

# Comments on non-supersymmetric orientifolds at strong coupling

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**ABSTRACT:** We consider several properties of a set of anti- $Dp$ -branes in the presence of orientifold  $p$ -planes in type-II theory. This system breaks all the supersymmetries of the theory, but is free of tachyons. In particular, we center on the case of a single anti- $Dp$ -brane stuck at a negatively charged orientifold  $p$ -plane, and study its strong coupling behaviour for  $p = 2, 3, 4$ . Interestingly enough, as the coupling increases the system undergoes a phase transition where an additional antibrane is created. We conclude with some remarks on the limit of large number of antibranes on top of orientifold planes.

**KEYWORDS:** D-branes, Brane Dynamics in Gauge Theories.

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## Contents

|                                     |           |
|-------------------------------------|-----------|
| <b>1. Introduction</b>              | <b>1</b>  |
| <b>2. Weak coupling description</b> | <b>2</b>  |
| <b>3. Strong coupling behaviour</b> | <b>4</b>  |
| 3.1 Orientifold 3-planes            | 5         |
| 3.2 Orientifold four-planes         | 7         |
| 3.3 T-duality relations             | 9         |
| 3.4 Orientifold two-planes          | 11        |
| <b>4. Final remarks</b>             | <b>12</b> |

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## 1. Introduction

In this paper we will be interested in the properties of a set of anti- $Dp$ -branes (denoted  $\overline{Dp}$ -branes) in the neighbourhood of an orientifold  $p$ -plane ( $Op$ -plane) in type-II string theory (see [1] for early discussions on orientifolds, and [2] for a review). Since the antibranes and the orientifold projection preserve different sets of supersymmetries, the system breaks all the supersymmetries of the theory, but it is free of tachyons.

The motivation to study these systems is two-fold. First, even though they are non-supersymmetric, they are relatively simple. For instance, supersymmetry is preserved on the closed string sector, and bulk physics reduces to that of type-II theory. These systems may therefore be a good laboratory to continue extending our limited understanding of string theory and string duality in non-supersymmetric situations. In fact, we will be able to extract information about the strong coupling behaviour of these systems in particular cases. The second motivation is that configurations with orientifold planes and antibranes appear in the non-supersymmetric (but tachyon-free) type-I compactifications in [3] (see also [4]). Models of this kind exhibit certain phenomenologically interesting features, and deserve further study. Our comments in the present paper constitute a small step towards dealing with some of the relevant issues in a simpler and more controlled situation.

|              | $SO(p-1)$ | $SO(9-p)$ | $SO(N) ; USp(N)$      |
|--------------|-----------|-----------|-----------------------|
| Gauge bosons | vector    | singlet   | $N(N-1)/2 ; N(N+1)/2$ |
| Scalars      | singlet   | vector    | $N(N-1)2 ; N(N+1)2$   |
| Fermions     | spinor    | spinor    | $N(N+1)2 ; N(N-1)2$   |

**Table 1:** World-volume field content for  $\overline{Dp}$ -branes on  $Op$ -planes. The two possible gauge groups in the last column correspond to the case of  $Op^-$ - and  $Op^+$ -planes, respectively.

This note is organized as follows. In section 2 we make some remarks on the perturbative properties of these configurations, and compute the leading contribution to the interaction between antibranes and the orientifold plane, which is relevant for the stability of the configurations. In sections 3 we study duality properties of these models, and in particular the strong coupling behaviour of a single  $\overline{Dp}$ -brane stuck on top of a negatively charged  $Op$ -plane, for  $p = 2, 3, 4$ . This is the simplest non-supersymmetric orientifold configuration within our framework. Using dual descriptions we show that at strong coupling an additional antibrane is created. Section 4 contains some final comments.

## 2. Weak coupling description

Recall the configuration of  $N$   $Dp$ -branes on top of an  $Op$ -plane, which preserves sixteen supersymmetries. There are two kinds of orientifold projections in string perturbation theory, which differ in the sign of the contribution of the  $\mathbb{RP}_2$  world-sheet topology, and hence in the RR charge of the corresponding  $Op$ -planes. We denote by  $Op^\pm$ -plane the orientifold plane with  $\pm 2^{p-4}$  units of  $Dp$ -brane charge (as counted in the covering space).<sup>1</sup> The massless open string modes produce a world-volume gauge group  $G$ , along with the scalars and fermions required to fill a vector multiplet of the corresponding supersymmetry. The group  $G$  is  $SO(N)$  or  $USp(N)$  for the case of  $Op^-$ -plane or  $Op^+$ -plane, respectively.

Let us consider instead a set of  $N$   $\overline{Dp}$ -branes on top of the  $Op$ -plane. Before the orientifold projection, the massless spectrum on the  $\overline{Dp}$ -brane world-volume consists of a  $U(N)$  vector multiplet with respect to the sixteen unbroken supersymmetries. As discussed in [4], the orientifold projection on the bosonic fields is just as for  $Dp$ -branes, while fermions pick up an additional minus sign.<sup>2</sup> The world-volume massless fields are given in table 1. The quantum numbers are with respect to the  $SO(p-1)$  Lorentz little group, a  $SO(9-p)$  global symmetry (arising from rotational invariance in the transverse space), and  $SO(N)$  or  $USp(N)$  gauge group for  $Op^-$ - or

<sup>1</sup>As further discussed in section 3, the  $Op^+$ -plane usually comes in two varieties, distinguished by the value of a RR flux. Since they are identical in perturbation theory, we will not distinguish them in the present section.

<sup>2</sup>This sign is related, by open-closed duality, to the fact that antibranes and branes carry opposite RR charges.

$Op^+$ -planes, respectively (Notice that the symmetric representation of  $SO(N)$  and the antisymmetric of  $USp(N)$  are actually reducible). As is manifest from the spectrum, the system breaks all the supersymmetries. However, and in contrast with the more familiar brane-antibrane configurations, the spectrum contains no tachyons, since no annihilation can take place.

Due to lack of supersymmetry, the flat directions of the scalar potential are not protected against further corrections, which therefore control the stability of the configuration. At leading order, they arise from the Möbius strip, which is the simplest world-sheet topology feeling the breaking of all supersymmetries. The corresponding piece in the partition function is related to the interaction energy between the  $Op$ -plane and the  $\overline{Dp}$ -branes. The answer expected from long-distance considerations (oppositely charged objects attract and equally charged objects repel) turns out to be correct even at short distance, as we sketch in the following.

Consider for simplicity a single  $\overline{Dp}$ -brane (and its image) located at a the position  $\vec{X}$  in the  $(9 - p)$ -dimensional transverse space. For a  $Dp$ -brane the Möbius strip contribution would be given by (see [2] for conventions)

$$\begin{aligned} \mathcal{A}_{\mathcal{M}} = & \pm V_{p+1} \int_0^\infty \frac{dt}{2t} (8\pi^2 \alpha' t)^{-\frac{p+1}{2}} e^{-\frac{2X^2 t}{\pi \alpha'}} q^{-\frac{2}{3}} \prod_{n=1}^\infty (1 - q^{2n} e^{-i\pi n})^{-8} \times \\ & \times \frac{1}{2} \left\{ -iq^{-\frac{1}{3}} \prod_{n=1}^\infty (1 + q^{2n-1} e^{-i\pi(n-1/2)})^8 + \right. \\ & \left. + iq^{-\frac{1}{3}} \prod_{n=1}^\infty (1 - q^{2n-1} e^{-i\pi(n-1/2)})^8 + 16q^{\frac{2}{3}} \prod_{n=1}^\infty (1 + q^{2n} e^{-i\pi n})^8 \right\}, \end{aligned} \tag{2.1}$$

where  $q = e^{-\pi t}$  and the  $\pm$  sign corresponds to the  $Op^\mp$ -plane case. Due to supersymmetry, the first two contributions in the bracket, arising from the NS sector, cancel the remaining one, from the R sector. The amplitude in the case of  $\overline{Dp}$ -branes differs just in the sign of the R sector contribution (due to the additional sign in the  $\Omega$  action on spacetime fermions). Therefore, it is given by minus two times the above R contribution, and can be written as

$$\mathcal{A}_{\mathcal{M}} = \mp V_{p+1} \int_0^\infty \frac{dt}{2t} (8\pi \alpha' t)^{-\frac{p+1}{2}} e^{-\frac{2X^2 t}{\pi \alpha'}} F(q^2), \tag{2.2}$$

with  $F(q^2) = f_2(q^2)f_4(q^2)/f_1(q^2)f_3(q^2)$ , and the functions  $f_i(x)$  defined as in [2]. For non-zero  $X$ , we can change variables to get

$$\mathcal{A}_{\mathcal{M}} = \mp \frac{1}{2} V_{p+1} (8\pi \alpha')^{-\frac{p+1}{2}} \left( \frac{\pi \alpha'}{2X^2} \right)^{-\frac{p+1}{2}} \int_0^\infty du u^{-\frac{p+3}{2}} e^{-u} F\left( e^{-\frac{\pi^2 \alpha' u}{X^2}} \right). \tag{2.3}$$

This integral converges for  $-1 < p < 7$ , as follows from the asymptotic behaviour

$$\begin{aligned} F(e^{-2\pi t}) & \xrightarrow{t \rightarrow \infty} 16 \\ F(e^{-2\pi t}) & \xrightarrow{t \rightarrow 0} 256 t^4. \end{aligned} \tag{2.4}$$

At large  $X$ , keeping only the leading term in  $F$ , the amplitude reads

$$\mathcal{A}_{\mathcal{M}} = \mp 2^{p-4} V_{p+1} 2\pi (4\pi^2 \alpha')^{3-p} G_{9-p}(X^2), \quad (2.5)$$

where  $G_{9-p}(X^2) = \frac{1}{4}\pi^{\frac{p-9}{2}}\Gamma(\frac{7-p}{2})|X|^{p-7}$  is the  $(9-p)$ -dimensional massless scalar Green's function. The force between the objects goes like  $d\mathcal{A}_{\mathcal{M}}/dx$ , hence the amplitude (2.5) corresponds to a repulsive (resp. attractive) interaction between  $\overline{Dp}$ -branes and  $Op^-$ -planes (resp.  $Op^+$ -planes), due to exchange of massless closed string modes in the transverse  $9-p$  directions. Comparing (2.5) with the brane-brane interactions in [2], the additional factor of  $\mp 2^{p-4}$  accounts for the orientifold charge and tension.

At small values of  $X$ , replacing  $F$  in (2.3) by its leading term does not give a good approximation to the complete integral. A better picture of the interaction is obtained by expanding the original expression (2.2) around  $X = 0$ ,

$$\mathcal{A}_{\mathcal{M}} = \mp [\Lambda - MX^2 + O(X^4)], \quad (2.6)$$

with positive coefficients

$$\begin{aligned} \Lambda &= V_{p+1} \int_0^\infty \frac{dt}{2t} (8\pi\alpha't)^{-\frac{p+1}{2}} F(q^2) \\ M &= V_{p+1} \frac{2}{\pi\alpha'} \int_0^\infty \frac{dt}{2} (8\pi\alpha't)^{-\frac{p+1}{2}} F(q^2). \end{aligned} \quad (2.7)$$

We see that also at short distances the interaction is repulsive (vs. attractive) for the  $Op^-$ -plane ( $Op^+$ -plane) case. Notice that the  $X^2$  contribution can be interpreted in the open string channel as a one-loop correction to the mass of the scalar  $X$ , which was massless at (open-string) tree level.

From our above comments, we learn that the configuration of  $N$   $\overline{Dp}$ -branes on top of an  $Op^+$ -plane is stable at this order, and so at sufficiently small coupling. On the other hand, the configuration of  $N$   $\overline{Dp}$ -branes on top of an  $Op^-$ -plane is unstable, with the exception of the case  $N = 1$ , where the brane is stuck on the orientifold even at tree level. One might worry about the consistency of the latter configuration, since it involves coincident charges of the same kind. However, at short distances the interaction arises from (2.6) rather than from the Coulomb-like (2.5), and is finite for  $X = 0$ . We would like to stress that the absence of short-distance divergences follows from the fact that the model contains no open string tachyons, in contrast with brane-antibrane systems, which are singular in that regime [5].

### 3. Strong coupling behaviour

In this Section we consider the strong coupling behaviour of the configurations of antibranes on top of orientifold planes. For obvious reasons we will be more interested in configurations which are at least perturbatively stable, and in particular we will center on the system of a single  $\overline{Dp}$ -brane stuck on an  $Op^-$ -plane.

| D-brane description        | $(\theta_{NS}, \theta_R)$ | RR charge  | World-volume |
|----------------------------|---------------------------|------------|--------------|
| $O3^- + 2P$ D3             | $(0, 0)$                  | $2P - 1/2$ | $SO(2P)$     |
| $O3^- + (2P+1)$ D3         | $(0, 1/2)$                | $2P + 1/2$ | $SO(2P + 1)$ |
| $O3^+ + 2P$ D3             | $(1/2, 0)$                | $2P + 1/2$ | $USp(2P)$    |
| $\widetilde{O3}^+ + 2P$ D3 | $(1/2, 1/2)$              | $2P + 1/2$ | $USp(2P)$    |

**Table 2:** Supersymmetric configurations of D3-branes on O3-planes.

We will base our arguments on string duality, which has been a useful tool in analyzing the strong coupling behaviour of supersymmetric configurations of  $Dp$ -branes and  $Op$ -planes. The case of O3-planes has been discussed in [6] using type-IIB self-duality (see also [7]), the M-theory lifts of O4-planes have been determined in [8] (see also [9]), and those of O2-planes and O0-planes have been considered in [10] and [11], respectively. Useful information about other values of  $p$  can be extracted from [12] for O5-planes, [13] for O6-planes and [14] for O7-planes. We expect these results to help in understanding duality properties in our non-supersymmetric models, since the bulk is still supersymmetric, and its duality properties may extend to the fixed points of the orientifold action.<sup>3</sup> In the following sections and for illustrative purposes, we center on the particular case of O3-, O4- and O2-planes.

### 3.1 Orientifold 3-planes

It will be useful to recall the situation for supersymmetric configurations of D3-branes on O3-planes, studied in [6]. There are four types of supersymmetric configurations, labeled by  $(\theta_{NS}, \theta_R)$ , where  $\theta_{NS}, \theta_R = 0, 1/2$  denote the field-strength flux of the type-IIB 2-forms in the transverse space (with the origin excised)  $\mathbb{RP}_5 \times \mathbb{R}$ . The map between the configurations and their fluxes is given in table 2, where the  $\widetilde{O3}^+$ -plane is an exotic variety of the  $O3^+$ -plane, differing from it in a RR-flux, and producing also a  $USp(2P)$  gauge theory. The above configurations come in multiplets of the type-IIB  $SL(2, \mathbb{Z})$  duality group. The  $SL(2, \mathbb{Z})$  action on the configurations follows from its action on the corresponding NS-NS and R-R fluxes, and underlies the Montonen-Olive duality properties of the  $\mathcal{N} = 4$  supersymmetric gauge theories in the last column. Setting  $P = 0$ ,  $SL(2, \mathbb{Z})$  also gives information about the non-perturbative properties of O3-planes. For instance, their behaviour at strong coupling can be extracted from their duals under the  $\tau \rightarrow -1/\tau$  transformation. We thus learn that in the strong coupling limit the  $O3^-$ -plane and the  $\widetilde{O3}^+$ -plane are unchanged, whereas the  $O3^+$ -plane turns into an  $O3^-$ -plane with a stuck D3-brane, and vice-versa (as proposed earlier in [7]).

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<sup>3</sup>In other words, the orientifolding action in our models belongs to family 2 in the classification in [15], where it was argued that the quotient theory retains the duality properties of the original theory, i.e. ‘orientifolding commutes with duality’.

| D-brane description                         | $(\theta_{NS}, \theta_R)$ | RR charge   | World-volume  |
|---|---------------------------|-------------|---------------|
| $O3^- + 2P \overline{D3}$                   | $(0, 0)$                  | $-2P - 1/2$ | $SO(2P)$      |
| $O3^- + (2P + 1) \overline{D3}$             | $(0, 1/2)$                | $-2P - 3/2$ | $SO(2P + 1)$  |
| $O3^+ + (2P + 2) \overline{D3}$             | $(1/2, 0)$                | $-2P - 3/2$ | $USp(2P + 2)$ |
| $\widetilde{O3}^+ + (2P + 2) \overline{D3}$ | $(1/2, 1/2)$              | $-2P - 3/2$ | $USp(2P + 2)$ |

**Table 3:** Non-supersymmetric configurations of  $\overline{D3}$ -branes on O3-planes.

Let us now turn to the non-supersymmetric case of  $\overline{D3}$ -branes on O3-planes. The classification of these configurations is analogous to that in the supersymmetric case, for the following reason. In the non-supersymmetric configurations, the transverse space (after excising the origin) is also  $\mathbb{RP}_5 \times \mathbb{R}$ . Moreover, as mentioned in section 2, the non-supersymmetric theories differ from the supersymmetric ones only in an additional minus sign in the orientifold action on fermions. This means that fermions pick up an additional minus sign in going along non-contractible 1-cycles in  $\mathbb{RP}_5$ , but the bosonic properties of the background are unchanged at the classical level, and so is the classification of fluxes for the 3-form fields strengths. Finally, since these fluxes are discrete, topological, this classification cannot be changed by quantum corrections, even in the non-supersymmetric situation. Therefore, we obtain four types of non-supersymmetric configurations, as in table 3. Recall that scalars transform in the adjoint of the gauge group, but fermions do not. As in the supersymmetric case, these configurations must appear in  $SL(2, \mathbb{Z})$  multiplets, and therefore transform according to their flux structure. Notice that in the above table we have arranged the number of  $\overline{D3}$ -branes so that configurations in the same  $SL(2, \mathbb{Z})$  multiplet have the same RR charge.

This leads to interesting proposals for the strong coupling behaviour of the configurations. For instance, a configuration of  $N$   $\overline{D3}$ -branes on a  $O3^+$ -plane, which is stable at weak coupling, becomes unstable at sufficiently strong coupling, since it is better described as a set of  $N + 1$   $\overline{D3}$ -branes on a  $O3^-$ -plane at weak string coupling. To stay on the safe side, in the following we center on a particular case which is stable at weak and strong coupling (in the hope that it behaves nicely also in between), namely the configuration of an  $O3^-$ -plane with a stuck  $\overline{D3}$ -brane. This object has charge  $-3/2$  under the RR four-form, and corresponds to fluxes  $(0, 1/2)$ . We propose that at strong coupling this configuration turns into a set of two  $\overline{D3}$ -branes on an  $O3^+$ -plane, which has the appropriate charge and flux structure. Notice that in the latter configuration, the  $\overline{D3}$ -branes are bound to the  $O3^+$ -plane due to the attractive interactions discussed in section 2, and only in the extreme strong coupling they are free to move off into the bulk (the dual coupling being strictly zero in this case). Hence this model presents an interesting transition between two mechanisms to bind antibranes to orientifold planes (stuck antibranes vs. attracted antibranes).

A further bit of information supporting this proposal follows from the world-volume perspective. Even though the dynamics of the relevant field theory is non-supersymmetric and therefore intractable beyond weak coupling, certain quantities, namely anomalies of global symmetries, should match in the weak and strong coupling limit [16]. In the present case, there is a classical  $SU(4)$  symmetry associated to rotations in the six-transverse dimensions. In the configuration of an  $O3^-$ -plane with an stuck  $\overline{D3}$ -brane, the world-volume contains no bosonic fields, but there is a fermion transforming in the fundamental representation of this  $SU(4)$ , and leading to an anomaly. In the configuration of an  $O3^+$ -plane with two  $\overline{D3}$ -branes, there is a gauge group  $USp(2)$ , under which scalars transform in the adjoint, but under which fermions are singlets. The latter transform in the fundamental of the  $SU(4)$  global symmetry. Hence the anomalies for both configurations match, making our strong coupling proposal plausible.

An intuitive explanation for the creation of an additional  $\overline{D3}$ -brane would be as follows. We start with one  $\overline{D3}$ -brane stuck at an  $O3^-$ -plane. As the string coupling becomes stronger, it becomes easier to nucleate  $D3$ - $\overline{D3}$ -brane pairs out of the vacuum. Since the ‘real’ and the ‘virtual’  $\overline{D3}$ -branes can pair up and move off slightly into the bulk, the  $D3$ -brane can be considered more tightly bound to the  $O3^-$ -plane than its companions. Eventually, the coupling is strong enough so that the compound made of one  $D3$ -brane and an  $O3^-$ -plane is better described as an  $O3^+$ -plane. Of course, this picture is rather heuristic, but gives answers consistent with all constraints in the system.

### 3.2 Orientifold four-planes

Supersymmetric configurations of  $D4$ -branes and  $O4$ -planes have been studied in [8], where their M-theory interpretation is provided (see also [9]). There are four kinds of configurations, which correspond to  $2P$   $D4$ -branes on an  $O4^-$ -plane,  $(2P + 1)$   $D4$ -branes on an  $O4^-$ -plane,  $2P$   $D4$ -branes on an  $O4^+$ -plane or  $2P$   $D4$ -branes on an  $\widetilde{O4}^+$ -plane. They differ in the choice of field-strength flux  $\theta_{NS}$  for the NS-NS 2-form, and in the possibility of embedding the orientifold projection as a  $\mathbb{Z}_2$  Wilson line  $w_R$  for the RR  $U(1)$  gauge field, as in [17]. This information, the charges under the RR 5-form, and the constraints of flux quantization in M-theory [18] are enough to provide the M-theory lifts of these configurations, and therefore to study their strong coupling limits. The result is given in table 4. The  $\mathbb{Z}_2$  acts by reflection of the coordinates of  $\mathbb{R}^5$ , and, in the second and fourth cases, by a half shift in the  $S^1$  coordinate. The M5-branes sit at the origin in the  $\mathbb{R}_5$  modded out by  $\mathbb{Z}_2$ . In matching the M5-brane charges with the  $D4$ -brane charges, one should take into account that fixed points  $\mathbb{R}^5/\mathbb{Z}_2$  in M-theory carry  $-1$  fivebrane charge [19], while smooth geometries carry no fivebrane charge. Also, one of the moduli in the M-theory configuration in the third line is frozen [8], by the mechanism explained in [9].



| D-brane description                | $(\theta_{\text{NS}}, w_R)$ | Charge   | World-volume | M-theory   |
|------------------------------------|-----------------------------|----------|--------------|--|
| $O4^- + 2P \text{ D4}$             | $(0, 0)$                    | $2P - 1$ | $SO(2P)$     | $\mathbb{R}^5 \times \mathbb{R}^5/\mathbb{Z}_2 \times S^1 + 2P \text{ M5}$         |
| $O4^- + (2P + 1) \text{ D4}$       | $(0, 1/2)$                  | $2P$     | $SO(2P + 1)$ | $\mathbb{R}^5 \times (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2 + 2P \text{ M5}$       |
| $O4^+ + 2P \text{ D4}$             | $(1/2, 0)$                  | $2P + 1$ | $USp(2P)$    | $\mathbb{R}^5 \times \mathbb{R}^5/\mathbb{Z}_2 \times S^1 + (2P + 2) \text{ M5}$   |
| $\widetilde{O4}^+ + 2P \text{ D4}$ | $(1/2, 1/2)$                | $2P + 1$ | $USp(2P)$    | $\mathbb{R}^5 \times (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2 + (2P + 1) \text{ M5}$ |

**Table 4:** Supersymmetric arrays of D4-branes on O4-planes, and their M-theory lifts.

| D-brane description                          | $(\theta_{\text{NS}}, w_R)$ | Charge    | World-volume | M-theory   |
|--|-----------------------------|-----------|--------------|--|
| $O4^- + 2P \overline{\text{D4}}$             | $(0, 0)$                    | $-2P - 1$ | $SO(2P)$     | $\mathbb{R}^5 \times \mathbb{R}^5/\mathbb{Z}_2 \times S^1 + 2P \overline{\text{M5}}$         |
| $O4^- + (2P + 1) \overline{\text{D4}}$       | $(0, 1/2)$                  | $-2P - 2$ | $SO(2P + 1)$ | $\mathbb{R}^5 \times (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2 + 2P + 2 \overline{\text{M5}}$   |
| $O4^+ + 2P \overline{\text{D4}}$             | $(1/2, 0)$                  | $-2P + 1$ | $USp(2P)$    | $\mathbb{R}^5 \times \mathbb{R}^5/\mathbb{Z}_2 \times S^1 + (2P - 2) \overline{\text{M5}}$   |
| $\widetilde{O4}^+ + 2P \overline{\text{D4}}$ | $(1/2, 1/2)$                | $-2P + 1$ | $USp(2P)$    | $\mathbb{R}^5 \times (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2 + (2P - 1) \overline{\text{M5}}$ |

**Table 5:** Non-supersymmetric arrays of  $\overline{\text{D4}}$ -branes on O4-planes, and their M-theory lifts.

We can repeat this exercise for the non-supersymmetric configurations of  $\overline{\text{D4}}$ -branes on O4-planes. As in the previous section, the classification in the supersymmetric case can be carried out for our non-supersymmetric models. We can also propose suitable M-theory configuration which reduce to these type-IIA models, and are consistent with flux quantization and other known properties of M-theory. It is meaningful to consider such M-theory lifts because away from the non-supersymmetric orientifold plane local physics is given by type-IIA physics. Table 5 should then be understood as providing the appropriate M-theory objects to be placed in the orientifold core region in the corresponding lifts.

This information provides the strong coupling description of the configurations of O4-planes and  $\overline{\text{D4}}$ -branes. However and as usual, the most meaningful statements are restricted to the stable systems, and in the following we center on the case of an  $O4^-$ -plane with one stuck  $\overline{\text{D4}}$ -brane. In the strong coupling limit, this configuration is better described in M-theory, as two  $\overline{\text{M5}}$ -branes in the background geometry  $\mathbb{R}^5 \times (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2$ . Notice that the naive lift as one  $\overline{\text{M5}}$ -brane in the background  $\mathbb{R}^5 \times \mathbb{R}^5/\mathbb{Z}_2 \times S^1$  is not consistent with the presence of non-zero  $w_R$  in the IIA model or with flux quantization in M-theory.

Notice that the two  $\overline{\text{M5}}$ -brane in M-theory are presumably bound to the origin in  $\mathbb{R}^5$  due to attractive interactions. These cannot be computed at short distances, given our ignorance about the fundamental degrees of freedom in M-theory, but at long distances they reduce to the exchange of massless supergravity fields.

Another interesting observation involving short-distance physics in M-theory is that a phase transition seems to occur between the large and small radius limits. Starting at large radius, and trying to reach the weakly coupled type-IIA limit, beyond a certain radius the geometry  $\mathbb{R}^5 \times (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2$  is better described as a type-IIA  $O4^-$ -plane with an stuck D4-brane. The latter can thus annihilate with one of the antibranes present from the beginning, leading to our familiar system of an  $O4^-$ -plane with one  $\overline{D4}$ -brane. This phase transition may imply that the M-theory lifts of these configurations are not useful in obtaining even qualitative features about gauge theories using brane configurations as in [20].

### 3.3 T-duality relations

In this section we would like to use T-duality to relate our proposals for the strong coupling behaviour of non-supersymmetric  $O3^-$  and  $O4^-$  planes, with an analysis inspired in [9]. Given the equivalence between type-IIB theory on a circle and M-theory on a 2-torus, one can find strong-weak coupling duals in type-IIB theory by obtaining two different degenerations of the M-theory 2-torus. We illustrate this technique in our non-supersymmetric orientifold context by considering type IIB on  $\mathbb{R}^4 \times \mathbb{R}^5 \times S^1$  modded out by  $\Omega(-1)^{F_L} I$ , where  $F_L$  is the left-handed world-sheet fermion number, and  $I$  inverts all coordinates of  $\mathbb{R}^5 \times S^1$ . The model contains two  $O3^-$  planes, which can be chosen of different type, and whose strong coupling behaviour can now be derived from the M-theory realization. In the following we consider several examples, with one  $O3^-$  plane of type  $O3^- + \overline{D3}$  and one supersymmetric  $O3^-$  plane.

- i) Consider an initial configuration with an  $O3^- + \overline{D3}$  system and an  $O3^-$ -plane, with a transverse circle  $S^1$ . Its M-theory lift can be obtained by first T-dualizing to a type-IIA model, and then growing the M-theory circle  $\tilde{S}^1$ . In this case, the IIA model is an  $O4^- + \overline{D4}$  wrapped on  $S^1$ , and the M-theory lift corresponds to two  $\overline{M5}$ -branes in the geometry  $\mathbb{R}^4 \times S^1 \times (\mathbb{R}^5 \times \tilde{S}^1)/\mathbb{Z}_2$ . A different type-IIB description, corresponding to the strong coupling limit of the initial one, can be now achieved by shrinking  $S^1$  first and then T-dualizing along  $\tilde{S}^1$ . Shrinking  $S^1$  yields two  $\overline{D4}$ -branes in the geometry  $\mathbb{R}^4 \times (\mathbb{R}^5 \times \tilde{S}^1)/\mathbb{Z}_2$ , with a  $\mathbb{Z}_2$  action not embedded as a  $U(1)_R$  Wilson line ( $w_R = 0$ ). The T-dual of this configuration is given by two  $\overline{D3}$ -branes and two oppositely charged  $O3^-$  planes (see [14]), whose overall  $\theta_R$  must be zero to agree with the vanishing type-IIA  $w_R$ . The T-dual configuration hence contains an  $O3^+ + 2\overline{D3}$  system and an  $O3^-$ -plane, which precisely is the proposed strong coupling limit for the initial configuration. Notice that the location of the  $\overline{D3}$ -branes on top of the  $O3^+$ -plane obeys dynamical reasons.
- ii) Let us start with an  $O3^- + \overline{D3}$  system and an  $O3^+$ -plane. The T-dual configuration corresponds to one  $\overline{D4}$ -brane in the geometry  $\mathbb{R}^4 \times (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2$ , with  $w_R = 1$ . Its M-theory lift is therefore one  $\overline{M5}$ -brane in the background

geometry  $\mathbb{R}^4 \times (\mathbb{R}^5 \times S^1 \times \tilde{S}^1)/\mathbb{Z}_2$ , with the  $\mathbb{Z}_2$  acting with a simultaneous half-shift on both circles. Notice this model is invariant under exchange of both circles, hence shriking  $S^1$  and T-dualizing along  $\tilde{S}^1$  takes us to a type IIB model isomorphic to the initial one. This self-duality is also obtained from our type-IIB analysis, but in a non-trivial fashion. The strong coupling of the initial configuration is given, according to section 3.1, by an  $O3^+ + 2\overline{D3}$  system and an  $O3^- + D3$  system. The two  $\overline{D3}$ -branes are attracted by the  $O3^+$ , but more strongly by the  $D3$ -brane (stuck at the  $O3^-$ -plane). Hence the true vacuum is achieved only after annihilating a  $D3$ - $\overline{D3}$  pair, and corresponds to an  $O3^+$ -plane and an  $O3^- + \overline{D3}$  system, as derived from the M-theory argument. Notice that the agreement in this case is surprisingly intricate.

- iii) Consider an initial configuration of an  $O3^- + \overline{D3}$  system and one  $\widetilde{O3}^+$ -plane. Its T-dual is given by one  $\overline{D4}$ -brane in the geometry  $\mathbb{R}^4 \times (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2$ , and with  $w_R = 0$ . Its M-theory lift is given by one  $\overline{M5}$ -brane in the geometry  $\mathbb{R}^4 \times \tilde{S}^1 \times (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2$ . Upon shrinking  $S^1$ , we recover an  $\widetilde{O4}^+ + 2\overline{D4}$  system (this is more easily understood by lifting the IIA configuration, and annihilating a  $M5$ - $\overline{M5}$  pair). Finally, T-dualizing to type IIB, we recover an  $O3^+ + 2\overline{D3}$  system and an  $\widetilde{O3}^+$ -plane, which agrees with our strong coupling proposal of the initial configuration.
- iv) Finally, consider an  $O3^- + \overline{D3}$  system and an  $O3^- + D3$  system. The T-dual IIA model corresponds to one  $O4^-$ -plane with one stuck  $D4$ -brane and one stuck  $\overline{D4}$ -brane, with different Wilson lines along the  $S^1$  they wrap, and with  $w_R = 0$ . In M-theory this is described as one  $M5$ -brane and one  $\overline{M5}$ -brane on  $\mathbb{R}^4 \times S^1 \times \mathbb{R}^5/\mathbb{Z}_2 \times \tilde{S}^1$ , with different ‘Wilson lines’ (actually, periods of the world-volume self-dual 2-form on the M-theory 2-torus). After shrinking  $S^1$ , we obtain a IIA configuration of an  $O4^+ + 2\overline{D4}$  system, with no Wilson lines (again this is easier to understand by lifting the IIA configuration, and annihilating a  $M5$ - $\overline{M5}$  pair with identical ‘Wilson lines’). The type-IIB T-dual contains an  $O3^+ + 2\overline{D3}$  system and an  $O3^+$ -plane, which agrees with the strong coupling proposal for the initial model.

Notice that the above arguments involve shriking circles in M-theory, whose treatment is not completely rigorous in the absence of supersymmetry and so of the BPS property. Therefore they rely in the assumption that supersymmetry away from the orientifold core is enough to allow taking such limits. Notice also that even though the duality chains are quite constrained from mere ‘kinematics’, namely matching of charges, fluxes, etc (that is actually the reason that allows us to match non-supersymmetric configurations) there is some role played by non-trivial dynamics, in particular in the form of brane-antibrane annihilations, and of uncanceled antibrane-

orientifold forces. Finally notice that in the above discussion we have ignored the issue of the dynamics of the modulus associated to the circle radius, which in a more detailed treatment should perhaps also be taken into account.

The above examples mainly center on our proposal for the strong coupling limit of the  $O3^- + \overline{D3}$  system. Other examples can be studied analogously.

### 3.4 Orientifold two-planes

We conclude this Section with a brief discussion on the strong coupling description of an  $O2^-$ -plane with one stuck  $\overline{D2}$ -brane. The only ingredients required from the supersymmetric case are the M-theory lifts of the  $O2^-$ -plane, the  $O2^+$ -plane and the  $\overline{D2}$  system, determined in [10]. They correspond to M-theory geometries of the form  $\mathbb{R}^3 \times (\mathbb{R}^7 \times S^1)/\mathbb{Z}_2$ , where the  $\mathbb{Z}_2$  action reverses all the coordinates of  $\mathbb{R}^7 \times S^1$ . The model contains two fixed points, locally of the form,  $\mathbb{R}^8/\mathbb{Z}_2$ , and each having one of two possible values of field-strength flux for the M-theory 3-form. The two possibilities endow the fixed point with different membrane charges:  $-1/8$  for a singularity with vanishing flux and  $3/8$  for a singularity with non-zero flux. The M-theory descriptions of the relevant  $O2$ -planes are in table 6.

In our context of non-supersymmetric configurations of  $O2$ -planes and  $\overline{D2}$ -branes, we center on the particular case of the  $O2^- + \overline{D2}$  system (other examples can be worked out analogously). Using our experience in similar systems

| IIA description | Charge | M-theory fixed points |
|-----------------|--------|-----------------------|
| $O2^-$          | $-1/4$ | $(-1/8, -1/8)$        |
| $D2$            | $3/4$  | $(3/8, 3/8)$          |
| $O2^+$          | $1/4$  | $(3/8, -1/8)$         |

**Table 6:** Supersymmetric configurations of  $D2$ -branes on  $O2$ -planes, and their M-theory lifts.

in other dimensions, we propose the correct M-theory lift is given by  $\mathbb{R}^3 \times (\mathbb{R}^7 \times S^1)/\mathbb{Z}_2$  with two fixed points of charge  $3/8$  and two  $\overline{M2}$ -branes.

In order to show that, we use T-duality with some familiar configurations of  $O3$ -planes and  $\overline{D3}$ -branes. Consider an  $O3^- + \overline{D3}$  system, wrapped on a longitudinal  $S^1$ , and perform a T-duality along it. The resulting type-IIA model contains an  $O2^-$ -plane and an  $O2^- + \overline{D2}$  system, with a transverse circle. The M-theory lift of this configuration is given by  $\mathbb{R}^3 \times (\mathbb{R}^6 \times \mathbb{T}^2)/\mathbb{Z}_2$ , with two  $\overline{M2}$ -branes. There are four fixed points, two of which have charge  $-1/8$  (from lifting the  $O2^-$ -plane) and two have charge  $3/8$  (from our proposed lift of the  $O2^- + \overline{D2}$  system). This M-theory lift can be confirmed by first S-dualizing the initial type-IIB model, and then lifting it to M-theory. In the strong coupling limit, the initial IIB configuration turns into an  $O3^+$  with two  $\overline{D3}$ -branes, wrapped on  $S^1$ . Its type-IIA T-dual contains two  $O2^+$ -planes and two  $\overline{D2}$ -branes. Its M-theory lift corresponds to  $\mathbb{R}^3 \times (\mathbb{R}^6 \times \mathbb{T}^2)/\mathbb{Z}_2$ , with two  $\overline{M2}$ -branes, two fixed points of charge  $-1/8$  and two of charge  $3/8$  (since each  $O2^+$  contributes with one fixed point of each kind). This agrees with the M-theory configuration found before, and supports our identification of the strong coupling limit of the  $O2^- + \overline{D2}$  system.

Notice that the naive M-theory lift corresponding to M-theory on  $\mathbb{R}^3 \times (\mathbb{R}^7 \times S^1)/\mathbb{Z}_2$  with two fixed points of charge  $-1/8$ , and with one  $\overline{\text{M2}}$ -brane is not correct. In fact, having M2-branes (or  $\overline{\text{M2}}$ -branes) stuck at  $\mathbb{R}^8/\mathbb{Z}_2$  singularities is not consistent with flux quantization conditions.

We hope this example suffices to illustrate the discussion of O2-planes, and spare the reader an exhaustive treatment, already performed in the analogous case of O3- and O4-planes.

#### 4. Final remarks

The purpose of this paper has been to explore some of the properties of systems of  $\overline{\text{D}p}$ -branes on  $\text{O}p$ -planes of different kinds. We believe these configurations show interesting features and non-supersymmetric dynamics, which nevertheless seem accessible to study (to a certain extent) due to the simplicity of the configuration.

A particular avenue, not directly exploited in this note, is to consider the limit of a large number  $N$  of  $\overline{\text{D}p}$ -branes on the orientifold planes.<sup>4</sup> This approach would be particularly useful to study the field theories in the  $\overline{\text{D}p}$ -brane world-volume, by computing in the dual supergravity background in the sense of the AdS/CFT correspondence (see [21] for a review). Some results in this direction have appeared in [22] for the case of  $\overline{\text{M2}}$ -branes. For reasons already explained, the bosonic parts of the corresponding supergravity backgrounds are identical to those in the supersymmetric cases. For instance, for  $\overline{\text{D3}}$ -brane on O3-planes, the background at leading order in  $N$  is given by  $AdS_5 \times \mathbb{R}P_5$ , just as in the supersymmetric case in [6]. The supersymmetry breaking effects arise because fermionic fields pick up an additional  $(-1)$  in going along non-contractible cycles in  $\mathbb{R}P_5$ , as compared with the supersymmetry-preserving case.<sup>5</sup> This effect is subleading in  $N$ , since it arises from the orientifold projection, which is suppressed in the large- $N$  limit [24]. It would be interesting to study  $1/N$  corrections to different quantities in these type of backgrounds (for instance, the subleading correction to the conformal anomaly, analyzed in [25] in the supersymmetric case (see also [26] for related computations)).

A different line of development would be to consider other values of  $p$ . In some cases this would require improving our understanding of the different orientifold planes even in the supersymmetric case. Finally, it would be interesting to consider more complicated configurations, by introducing new objects (e.g. additional branes and/or antibranes) or orbifold projections. This would require further developments in the study of the stability of additional moduli in the former case, and the appropriate treatment of uncancelled twisted NS tadpoles in the latter.

In any event, we hope our observations on these systems are useful as a starting point for the investigation of these models.

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<sup>4</sup>I thank G. Mandal for conversations on this point.

<sup>5</sup>This is equivalent to the orientation-reversing introduced in [23] in more general supergravity backgrounds.

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