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GENERAL RELATIVISTIC THERMOELECTRIC EFFECTS IN SUPERCONDUCTORS

B.J. Ahmedov
Institute of Nuclear Physics, Ulughbek, Tashkent 702132, Uzbekistan
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

Abstract

We discuss the general-relativistic contributions which occur in the electromagnetic properties of a superconductor with a heat flow. The appearance of a general-relativistic contribution to the magnetic flux through a superconducting thermoelectric bimetallic circuit is shown. The response of the Josephson junctions to a heat flow is investigated in the general-relativistic framework. Some gravitothermoelectric effects which are observable in the superconducting state in the Earth's gravitational field are considered.

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1. Introduction

Effects of fields of gravity and inertia on a superconductor have been investigated by a number of authors starting with DeWitt [1] and Papini [2]. More recently, a general relativistic treatment of electromagnetic effects in normal conductors with a heat flow and superconductors without gradient of temperature have been given. In particular, several superconducting devices that can, in principle, detect a gravitational field have been presented by Anandan [3]. Further, one of these effects has been emphasized in the experiment [4] which tested the equivalence principle for Cooper pairs.

From our point of view, a study of thermoelectromagnetic relativistic gravitational effects in superconductors with nonzero gradient of temperature is of fundamental interest for the following two reasons.

Thermoelectric effects do not vanish, in principle, in inhomogeneous and anisotropic superconductors and recently attract an increasing interest [5,6] especially due to the discovery of high temperature superconducting materials which combine both anisotropy and inhomogeneity properties and in this connection which are favourable for measurement of thermal effects. Since superconductors provide extremely sensitive and accurate measurements, there is hope that the very weak general relativistic contributions to the thermoelectromagnetic effects due to the inhomogeneity arising from the Earth's gravitational field might be detectable.

On the other hand, according to the recent theoretical models, the core of neutron stars forms matter in a superconducting state with a thermal distribution and heat flow. It suggests that thermoelectromagnetic effects might explain the origin and evolution of the magnetic field inside the core of supermassive pulsars for which the dimensionless general relativistic parameter $\frac{\alpha}{R_s}$ reaches about 0.5 [7] (α and R_s are the gravitational radius and radius of star, respectively). In addition thermoelectromagnetic effects in Josephson contacts could be responsible for electromagnetic radiation arising from possible superconductor-normal metal-superconductor layers in the intermediate boundary between conducting crust and superconducting core inside neutron star.

2. The General Relativistic Correction to the Magnetic Flux Induced by a Heat Flow in Superconducting Thermocouple

Consider a superconductor with a heat flow in an external stationary gravitational field. According to the two-component model, two currents flow in this superconductor: the superconducting current of density $\hat{\jmath}_{(s)\alpha}$ and the normal current of density $\hat{\jmath}_{(n)\alpha}$. The normal current is carried by 'normal' electrons (exitations) and it does not differ essentially from the current in the normal state of a metal [8]

$$F_{\alpha\beta}u^{\beta} = \frac{1}{\lambda}\hat{\jmath}_{(n)\alpha} + R_H(F_{\nu\alpha} + u_{\alpha}u^{\sigma}F_{\nu\sigma})\hat{\jmath}^{\nu}_{(n)} + \Lambda^{-1/2}\overset{\perp}{\nabla}_{\alpha}\tilde{\mu} - \beta\Lambda^{-1/2}\overset{\perp}{\nabla}_{\alpha}\tilde{T} - b\hat{\jmath}^{\beta}_{(n)}A_{\alpha\beta}, \qquad (1)$$

where $\tilde{T} = \Lambda^{1/2}T, \tilde{\mu} = \Lambda^{1/2}\mu$, μ is the chemical potential per unit charge, T is the temperature, λ is the conductivity, $\Lambda = -\xi^\alpha \xi_\alpha$, ξ_α is a timelike Killing vector being parallel to the four velocity of the conductor u^α, β is the normal differential thermopower coefficient, $F_{\alpha\beta} = A_{\beta,\alpha} - A_{\alpha,\beta}$ is the electromagnetic field tensor, A_α is the vector potential, R_H is the Hall constant, $A_{\beta\alpha} = u_{[\alpha,\beta]} + u_{[\beta}w_{\alpha]}$ is the relativistic rate of rotation, $w_\alpha = u_{\alpha;\beta}u^\beta$ is the absolute acceleration and b is the parameter for the conductor, $\overset{\perp}{\nabla}_\alpha$ and $[\cdots]$ denote the spatial part of covariant derivative and antisymmetrization.

Thus in the general case the conduction current can arise due to the following reasons (with general relativistic corrections and contributions) - (a) the electric field, (b) the Hall effect, (c) nonequilibrium effects and (d) the Coriolis (gravitomagnetic) force effects described by the last term on the right hand side of equation (1).

If the wavefunction of the Cooper pairs is $\psi = n_s^{1/2}e^{i\vartheta}$ then four-vector of supercurrent density is [3]

$$j_{(s)\alpha} = \frac{ie\hbar}{m_s}\{\psi^*(\partial_\alpha - i\frac{2e}{\hbar c}A_\alpha)\psi - \psi(\partial_\alpha + i\frac{2e}{\hbar c}A_\alpha)\psi^*\}, \tag{2}$$

which satisfies the equation

$$j_{(s)\alpha} = \frac{2n_s e}{m_s}\{-\hbar\partial_\alpha\vartheta + 2e/cA_\alpha\} = \frac{2n_s e}{m_s}P_\alpha \tag{3}$$

where n_s represents the density of Cooper pairs, m_s and $P_\alpha = p_\alpha + \frac{2e}{c}A_\alpha$ are the mass and the generalized momentum of the Cooper pair and ϑ is the phase of superconducting wavefunction.

Since any supercurrent flows only on the surface within the penetration depth, in the interior of the superconductor supercurrent (3) is parallel to the four-velocity u^α i.e.

$$P^\alpha = -\frac{2e}{c}\mu u^\alpha. \tag{3a}$$

In order to specify the unknown coefficient of proportionality μ one can consider the flat space-time value of equation (3a) when $u^\alpha = \{1,0,0,0\}$. Then the zeroth component of (3a)

$$-\hbar\frac{\partial\vartheta}{\partial t} = 2e(\mu - A_0)$$

is in agreement with one of the well-known Josephson equations (nonstationary one) provided μ is the chemical potential (per unit charge of electron) including the rest mass-energy.

It follows from (3a) that in the interior of the superconductor

$$F_{\alpha\beta} = 2\mu_{[,\beta}u_{\alpha]} + 2\mu\partial_{[\beta}u_{\alpha]}$$

and after multiplying it by u^α one can get that [3]

$$E_\beta - \Lambda^{-1/2}\partial_\beta\tilde{\mu} = 0 \tag{4}$$

everywhere inside the superconductor in the steady state. Here $E_\beta = F_{\beta\alpha}u^\alpha$ is the electric field as seen by an observer who is at rest with respect to the superconductor.

Formula (4) can be also derived from the general relativistic London equations [9]

$$\hat{\jmath}_{(s)[\beta;\alpha]} - \hat{\jmath}_{(s)[\alpha}(\ln n_s)_{,\beta]} + \frac{2n_s e^2}{m_s c} F_{\alpha\beta} - 2cen_s(A_{\alpha\beta} + u_{[\alpha}w_{\beta]}) = 0, \tag{5}$$

which have been obtained by requiring, that inside a superconducting medium, the motion of the Cooper pairs is free of resistance.

Suppose that two ends of a bulk piece of superconductor are at different temperatures, T_1 and T_2. The temperature gradient will produce a force on the normal exitations of the superconductor, initiating a current of the normal exitations

$$\hat{\jmath}_{(n)\alpha} = \Lambda^{-1/2}\lambda\beta\partial_\alpha\tilde{T} - \lambda R_H(F_{\nu\alpha} + u_\alpha u^\sigma F_{\nu\sigma})\hat{\jmath}^{\nu}_{(n)} + \lambda b\hat{\jmath}^{\beta}_{(n)}A_{\alpha\beta} \tag{6}$$

as a consequence of Ohm's law (1) under the conditions corresponding to equation (4).

It therefore follows that below T_c, under steady-state conditions, when equation (4) should be obeyed, if $\partial_\nu\tilde{T} \neq 0$, the density of the normal current $\hat{\jmath}_{(s)\alpha}$ should be finite because of equation (6). However, if the circuit is open, the total current will be zero and in the simplest case the density of the total current also vanishes: $\hat{\jmath}_\alpha = \hat{\jmath}_{(s)\alpha} + \hat{\jmath}_{(n)\alpha} = 0$, i.e. the normal current density $\hat{\jmath}_{(n)}$ is cancelled locally by a counterflow of supercurrent density $\hat{\jmath}_{(s)}$

$$\hat{\jmath}_{(s)\alpha} = -\hat{\jmath}_{(n)\alpha} = -\Lambda^{-1/2}\lambda\beta\partial_\alpha\tilde{T} + \lambda R_H(F_{\nu\alpha} + u_\alpha u^\sigma F_{\nu\sigma})\hat{\jmath}^{\nu}_{(n)} - \lambda b\hat{\jmath}^{\beta}_{(n)}A_{\alpha\beta}. \tag{7}$$

Suppose the superconductor is embedded in the Schwarzschild space-time

$$ds^2 = -(1-\alpha/r)(dx^\circ)^2 + (1-\alpha/r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{8}$$

where the timelike Killing vector ξ^α can be chosen so that $\Lambda = 1 - \alpha/r$. If curvature effects are negligible, than the apparatus may be regarded as having an acceleration, g, relative to a local inertial frame, and thus, $\Lambda = (1 + 2gH/c^2)^2$, where H is the height above some fixed point.

Because of the cancellation of the thermoelectric current in the superconductor, schemes for measuring the thermoelectric effects, based on inhomogeneous or anisotropic superconductor configurations, become necessary. A review of various experiments, as well as a bimetallic superconducting ring has been given e.g. by Van Harlingen [10].

We shall now consider two bulk specimens of two dissimilar superconductors, S_I and S_{II}, which are brought into contact in such a way that together they form a closed ring, as in fig.1. Suppose that the temperatures of the upper and lower contacts are kept at the different temperatures T_1 and T_2 respectively. The presence of an open gap in a massive circuit when its thickness d (for example, the diameter of the wire forming the circuit) is much bigger than the depth of penetration δ of the field allows us to calculate the magnetic flux Φ_b across the gap without solving the problem completely. As was shown above, in the bulk of a superconductor $\hat{\jmath}_\alpha = \hat{\jmath}_{(s)\alpha} + \hat{\jmath}_{(n)\alpha} = 0$, so that

$$\hat{\jmath}_{(n)\nu} = -\Lambda^{-1/2}\lambda\beta\partial_\nu\tilde{T} = -\hat{\jmath}_{(s)\nu} = \frac{2n_s e}{m_s}[\hbar\partial_\nu\vartheta - \frac{2e}{c}A_\nu]. \tag{9}$$

For the sample embedded in the Schwarzschild space-time the last two terms in formula (7) disappear since in this case the magnetic field does not penetrate inside the superconductor and all components of the relativistic rate of rotation are zero.

Integrating equation (9) along the contour C, which is in the bulk of the superconductor and using that $\oint A_\alpha dx^\alpha = \frac{1}{2}\int F_{\alpha\beta} dS^{\alpha\beta} = \Phi_b$ and $\oint \partial_\nu \vartheta dx^\nu = 2\pi n$, where $n = 0, 1, 2, ...$ yields directly

$$\Phi_b = n\Phi_0 + \frac{m_s c}{4n_s e^2}\int \Lambda^{-1/2}\lambda\beta\partial_\nu \tilde{T} dx^\nu, \tag{10}$$

where $m_s/e^2 n_s = \Lambda_0 = 4\pi\delta^2/c^2$ and $\Phi_0 = \pi\hbar c/e = 2\times 10^{-7} Gauss \cdot cm^2$ is quantum of the magnetic flux. The current I_s which leads to the appearance of a flux Φ_b flows on the internal surface of the circuit in a layer of thickness of the order of δ.

If $\Lambda = \Lambda(0)$ at 0 then from (10), the magnetic flux through contour is

$$\Phi_b = n\Phi_0 + \Lambda(0)^{-1/2}\frac{m_s c}{4n_s e^2}\oint \beta\lambda\partial_\nu \tilde{T} dx^\nu =$$

$$n\Phi_0 + \frac{m_s c}{4n_s e^2}((\beta\lambda)_I - (\beta\lambda)_{II})[T_1(1+gH_1/c^2) - T_2(1+gH_2/c^2)], \tag{11}$$

where β_I and β_{II} are the values of β for the two metals, H_1 and H_2 are the heights of the junctions above the Earth's surface. This is the general relativistic generalization of the thermoelectric effects in the inhomogeneous (bimetallic) superconductor.

If, for the sake of simplicity, we assume that $(\beta\lambda)_{II} \gg (\beta\lambda)_I$ we then find from equation (11) that

$$\Phi_T = \Phi_b - n\Phi_0 \approx \frac{m_s c}{4n_s e^2}(\beta\lambda)_{II}\Delta T + \frac{gH}{c^2}\frac{m_s c}{4n_s e^2}(\beta\lambda)_{II}\Delta T. \tag{12}$$

When the apparatus is horizontal, the magnetic flux in the circuit would remain at the flat space-time value and last term in formula (12) will disappear. In this case equation (12) describes unquantized thermoelectric flux Φ_T in the presence of a temperature gradient. For typical parameters of the superconductor, the thermal-current-related flux (first term in (12)) is expected to be of order of $10^{-2}\Phi_0$. In actual fact, however, much - indeed orders of magnitude - stronger fluxes of hundreds of Φ_0 have been measured experimentally [11].

However if the apparatus is brought into a vertical plane, then the magnetic flux would be changed according to the formula for Φ_T. The height between two junctions is changed by $H = H_1 - H_2$ and the magnetic flux undergoes a fractional change gH/c^2. Then the general relativistic contribution in Φ when $H = 10cm$, in the Earth's gravitational field, will be proportional to a small dimensionless parameter 10^{-17}.

There is no doubt that the flux production mechanism discussed in this section may be significant and relevant to the problem of origin and evolution of magnetic fields in isolated neutron stars since their radius can be only $1.4 - 3.5$ times larger than gravitational radius α and their substance may be superconducting of II-type or superfluid at high densities (see e.g. [12]).

3. Thermoelectric Effects in SNS Junctions in the External Gravitational Field

In this section we shall first consider the behaviour of an superconductor-normal metal-superconductor (SNS) junction when its S electrodes have different temperatures or, in other words, when there is a heat flow through the junction placed in a gravitational field. After that, we shall discuss the thermoelectric effects arising when, in addition to carrying a heat flow, the junction is placed in both magnetic and gravitational fields.

Let us suppose that there is a temperature difference ΔT between superconducting electrodes of the SNS Josephson junction placed in the external gravitational field (8) (see fig.2). Due to Kirchoff's first law

$$\hat{\jmath}_{(s)\alpha} = \hat{\jmath}_{(n)\alpha}. \tag{13}$$

According to Ohm's law (1) a normal component of a current in the junction is

$$\hat{\jmath}_{(n)\alpha} = \lambda E_\alpha - \lambda \Lambda^{-1/2} \partial_\alpha \tilde{\mu} + \beta \lambda \Lambda^{-1/2} \partial_\alpha \tilde{T}. \tag{14}$$

Density of the superconducting current flowing through the junction is related to the phase difference $\phi = \Delta \vartheta$ across the junction by

$$\hat{\jmath}_{(s)\alpha} = \hat{\jmath}_{(c)\alpha} \sin \phi \tag{15}$$

where $\hat{\jmath}_{(c)\alpha}$ is the critical value of electric current density.

Using formulae (13)-(15) and Josephson equation derived from (3a)

$$-\hbar \frac{\partial \phi}{\partial \tau} = \frac{2e}{c} \mu - \frac{2e}{c} A_\mu \xi^\mu \Lambda^{-1/2} \tag{16}$$

one can obtain

$$\hat{\jmath}_{(c)\alpha} \sin \phi = \lambda \Lambda^{-1/2} \{ -\partial_\alpha (\frac{\hbar c}{2e} \frac{\partial \phi}{\partial \tau} \Lambda^{1/2}) + \beta \partial_\alpha \tilde{T} \}$$

which after integration on $n^\alpha dS$ will give

$$\Lambda(0)^{1/2} R I_{(c)} \sin \phi = -\frac{\hbar c}{2e} \frac{\partial \phi}{\partial \tau} \Lambda^{1/2} + \beta \Delta \tilde{T}. \tag{17}$$

Here $R = \int \frac{dl}{\rho dS}$ is the resistance of the normal layer with length dl, $I_{(c)} = \int \hat{\jmath}_{(c)\alpha} n^\alpha dS$ is electric current, n^α is normal vector to the cross section of wire dS, $\partial \phi / \partial \tau = \phi_{,\alpha} u^\alpha$.

Therefore if the thermoelectric current exceeds the critical current of the Josephson junction, then as a consequence of (15) and (17), an alternating current (ac) of frequency

$$\omega = \frac{2e}{\hbar} \beta \Lambda^{-1/2} \Delta \tilde{T} \tag{18}$$

is produced and the junction emits radiation with the frequency ω measured by an observer at rest with respect to the junction. Formula (18) is the general relativistic generalization of thermoelectric ac Josephson effect according to which a temperature difference ΔT across the SNS junction results in electromagnetic radiation, which has been predicted in [13] and experimentally confirmed [14].

Now we would like to show with the help of a simple example that general relativistic thermoelectric effects may have astrophysical applications. Suppose that SNS junctions can be realized in the intermediate boundary between the conducting crust and the superconducting core inside neutron star. Then as a consequence of equation (18), that is, of the existence of thermal analog of the ac Josephson effect, we can predict a new mechanism for electromagnetic radiation production from pulsars: a steady heat flow through SNS junctions in intermediate boundary gives rise to Josephson radiation with frequency ω. Let us roughly estimate the frequency of expecting radiation. If thermoelectric power of conducting crust $\beta(T \sim 10^5 K) \sim 10^{-2} V/K$, then $\omega \approx 3 \times 10^{13} \Lambda^{-1/2}(T_2 - T_1)K$, and at a temperature difference $10^{-7}K$ we have $\omega \sim 3 \times 10^5 Hz$.

According to (18) the frequency of the junction depends on the altitude in the gravitational field and hence frequences ω_1 and ω_2 of the junction at H_1 and H_2 ($H = H_2 - H_1; H_1 < H_2$) are connected through $\omega(H_2) = \omega(H_1)(1 - gH/c^2)$. If $\beta \approx 10^{-5} V \cdot K^{-1}$ and $H = 10m$, then the general relativistic red-shift for the frequency is

$$\omega_{gr}(Hz) = \frac{gH}{c^2}\frac{2e}{\hbar}\beta\Delta T \approx 3 \times 10^{-3} \Delta T(K). \qquad (19)$$

This quantity is weak since it is experimentally very difficult to obtain a great temperature difference in the considered superconducting system due to the thinness of the normal layer in the junction. Probably comparatively good experimental results could be achieved by using the superconducting materials with the long coherence length which allow the thick normal layer. If the radiations are detected by one device, there would be no frequency difference between them due to the gravitational redshift effect for frequency which will compensate correction (19). Hence this null experiment would confirm that the temperature differences, ΔT_2 and ΔT_1, at the upper and low altitudes are connected through

$$\Delta T_1 = \Delta T_2 (1 - gH/c^2) \qquad (20)$$

and varying with height.

Consider two dissimilar SNS junctions, separated by a height H and connected in parallel by superconducting wires to a common heater source. Suppose that β_{II} and β_I are the thermoelectric power of the upper and lower junctions, respectively. Now we integrate equation (3) over the contour which passes through the interior of superconducting ring with two Josephson contacts. Then

$$\Phi_b = n\Phi_0 + \frac{\hbar c}{2e}(\phi_{II} - \phi_I),$$

where ϕ_{II} and ϕ_I are the contributions due to the phase discontinuities at the Josephson junctions.

The rate of change of magnetic flux is related to $\Delta\omega \equiv \omega_{II} - \omega_I$ by

$$\frac{d\Phi_b}{d\tau} = \frac{dn}{d\tau}\Phi_0 + \frac{\hbar c}{2e}\Delta\omega,$$

so if $\Delta\omega$ is not equal to zero, a magnetic flux $\Delta\Phi_b \neq 0$ will be induced. As long as $\Delta\Phi_b < \Phi_0$, n will remain constant and $\Delta\Phi_b$ will increase linearly with time until $\Delta\Phi_b = \Phi_0$, then the order of the step n will change as flux quantum enters the loop. Thus due to the effect of gradient of temperature on the junctions with unequal thermoelectric powers is equivalent to having a time dependent flux, as given by the last term on the right hand side of the last equation. Then using equations (18) and (20) one can derive that the change in magnetic field inside the circuit during the time interval $[0, \tau]$ is

$$\Delta\Phi_b = c\int_0^\tau [\Delta T_1\beta_I - \Delta T_2\beta_{II}(1 - \frac{gH}{c^2})]d\tau + \Delta n\Phi_0 = c\int_0^\tau \Delta T_1(\beta_I - \beta_{II})d\tau + \Delta n\Phi_0. \quad (21)$$

Thus this particular loop is sensitive to the frequency and in this connection to the thermoelectric power difference between the junctions. The independence of the magnetic flux (21) from the gravitational field g confirms the validity of formula (18) for the temperature, that is in a gravitational field, during thermal equilibrium, the related quantity $\tilde{T}$ (rather T) is constant along the sample. In addition, a new experiment, in which the thermoelectric response creates a flux (21) changing with time will yield one more possibility of measurement of thermoelectric effects in superconductors. It is more or less important, not only from general-relativistic point of view, but also for the new proposals [15] for confirmation of some aspects of thermoelectric transport theory.

When the current exceeds the critical value potential difference $\tilde{V} = \beta\Delta\tilde{T}$ appears across the junction due to the thermoelectric effects. Since the thermoelectric power of the junction II differs from that at the junction I, the potential differences across the first and second junctions, $\tilde{V}_{II}$ and $\tilde{V}_I$, respectively, will differ so that $\Delta\tilde{V} = \tilde{V}_{II} - \tilde{V}_I = (\beta_{II} - \beta_I)\Delta\tilde{T}$. The basic technique for the detection of extremely small voltage differences between two Josephson junctions by monitoring of magnetic flux change was first developed by Clarke [16].

In the absence of any additional effects on the Cooper pairs, one would thus expect the net EMF in the loop containing the junctions to be $(\beta_{II} - \beta_I)\Delta\tilde{T} \sim 10^{-11}V$ for the typical values of parameters $(\beta_{II} - \beta_I) \sim 10^{-6}V/K$ and $\Delta\tilde{T} \sim 10^{-5}K$. For the loop of inductance L the evolution of magnetic field is approximately governed by law $\frac{d\Phi_b}{d\tau} = -L\frac{dI_l}{d\tau}$. In this connection a nonvanishing value for ΔV would lead, according to (18) and (21), to a time varying current I_l (from zero to the critical maximum value in the range of one number of the step n): $\frac{dI_l}{cd\tau} = -\frac{1}{L}\Delta V$, which will induce the above discussed flux $\Delta\Phi_b = c\int \Delta V d\tau$ through the loop in the linear regime.

For the measurement of gravitational contribution we can propose the following change. Connect the Josephson junctions to two independent heaters which have different temperatures, such that $\Delta T_2 = A\Delta T_1$ and A is constant. In this case the flux (21) takes form

$$\Delta\Phi_b = \Delta n\Phi_0 + c\int_0^\tau \Delta T_1(\beta_I - A\beta_{II})d\tau + Ac\int_0^\tau \Delta T_1\beta_{II}\frac{gH}{c^2}d\tau. \tag{22}$$

If the apparatus is horizontal then $H = 0$ and therefore the last term in the magnetic flux change should disappear. When the coil is vertical the last term will increase the rate of change of flux according to (22). By detecting this contribution, the gravitational corrections to thermoelectric effects can be measured. Taking $A\beta \sim 10^{-7}cm^{1/2}\cdot g^{1/2}\cdot s^{-1}/K, \Delta T_1 \sim 10^{-4}K$ and $H = 10cm$ we obtain for $\Delta\Phi_G(Gauss\cdot cm^2) \sim 3\times 10^{-19}\cdot\Delta\tau(s)$. Measuring such tiny variations of magnetic field for large $\Delta\tau$ is near to the limit of SQUID sensitivity.

The main problem in observing the flux (21) and (22) will be connected with generating thermal current $\vec{\jmath}_s = -\Lambda^{-1/2}\lambda\beta grad\tilde{T}$ comparable in magnitude with its critical value $\vec{\jmath}_c$, since the temperature difference across the junction is limited by a low temperature T_c and small sizes of the junction.

The similar method of measurement has been used by Jain et al [4] in null result experiment on confirmation of the strong equivalence principle for a charged massive particle. In their experiment the phase of Josephson contacts has been locked to an external microwave source and is shown to be technically feasible to measure a voltage drop $10^{-22}V$.

It is interesting to mention that the predicted mechanism for production of magnetic field and current changing with time can be of crucial importance in astrophysics as an additional way (to the existed ones [17]) of generating electromagnetic radiation from pulsars. According to the recent theoretical models [7], a neutron star is the relativistic compact object consisting of the conducting crust and superfluid core. In the inner crust of the neutron star the superfluid coexists with a crystal lattice and in its core, at densities above $2\times 10^{14}gm/cm^3$ there is a homogeneous mixture of superfluid neutrons and superconducting protons.

An important fact is that the thermoelectric power β is the function of temperature as $T^{3/2}$ and in this connection can reach large numbers since superconductivity in the stars takes place at the temperatures $10^6 - 10^7K$. So if one accepts that the SNS structures are realized in the intermediate boundary between conducting crust and superconducting core inside the neutron star then the strong heat fluxes in these SNS junctions can lead to the generation of time-varying magnetic field (i.e. electromagnetic radiation) due to the thermoelectric effect described by the basic formula (21).

Suppose that, in addition to the heat flow, SNS Josephson junction is placed in a magnetic field parallel to the plane of the junction (the xy plane). It is well-known that the maximal current density, $\hat{\jmath}_{(s)max\alpha}$, which can pass through the junction is

$$\hat{\jmath}_{(s)max\alpha} = \hat{\jmath}_{(c)\alpha}\left|\frac{\sin\pi\Phi_b/\Phi_0}{\pi\Phi_b/\Phi_0}\right|, \tag{23}$$

where Φ_b is the magnetic flux through the junction.

Taking into account that $\hat{\jmath}_{(s)\alpha} = \lambda\beta\Lambda^{-1/2}\partial_\alpha\tilde{T}$ we can find that the critical temperature difference, corresponding to appearance of voltage across the junction, is

$$(\Delta\tilde{T})_c = \Lambda(0)^{1/2}\frac{I_c R}{\beta}|\frac{\sin\pi\Phi_b/\Phi_0}{\pi\Phi_b/\Phi_0}| \tag{24}$$

and depends on gravitational field, where $\Lambda = \Lambda(0)$ at the junction. This is the general relativistic generalization of a thermal analog of the dc Josephson effect [18,19] according to which the critical value of heat flow through a SNS junction is a nonmonotonic function of the magnetic flux Φ_b.

Thus the gravitothermoelectric phenomena in superconductors considered here allow us, in principle, to detect the general relativistic effects. But we would like to emphasize that we concentrated only on the gravitothermoelectric phenomena in superconductors of I type. Nevertheless, recently several thermoelectric effects were observed in high temperature superconducting materials of II type (see, for review, [5,6]). In this connection further investigation is needed to take into account gravitational corrections for thermoelectric effects in II type superconductors.

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Figure Captions

Fig.1. Totally superconducting circuit made of two metals I and II in the Earth's vertical gravitational field. The magnetic field Φ_T through the ring has gravitational contribution proportional to gH/c^2.

Fig.2. Equivalent circuit for SNS junction with heat flow: $\vec{\jmath}_T = \lambda\beta\Lambda^{-1/2} grad\tilde{T}$ is the current due to the heat flow across the normal layer.