HEP'99 # 1_572
Submitted to Pa 1
Pl 1

DELPHI 99-128 CONF 315
15 June 1999

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Preliminary

DELPHI Collaboration

Paper submitted to the HEP'99 Conference
Tampere, Finland, July 15-21

DELPHI Collaboration DELPHI 99-128 CONF 315
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An Upper Limit for BR(Z $\rightarrow$ ggg) from Two and Three Jet Correlations in 3-jet Z Hadronic Decays

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Abstract

An upper limit for $Br(Z^0 \rightarrow 3g)$ is obtained from a correlation method, which distinguishes statistically between quark and gluon jets by using the difference in their charged particle multiplicity distributions. Mirror symmetric and threefold symmetric three jet events collected by the DELPHI experiment at LEP during 1991-1995 have been analysed. From the sample of threefold symmetric three jet events the 95% confidence level upper limit is obtained to be: $Br(Z^0 \rightarrow 3g) \leq 3.9 \times 10^{-3}$.

1 Introduction

The measurement of branching ratio for the decay of the Z^0–boson into three gluons is a good test for the Standard Model which predicts a very small branching ratio for the decay $Z^0 \to 3g$ from quark loops [1]:

$$Br^{SM}(Z \to 3g) \ \simeq 2.0 \times 10^{-6} \tag{1}$$

and for the Model of Compositeness of the Z–boson which would induce new couplings and decay modes and a predicted branching ratio [2]:

$$Br(Z \to 3g) \ \leq 2.0 \times 10^{-3}, \tag{2}$$

much larger than the standard model expectation.

In recent DELPHI paper [3] an upper limit for $Br(Z \to 3g)$ has been determined from a sample of threefold symmetric 3 jet events in which the angles between jets are in the range $120 \pm 20°$ (referred to below as M–events). The analysis is based on the difference between the charged particle multiplicity distributions of quark and gluon jets. This difference is exploited by comparing the correlations present between the jet multiplicities in symmetric 3 jet events, in general consisting of two quark jets and one gluon jet, to those in uncorrelated fake events constructed by mixing jets from different real events. The upper limit for $Br(Z^0 \to 3g)$ is found to be equal to 1.6×10^{-2}. This method, generally referred to as the correlation method, has also been applied to the study of the ratio of the mean charged particle multiplicities in gluon and quark jets in symmetric 3 jet events [4].

In present letter the correlation method is applied to the more abundant sample of mirror symmetric 3 jet events in which the two smaller energy jets have approximately equal energy in the range 25 ± 5 GeV (referred to below as Y–events). The method is modified in such a way that only correlation between charged particle multiplicities of smaller energy jets is considered.

In addition, the final sample of M–events is processed by using the procedure described in previous work[3] but greater precision is obtained by applying an additional constraint in the final fit.

The data used were collected by the DELPHI experiment at LEP in the years 1991 to 1995 at centre-of-mass energies around 91.2 GeV. They consist of about 3.5 million hadronic Z^0 decays.

2 The correlation method

2.1 Application to the Y–event sample

The multiplicity correlation function for the Y–event sample is defined as :

$$C(n_2, n_3) = \frac{P(n_2, n_3)}{P_{uncor}(n_2, n_3)}, \tag{3}$$

where $P(n_2, n_3)$ is the probability of observing a 3 jet event in which the charged particle multiplicities of the smaller energy jets are equal to n_2 and n_3. Jets will always be numbered such that $n_2 \geq n_3$. The charged particle multiplicity of the biggest energy jet, n_1, is ignored. $P_{uncor}(n_2, n_3)$ is the corresponding probability for uncorrelated jets constructed using the mixed event technique: one mixed Y–event was obtained from three different real 3 jet events by selecting one jet at random from each event.

Assuming the multiplicities of the individual jets in a real event to be uncorrelated, the probability $P(n_2, n_3)$ can be expressed through the multiplicity distributions for gluon jets, $G(n)$, light (uds) quark jets, $Q(n)$, c–quark jets, $O(n)$, and b–quark jets, $B(n)$, respectively:

$$\begin{aligned} P(n_2, n_3) = \tfrac{1-\beta-\zeta}{2}\{(1 - R_c - R_b)[G(n_2)Q(n_3) + G(n_3)Q(n_2)] + \\ +R_c[G(n_2)O(n_3) + G(n_3)O(n_2)] + R_b[G(n_2)B(n_3) + G(n_3)B(n_2)]\} + \\ +\zeta\{(1 - R_b - R_c)Q(n_2)Q(n_3) + R_bB(n_2)B(n_3) + R_cO(n_2)O(n_3)\} + \beta G(n_2)G(n_3), \end{aligned} \tag{4}$$

where $\beta = N^{sym}_{ggg}/N^{sym}_{3jet}$ is the fraction of three–gluon events and $1 - \beta$ the fraction of $Z \to q\bar{q}g$ events in the symmetric 3 jet event sample, ζ is the fraction of events with the gluon jet carrying the highest energy, and $R_c = \Gamma_{c\bar{c}}/\Gamma_{had}$ and $R_b = \Gamma_{b\bar{b}}/\Gamma_{had}$ are the $Z \to c\bar{c}$ and $Z \to b\bar{b}$ branching fractions.

By construction, jets in the mixed event sample are completely uncorrelated. Therefore:

$$P_{uncor}(n_2, n_3) = J(n_2)J(n_3), \tag{5}$$

where

$$\begin{aligned} J(n) = \tfrac{1-\beta-\zeta}{2}[G(n) + (1 - R_c - R_b)Q(n) + R_cO(n) + R_bB(n)] + \\ +\zeta[(1 - R_c - R_b)Q(n) + R_cO(n) + R_bB(n)] + \beta G(n). \end{aligned} \tag{6}$$

The experimental correlation function $C(n_2, n_3)$ is determined by dividing the number of measured events with given n_2 and n_3 by the normalized number of such events from the mixed event sample.

2.2 Application to the M–event sample

The multiplicity correlation function for the M–event sample is defined as

$$C(n_1, n_2, n_3) = \frac{P(n_1, n_2, n_3)}{P_{uncor}(n_1, n_2, n_3)}, \tag{7}$$

where $P(n_1, n_2, n_3)$ is the probability of observing a M–event in which the charged particle multiplicities of the jets are equal to n_1, n_2 and n_3. Jets will always be numbered such that $n_1 \geq n_2 \geq n_3$. $P_{uncor}(n_1, n_2, n_3)$ is the corresponding probability for uncorrelated jets constructed using the mixed event technique.

Under the assumption that the multiplicities of the individual jets in a real event are uncorrelated, the probability $P(n_1, n_2, n_3)$ can be expressed through the multiplicity distributions for jets:

$$\begin{aligned} P(n_1, n_2, n_3) = \beta G(n_1)G(n_2)G(n_3) + \tfrac{1-\beta}{3} \times \\ \times\{(1 - R_c - R_b)[G(n_1)Q(n_2)Q(n_3) + Q(n_1)G(n_2)Q(n_3) + Q(n_1)Q(n_2)G(n_3)] + \\ +R_c[G(n_1)O(n_2)O(n_3) + O(n_1)G(n_2)O(n_3) + O(n_1)O(n_2)G(n_3)] + \\ +R_b[G(n_1)B(n_2)B(n_3) + B(n_1)G(n_2)B(n_3) + B(n_1)B(n_2)G(n_3)]\}. \end{aligned} \tag{8}$$

The probability of generating the mixed event with n_1, n_2 and n_3 is given by the following formula:

$$P_{uncor}(n_1, n_2, n_3) = J(n_1)J(n_2)J(n_3), \tag{9}$$

where

$$J(n) = \frac{1-\beta}{3}\{G(n) + 2[(1 - R_c - R_b)Q(n) + R_cO(n) + R_bB(n)]\} + \beta G(n). \tag{10}$$

The experimental correlation function $C(n_1, n_2, n_3)$ is determined by dividing the number of measured events with given n_1, n_2 and n_3 by the normalized number of such events from the mixed event sample.

2.3 Features common to both analyses

In both analyses, the particle multiplicity distributions of gluon and quark jets, $G(n)$, $Q(n)$, $O(n)$ and $B(n)$, are assumed to be described by Negative Binomial Distributions (NBD)[6]:

$$P(n|\langle n\rangle, k) = \frac{(n+k-1)!}{n!(k-1)!}\left(\frac{\langle n\rangle/k}{1+\langle n\rangle/k}\right)^n \frac{1}{(1+\langle n\rangle/k)^k}, \tag{11}$$

where $\langle n\rangle$ is the mean multiplicity and k is the width parameter related to the dispersion of the distribution.

To cross–check that the results are not unduly sensitive to this assumption, a Poissonian parameterization (PD) of the shapes of the multiplicity distributions was also tried.

The NBD parameters of b–quark jets, $\langle n\rangle_b$ and k_b, were obtained from a separate fit to the charged particle multiplicity distribution of the smaller energy b–tagged[5] jets in Y– and M–event samples.

The NBD parameters of gluon jets, $\langle n\rangle_g$ and k_g, were obtained from a fit to the charged particle multiplicity distribution of the smaller energy jet not tagged as a b jet in the events with the second smaller energy jet tagged as a b jet.

The NBD parameters of light–quark jets, $\langle n\rangle_l$ and k_l, were obtained from a fit to the charged particle multiplicity distribution of the smaller energy jets in uds–tagged events assuming them to consist of equal numbers of gluon and light–quark jets.

The parameter corresponding to the difference in mean multiplicity between c-quark and light quark jets was fixed according to the published data[7, 8, 9, 10, 11]. The NBD width parameter of c–quark jets, k_c, was assumed to be equal to that of light–quark jets, k_l.

The fraction of 3–gluon events, β, was then determined from a fit of the parametrized correlation function $C(n_2, n_3)$ ($C(n_1, n_2, n_3)$) as defined by equations 3–6 (7–10) to the measured one.

3 Experiment and data selection

A detailed description of the DELPHI detector can be found elsewhere [12]. In this analysis only charged particles were used. Their momenta were measured in the 1.2 T solenoidal magnetic field by combining the information from the Micro Vertex Detector, the Inner Detector, the Time Projection Chamber (TPC, the principal tracking device of DELPHI), the Outer Detector and the Forward Chambers A and B.

A charged particle was required to satisfy the following criteria :

- momentum, p, greater than 0.2 GeV/c;
- error on $p < p$;
- polar angle, θ, with respect to the beam between $25°$ and $155°$;
- measured track length in the TPC greater than 50 cm;
- impact parameter with respect to the nominal beam crossing point within 5 cm in the transverse xy plane and 10 cm along the beam direction (z-axis).

Hadronic events from Z^0 decays were then selected if

- there were at least 5 charged particles;
- the total energy of charged particles (assuming a pion mass) in each of the two hemispheres defined with respect to the beam direction exceeded 3 GeV;
- the total energy of all charged particles was greater than 15 GeV.

A total of 3.5×10^6 events satisfied these cuts. The contamination from events due to beam-gas scattering and to $\gamma\gamma$ interactions was estimated to be less than 0.1% and the background from $\tau^+\tau^-$ events to be less than 0.3% of the accepted events [14].

Samples of events with three jets were selected by applying the DURHAM jet–finder (also known as the $k_\perp$ algorithm), with jet resolution parameter y_{min}=0.015 or 0.035. The DURHAM jet–finder is well defined in perturbation theory, allowing calculations to incorporate leading terms to all orders , and is widely used in experimental work. The value y_{min}=0.035 has an advantage with respect to smaller values of y_{min} because it gives a symmetric 3 jet event sample which is less contaminated by the events without hard gluon emission artificially split into 3 jets by the jet–finder.

Each reconstructed jet was required to contain at least 2 charged particles, to have the jet axis lying in the region $|\cos\theta| \leq 0.7$ and to have a visible energy larger than 2 GeV. To eliminate non-planar events, the sum of the angles between the three jets was required to exceed $357°$.

Mirror symmetric 3–jet events (Y–events) were required to have two jets with energy calculated from the angular relation in the range 25 ± 5 GeV and one with greater than 30 GeV. The total number of Y–events obtained using the DURHAM algorithm are 82994 at $y_{min} = 0.015$ and 54371 at $y_{min} = 0.035$.

Threefold symmetric 3–jet events of M–type were selected by projecting the jets into the 3–jet event plane and requiring the angles between them to be in the range $100°$ to $140°$. The total numbers of events are 12030 at $y_{min} = 0.015$ and 13702 at $y_{min} = 0.035$.

The tagging of events and jets containing b–hadrons used in this analysis [5, 14] is based on the fact that, due to their long lifetimes and large masses, these hadrons have many decay products with large positive impact parameters while tracks from the primary interaction have impact parameters which are smaller in absolute value and are equally likely to be positive or negative. The impact parameter is defined as the distance of closest approach of a charged particle to the reconstructed primary vertex. The sign of the impact parameter is defined with respect to the jet direction: it is positive if the vector joining the primary vertex to the point of closest approach of the track is less than 90° from the direction of the jet to which the track belongs. The tagging variable P_N gives the probability for a group of N tracks with the observed values of impact parameters for the hypothesis that they all come from the primary vertex.

4 Results

4.1 Determination of the NBD parameters

As outlined in Section 2.3, the values of the NBD parameters for charged particle multiplicity distributions in different types of jet were obtained by fitting distributions for jets selected using the b–tagging technique.

The observed charged particle multiplicity distributions were fitted with the convolution of the NBD with the acceptance matrix:

$$f(n) = \sum_{m=m_{min}}^{m_{max}} A_{nm} P_m^{NBD}, \tag{12}$$

where A_{nm} is the probability to observe n charged particles in the jet when its true charged multiplicity is m and was calculated as the fraction of jets generated by JETSET[15] with multiplicity m that were reconstructed with multiplicity n after the DELPHI detector simulation program DELSIM[14]. Separate acceptance matrices were calculated for Y– and M–event samples.

The NBD parameters of b–quark jets were obtained from an NBD fit to the charged particle multiplicity distribution of the smaller energy jets with Negative Logarithm of Positive Probability for the jet, P_J^+, greater than 4 [14]. The purity of the sample of b jets was 92%.

The NBD parameters of gluon jets were obtained from an NBD fit to the charged particle multiplicity distribution of the smaller energy jets with P_J^+ less than 1 when the second smaller energy jet was tagged as a b jet. To cross–check the results of the fit, the sample of events with both b jets tagged was also used but due to low statistics the errors were too big. The purity of the gluon jets sample was the same as that of the b jets sample.

The NBD parameters of light–quark jets were obtained from a fit to the charged particle multiplicity distribution of the smaller energy jets in uds–tagged events assuming that distribution to be an equal superposition of gluon and light–quark jets. The sample of uds events was obtained requiring the maximum P_J^+ in the event to be less than 1. The sample obtained with this cut consist of 83% uds, 14% c and 3% b events.

The differential dependence of the purity of the sample on maximum P_J^+ is shown in fig.1. The charged particle multiplicity distributions for b–quark, gluon and superposition of uds and gluon jets are presented in fig.2 with y_{min} equal to 0.035. The curves in fig.2 shows the result of the fit by using the formula (12).

The resulting values of the multiplicity distribution parameters for b, uds and gluon jets are presented in table 1. An unexpected feature of table 1 is a decreasing of the average particle multiplicity for light quark jets, $\langle n \rangle_l$, with increasing jet energy. This effect can be explained as a consequence of jet–finder imperfection. The DURHAM algorithm assigns on average too many to the low multiplicity jet and depletes the high multiplicity jet, and this effect is stronger when the angular separation of the jets is smaller. For the M–event sample, the angular separation of the jets is the maximum possible.

It is worthwhile to note that the variance of the multiplicity distribution, D, related with $\langle n \rangle$ and k through the formula:

$$\frac{D^2}{\langle n \rangle^2} = \frac{1}{\langle n \rangle} + \frac{1}{k}, \tag{13}$$

in the M–event samples at least is greater for gluon jets than for quark jets , as expected from the oscillations of cumulant moments of parton multiplicity distributions inside a jet[13].

The average value of the difference between the mean charged particle multiplicity in c–quark jets and that in light quark jets, δ_{cl}, was taken to be equal to 0.44±0.21, the weighted average of the measurement by OPAL[8] and SLD[11].

The ζ parameter in eq. (4) was determined from HERWIG[17] generated events and found to be equal 0.05±0.01.

Table 1: Number of events, average energy of jet and NBD parameters of charged particle multiplicity distribution in b, g and uds jets for jet energy intervals $20 \le E_{jet} \le 30 GeV$ (upper sets) and $25 \le E_{jet} \le 35 GeV$ (lower sets) for the two different y_{min} values used.

Jet	N_{ev}	E_{jet}, GeV	$\langle n \rangle \pm \sigma_{stat} \pm \sigma_{syst}$	$1/k \pm \sigma_{stat} \pm \sigma_{syst}$	$D \pm \sigma$	$P(\chi^2)$
		DURHAM, $y_{min} = 0.015$				
b	9091	25.8 ± 2.8	8.80±0.04 ± 0.05	0.012±0.002 ± 0.002	3.12±0.04	0.20
g	2172	24.0 ± 2.8	9.32±0.08 ± 0.05	0.017±0.005 ± 0.004	3.29±0.09	0.51
$l = uds$	24851	25.0 ± 2.9	6.83±0.05 ± 0.05	0.051±0.006 ± 0.005	3.04±0.06	0.07
b	7755	30.5 ± 2.8	9.25±0.04 ± 0.08	0.013±0.003 ± 0.002	3.23±0.05	0.28
g	2470	29.0 ± 2.8	10.22±0.08 ± 0.05	0.033±0.004 ± 0.002	3.70±0.07	0.69
$l = uds$	13082	30.0 ± 2.9	6.25±0.06 ± 0.05	0.014±0.010 ± 0.006	2.61±0.09	0.67
		DURHAM, $y_{min} = 0.035$				
b	5938	25.9 ± 2.8	9.22±0.05 ± 0.05	0.013±0.004 ± 0.002	3.22±0.06	0.42
g	1545	24.4 ± 2.8	9.92±0.07 ± 0.05	0.010±0.009 ± 0.002	3.31±0.14	0.15
$l = uds$	8351	25.1 ± 2.9	7.02±0.07 ± 0.05	0.050±0.011 ± 0.006	3.08±0.11	0.07
b	5216	30.4 ± 2.8	9.63±0.05 ± 0.04	0.012±0.004 ± 0.001	3.27±0.06	0.77
g	2039	29.1 ± 2.7	10.79±0.09 ± 0.06	0.036±0.006 ± 0.005	3.87±0.12	0.93
$l = uds$	8538	29.9 ± 2.8	6.66±0.07 ± 0.05	0.025±0.010 ± 0.007	2.79±0.10	0.17

4.2 Result from Y–event sample

In order to correct for the influence of imperfections of the DELPHI detector, the correlation method was applied to the samples of simulated events from the DELPHI detector simulation program DELSIM [14]. In DELSIM, events were generated using the JETSET 7.3 PS program [15] with DELPHI default parameters [16]. Particles were followed through the detector and the resulting simulated digitizations were processed with the same reconstruction programs as the experimental data.

Detector imperfections introduce a systematic difference between $C_J(n_2, n_3)$ for the events generated by JETSET and $C_D(n_2, n_3)$ for the events reconstructed after DELSIM (i.e. after the detector simulation). In order to correct for this influence of the detector, the correlation function $C(n_2, n_3)$ observed for uncorrected data was multiplied by the ratio $K(n_2, n_3) = C_J(n_2, n_3)/C_D(n_2, n_3)$.

In order to take into account the imperfections of the jet finder algorithms, a further correction factor was introduced. It was calculated as a ratio $N(n_2, n_3) = C_{expected}(n_2, n_3)/C_{observed}(n_2, n_3)$ for a normalisation sample of events obtained by generating symmetric $Z^0 \to q\bar{q}g$ decays using JETSET. This correction is based on the fundamental property that the correlation function should equal unity, i.e. $C_{expected}(n_2, n_3) = 1$, when the mixed events are constructed from the same numbers of quarks and gluons as real events. Indeed the probabilities $P(n_2, n_3)$ and $P_{uncor}(n_2, n_3)$ both are described by the formula (4) in this case. The total correction factor $K \cdot N$ is typically between 0.9 and 1.1.

The numerical results of the fit to the corrected correlation function $C(n_2, n_3)$ are as follows. The value of β is equal to -0.010±0.022 with probability of the fit equal to 0.14 for 272 experimental points for y_{min} equal to 0.015, and -0.032±0.037 with probability of the fit equal to 0.041 for 251 experimental points for y_{min} equal to 0.035.

In order to estimate the systematic errors due to the uncertainties in the values of the fixed parameters, the fit was also performed for the central values of these parameters plus or minus one standard deviation. The corresponding systematic errors in β are detailed in Tables 2 and 3.

Table 2: Contributions to the systematic error in β from the uncertainties in the parameters fixed in the fits for the Y–event sample at $y_{min} = 0.015$.

Parameter value ± error	σ_{syst} NBD	σ_{syst} PD
$\langle n_g\rangle/\langle n_l = 1.37 \pm 0.02$	$^{+0.048}_{-0.054}$	$^{+0.040}_{-0.044}$
$\delta_{bl} = 1.97 \pm 0.09$	± 0.013	± 0.010
$\delta_{cl} = 0.44 \pm 0.21$	± 0.018	± 0.016
$1/k_l = 0.051 \pm 0.008$	± 0.011	
$1/k_g = 0.017 \pm 0.006$	± 0.013	
$1/k_b = 0.012 \pm 0.003$	± 0.001	
Total	$^{+0.056}_{-0.060}$	$^{+0.044}_{-0.048}$

Further systematic errors were estimated taking into account the variation of the results obtained with different cuts on the jet multiplicity n_2 and the uncertainty in the values of the total correction coefficients. The resulting systematic bias in the values of β does not exceed 0.004 and 0.002 for y_{min} equal to 0.015 and 0.035 respectively.

Including the systematic errors in β leads to the following final results for β:

$$\beta \;=\; -0.010 \pm 0.022(stat.) \pm^{0.056}_{0.060} (syst.) \quad (y_{min} = 0.015)$$

Table 3: Contributions to the systematic error in β from the uncertainties in the parameters fixed in the fits for the Y–event sample at $y_{min} = 0.035$.

Parameter value $\pm$ error	σ_{syst} NBD	σ_{syst} PD
$\langle n_g \rangle / \langle n_l \rangle = 1.41 \pm 0.02$	$^{+0.045}_{-0.052}$	$^{+0.047}_{-0.041}$
$\delta_{bl} = 2.20 \pm 0.11$	± 0.013	± 0.011
$\delta_{cl} = 0.44 \pm 0.21$	± 0.015	± 0.013
$1/k_l = 0.050 \pm 0.013$	± 0.015	
$1/k_g = 0.010 \pm 0.009$	± 0.018	
$1/k_b = 0.013 \pm 0.004$	± 0.001	
Total	$^{+0.054}_{-0.060}$	$^{+0.050}_{-0.044}$

$$\beta = -0.032 \pm 0.037(stat.) \pm^{0.054}_{0.060} (syst.) \quad (y_{min} = 0.035).$$

The branching fraction $Br(Z^0 \to ggg)$ is calculated from β using the formula

$$Br(Z^0 \to 3g) = \beta \cdot Br(Z^0 \to hadr) \cdot \frac{N_{3jet}^{sym}}{N_{hadr}} \cdot \frac{N_\Upsilon}{N_\Upsilon^{sym}}, \tag{14}$$

where N_{3jet}^{sym}/N_{hadr} is the fraction of symmetric 3 jet events in the hadronic event sample and $N_\Upsilon^{sym}/N_\Upsilon$ is the fraction of symmetric decays in an Υ–like 1^{--} quarkonium state to three gluons. The latter ratio was calculated using JETSET 7.3. The mass of the pseudo-onium was chosen to be equal to the Z mass. Due to the identical helicity structure of $Z^0 \to ggg$ and $\Upsilon \to ggg$ decays, the angular distributions for jets from the two sources are expected to be identical. Thus N_{ggg}^{sym}/N_{ggg} should equal $N_\Upsilon^{sym}/N_\Upsilon$. The numerical value of the factor relating $Br(Z \to 3g)$ to β in eq.(14) was found to be 0.263 at y_{min}=0.015 and 0.184 at y_{min}=0.035, leading to

$$Br(Z \to 3g) = -0.003 \pm 0.017 \quad (y_{min} = 0.015)$$

and

$$Br(Z \to 3g) = -0.006 \pm 0.013 \quad (y_{min} = 0.035).$$

The cross-check of using the Poissonian parametrisation of the multiplicity distributions gave similar values, namely 0.028$\pm$0.013 and 0.013$\pm$0.011 respectively, with fit probability at the level of 3×10^{-3} for $y_{min} = 0.015$ and 0.07 for $y_{min} = 0.035$.

4.3 Result from M–event sample

In the analysis of the increased statistics of M–events, in contrast with previous work[3], the difference in average charged particle multiplicities in c–quark jets and light quark jets was taken into account according to eqn.7-10.

The contributions to the systematic error in β from the uncertainties in the parameters fixed in the fits are collected in Tables 4 and 5.

The fit and the systematic errors in β give the following final results for β:

$$\beta = -0.009 \pm 0.014(stat.) \pm^{0.016}_{0.017} (syst.) \quad (y_{min} = 0.015)$$
$$\beta = -0.003 \pm 0.013(stat.) \pm^{0.015}_{0.019} (syst.) \quad (y_{min} = 0.035).$$

Table 4: Contributions to the systematic error in β from the uncertainties in the parameters fixed in the fits for the M–event sample at $y_{min} = 0.015$.

Parameter value $\pm$ error	σ_{syst}	
	NBD	PD
$\langle n_g \rangle / \langle n_l \rangle = 1.64 \pm 0.02$	${}^{+0.010}_{-0.011}$	${}^{+0.003}_{-0.004}$
$\delta_{bl} = 3.00 \pm 0.12$	± 0.012	± 0.004
$\delta_{cl} = 0.44 \pm 0.21$	± 0.002	± 0.002
$1/k_l = 0.014 \pm 0.012$	± 0.001	
$1/k_g = 0.033 \pm 0.004$	± 0.004	
$1/k_b = 0.013 \pm 0.004$	± 0.001	
Total	${}^{+0.016}_{-0.017}$	${}^{+0.005}_{-0.005}$

Table 5: Contributions to the systematic error in β from the uncertainties in the parameters fixed in the fits for the M–event sample at $y_{min} = 0.035$.

Parameter value $\pm$ error	σ_{syst}	
	NBD	PD
$\langle n_g \rangle / \langle n_l \rangle = 1.62 \pm 0.03$	${}^{+0.011}_{-0.014}$	${}^{+0.011}_{-0.013}$
$\delta_{bl} = 2.97 \pm 0.11$	± 0.009	± 0.008
$\delta_{cl} = 0.44 \pm 0.21$	± 0.002	± 0.002
$1/k_l = 0.025 \pm 0.012$	± 0.002	
$1/k_g = 0.036 \pm 0.008$	± 0.003	
$1/k_b = 0.012 \pm 0.004$	± 0.001	
Total	${}^{+0.015}_{-0.019}$	${}^{+0.014}_{-0.016}$

The corrected correlation function $C(n_1, n_2, n_3)$ is presented as a function of n_3 in Fig.3 for the DURHAM jet–finder with $y_{min} = 0.035$ for several n_1 values. The curves in Fig.3 are the results of the fit for all values of $2 \leq n_1 \leq 25$. The probability of the fit is equal to 0.97 for 235 experimental points for y_{min} equal to 0.015 and 0.98 for 242 experimental points for y_{min} equal to 0.035.

The numerical value of the factor relating $Br(Z \to 3g)$ to β in eq.(14) was found to be 0.144 at y_{min}=0.015 and 0.100 at y_{min}=0.035, leading to

$$Br(Z^0 \to 3g) = -0.0013 \pm 0.0030 \quad (y_{min} = 0.015)$$

and

$$Br(Z^0 \to 3g) = -0.0003 \pm 0.0020 \quad (y_{min} = 0.035).$$

The cross-check of using the Poissonian parametrisation of the multiplicity distributions gave similar estimates of the upper limit, namely 0.0014$\pm$0.0019 and 0.0006$\pm$0.0017 respectively with an acceptable fit probability in both cases.

5 Summary

The parameters of the multiplicity distributions for b, uds, and gluon jets are obtained for two topologies of 3-jet events. The ratio of the average charged particle multiplicities in gluon and light quark jets is found to be equal to 1.64 ± 0.02 and 1.62 ± 0.03 depending on the y_{min} cut for the M–type events where the assigning of the particles to the one or another jet is more reliable then for other angular configurations. The variance of multiplicity distribution for gluon jets is greater than for quark jets, at least in the M–events sample, as it is expected from the oscillations of cumulant moments of parton multiplicity distributions inside a jet.

By using a correlation method based on the difference between the particle multiplicity distributions of quark and gluon jets, the $Z^0 \to ggg$ branching ratio has been measured from the sample of Y–events to be

$$Br(Z^0 \to 3g) = -0.003 \pm 0.017$$

for the DURHAM jet–finder with y_{min}=0.015 and

$$Br(Z^0 \to 3g) = -0.006 \pm 0.013$$

with y_{min}=0.035.

The correlation method modified for the case of two jet correlations described in present paper can be applied in principle to estimate the possible contribution in $2jet + \gamma$ event sample from the process $Z^0 \to \gamma gg$ which has the one order of magnitude larger cross section than that for $Z^0 \to 3g$ in the compositeness model.

The correlations between multiplicities in M–events give more precise values:

$$Br(Z \to 3g) = -0.0013 \pm 0.0030$$

for the DURHAM jet–finder with y_{min}=0.015 and

$$Br(Z \to 3g) = -0.0003 \pm 0.0020$$

with y_{min}=0.035.

At the present level of statistics, no signal of the decay $Z^0 \to ggg$ expected from the compositeness model is observed.

The most precise measurement is that from the M–event sample with y_{min}=0.035. From this measurement we deduce an upper limit at 95% confidence level

$$Br(Z \to 3g) \leq 3.9 \times 10^{-3}$$

after setting the unphysical negative central value to zero and accounting for both statistical and systematic errors. The upper limit is four times lower than that published previously[3] due to the increased statistics and to the reduced number of free parameters of the fit[1].

[1] In the previous analysis the parameter k_g was determined from the final fit to the correlation function together with β

Acknowledgements

We are greatly indebted to our technical staff and collaborators and our funding agencies for their support in building and operating the DELPHI detector, and to the members of the CERN-SL Division for the excellent performance of the LEP collider.

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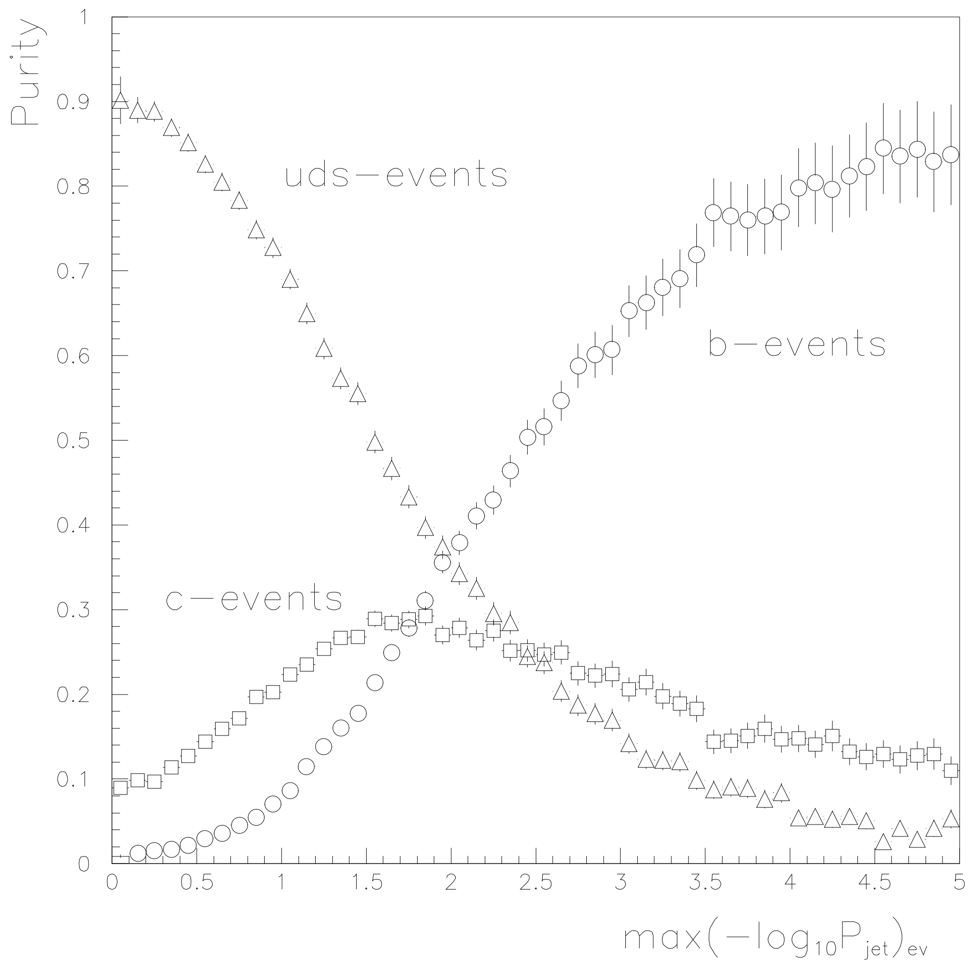

Figure 1: The purity of the sample of events as a function of the maximum of the negative logarithm of the positive probability for jets P_J^+ in the event.

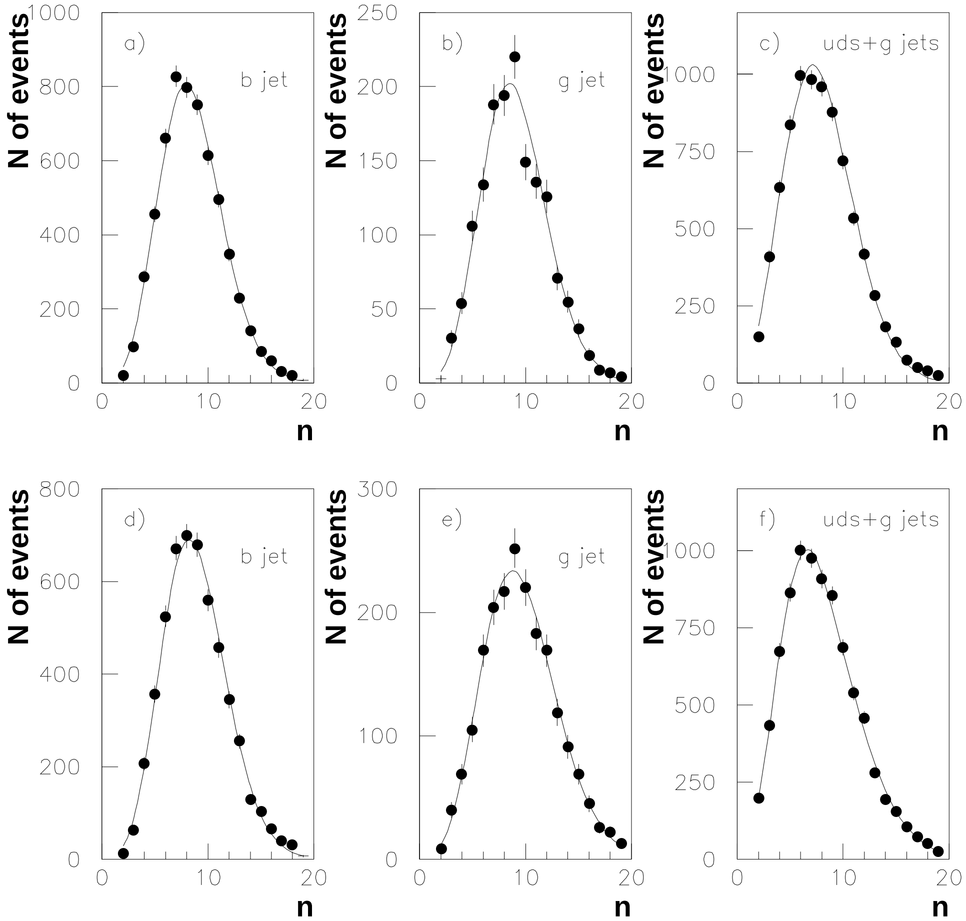

Figure 2: The charged particle multiplicity distributions for b–quark, gluon and superposition of uds and gluon jets with y_{min} equal to 0.035 in Y–event (a-c) and M–event (d-f) samples. The curves are the result of the fit (see the text).

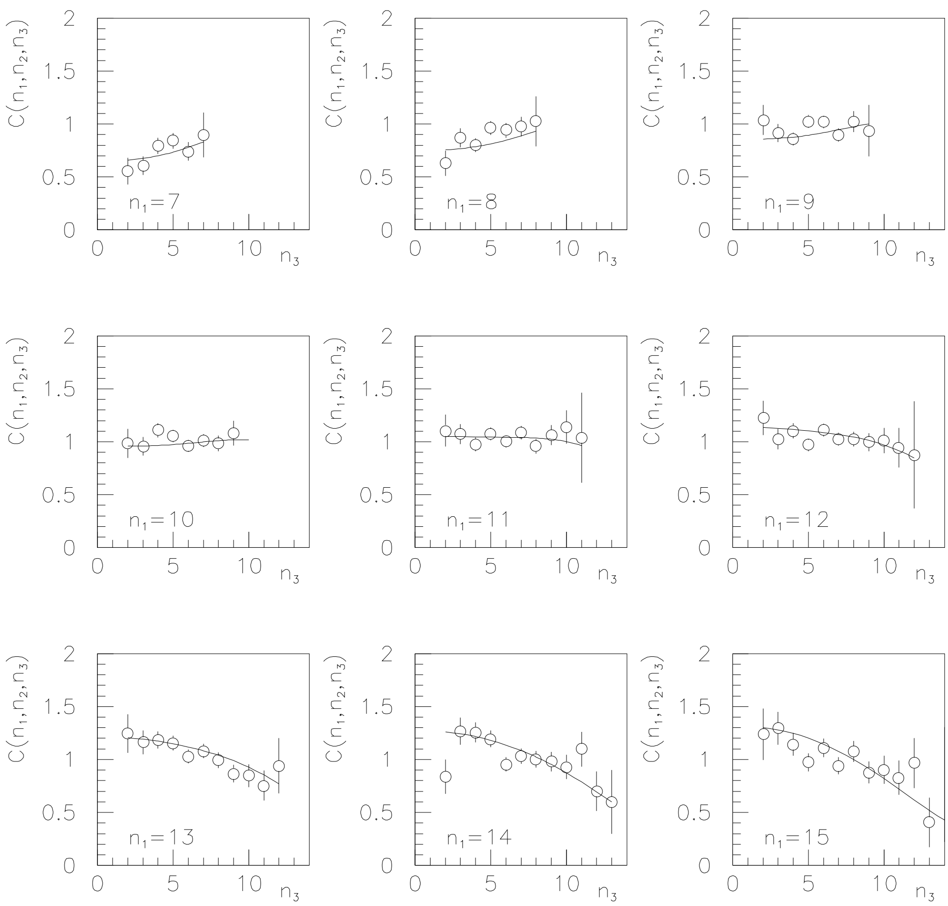

Figure 3: The corrected correlation function $C(n_1, n_2, n_3)$ as a function of the smallest jet multiplicity n_3 for different values of the jet multiplicity n_1 (always $n_1 \geq n_2 \geq n_3$). Threefold symmetric 3–jet events are selected from the sample of DELPHI data by using the DURHAM jet–finder with y_{min} equal to 0.035. The curves are the result of the fit.