictions for $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$, etc. at energies now available[6]. Accurate QED experiments are therefore very interesting since they can be used to probe (dispersively) the behaviour of R at higher energies[6,13].

Proponents of asymptotic models which fit/explain/postdict the CEA-SPEAR data must show that their models do not violate QED tests already performed.

4.1.2 The energy crisis

(i.e. the large and rising fraction of the energy which seems to go into neutrals.) Conventionally it is supposed that the photon has $I^G = 0^- + 1^+$. In this case when $e^+e^- \to \gamma \to N\pi$ with N odd, the isospin must be zero; there is therefore complete isotropy in isospin space and

$$d\sigma(\pi^0) = d\sigma(\pi^\pm).$$

For N even, the isospin is one and the following optimal bounds can be obtained[14 15]:

$$\frac{N-2}{4N+2} \leqslant \Gamma(N) = \frac{N_{\pi^0}}{N_{\pi^+}+N_{\pi^-}} \leqslant \frac{3N-2}{2N+2}$$

for $N \gtrsim 4$. These bounds are for the numbers, not for the energy, so strictly speaking they are not threatened at present. However, it would be curious if the mean energy was substantially greater for neutral than for charged pions. (These bounds only hold if the final state consists exclusively of pions; the evidence suggests that this may be a good approximation. If many η's are produced as well as π's the neutral energy can be enhanced but assuming $\langle E_{\pi^0}\rangle = \langle E_{\pi^+}\rangle = \langle E_{\pi^-}\rangle$ it requires $\langle E_\eta\rangle \approx 1.7\langle E_{\pi^0}\rangle$ to get $\langle E_{\text{neutral}}\rangle = \langle E_{\text{charged}}\rangle$.)

These number bounds involve the properties of identical particles which are reflected in the possible symmetries of the wave function; if they are saturated one class of wave functions is singled out and further constraints can be derived[16]. Note that in models which are statistical, in the sense that all possible isospin structures occur with equal probability, $\Gamma(N)$ tends rapidly to $\frac{1}{2}$ as N increases[14) $[\Gamma(4) = 7/13,\ \Gamma(6) = 1/2,\ \Gamma(8) = 77/155,\ \ldots]$.

Weaker bounds have been obtained by Di Giacomo using more elementary methods[17] which do not take into account the identity of the particles. With a very plausible dynamical assumption such methods can be used to get an energy bound[18]:

Denote the three I = 1 multipion states by $|+\rangle$, $|0\rangle$, and $|-\rangle$ and consider the isotensor number operator:

$$\langle 0|2n_0 - n_+ - n_-|0\rangle = -2\langle +|2n_0 - n_+ - n_-|+\rangle$$

$$= 2\langle +|\bar{a}_+ a_-|-\rangle \leq 2\,|\langle +|n_+|+\rangle|$$

(where the Clebsch-Gordan coefficients can be obtained trivially by considering the one-pion state and we have used the Schwartz inequality). Now

$$2\langle +|n_+|+\rangle = \langle +|n_+ + n_- + n_+ - n_-|+\rangle$$

$$= \frac{1}{3}\langle +|n_+ + n_- - 2n_0 + 2(n_+ + n_- + n_0)|+\rangle + \langle +|n_+ - n_-|+\rangle$$

$$= \frac{1}{6}\langle 0|2n_0 - n_+ - n_- + 4(n_+ + n_- + n_0)|0\rangle + \langle +|n_+ - n_-|+\rangle$$

Hence:

$$\langle 0|n_0|0\rangle \leq \frac{3}{2}\langle 0|n_+ + n_-|0\rangle + \langle +|n_+ - n_-|+\rangle.$$

This equation is true locally -- i.e. for the number of π's with a given momentum. (On integration: $\langle 0|n_i|0\rangle \to N_i$, $\langle +|n_+ - n_-|+\rangle \to 1$, and we get $2N_0 \leq 3N_c + 2$ -- which is Di Giacomo's bound to be compared to the best bound $2N_0 \leq 3N_c - 2$ for $N_0 + N_c = N$ even). Weighting with the energy, writing $E_c = E_+ + E_- = E - E_0$ and integrating we get

$$\frac{E_0}{E} \lesssim \frac{3}{5} + \frac{\langle +|E_+ - E_-|+\rangle}{E}$$

It seems plausible to assume that for large multiplicities the last term is negligible, in which case we get:

$$E_0 \lesssim \frac{3E_c}{2}.$$

It is possible that by using more detailed properties of multipion systems, rigorous energy bounds can be obtained. In any case, if the energy crisis persists it will surely give strong clues about the underlying dynamics (or even possibly force us to revise our ideas about the isospin of the electromagnetic current).

4.2 Models

4.2.1 Mundane models[19-21]

By this I mean models which only invoke particles to be found in the Particle Data Group's tables. I will not discuss them beyond remarking that they tend to make very specific predictions for the structure of the final state and to have trouble accounting for the energy crisis.

4.2.2 Parton models

These models made many predictions which have not (so far?) been verified (see, e.g. Refs. 6 and 7): $R \xrightarrow[s\to\infty]{} \text{const.} = \Sigma_i Q_i^2$ (for spin-½ partons) = = 2 (for the fashionable coloured quarks), $s(d\sigma/dx) = f(x)$ etc. Suppose that at higher energies:

a) $R \not\to$ const. Then we must ask whether the "successes" of the parton model for space-like q^2 were fortuitous.

b) $R \to$ const. Why is scaling good for $Q^2 \gtrsim 1$ GeV^2 (space-like) but only for $Q^2 > 25$ GeV^2 (time-like)?

c) $R \to$ const. $\neq 2$(or $^2/_3$). Were the "successes" of the quark parton model fortuitous?

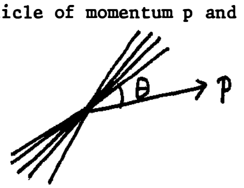
The parton model can provide answers (or excuses) in the different cases:

a) R $\not\to$ const.

Chanowitz and Drell[22], Pavkovic[23], and West[24] have pointed out that this possibility can be accommodated by giving the partons structure and that it is possible to arrange for a damping in the space-like region and an enhancement in the time-like region. The simplest example is a parton form factor[22] $\sim (q^2 - \lambda^2)^{-1}$; however, the SLAC data require $\lambda \gtrsim 12$ GeV, in which case the time-like enhancement is not big enough. With an anomalous moment as well as parton form factors, however, there is enough freedom to arrange for negligible scaling violation in the SLAC region but a sufficient enhancement for time-like q^2 [24]. Eventually the form factors will tend to win giving R $\xrightarrow{s\to\infty}$ 0. These models have the virtue that when fitted to existing data they then make specific predictions about the behaviour in unexplored regions of q^2. However, if the partons are supposed to be quarks (or more generally have $I \leq \frac{1}{2}$), and we believe the parton folklore about the incoherence of the evolution of quarks into hadrons in the final states, these models will give[25] $d\sigma(\pi^0) = d\sigma(\pi^+) = d\sigma(\pi^-)$ and hence face severe difficulties with the energy crisis.

b) R $\to$ const.

In models, such as the parton model, in which the particles are supposed to be produced in jets it is possible to argue that the scaling limit should be approached very slowly in e^+e^- annihilation. Suppose the inclusive cross-section factorizes into a function of s and of the longitudinal momentum along the jet axis and a rapidly falling function of the momentum transverse to the jet axis. If Θ is the angle between the direction of the observed particle of momentum p and the jet axis:



then averaging over all jet directions:

$$s \frac{d\sigma}{dx} = \int f(p_L, s)\, g(p_T)\, d(\cos\theta)$$

where $$x = 2p/\sqrt{s},$$

$$p_L = p\cos\theta \quad , \quad p_T = p\sin\theta .$$

For definiteness we put $g = e^{-\alpha p_T^2}$, although actually the conclusion holds for any very rapidly falling function. Writing

$$f(p_L, s) = 4p_L^2\, F\left(\frac{2p_L}{\sqrt{s}}, s\right)$$

and introducting $z = \sin^2 \Theta$ we obtain:

$$s\frac{d\sigma}{dx} = \int F(x\sqrt{1-z}, s)\, x^2 s\sqrt{1-z}\; e^{-\alpha s x^2 z}\, dz$$

$$\xrightarrow[s\to\infty]{} \frac{F(x, s)}{\alpha}.$$

To obtain scaling asymptotically, F must depend on x only. It is clear by inspection that there are scaling corrections of order $1/x^2 s$ [26]. Therefore scaling for $s(d\sigma/dx)$ will be approached extremely slowly for small x; since this region is responsible for a large fraction of the cross-section, σ will also approach its scaling limit very slowly. A similar result is suggested by a light-cone analysis[27].

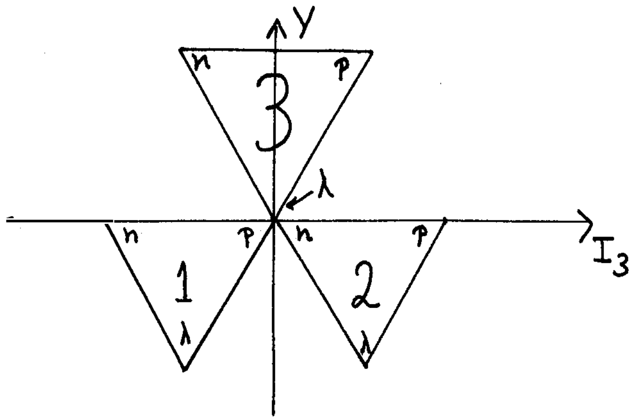
c) R $\rightarrow$ const. $\neq$ 2 or $^2/_3$

If we believe the parton prediction

$$R = \sum_{i,\,\mathrm{Spin}\,1/2} Q_i^2 + \frac{1}{4}\sum_{j,\,\mathrm{Spin}\,0} Q_j^2$$

then the value of R reflects a fundamental property of the building blocks of matter.

Of the limitless possibilities, we consider here only the case of three triplets of Han-Nambu quarks, which reproduce the usual low-energy quark model results, give R $\rightarrow$ 4 [which may be compatible with the data[28]], and may be able to account for the "energy crisis". The quantum numbers of the three triplets are



There is an SU(3) symmetry (which mixes p, n and λ inside each triplet) and an SU(3)′ symmetry (which mixes the different triplets). The low-lying hadrons are supposed to be SU(3)′ singlets. The electromagnetic current

$$J_\mu = \bar{p}_3\gamma_\mu p_3 + \bar{p}_2\gamma_\mu p_2 - \bar{n}_1\gamma_\mu n_1 - \bar{\lambda}_1\gamma_\mu\lambda_1$$

may be split into a singlet and an octet part under SU(3)′:

$$J_\mu = J_\mu^0 + J_\mu^8$$

with

$$J_\mu^0 = \frac{2}{3}\sum_i \bar{p}_i\gamma_\mu p_i - \frac{1}{3}\sum_i (\bar{n}_i\gamma_\mu n_i + \bar{\lambda}_i\gamma_\mu\lambda_i).$$

Between singlet states only J^0 plays a role, and the effective quark charges are the same as in the usual (coloured) quark model -- all of whose results are therefore reproduced including R = 2 (if the parton idea applies at these energies). When the threshold for producing SU(3)′ octet states is reached there is a "colour thaw" and R increases to its asymptotic value of 4 (a behaviour which can be imagined in the data). The octet states decay electromagnetically, emitting a single photon, into the SU(3)′ singlet π's, K's, N's, etc. observed asymptotically -- which could account for the energy crisis[6)]. The principal drawback in invoking this explanation of the e^+e^- data is that it implies that the "colour threshold" is at $\sqrt{s} \sim 3$ GeV and no striking threshold effects have been reported in ep → e + ... , or γp → ... at this energy[29)] (or in pp → ... , etc., or other strong processes, at the threshold for production of two octets) nor, as far as I know, is there evidence for excess photon production in γp reactions at high energy (although this might have been missed in hydrogen bubble chamber experiments).

If R tends to a theoretically undesirable value it can be [and has been[30,31)]] argued that the parton prediction for the value of R is the weakest of all parton predictions and should not necessarily be believed. If partons are quarks, or other objects which are not believed to be present in the final state, we can not believe the parton model to be literally true since it predicts parton production. The problem then is how can we contain the quarks without destroying all the predictions and which ones should we believe?

First we consider a rather general argument which suggests that the predictions for $eN \to e + \ldots$, and $e\bar{e} \to N + \ldots$, may be on a very different footing if the target N is a bound state. If electron scattering on protons and neutrons scales, presumably scattering on uranium $eU \to e + \ldots$ will scale also. However, it would not seem reasonable to expect that there is any simple connection between $d\sigma(eU \to e + \ldots)$ and $d\sigma(e\bar{e} \to U + \ldots)$ [32)]. In the first case we are given that the nucleons form a nucleus whereas the amplitude for this to occur plays a role in the second.

Of course this example of nucleons in a finite potential wall may not be very relevant to the fashionable picture of quarks bound in an infinite potential well. In such a picture we would expect $(^{\nu}_{e})N \to (^{\mu}_{e}) + \ldots$ to scale if the potential is soft near the origin and, in addition, the parton/light-cone sum rules should hold since the calculation of $d\sigma$ is normalized by the known Y, I_3, B, and Q of the target. We also expect that $\sigma(e^+e^- \to \text{all})$ will scale (although here we do not have non-relativistic models to test this idea); however, there is no exactly known property of the vacuum which normalizes the calculation since it has $Q = Y = I_3 = B = 0$.

The same is true formally. Naive operator manipulations give scaling and sum rules for $(^{\nu}_{e})N \to (^{\mu}_{e}) + \ldots$. For e^+e^- they give scaling but not the value of R, except by the assumption that the disconnected part of the light-cone algebra [or, equivalently, $\langle 0|\bar{\psi}(x)Q^2\psi(0)|0\rangle$] is the same as in free-field theory; this goes beyond the minimal light-cone assumptions needed to get scaling and sum rules in the space-like region, which are statements about the algebraic properties of operators, since it assumes something about the properties of the vacuum. If we reject this extra assumption, we can still (using some mild technical assumptions) relate R to some other vacuum expectation values.

For example, Crewther[33,34] has derived a relation between $\pi^0 \to \gamma\gamma$ (which, by PCAC, is related to a vacuum expectation value of three currents), R and the asymmetry in the scattering of polarized electrons on a polarized target. Using the quark parton/light-cone value for the latter quantity, the relation gives $R \simeq 2$ -- so we are back where we started with coloured quarks[35]! Hence if $R \neq 2$, we will have to abandon either Crewther's argument or the part of the quark algebra which determines polarized scattering or add new (charmed) currents which would change the relation. Another example is due to Terazawa[36] who (using earlier work by Crewther, Ellis and Ellis and Chanowitz) making the relatively strong assumption that the ε dominates the normal part of the trace of the energy momentum tensor, has related R to $\Gamma_{\varepsilon\to\gamma\gamma}$ and hence, using existing data, obtained $R < 15$.

To summarize: although superficially the data seem to disagree completely with (quark) parton predictions, it is possible to argue that scaling should be approached very slowly in e^+e^- and that the prediction $R = \Sigma\, Q^2$ should be distrusted. Unfortunately this was not widely stressed before the data were available. To achieve credibility *a posteriori*, those who support this view will have to be very convincing (they will presumably have trouble accounting for the energy crisis if it persists into the anticipated scaling region); it would be nice if they could account qualitatively for the sign and rate of the approach to scaling. It seems that this can be done in Preparata's model in which it is related to the approach to scaling in the central region in pp collisions; we refer to a forthcoming paper by Gatto and Preparata for details[37].

4.2.3 Renormalization group

Before much data was available, there was great excitement about the fact that the renormalization group gave $R = \Sigma\, Q_i^2$ and seemed to justify the parton result (it also predicted that R would approach its limit from above)[38]. Now Kogut[39] (following arguments due to Wilson) has used the renormalization group as a phenomenological tool which can accommodate scaling at SLAC energies in $eN \to e + \ldots$ and $R \simeq 2$ for $Q^2 \lesssim 10$ and link the rise in R to a scaling breakdown at higher space-like q^2. This will be discussed further by Rubinstein at this meeting[40].

4.2.4 Thermodynamic/hydrodynamic models

Lots of work is in progress on the properties of these models[41] which seem able to account for many features of the e^+e^- and also purely hadronic data[42] [however, these models can only describe $(1/\sigma)d\sigma$ and always leave the over-all behaviour unspecified]. I have neither time nor competence to describe this work and refer the interested reader to the literature.

4.2.5 An amusing philosophical remark due to Amati and Fubini[43] is worth mentioning. They consider first pp → h + ... For small p_T they call this a two-dimensional process (energy and one momentum); the one-particle distribution is supposed to obey the scaling law

$$\frac{1}{s^a}\, f\left(\frac{p_L}{\sqrt{s}}, p_T\right)$$

with a rapid fall-off in p_T. As p_T increases beyond the scale which characterizes the fall-off, there is a transition to a new three-dimensional regime and a new scaling law is supposed to set in:

$$\frac{1}{s^b}\, g\left(\frac{p_L}{\sqrt{s}}, \frac{p_T}{\sqrt{s}}\right).$$

The price for exciting more dimensions is a more rapid fall-off in s -- i.e. b > a. This can easily be generalized for two- or many-particle distributions. In $e^+e^- \to H(p) + \ldots$, the process is one-dimensional for small p so they expect

$$\frac{1}{s^A}\, F(p)$$

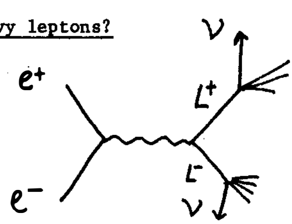
where F(p) falls rapidly with p (with the same scale as the p_T fall-off in hadronic collisions). When p becomes sufficiently large, the two-dimensional scaling law

$$\frac{1}{s^B}\, G\left(\frac{p}{\sqrt{s}}\right)$$

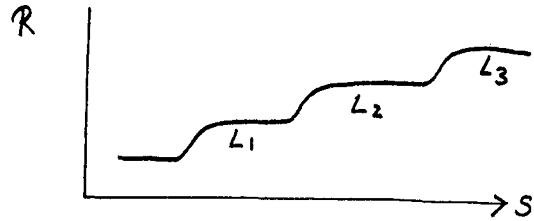
sets in with B > A. This certainly agrees with the trend of the data.

5. SOMETHING ELSE?

5.1 Heavy leptons?



A sequence of heavy leptons would give:



The neutrinos could resolve the energy crisis. However, according to conventional ideas, there should be a substantial branching ratio for $L \to \nu\mu\nu$ and $L \to \nu e\nu$, which would presumably have been noticed if L-production accounts for the bulk of the data. In addition, for light L's one would expect the dominant semi-leptonic decays to be $L^+ \to \nu\pi^+$ and $L^+ \to \nu\rho^+ (\to \pi^+\pi^0)$, in which case L-dominance would be hard to reconcile with a charged multiplicity of four.

5.2 Vector mesons?

Vector-meson production would give a rising R. However, the vectors could not be the usual weak W's (although they might be other weak ones) which are known to be heavier than 8 GeV and are expected to decay frequently into $\mu\nu$ and $e\nu$. Could they be charged spin-one partons which have so far escaped detection in $e(\nu)N$ reactions because they are hiding at small x [44]?

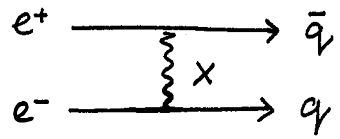
5.3 Neutral currents?

Although it might seem ridiculous to suppose that neutral currents could account for the rise in R, it is not immediately obvious that existing data cannot accommodate a comparatively light Z coupled sufficiently weakly to electrons not to spoil the successes of QED but sufficiently strongly to hadrons to give a pronounced resonance in e^+e^-. In fact a detailed analysis[45] shows that (without special pleading) this is hard to arrange while respecting the neutral current data (unless, contrary to present evidence, it turns out that $\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) \ll \sigma_{V-A}$). In any case this absurd suggestion can be ruled out[46] by showing that R does not depend sensitively on the longitudinal polarization of the beam; this might be worth showing anyway -- if only it could be done easily!

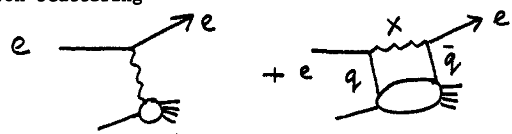
5.4 "A new phenomenon"?

(Possibly we can invent something which only contributes at small x, leaving the conventional scaling parton part, which on its own gives R = 2, exposed at large x!) We consider two suggestions:

a) Pati and Salam[47] have discussed the possibility that there are heavy exotic spin-1 objects which give:



where the quark-antiquark ($q\bar{q}$) state subsequently evolves into hadrons. For $S \ll M_X^2$ this would give an isotropic cross-section which could rise with S and an X-γ interference piece which could be constant. This model would lead (among other things) to unexpected states in e^+e^- annihilation (e.g. C even, $J^P = 1^+, 0^+, 0^-$) as well as giving scaling breakdown in inelastic electron scattering



characterized in general by parity violation, which will also occur in e^+e^-. Such exotic objects as the X do occur in some gauge theories.

b) Richter, when presenting the SPEAR data for the first time in his Irvine talk[3)], remarked that it could be understood if the electron had a strong interaction radius of 10^{-16} cm. This idea has been formalized by Greenberg and Yodh[48)] and by Nanopoulos and Vlassopoulos[49)]. It is supposed that in processes such as $e^+e^- \to e^+e^-$ there is a pure imaginary diffractive part in addition to the usual photon-exchange contributions. If this is of the same order as the imaginary part of the on-photon-exchange piece, it can play an important role in $\sigma^{tot}_{e^+e^-}$ but it will only give an order α correction to Bhabha scattering at present energies[50)]. Assuming this piece dominates $\sigma^{tot}_{e^+e^-}$ and using Muller-Regge phenomenology, this picture can fit the data for $e^+e^- \to \pi + \ldots$. However, it predicts that the one-particle distribution should have a sharp forward and backward peak which can be tested in the near future.

6. VITAL QUESTIONS

I finish by listing some of the obvious but fascinating experimental questions which may be answered during the next year:

1) What happens at higher energies?

2) Is the energy crisis real? If so, where is the missing energy? In ν's? In γ's? Are the γ's correlated into π^0's? Is there much η production?

3) What is the angular distribution in $e^+e^- \to H(p) + \ldots$? Is it $A(s,p) + B(s,p) \cos^2 \Theta$ with $|B| \leq A$ as required by one-photon exchange or are new phenomena at work?

4) Are there surprises in e^-e^- also? (This can be answered at the storage rings DORIS at DESY.)

Acknowledgements

I am grateful to many of my colleagues for discussion about e^+e^- annihilation. I wish to thank G. Karl and J. Prentki for comments on the manuscript.

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REFERENCES AND FOOTNOTES

1) The centre-of-mass energy range 3-5 GeV was first explored in experiments at CEA (Ref. 2). These experiments have been confirmed in an experiment employing a magnetic detector at the higher luminosity machine SPEAR, by a SLAC-LBL group, which has given much more detail (Refs. 3 and 4). The results fit onto the very interesting lower-energy data obtained with ADONE at Frascati (these results are reviewed by K. Strauch, Ref. 2, and in Ref. 5).

2) A. Litke et al., Phys. Rev. Letters 30, 1189 (1973).
G. Tarnopolosky et al., Phys. Rev. Letters 32, 432 (1974).
K. Strauch, to be published by North Holland Press *in* Proc. 1973 Bonn Conference.

3) B. Richter, Invited Talks, presented at the Irvine Conference (December 1973) and the Chicago APS Meeting (February 1974).

4) H. Lynch, Proc. of this Meeting.

5) V. Silvestrini, *in* Proc. XVI Int. Conf. on High-Energy Physics, NAL (1972).

6) J.D. Bjorken, SLAC PUB 1318, to be published by North Holland Press *in* Proc. 1973 Bonn Conference.

7) R.H. Cahn and J. Ellis, SLAC PUB 1384 (1973).

8) Apart from one figure in Physics Today and the CERN Courier for March 1974, the data from SPEAR is not published. This summary is therefore needed to make this paper comprehensible to those who did not hear Refs. 3 or 4 or other talks on the subject. For obvious reasons I give no details of the set-up, the definition of hadronic events, the cuts, etc., etc., although these are needed to evaluate the significance of the results.

9) This result takes into account the K/π ratio in going from mean momentum to mean energy per charged track (G. Goldhaber, private communication). Like all the results, it relies on a model (invariant phase space) to account for the 25% of phase space which is not observed. However, if annihilation via one photon dominates, the mean momentum cannot be very badly distorted since $|\cos\Theta| < 0.77$ is observed; the worst case would be if fast particles are as forward peaked as allowed by one-photon exchange ($\sim 1+\cos^2\Theta$) and slow ones are maximally peaked at 90° ($\sim 1-\cos^2\Theta$). Likewise the multiplicity is unlikely to be badly wrong.

10) M. Greco, Frascati preprint LNF-74/2(P), and contribution to this Conference.

11) N. Cabibbo and R. Gatto, Phys. Rev. 124, 1577 (1961).

12) For example, the hadronic contribution to g-2, which is

$$\Delta(g-2)_\mu = \frac{m_\mu^2}{6\pi^3}\int \frac{\sigma(e^+e^-\to\gamma\to \mathrm{Had.})}{s}\,ds$$

to lowest order, would involve an arbitrary subtraction constant; we would have to understand why it is small and does not spoil the agreement between QED and experiment.

13) For example, if

$$\sigma(e^+e^-\to\gamma\to \mathrm{Had}) = 20\,\mathrm{nb} \quad \text{for } s_0<s<s_1$$

and

$$= 20\,\mathrm{nb}\left(\frac{s_1}{s}\right) \quad \text{for } s>s_1,$$

I find that for $q^2 \ll s_1$:

$$\frac{\sigma(e^+e^-\to\mu^+\mu^-)}{\sigma_{QED}} = 1 + 4\times10^{-3}\left(\frac{E}{M_P}\right)^2\left[1+\ln\left(\frac{q^2-s_0}{s_1}\right)\right]$$

where E is the energy/beam. With certain additional assumptions it can be argued that the formula for corrections to $\sigma(e^+e^- \to \mu^+\mu^-)$ is valid to all orders in α (G. Karl, CERN TH 1813, 1974).

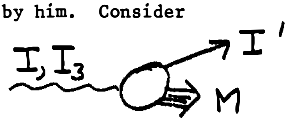
14) C.H. Llewellyn Smith and A. Pais, Phys. Rev. D6, 2625 (1972).

15) The results of Ref. 14 have been re-derived using a somewhat more elegant technique by E. Chacon and M. Moshinsky, Phys. Rev. D7, 2783 (1973).

16) A. Pais, Rockefeller preprint COO-2232B-48 (1974).

17) A. Di Giacomo, Phys. Letters 40 B, 569 (1972).
For N even Di Giacomo's lower bound is the same as that obtained in Ref. 14 but the upper bound is weaker. For an odd number of pions coupled to I = 1 the situation is reversed; Di Giacomo's method gives the upper bound of Ref. 14, but a weaker lower bound [in the cases in which both methods give the same bound, the symmetry of the wave functions which saturate the bounds is characterized by a Young

tableau with at most two rows and the full influence of the identity of the particles, which is taken into account in Ref. 14, is dormant]. Di Giacomo's results can be obtained in a less sophisticated way than that employed by him. Consider



Take the isospin of the missing mass M to be diagonal (I_M) and define "reduced cross-sections" $d\sigma_{I_M}$ and related quantities

$$A_{I_M} = \frac{1}{\sigma} \int d\sigma_{I_M} \,.$$

The I_3 conservation sum rule may be written

$$I_3 = \sum_{I_M} A_{I_M} \sum_{T_3} T_3 \left| \left(I, I_3 \middle| I', T_3 ; I_M, I_3 - T_3 \right) \right|^2 .$$

This is trivially satisfied for $I_3 = 0$, which is the case of direct interest here. However, the A_{I_M} are independent of I_3 and it gives a non-trivial relation between them for $I_3 \neq 0$ which then constrains the $I_3 = 0$ case. Using the "Landé g factor" relation

$$\sum_{T_3} T_3 \left| \left(I, I_3 \middle| I', T_3 ; I_M, I_3 - T_3 \right) \right|^2 = \frac{I_3 \left(I^2 + I'^2 - I_M^2 \right)}{2 I^2}$$

we obtain

$$2 I^2 = \sum_{I_M} A_{I_M} \left(I^2 + I'^2 - I_M^2 \right)$$

which is Di Giacomo's main result (in a slightly different notation). Using

$$N_{\pi^0} = A_0 + \frac{2}{5} A_2$$

$$N_{\pi^+} = N_{\pi^-} = \frac{1}{2} A_1 + \frac{3}{10} A_2 \,,$$

and $A_I \geq 0$ it leads to his bounds.

18) The method employed here and the subsequent discussion emerged from conversations with J.S. Bell and G. Karl. Curiously this formalism yields Di Giacomo's upper bound but a weaker lower bound.

19) C. Ferro Fontan and H.R. Rubinstein, CERN TH-1810 (1974), and H.R. Rubinstein, Proc. of this Meeting.

20) F.M. Renard, Montpellier preprint PM/7415 and contribution to this Conference.

21) N.S. Craigie and K.D. Rothe, CERN TH-1777 (1973), TH-1820 and 1821 (1974).

22) M. Chanowitz and S.D. Drell, Phys. Rev. Letters 30, 807 (1973), and SLAC PUB 1315 (1973).

23) M. Pavkovic, Phys. Letters 46 B, 435 (1973).

24) G.B. West, Stanford preprint ITP 454 (1974).

25) Of course this does not hold for π's from the isospin violating η decays. I believe that this result was first stated by J. Ellis and C.H. Llewellyn Smith -- footnote 36 of C.H. Llewellyn Smith "Theory of Lepton-Hadron Interactions" *in* Proc. IV Int. Conf. on High-Energy Collisions, Oxford, 1972. RHEL-72-001, Vol. 1 (ed. J.R. Smith).

26) This was pointed out to me by J. Ellis. I have subsequently discovered that the connection between a p_T cut-off and a slow approach to scaling for small x was discussed by T.F. Walsh and P. Zerwas (DESY preprint 73/34); the fact that the corrections are expected to be of order $(x^2 s)^{-1}$ was first pointed out by A.I. Sanda (NAL-Pub 74/16-THY).

27) J. Ellis and Y. Frishman, Phys. Rev. Letters 31, 135 (1973).

28) Especially if there is a 20% contribution from two-photon exchange. The Han Nambu model has other virtues -- for a recent discussion and references see Y. Nambu and M.Y. Han, Chicago preprint EF173/27.

29) Bjorken (Ref. 6) has argued that it would take large q^2 (for example, $Q^2 > 20$ GeV^2), as well as s above threshold, to produce octet states. This would protect existing data which is supposed to be described well by the Gell-Mann Zweig model (and hence also by the singlet part of J_μ in the Han-Nambu model). Naively one would expect the structure functions to increase by a factor of about two above threshold in eN; the effect in νN depends on the structure of the weak current which is not unique.

30) See, for example, the paper by me cited in footnote 25.

31) G. Preparata and R. Gatto, Nuclear Phys. B67, 362 (1973).

32) I believe that this example is due to Bjorken.

33) R.J. Crewther, Phys. Rev. Letters 28, 1421 (1972).

34) A simplified deviation has been given by W.A. Bardeen, H. Fritzsch and M. Gell-Mann, CERN TH-1538 (1972), published in "Scale and Conformal Symmetry in Hadron Physics", J. Wiley and Son (1973).

35) Alternatively the coloured quark model gives the correct value of $\pi^0 \to \gamma\gamma$ if we believe $R = \Sigma Q_i^2$. It can also be used to predict $\pi^0 \to \gamma\gamma$ directly if we dare to abstract the value of the Adler-Bell-Jackiw anomaly in $\partial_\mu A^\mu$ from all orders of perturbation theory.

36) H. Terazawa, Rochefeller preprint COO-2232B-38 (1974).

37) R. Gatto and G. Preparata, CERN preprint in preparation.

38) A. Zee, Phys. Rev. D8, 4038 (1973).
T. Appelquist and H. Georgi, Phys. Rev. D8, 4000 (1973).

39) J. Kogut, Cornell Preprint CLNS-259 (1974).

40) H.R. Rubinstein, Proc. of this Meeting.

41) See, for example, J. Engels, H. Saltz and K. Schilling, Nuovo Cimento 17 A, 535 (1973), and Bielefeld preprint Bi 74/07.
F. Cooper, G. Frye and E. Schonberg, Yeshiva preprint, "e^+e^- annihilation into hadrons and Landau's hydrodynamic model".
J. Baacke, Dortmund preprint, "Predictions of the hydrodynamical model for e^+e^- annihilation into hadrons".

42) For a review of recent work (mainly by Carruthers and Minh Duong Van) see P. Carruthers, Cornell preprint CLNS 219 (1973).

43) D. Amati and S. Fubini, CERN TH-1823 (1974).

44) J. Cleymans and G. Komen, CERN preprint in preparation.

45) C.H. Llewellyn Smith and D.V. Nanopoulos, CERN preprint in preparation.

46) Provided the $Ze\bar{e}$ coupling is not pure V or pure A.

47) J.C. Pati and A. Salam, Maryland preprint, "Are there anomalous lepton-hadron interactions?"

48) O.W. Greenberg and G.B. Yodh, Maryland preprint, "Diffractive lepton scattering and $\sigma_{e^+e^-}$; a new regime in lepton physics".

49) D.V. Nanopoulos and S. Vlassopoulos, CERN TH 1842 (1974).

50) A diffractive piece

$$i\, g(s)\, \frac{f(t)}{f(0)}$$

in the amplitude contributes

$$\sigma' = \frac{8\pi}{\sqrt{s}}\, g(s) \quad \text{to} \quad \sigma^{tot.}_{e^+e^-}$$

and

$$\frac{s}{(8\pi)^2}\,(\sigma')^2 \left(\frac{f(t)}{f(0)}\right)^2 \quad \text{to} \quad \left(\frac{d\sigma}{d\Omega}\right)_{Bhabha}$$

Assuming the diffractive part is due to factorizable Pomeron exchange, it can be shown that it plays a negligible role in $eN \to e + \ldots$ at present energies.