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Three- and four-jet heavy b-quark production rates in e^+e^- annihilation at the Z peak

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Three- and four-jet heavy b-quark production rates in e^+e^- annihilation at the Z peak

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Abstract

Three- and four-jet topologies are studied for hadronic events originated by b (massive) and uds (light) quarks using the DELPHI detector at LEP. The data were collected during the years 1994 and 1995 at a center of mass energy corresponding to $\sqrt{s} \approx M_Z$. The experimental results are compared to theoretical predictions including NLO radiative corrections with mass effects. As a result of this study and using CAMBRIDGE as the algorithm for reconstructing jets, the measured ratio of the normalized three jet rate of b quark with respect to that of light (ℓ) quark events is

$$R_3^{b\ell}(y_c = 0.005) = 0.965 \pm 0.004 \text{ (stat.)} \pm 0.011 \text{ (frag.)} \pm 0.001 \text{ (tag.)}$$

This ratio is in agreement with the theoretical predictions based on the running b-quark mass, $m_b(M_Z)$, at both LO and NLO, whereas only a reasonable agreement is found at NLO when the prediction is based on the pole b-quark mass, M_b. The LO prediction in terms of M_b is almost 5σ away from the measured $R_3^{b\ell}$.

The value for the b-quark mass

$$m_b(M_Z) = 2.61 \pm 0.18 \text{ (stat)}^{+0.45}_{-0.49} \text{ (frag.)} \pm 0.04 \text{ (tag.)} \pm 0.07 \text{ (theo.)}\ GeV/c^2$$

is extracted from the NLO massive calculations of the $R_3^{b\ell}$ and its measurement.

A test on the universality of the strong coupling constant is also performed with high precision leading to

$$\frac{\alpha_s^b}{\alpha_s^\ell} = 1.005 \pm 0.012 \text{ (stat. + frag. + theo.)}$$

1 Introduction

The effects of the b-quark mass in the production of three jet event topologies at the Z peak in e^+e^- annihilations have recently been observed [1, 2, 3]. There exist also theoretical studies on this topic based on Next-to-Leading Order (NLO) calculations including mass corrections [4, 5, 6]. These analyses [1, 2, 3] made possible to perform an improved experimental test of the universality of strong interactions and the first measurement of the b-quark mass far from the $b\bar{b}$ production threshold [1, 7].

The previous DELPHI analysis on the subject [1] was based on the DURHAM jet reconstruction algorithm [8] whereas the present one uses also the more recent CAMBRIDGE jet reconstruction procedure [9] with improved understanding of the soft gluon emission. The normalized three- and four-jet production rates of b-quarks with respect to that of uds-quarks (ℓ) using this latter algorithm has been recently studied in reference [10]. The CAMBRIDGE jet finder has smaller NLO corrections as well as a reasonable stable hadronization correction as a function of the jet resolution parameter down to very low values.

This study presents the analysis of the DELPHI data collected during the years 1994 and 1995 at a center of mass energy of $\sqrt{s} \approx M_Z$. The experimental results are compared to the predictions described in [10] for the observables:

$$R_j^{b\ell}(y_c) = \frac{[\Gamma_j(y_c)/\Gamma_{tot}]^{Z\to b\bar{b}}}{[\Gamma_j(y_c)/\Gamma_{tot}]^{Z\to \ell\bar{\ell}}} \qquad j = 3, 4 \text{ jets} \tag{1}$$

where $[\Gamma_j(y_c)/\Gamma_{tot}]^{Z\to b\bar{b}}$ and $[\Gamma_j(y_c)/\Gamma_{tot}]^{Z\to \ell\bar{\ell}}$ represent the normalized three- ($j = 3$) and four- ($j = 4$) jet cross sections for b- and ℓ-quarks.

The theoretical predictions include up to $O(\alpha_s^2)$ terms which means that NLO corrections are considered for the 3-jet rates, while only LO terms enter on 4-jet rates.

2 The selection procedure

The present study was based on a sample of $\sim 1.33 \times 10^6$ hadronic decays of the Z^0 boson recorded with the DELPHI detector [11] at LEP during the years 1994 and 1995 and corresponded to centre of mass energies of $\sqrt{s} \approx M_Z$.

2.1 Event selection

In the first stage of the selection procedure quality cuts were applied to select charged and neutral particles in order to ensure a reliable determination of their kinematic variables: momenta and energies. Hadronic Z^0 decays were selected demanding at least 5 well measured ($\Delta(p)/p \leq 1$) charged particles and 15 GeV of visible energy carried by charge particles. Events having poorly measured particles were reduced with a charged balance. Those events containing particles with an energy higher than 40 GeV were discarded in order to avoid the participation of those particles in the thrust or jet axis determination. The retained data sample contained $\sim 1.33 \times 10^6$ hadronic Z^0 decays ($\sim 9 \times 10^5$ in the year 1994 and $\sim 4.3 \times 10^5$ in 1995) with a small contamination from $\tau^+\tau^-$ pairs ($\sim$0.1%) and a negligible background from beam-gas scattering and $\gamma\gamma$ interactions.

2.2 Jet reconstruction

The jet clustering algorithm DURHAM, the most extended in LEP physics, and the recently introduced CAMBRIDGE jet algorithm were applied to group the selected charge and neutral particles in jets. The new ingredients in the CAMBRIDGE method: the angular ordering and the *soft freezing* improve the understanding of soft gluon radiation. For each pair of particles ij, the ordering variable v_{ij} and the resolution variable y_{ij} were calculated from their respective four-momentum vectors. The pair with the smallest v_{ij} was combined to form a new pseudo-particle with four-momentum $p_k = p_i + p_j$ if the resolution variable y_{ij} did not exceed the jet resolution parameter y_c, otherwise the *soft freezing* mechanism would be activated: the particle i with $E_i < E_j$ would constitute a jet and the particle j would return to the binary procedure. The *soft freezing* eliminated the tendency of soft 'resolved' jets attracting extra wide-angle particles. The procedure was iterated until no further pairs of particles or pseudo-particles could be recombined. The number of remaining objects added to the jets formed by *soft freezing* determined the class of the event: two-jet, three-jet, etc.

The DURHAM sequence of clustering was independent of the external y_c parameter because both the ordering variable and the resolution variable were the same (see table 1) and the *soft freezing* mechanism was absent. The number of jets was monotonically decreasing for increasing y_c. The transition values $y^{n \leftarrow n+1}$ or y_c parameter values of change between a $n+1$ jet configuration to a n jet configuration were determined for $n = 3, 4$ in each hadronic event to classify the event as a three- or four-jet event and to study the b-quark mass effects in the jet production rates R_3 and R_4. For the CAMBRIDGE reconstruction algorithm the number of jets is not necessarily monotonically decreasing for increasing y_c. This is due to its definition and in some circumstances certain jet topologies were not present for a specific event. In the case of three jets this affected ∼1% of the events in the range $y_c \geq 0.01$. The quantities R_n were properly normalized in all cases and this property was also considered in the theoretical calculations.

Algorithm	Resolution	Ordering	Recombination
DURHAM [8]	$y_{ij} = \frac{2 \cdot \min(E_i^2, E_j^2) \cdot (1 - \cos\theta_{ij})}{E_{vis}^2}$	$v_{ij} = y_{ij}$	$p_k = p_i + p_j$
CAMBRIDGE [9]	$y_{ij} = \frac{2 \cdot \min(E_i^2, E_j^2) \cdot (1 - \cos\theta_{ij})}{E_{vis}^2}$	$v_{ij} = 2 \cdot (1 - \cos\theta_{ij})$	$p_k = p_i + p_j$

Table 1: Definition of the jet resolution variable y_{ij}, ordering variable and recombination procedure of the DURHAM and CAMBRIDGE jet finders. E_{vis} is the total visible energy of the event, $p_i \equiv (E_i, \vec{p}_i)$ denotes a 4-vector and θ_{ij} is the angle between $\vec{p}_i$ and $\vec{p}_j$.

2.3 Selection on the reconstruction event quality

In order to ensure a good energy balance in the event all of them were reconstructed as three-jet and additional cuts were applied using the reconstructed jet information. For those cases were CAMBRIDGE was unable to find exactly three jets (5% of the cases), DURHAM was used instead. All the three jets in the event were demanded to comply with a minimum charge multiplicity per jet of $N_{ch} \geq 1$, at least 1 GeV of visible energy carried by charged particles belonging to the jet, the jet polar angle to be well contained within the detector volume and the three jet axis to be in a planar configuration. A total of $\sim 1.15 \times 10^6$ hadronic events passed these criteria ($\sim 7.7 \times 10^5$ in 1994 year and $\sim 3.7 \times 10^5$ in 1995 year).

2.4 The quark flavour tag

The selection of $\ell = uds$ and b quark initiated events was performed using two different methods: the signed impact parameter of all charge particles in the event [12] and the combined tagging technique developed by DELPHI [13].

The first method was based on the construction of a function, P_E^+, in order to estimate the probability of having all particles compatible with being generated in the events' Interaction Point (IP). This method was already used in our former publication [1]. The decays of long lived B hadrons led to particles generated in secondary vertices far away from the IP, biasing P_E^+ towards low values, while uds events have an uniform distribution of P_E^+. Consequently, b-quark events were selected by requiring $P_E^+ < 5{\cdot}10^3$ and ℓ-quark events with $P_E^+ > 0.2$. The P_E^+ distribution and the flavour tagging regions are presented in figure 1.

In the second method, an optimal combination of a set of discriminating variables defined for each reconstructed jet was performed. The secondary vertices reconstructed in each jet were required to have at least two tracks not compatible with the primary vertex and to have $L/\sigma_L > 4$ where L is the distance from primary to secondary vertex and σ_L is its error. The quantities combined per jet were: the jet lifetime probability, P_j^+; the effective mass distribution of particles included in the secondary vertex, M_s; the rapidity distribution of tracks included in the secondary vertex with respect to the jet direction, R_s^{tr}, and the fraction of the charged energy distribution of a jet included in the secondary vertex, X_s^{ch}. For a more detailed description of the method see reference [13]. Finally the discriminating variables per jet were combined in a single variable per event X_{effev}. Candidates for b-quark events were selected by requiring $X_{\text{effev}} > -0.2$ and ℓ-quark events with $X_{\text{effev}} < -1$. Figure 2 presents the X_{effev} distribution as well as the flavour tagging regions.

Each value of P_E^+ and X_{effev} corresponded to a well determined combination of purity and efficiency. The ℓ-quark selection efficiency obtained with both methods was ∼60%. The b-quark selection efficiency attained was ∼67% for the impact parameter method and ∼53% with the combined method of tagging. The combined method reached a higher purity ∼85% in the b sample in relation to the impact parameter method ∼81% with a lower level of contamination for c-quark ∼10% while maintaining at the same time enough efficiency (see table 2).

b-tag	# ev. in data	q-type	$\ell \to q$-type (%)	$c \to q$-type (%)	$b \to q$-type (%)
Imp. par.	606831	ℓ	85.	13.	2.
Imp. par.	262252	b	4.	15.	81.
Combined	754729	ℓ	82.	15.5	2.5
Combined	245124	b	4.	10.	86.

Table 2: Event statistics and flavour compositions of the samples tagged as light-quark ($\ell \equiv u, d, s$) and b-quark events in real data and for each tagging technique.

3 Experimental three- and four-jet event rates

The raw distributions of the $R_3^{b\ell}$ and $R_4^{b\ell}$ observables (eq. 1) were corrected using a sample of $\sim 4.3 \times 10^6$ events generated with JETSET 7.3 Parton Shower (PS) Monte Carlo [14]. These Monte Carlo events were processed by the full DELPHI simulation program, and then passed through the same reconstruction and analysis chain than the real data events.

The flavour assignment of the simulated events was defined to be that of the pair of quarks coupled to the Z which initiated the parton shower. The same convention was considered in the theoretical calculation [4, 10] thus allowing a consistent comparison.

The experimental method to correct the measured $R_3^{b\ell}$ and $R_4^{b\ell}$ quantities for detector acceptance effects, kinematic biases introduced in the two tagging procedures, and the hadronization process was the same as described in [1]. Three- and four- jet topologies had different hadronization correction factors as well as detector, acceptance and tagging correction factors, but the same flavour compositions were used in the correction procedure.

Both tagging methods described in the previous section have been considered in the analysis. Therefore the average of the experimental results of $R_3^{b\ell}$ obtained with both methods has been taken as the experimental result of the partonic $R_3^{b\ell}$. A new additional systematic uncertainty is assigned to $R_3^{b\ell}$ as half of the difference between the results with both methods and referred to as *tag*.

The $R_3^{b\ell}$ corrected experimental result is shown for both algorithms Durham and Cambridge in figures 3 and 4 respectively.

Figure 3 shows that when using Durham as jet finder, neither the LO calculations in terms of the pole mass ($M_b = 4.6\ GeV/c^2$) nor those in terms of the running mass ($m_b(M_Z) = 2.8\ GeV/c^2$)[1] provide an acceptable description of the data points. Therefore, and as already indicated by the previous DELPHI analysis [1], QCD radiative corrections including mass effects are necessary to properly describe the data. As shown in figure 3, the Durham NLO corrections are larger in terms of the running mass $m_b(M_Z)$ than in terms of the pole mass M_b (figure 3), however the data points are closer to the prediction based on the running mass. This fact indicates that in this scheme the convergence of the higher order terms is faster. This was observed for the first time in the previous DELPHI publication [1] using data from the years 1992-1994 and quantified as a 2-3σ effect. The same conclusion can be draw from the present analysis, which uses different

[1]These values for M_b and $m_b(M_Z)$ are taken from analyses based on data collected at the Υ production threshold as published in references [15]

sets of data (collected during the years 1994 and 1995) analyzed with two different quark tagging methods.

The $R_3^{b\ell}$ NLO corrections for different jet algorithms have been studied in a recent work [16]. There the NLO corrections are compared for DURHAM and JADE-LIKE algorithms. In reference [10] the CAMBRIDGE algorithm NLO corrections have been studied and found to be smaller when the calculation is expressed in terms of $m_b(M_Z)$ instead of M_b. The question then is whether the data follows (or not) this behaviour as this would be the first time in which theory and experiment would prefer (or not) a description of jet production rates based on the running mass. This may be an scenario in which the LO calculation as a function of $m_b(M_Z)$ would give a reasonable description of the experimental results. For this purpose in figure 4, the $R_3^{b\ell}$ LO and NLO predictions using the CAMBRIDGE algorithm are compared to the data points. The NLO calculations in terms of the running mass match the experimental points more accurately than the NLO calculations in terms of the pole mass do. At LO only the $m_b(M_Z)$ provides a rough description of the experimental results, as the curve based on M_b is 4-5σ away from data.

The b-quark mass effects have been observed also in 4 jets topologies at the Z peak. In the case of 4 jets topologies, theoretical calculations including b-quark mass effects are available only at LO [17]. Figure 5 presents the corrected $R_4^{b\ell}$ observable as a function of the DURHAM y_c. The massive LO calculations with $m_b = 2.8\ GeV/c^2$ and 4.6 GeV/c^2 are included for comparison. Since none of them provides a successful description of the data, QCD radiative corrections with mass effects are required to describe the results. NLO calculations would be needed if one plans to use the mass effects in the 4-jet rate in order to extract a reliable m_b value. $R_4^{b\ell}$ data points lay in the band limited by the LO massive calculations with $m_b = 2.8\ GeV/c^2$ and 4.6 GeV/c^2. This situation is analogous to that found in the case of $R_3^{b\ell}$ (figure 3). However for the $R_4^{b\ell}$ the net effect of the mass is considerably larger ($\sim$8% for $y_c = 0.02$ compared to a $\sim$3% for $R_3^{b\ell}$ at the same y_c value), so the mass effects become more apparent. A similar study using the CAMBRIDGE algorithm will follow.

4 Hadronization corrections and systematic uncertainties

The impact of the fragmentation process on the observable $R_3^{b\ell}$ was studied and quantified by adding in quadrature two different source of errors: σ_{tun}, uncertainty with origin in the lack of an exact knowledge of the main fragmentation parameters in JETSET [14] and σ_{mod}, uncertainty due to the dependence of the hadronization correction factors with the two fragmentation models considered: cluster fragmentation in HERWIG [18] and Lund string fragmentation in JETSET. Complete details of the evaluation of these two uncertainties are given in reference [1]. The total error due to the lack of knowledge on the hadronization process can be expressed as:

$$\sigma_{had}(y_c) = \sqrt{\sigma_{tun}^2(y_c) + \sigma_{mod}^2(y_c)} \quad (2)$$

Figure 6 presents the size of the hadronization correction uncertainty for DURHAM and CAMBRIDGE algorithms. This figure shows a larger flat y_c-region in the case of CAMBRIDGE with respect to DURHAM which can be extended down to $y_c = 0.004$ while keeping the four jet contribution $\leq$10%. However, in this flat region the total hadronization error of the CAMBRIDGE algorithm is higher than in the plateau reached with DURHAM algorithm.

Nevertheless the relative sensitivity to the mass correction is larger for CAMBRIDGE at y_c values close to 0.005 than for DURHAM at y_c values close to 0.02. For comparison purposes the difference between the theoretical prediction of R_3^{bd} at LO in terms of the pole mass, $M_b = 4.6$ GeV/c^2 with respect to that obtained using the running mass $m_b(M_Z) = 2.8$ GeV/c^2 is also shown. A higher sensitivity to this difference is again found for the new CAMBRIDGE jet algorithm in the valid, flat, y_c-region ($y_c > 0.004$) thus enabling a more significant test of the mass effects in the jet rates.

5 Results and discussion

The values of $R_3^{b\ell}$ and $m_b(M_Z)$ obtained with DURHAM algorithm at $y_c = 0.02$ and the breakdown of all affecting uncertainties to both quantities are summarized in table 3. The result of the $R_3^{b\ell}$ observable and $m_b(M_Z)$ are fully compatible with the previous results published in [1] (see table 5 for $R_3^{b\ell}$ and table 6 for $m_b(M_Z)$).

The argument followed for the election of the y_c value to perform the $m_b(M_Z)$ measurement is such that allows a more accurate determination of $R_3^{b\ell}$, thus the optimization of all the statistical and systematic uncertainties.

The values of $R_3^{b\ell}$ and $m_b(M_Z)$ obtained with CAMBRIDGE algorithm at $y_c = 0.005$ and the listing of all affecting uncertainties to both quantities are summarized in table 4. The total error of $m_b(M_Z)$ is $\sim$0.52 GeV/c^2 at $y_c = 0.005$.

The experimental result obtained with CAMBRIDGE algorithm at $y_c = 0.005$ for the $R_3^{b\ell}$ observable is: 0.965 ± 0.012 (stat. + syst.). This value compared with the rough LO prediction with a pole mass $M_b = 4.6$ GeV/c^2, $R_3^{b\ell}(y_c = 0.005) = 0.910$ differs by almost 5σ. As the experimental value is in agreement with the LO and the NLO calculation in terms of the running b-quark mass, the present result exhibits the effects of the running mass of the b-quark in the 3-jet rate. This represents a net improvement with respect to the former [1] and current results with the DURHAM algorithm where only a 2-3σ deviation was seen.

Using NLO calculations [10] and the $R_3^{b\ell}$ measurement, the $m_b(M_Z)$ value extracted with CAMBRIDGE algorithm at $y_c = 0.005$ is:

$$m_b(M_Z) = 2.61 \pm 0.18 \text{ (stat)}^{+0.45}_{-0.49} \text{ (frag.)} \pm 0.04 \text{ (tag.)} \pm 0.07 \text{ (theo.) GeV/c}^2 \qquad (3)$$

A high stability of the $m_b(M_Z)$ value versus y_c is obtained with the CAMBRIDGE algorithm, given that none of the $m_b(M_Z)$ values extracted from $R_3^{b\ell}$ in the range $0.005 \leq y_c \leq 0.025$ differs from the reference value by a quantity greater than 0.15 GeV/c^2.

A direct measurement of the pole mass of the b-quark from the NLO prediction gives $M_b = 4.1 \pm 0.5$(stat.+frag.+theo.) GeV/c^2 in reasonable agreement with that obtained at the Υ ($M_b = 4.6$ GeV/c^2) [15]. The conversion of the value of running b-quark mass into the pole b-mass is $M_b = 4.3 \pm 0.5$ (stat.+frag.+theo.)GeV/c^2 which is compatible with the direct measurement. These results reveal a coherent picture within errors of the mass effects at NLO at the M_Z scale. The use of the running b-quark mass shows its advantage.

Note that the theoretical error associated with the $m_b(M_Z)$ measurement is smaller when using CAMBRIDGE (table 4) instead of DURHAM (table 3). This is due to the weak dependence of the $R_3^{b\ell}$ with the scale (μ) of the process [10].

The present result of $m_b(M_Z)$ (eq. 3) is nice agreement with the former DELPHI measurement [1]. It also agrees with the result from the reference [2] extracted from SLD data of $m_b(M_Z)$ obtained with 6 jet algorithms (E,E0,P, P0,DURHAM and GENEVA) and taking into account statistical correlations as well as correlations in the systematic error and hadronization uncertainties:

$$[2]: \quad m_b(M_Z) = 2.52 \pm 0.27 \text{ (stat.)}^{+0.33}_{-0.47}\text{(frag.)}^{+0.54}_{-1.46}\text{(theo.) GeV/c}^2 \qquad (4)$$

A net change in the value of the running b-quark mass between the scales $\mu_1 = M_Z$ and $\mu_2 = M_{\Upsilon/2}$ is observed with almost 3 standard deviations $m_b(m_\Upsilon/2) - m_b(M_Z) = 1.55 \pm 0.54$ GeV/c^2 (see figure 7). This result is in good agreement with that predicted from the QCD evolution.

The test of flavour independence of α_s is performed with the experimental result of $R_3^{b\ell}$ using the CAMBRIDGE algorithm at $y_c = 0.005$ and the NLO calculations [10] and following the same method as in [1]. Taking as an input now the hypothesis that the QCD prediction for $R_3^{b\ell}$ is governed by $m_b(M_Z) = 2.8$ GeV/c^2 the result is:

$$\frac{\alpha_s^b}{\alpha_s^\ell} = 1.005 \pm 0.012 \text{ (stat. + frag. + theo.)}, \qquad (5)$$

which verifies the flavour independence of the strong coupling constant for b and light quarks.

The b-quark mass effects have been also seen in the 4-jet rate and compared with the LO massive calculations.

A better understanding of the hadronization effects would lead to very competitive values of the b mass with respect to those measured from the Υ resonance production with the additional benefit of being extracted far from the $b\bar{b}$ production threshold.

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Table 3: Values of $R_3^{b\ell}$ and $m_b(M_Z)$ obtained with DURHAM algorithm and break-down of their associated errors (statistical and systematic) for $y_c = 0.02$

DURHAM (94+95)	$R_3^{bl}(y_c = 0.02)$	$m_b(M_Z)$ GeV/c^2
Value	0.968	2.81
Statistical error	± 0.004	± 0.20
Flavour tag error	± 0.001	± 0.04
Simulation error	± 0.003	± 0.15
Fragmentation Model error	± 0.006	± 0.30
Fragmentation Tuning error	± 0.003	± 0.15
Mass Ambiguity error	–	± 0.25
μ-scale error ($0.5 \leq \mu/M_Z \leq 2$)	–	± 0.10

Table 4: Values of $R_3^{b\ell}$ and $m_b(M_Z)$ obtained with CAMBRIDGE algorithm and break-down of their associated errors (statistical and systematic) for $y_c = 0.005$

CAMBRIDGE (94+95)	$R_3^{bl}(y_c = 0.005)$	$m_b(M_Z)$ GeV/c^2
Value	0.965	2.61
Statistical error	± 0.003	± 0.13
Flavour tag error	± 0.001	± 0.04
Simulation error	± 0.003	± 0.13
Fragmentation Model error	± 0.011	$^{+0.45}_{-0.49}$
Fragmentation Tuning error	± 0.003	± 0.12
Mass Ambiguity error	–	± 0.07
μ-scale error ($0.1 \leq \mu/M_Z \leq 1$)	–	± 0.02

Table 5: Results of $R_3^{b\ell}$ obtained with Durham and Cambridge algorithms.

y_c	Algorithm	$R_3^{b\ell}$	σ_{stat}	σ_{frag}	σ_{b-tag}	Reference
0.02	Durham	0.971	0.005	0.007	-	[1]
0.02	Durham	0.968	0.005	0.007	0.001	present study
0.005	Cambridge	0.965	0.004	0.011	0.001	present study

Table 6: Results of $m_b(M_Z)$ obtained with Durham and Cambridge algorithms.

y_c	Algorithm	$m_b(M_Z)$	σ_{stat}	σ_{frag}	σ_{b-tag}	σ_{theo}	Reference
0.02	Durham	2.67	0.25	0.34	-	0.27	[1]
0.02	Durham	2.81	0.25	0.34	0.04	0.27	present study
0.005	Cambridge	2.61	0.18	$^{+0.45}_{-0.49}$	0.04	0.07	present study

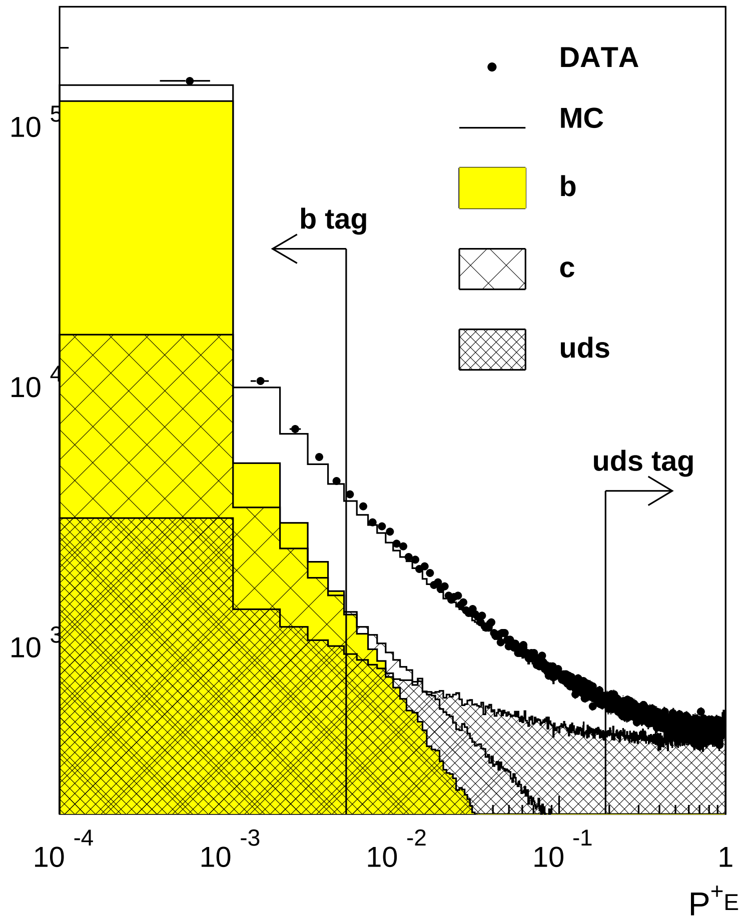

Figure 1: Event distribution of the probability P to contain no secondary vertices. The data (points) and the Monte Carlo (histogram) are compared. The specific contribution of each quark flavour is displayed as derived from the Monte Carlo. The cuts used to tag the b-quark and ℓ-quark ($\ell \equiv uds$ samples are also indicated).

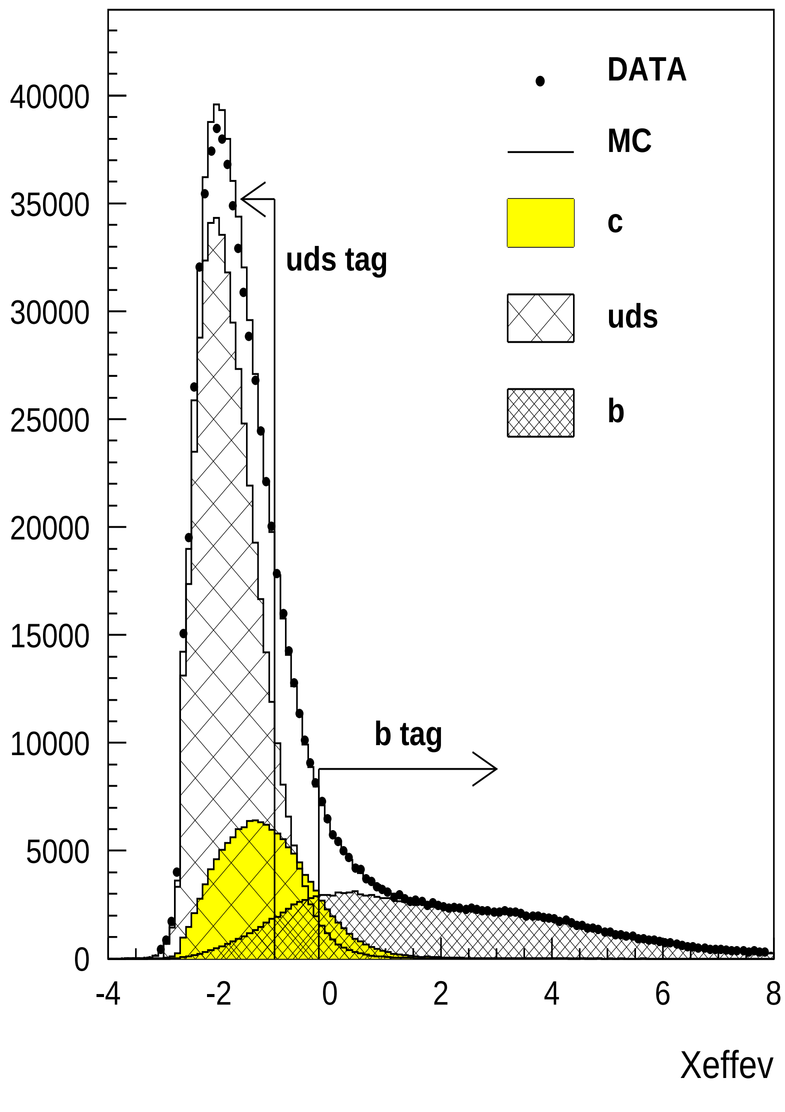

Figure 2: Event distribution of the combined variable X_{effev}. The data (points) and the Monte Carlo (histogram) are compared. The specific contribution of each quark flavour is displayed as derived from the Monte Carlo. The cuts used to tag the b-quark and ℓ-quark ($\ell \equiv u, d, s$) samples are also indicated

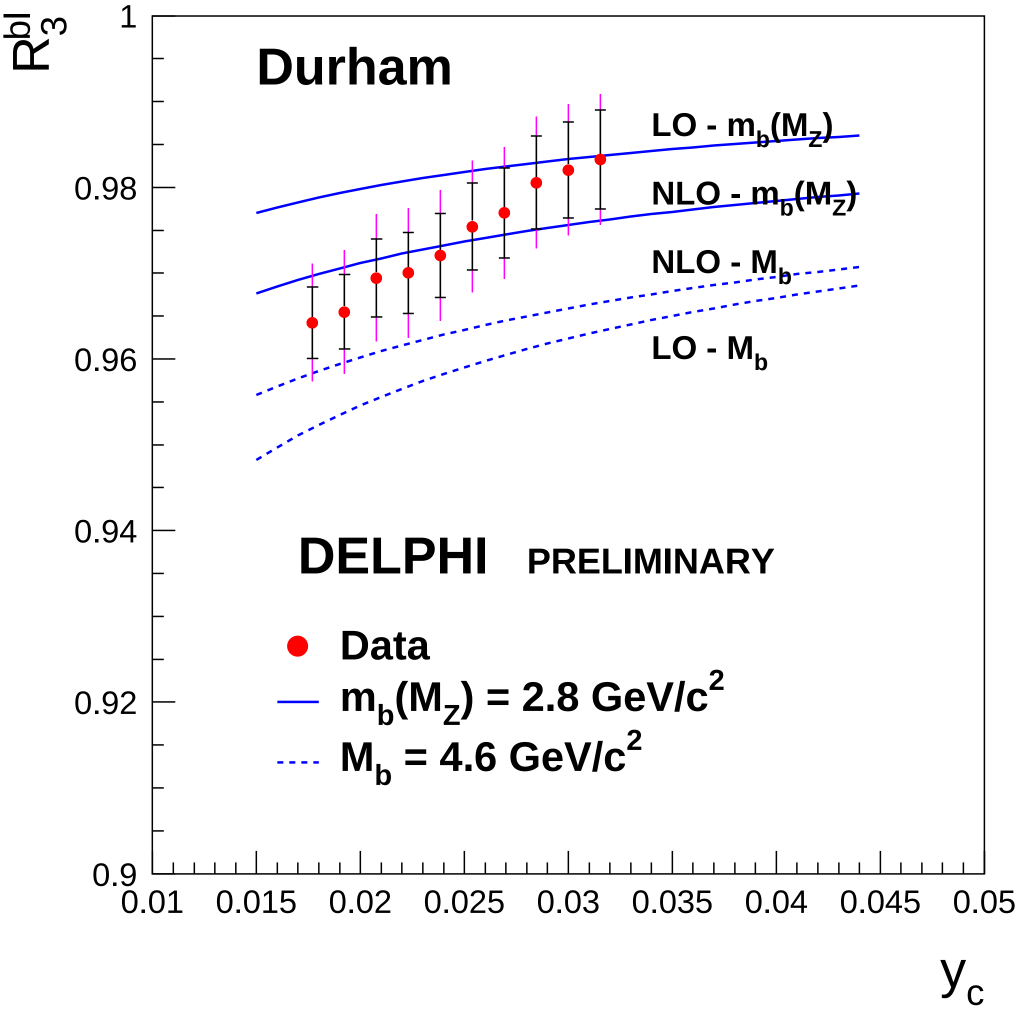

Figure 3: Corrected data values of $R_3^{b\ell}$ using DURHAM algorithm compared with the theoretical predictions from reference [10] at LO and NLO in terms of the pole mass $M_b = 4.6\ GeV/c^2$ (dashed lines) and in terms of the running mass $m_b(M_Z) = 2.8\ GeV/c^2$ (solid lines).

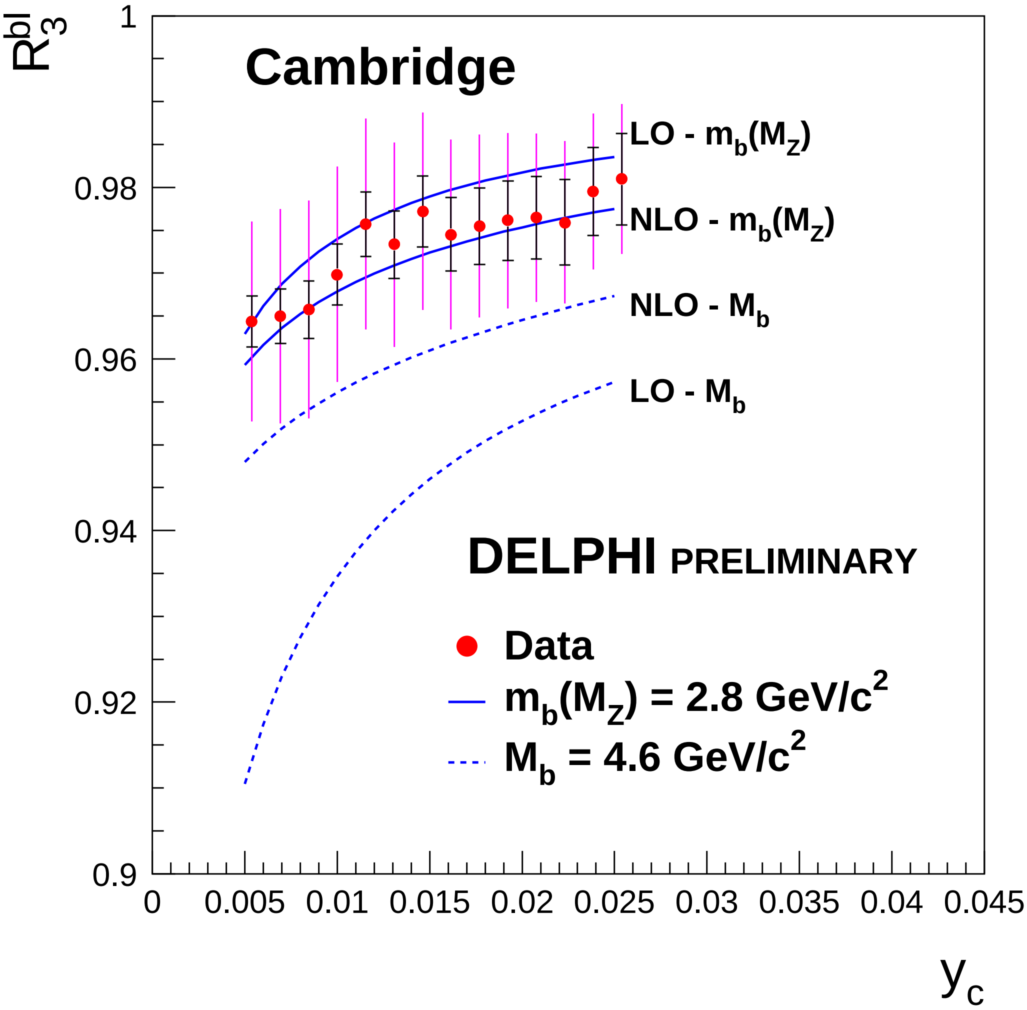

Figure 4: Corrected data values of $R_3^{b\ell}$ using CAMBRIDGE algorithm compared with the theoretical predictions from reference [10] at LO and NLO in terms of the pole mass $M_b = 4.6\ GeV/c^2$ (dashed lines) and in terms of the running mass $m_b(M_Z) = 2.8\ GeV/c^2$ (solid lines).

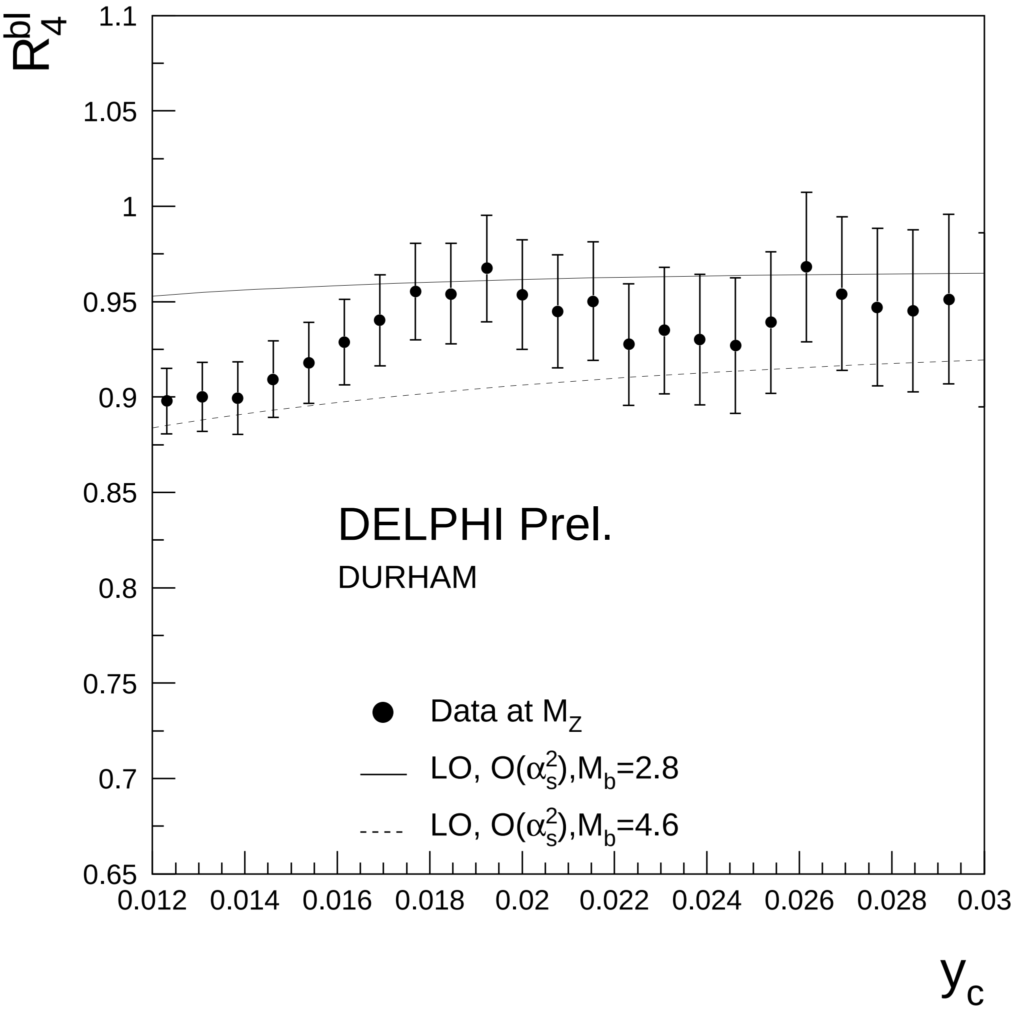

Figure 5: Corrected data values of $R_4^{b\ell}$ with their statistical errors using Durham algorithm compared with the theoretical predictions from reference [17] at LO with $M_b = 2.8\ GeV/c^2$ (solid line) and $M_b = 4.6\ GeV/c^2$ (dashed line).

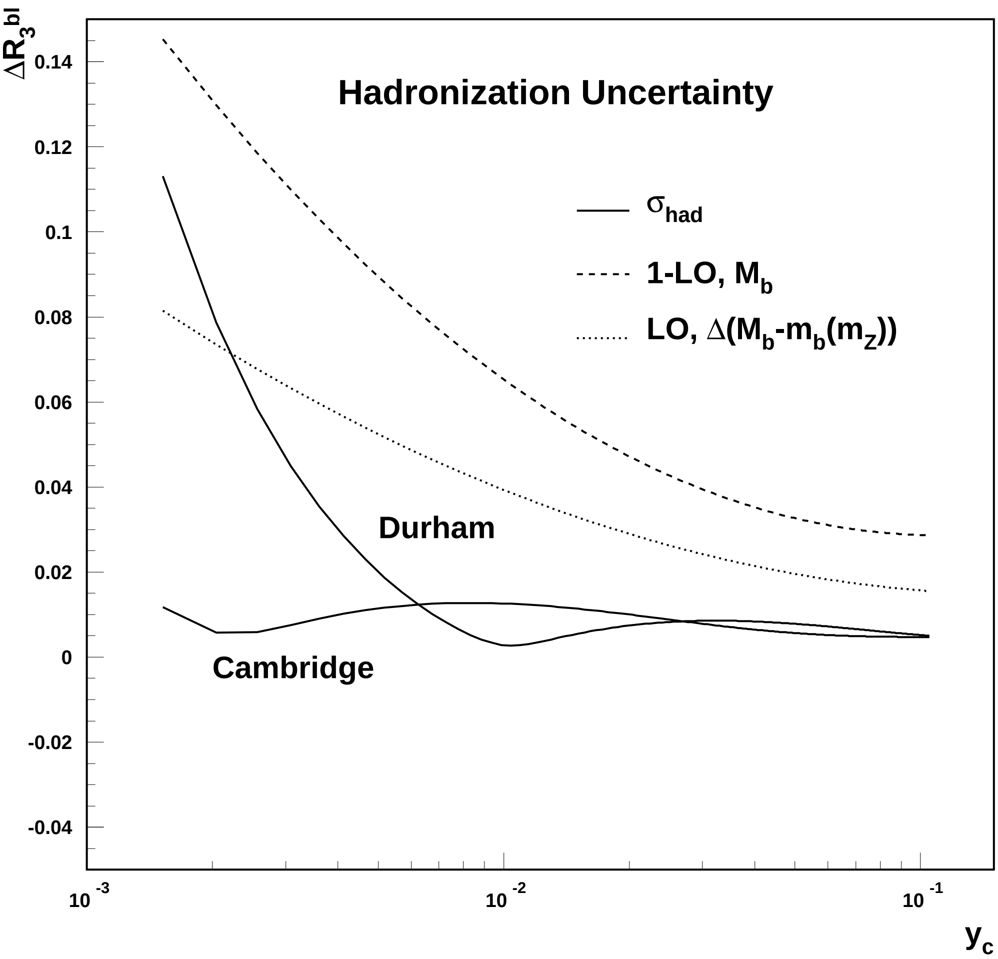

Figure 6: Evolution of the total hadronization error of $R_3^{b\ell}$ with the resolution parameter y_c in comparison with the LO prediction in terms of the pole mass, $M_b = 4.6\ GeV/c^2$ and the difference between the LO prediction using $M_b = 4.6\ GeV/c^2$ and $m_b(M_Z) = 2.8\ GeV/c^2$.

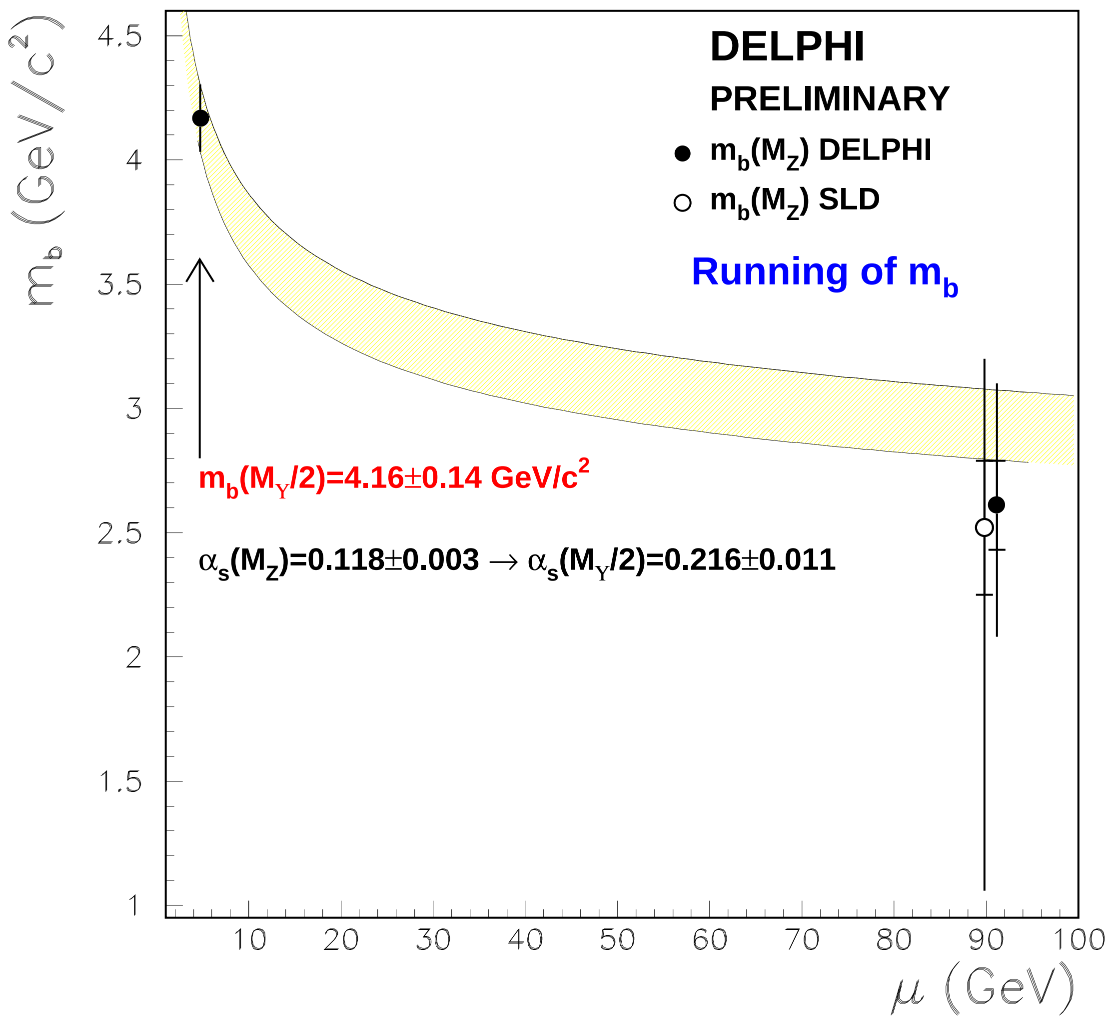

Figure 7: The running of $m_b(\mu)$ from the scale $M_\Upsilon/2$ up to the M_Z scale using the QCD renormalization group equations. The $m_b(M_Z)$ value obtained by DELPHI and the value from reference [2] using SLD results are displayed together with the statistical and total errors.