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CP VIOLATING BUBBLE WALL PROFILES

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Abstract

We solve the equations of motion for a CP violating phase between the two Higgs doublets at the bubble wall of the MSSM electroweak phase transition. Contrary to earlier suggestions, we do not find indications of spontaneous “transitional” CP violation in the MSSM. On the other hand, in case there is explicit CP violation in the stop and chargino/neutralino sectors, the relative phase between the Higgses does become space dependent, but only mildly even in the maximal case. We also demonstrate that spontaneous CP violation within the bubble wall could occur, e.g., if the Higgs sector of the MSSM were supplemented by a singlet. Finally we point out some implications for baryogenesis computations.

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Introduction. For producing the baryon asymmetry of the Universe, the Sakharov conditions demand three properties of a model. The first one, baryon number non-conservation, is already present in the Standard Model. The second one, deviation from thermal equilibrium, can be potentially realized by a strongly first order electroweak phase transition. For the Higgs masses allowed this requires extensions of the Standard Model [1], and the simplest such possibility appears to be the MSSM with a lightest stop lighter than the top [2]–[7]⁵. The third requirement, CP violation, comes into the game when actually computing the baryon asymmetry. Many computations suggest that new CP violating phases as large as $\mathcal{O}(10^{-1})$ are required for generating the observed baryon asymmetry [11]–[15], and phases of this order of magnitude might potentially be in conflict with constraints coming from the electric dipole moment experiments [16]⁶.

Whether or not explicit phases are eventually a problem, it is in any case interesting to note that the MSSM may also offer a mechanism for generating enough CP violation for baryogenesis, without conflicting with *any* experimental constraints. Indeed, due to the fact that there are two Higgs doublets, one can in principle have a spontaneously generated CP violating phase between them [19]. While spontaneous CP violation is excluded at $T = 0$ for the experimentally allowed parameter values [20], there is a suggestion that it might be more easily realized at finite temperatures [21], or even only in the phase boundary between the symmetric and broken phases [22, 23]. Such a profile could conceivably be quite useful for electroweak baryogenesis [24].

Let us stress that even if explicit CP phases are present, it is important to know whether there is some dynamics present in the system which intensifies or suppresses the explicit effects around the phase transition. Thus we need to solve for the profiles of the bubbles.

In order to really compute the profiles and the baryon number produced at the electroweak phase transition, we should follow the history of bubbles from the moment of nucleation, until the time the broken phase fills the Universe. After nucleation, there is in general a long period of stationary growth at a relativistic velocity, and then, if the latent heat of the transition is large enough to reheat the Universe back to the critical temperature T_c , another period of slower growth at a rate determined by the expansion of the Universe [25]. The stage of stationary fast growth is characterized by a non-trivial hydrodynamical temperature and velocity profile affecting also the Higgs field profiles [26]. We will not consider this problem here, but concentrate rather on the profile of a (nearly equilibrium) planar phase boundary at T_c after the assumed

⁵Among alternative scenarios leading to a strong transition is the MSSM augmented by a gauge singlet (NMSSM) [8]–[10].

⁶There is a recent interest in scenarios where this conclusion can be avoided; for a discussion and references see, e.g., [17]. We may note, in particular, that it appears sufficient to have the 1st and 2nd generation scalar partners heavy [18], an assumption often made in electroweak phase transition studies anyway.

reheating.

Previously, the moduli of the two Higgs doublets around the phase boundary at T_c (and also for the newly nucleated bubble at $T < T_c$ before the setup of a stationary hydrodynamical solution [27]) have been determined from the 2-loop effective potential [4, 27, 28]. The CP violating phase between the two Higgs doublets has been addressed in [23]. Both problems can in principle also be studied non-perturbatively with lattice simulations [29, 30].

The purpose of this paper is to present the first complete solution of the equations of motion for the phase between the two Higgs doublets within the MSSM, utilizing a perturbative effective potential, but without restricting it to the effective quartic couplings. Our conclusions will differ from those obtained earlier on.

Solving for the CP violating phase. We parameterize the two Higgs doublets of the MSSM as

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} h_1 e^{i\theta_1} \\ 0 \end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h_2 e^{i\theta_2} \end{pmatrix}. \quad (1)$$

Since we want to use equations of motion involving only the Higgs degrees of freedom, we have to make sure that no source terms are generated for the gauge fields. The form of Eq. (1) guarantees that this is true for W^\pm and the photon, and to remove also the source terms for Z , we need to impose the constraint

$$h_1^2 \partial_\mu \theta_1 = h_2^2 \partial_\mu \theta_2. \quad (2)$$

In addition, because of gauge invariance, the effective Higgs potential depends on the phases only via $\theta = \theta_1 + \theta_2$. We can then concentrate on θ , and using Eq. (2) as well as assuming tree-level kinetic terms and moving to a frame where the bubble wall is static and planar, the action to be minimized is

$$S \propto \int dz \left[\frac{1}{2} (\partial_z h_1)^2 + \frac{1}{2} (\partial_z h_2)^2 + \frac{1}{2} \frac{h_1^2 h_2^2}{h_1^2 + h_2^2} (\partial_z \theta)^2 + V_T(h_1, h_2, \theta) \right], \quad (3)$$

where $V_T(h_1, h_2, \theta)$ is the finite temperature effective potential for h_1, h_2, θ . In general, we are solving the equations of motion for h_1, h_2, θ following from this action. In the numerical solution we use the method outlined in [28] which deals with the minimization of a functional of the squared equations of motion.

At the first stage, we consider the case with no explicit CP phases, and ask whether a particular solution without CP violation ($\theta = 0, \pi$), is in fact a local minimum of the action or not. Clearly, it is not if

$$m_3^2(h_1, h_2) \equiv \frac{1}{|h_1 h_2|} \left. \frac{\partial^2 V_T(h_1, h_2, \theta)}{\partial \theta^2} \right|_{\theta=0} < 0, \quad (4)$$

where we have divided by $|h_1 h_2|$, assuming that this is non-zero. Eq. (4) is to be evaluated along the path found by solving the equations of motion for h_1, h_2 . We have chosen the convention that h_1 can have either sign, allowing us to consider only $\theta = 0$. For the case of the most general quartic two Higgs doublet potential, Eq. (4) agrees with the constraint first written down by Lee [19], on which most of the investigations of spontaneous CP violation are based [20]–[23]. However, Eq. (4) is true more generally, independent of the form of the potential $V_T(h_1, h_2, \theta)$.

Now, the tree-level potential of the theory is

$$V_{\text{tree}} = \frac{1}{2}m_1^2 h_1^2 + \frac{1}{2}m_2^2 h_2^2 + m_{12}^2 h_1 h_2 \cos \theta + \frac{1}{32}(g^2 + g'^2)(h_1^2 - h_2^2)^2, \quad (5)$$

where g, g' are the SU(2) and U(1) gauge couplings, and at tree-level

$$m_{12}^2 = -\frac{1}{2}m_A^2 \sin 2\beta. \quad (6)$$

It follows that $m_3^2(h_1, h_2) = (1/2)m_A^2 \sin 2\beta > 0$, so that the minimum of the potential in the θ direction is at $\theta = 0$. Thus, in order to get spontaneous CP violation one needs radiative corrections which can overcome the tree-level term.

There are various mechanisms by which $m_3^2(h_1, h_2)$ might decrease. One potentially useful correction can be obtained at finite temperatures in the limit $m_U^2 \ll (2\pi T)^2 \ll m_Q^2$, where m_U^2, m_Q^2 are squark mass parameters. Then [29],

$$m_3^2(0, 0) \rightarrow \frac{1}{2}m_A^2 \sin 2\beta - \frac{1}{4} \frac{A_t \mu}{m_Q^2} h_t^2 T^2. \quad (7)$$

Here $h_t \approx 1$ is the top Yukawa coupling, and A_t, μ are squark mixing parameters, assumed real for the moment. The temperature correction is seen to reduce m_3^2 for $A_t \mu > 0$. However, this alone does not improve the situation very much, since $A_t \mu / m_Q^2$ is constrained by stability bounds to be $\ll 1$, particularly for small m_U^2 (see, e.g., [31]).

Another possibility is to radiatively generate quartic couplings which then effectively modify $m_3^2(h_1, h_2)$ at finite h_1, h_2 [21]–[23]. However, these effects are 1-loop suppressed in magnitude and are thus also typically relatively small. Moreover, one must take into account that h_1, h_2 are not free parameters but are determined by the couplings of the theory and by the temperature: they should be solved for from the equations of motion. In particular, it can happen that tuning $m_3^2(h_1, h_2)$ towards zero at the same time takes $h_1 h_2$ to zero [29]: then the division carried out in Eq. (4) is not defined and the effective potential itself, where $h_1 h_2$ multiplies $m_3^2(h_1, h_2)$, need not become more favourable to spontaneous CP violation. In fact, taking the solution of the equations of motion into account and considering radiatively generated quartic couplings, it was argued in [29] that there is effectively a much stronger constraint for obtaining spontaneous CP violation in the MSSM: $m_A^2 + \#T^2 \lesssim \lambda_5 (h_1^2 + h_2^2)$ with λ_5 an effective quartic coupling of magnitude $\lesssim 0.01$. This constraint cannot be satisfied in practice.

Nevertheless, these considerations are based on the approximation to the effective potential where only the quadratic and quartic operators are considered. At finite temperatures around the electroweak phase transition, important contributions come from infrared sensitive non-analytic contributions which are not of this form, and can affect spontaneous CP violation [29]. Thus, it is important to solve the equations of motion more generally for the full effective potential.

In this work, we will consider the full finite temperature 1-loop effective potential of the MSSM. It is known that 2-loop corrections are very important in the MSSM in general [32], and bring the perturbative results [2]–[5] rather close to the lattice results [7], allowing for larger values of h_1, h_2 in the broken phase. Nevertheless, for the present problem we find that even 1-loop effects are in most cases very small, so we do not expect qualitative changes from the 2-loop effects. Eventually, the problem can be studied non-perturbatively with lattice simulations [29, 30].

A scan for spontaneous transitional CP violation. The tree-level part of the effective potential $V_T(h_1, h_2, \theta)$ is in Eq. (5). In the resummed 1-loop contribution to $V_T(h_1, h_2, \theta)$, we include the same particle species as, e.g., in [23]: gauge bosons, stops, charginos and neutralinos. This introduces dependences on the trilinear squark mixing parameters A_t and μ as well as on the squark mass parameters m_Q^2, m_U^2 , and the U(1), SU(2) gaugino parameters M_1 and M_2 . We work in the $\overline{\text{DR}}$ scheme and the Landau gauge, choosing as the renormalization scale $\bar{\mu} = 246$ GeV. The parameters m_1^2, m_2^2 of the $T = 0$ potential are renormalized such that the minimum is at $(v_1, v_2) = (\cos\beta, \sin\beta) \times 246$ GeV. The parameter m_{12}^2 can be expressed in terms of the CP odd Higgs mass m_A in the standard way, including 1-loop corrections.

We do not make a finite temperature expansion as some of the particles can be heavy. Rather, the 1-loop temperature part of the effective potential,

$$V_1(T \neq 0) = \frac{T^4}{2\pi^2} \sum_i n_i \int_0^\infty dx x^2 \ln \left(1 \mp e^{-\sqrt{x^2+z_i}} \right), \quad (8)$$

where $z_i = m_i^2/T^2$ and n_i counts the degrees of freedom (negative for fermions), is evaluated using a spline interpolation between the high and low temperature regions.

We now wish to see whether the constraint in Eq. (4) can be satisfied at the bubble wall between the symmetric and broken phases. To do so, we have to search for each parameter set for the critical temperature T_c , solve the equations of motion for (h_1, h_2) between the minima, and evaluate $m_3^2(h_1, h_2)$ along this path. Since this is quite time-consuming, we proceed in two steps.

1. At the first stage, we do not solve for h_1, h_2, T_c , but rather take them as free parameters in the ranges $h_1/T = -2..2$ and $h_2/T = 0..2$, $T = 80..120$ GeV. The zero temperature parameters are varied in the wide ranges

$$\tan\beta = 2..20, \quad m_A = 0..400 \text{ GeV},$$

$$\begin{aligned}
m_U &= -50\dots 800 \text{ GeV}, & m_Q &= 50\dots 800 \text{ GeV}, \\
\mu, A_t, M_1, M_2 &= -800\dots 800 \text{ GeV}.
\end{aligned}
\tag{9}$$

Here a negative m_U means in fact a negative right-handed stop mass parameter, $-|m_U^2|$. We have also studied separately the (dangerous [6]) region where the transition is very strong [2]–[7], corresponding to $m_U \sim -70\dots -50$ GeV.

Note that since we do not solve for the equations of motion at this stage but allow for $h_1 = \pm|h_1|$, we have to divide in Eq. (4) by $h_1 h_2$ instead of $|h_1 h_2|$: this leads in general to positive values due to the tree-level form of the potential, Eqs. (5),(6). A signal of a potentially promising region is then a small absolute value of the result, since this means that we are close to a point where $\partial_\theta^2 V_T(h_1, h_2, \theta)$ crosses zero.

2. At the second stage, we study the most favourable parameter region thus found in more detail. First of all, we search for the critical temperature. Then, we solve the equations of motion for (h_1, h_2) . By comparing with the exact numerical solution in several cases, we find that a sufficient accuracy can be obtained in practice by searching for the “ridge” as an approximation to the wall profile. It is determined as the line of maxima of the potential in the direction perpendicular to the straight line between the minima. Finally, we look for the minimum of $V_T(h_1, h_2, \theta)$ at fixed (h_1, h_2) : this is a fast and reliable approximation for the full solution in the case that θ is small (i.e., just starts to deviate from zero), and corresponds to Eq. (4).

For the first stage, we perform a Monte Carlo scan with about 10^9 configurations. Small values of $m_3^2(h_1, h_2)$ are scarce, and even then do not necessarily correspond to the desired phenomenon of spontaneous CP violation: they could also be points far from the actual wall. This will be clarified at stage 2.

The parameter region found depends most strongly on m_A , $\tan\beta$, with a preference on small values of m_A and large of $\tan\beta$, such that m_{12}^2 in Eq. (6) is small. (It can be noted that this requirement is not favourable for a strong phase transition [2]–[5]). There is also a relatively strong dependence on A_t and μ : the region favoured is shown in Fig. 1. The dependences on the other parameters are less significant; for m_U and m_Q small values are preferred. The region found is in rough agreement with those found in [21, 22, 23, 29].

At the second stage, we make further restrictions. For instance, we exclude the cases leading to non-physical negative mass parameters, e.g. a lightest stop $m_t^2 < 0$ (a stronger restriction could be obtained by excluding regions leading to a charge and colour breaking minimum at some stage of the Universe expansion [6]). We exclude cases leading to $T = 0$ spontaneous CP violation in the broken phase: this phenomenon requires very small values of m_A [20]. We also discard phase transitions which are exceedingly weak, $v/T \ll 0.1$.

In any case, even before taking into account the experimental lower limits on the Higgs masses, $m_H, m_A \gtrsim 80$ GeV, we cannot find any promising case in the sample of $\sim 2 \times 10^6$ configurations of stage 2, with the desired property of a temperature induced

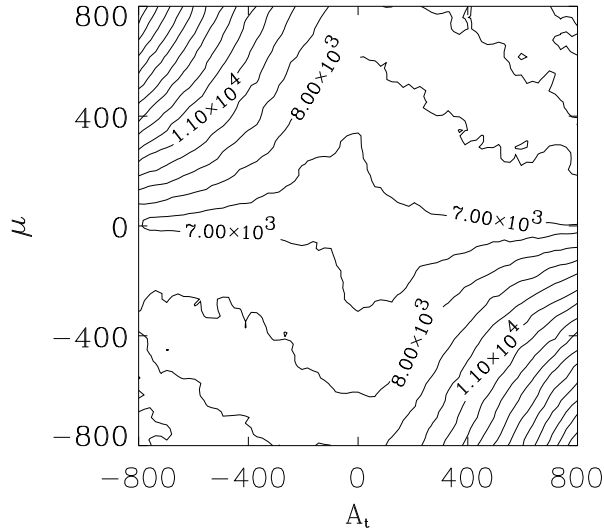


Figure 1: The average value of m_3^2 versus μ and A_t . We observe that small values of m_3^2 are not typical in any part of the plane but are on the average more likely for small μ, A_t , and that the distribution is wider (and thus more favourable) for like signs of μ, A_t , as shown by the noisy contours obtained with a finite amount of statistics.

transitional CP violation within the bubble wall in the MSSM.

In [23], the special point $m_{\tilde{t}}^2 \approx 0$ was considered. Due to the fact that in [23] thermal mass corrections were neglected for $m_{\tilde{t}}^2$, this point corresponds in the physical MSSM to the case where the thermally corrected stop mass parameter vanishes, $m_{\tilde{t}}^2 + \#T^2 \sim 0$. Expanding the 1-loop cubic term from the stops to a finite order in v_1/v_2 , it was suggested that transitional spontaneous CP violation can take place. This region is quite dangerous due to the vicinity of a charge and colour breaking minimum [6], and furthermore, perturbation theory is not reliable. In any case, without expanding the 1-loop contribution in v_1/v_2 , we cannot reproduce the behaviour proposed in [23].

We conclude that after taking into account the infrared sensitive effects inherent in the 1-loop effective potential, coming from a light stop and gauge bosons, and solving for the wall profile from the equations of motion, spontaneous CP violation does not take place in the physical MSSM bubble wall.

Explicit CP violation. We next turn to explicit CP violation. For the moment we ignore the experimental constraints on the magnitude of the explicit CP phases. Assuming universality in the gaugino sector, there are in principle four independent parameters which could carry a phase: m_{12}^2, A_t, μ, M_2 . However, only two of the phases are physical after field redefinitions: $\theta_A = \theta_{m_{12}^2} + \theta_{A_t} + \theta_\mu$ appearing in the stop matrix, and $\theta_C = \theta_{m_{12}^2} + \theta_\mu + \theta_{M_2}$ appearing in the chargino and neutralino matrices. In addition there is the dynamical phase between the two doublets, θ . The mass eigenvalues

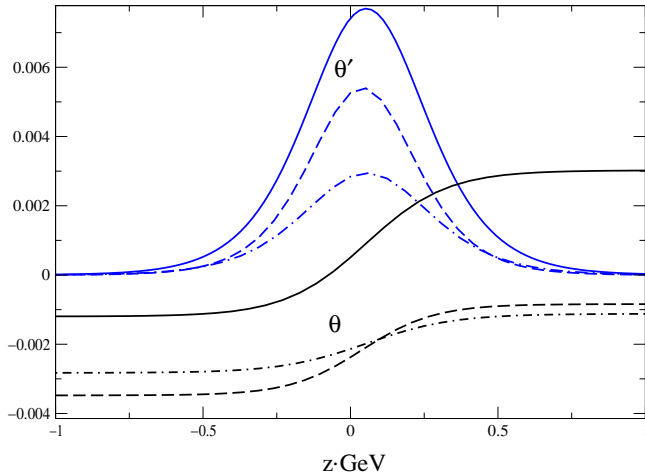


Figure 2: The phase θ and its derivative θ' in the case of large explicit phases (see the text) for three sets of $m_A, \tan\beta$: $m_A = 80$ GeV, $\tan\beta = 2.0$ (solid); $m_A = 120$ GeV, $\tan\beta = 2.0$ (dashed); $m_A = 120$ GeV, $\tan\beta = 3.0$ (dot-dashed). We have $m_U = 0$ GeV, $T_c \approx 100$ GeV.

entering the 1-loop effective potential now have to be computed in the presence of these complex phases.

We can again search for the minima along the θ direction, as we have verified that this is a good approximation to the full solution for the small values of θ we shall find. Indeed, even for maximal phases $\theta_A = \pi/2$ and $\theta_C = \pi/2$, we find a strongly suppressed CP phase θ in the broken Higgs phase. For $m_A \gtrsim 80$ GeV, $\theta(x)$ is of order $10^{-2} \dots 10^{-3}$, and varies relatively mildly within the wall (Fig. 2)⁷. Only for experimentally excluded small values of m_A do we obtain phases up to order unity. For explicit phases of order $\mathcal{O}(10^{-1})$, the dynamical phase θ generated is typically very small, $\mathcal{O}(10^{-3} - 10^{-4})$, and thus, from the baryogenesis point of view, has an effect inferior to those arising from the explicit phases, $\mathcal{O}(10^{-1})$ [12]–[15].

Transitional CP violation in the NMSSM. Finally, we will consider the case of the singlet extension of the MSSM, called the NMSSM. In this case there is a large parameter region allowing for a strong phase transition [8]–[10].

The most general superpotential containing the two Higgs doublets which are already

⁷As a technical point, it should be noted that the determination of $\theta(x)$ is more difficult (and less meaningful) closer to the symmetric phase. In Fig. 2, the solution has been obtained by using a tanh-ansatz for $\theta(x)$, which turns out to compare very well with the more precise solution in the middle of the wall, where the full equations can be more easily solved.

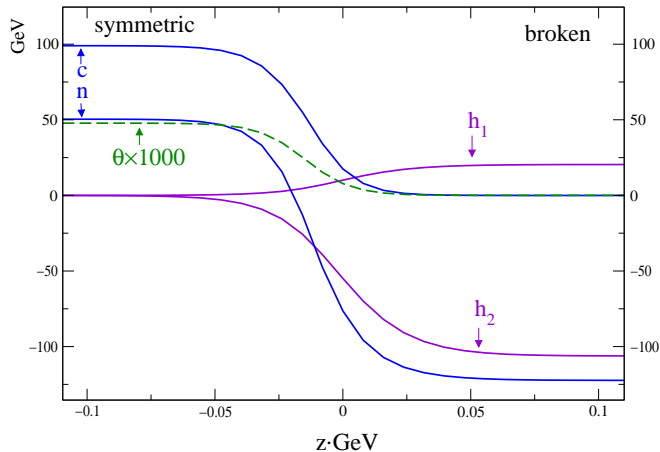


Figure 3: Spontaneous transitional CP violation in the bubble wall. A full solution with 5 fields is shown. The phase θ varies from a finite value in the symmetric phase to zero in the broken phase.

present in the MSSM, and the gauge singlet field S , can be written as [33]

$$W = \mu H_1 H_2 + \lambda S H_1 H_2 - \frac{k}{3} S^3 - r S. \quad (10)$$

In addition, there are soft SUSY breaking terms. As a result, the mass parameter m_{12}^2 of the MSSM gets effectively replaced by a dynamical variable,

$$m_{12}^2 H_1 H_2 + \text{H.c.} \rightarrow (m_{12}^2 + \lambda A_\lambda S + \lambda k^* S^{2*}) H_1 H_2 + \text{H.c.} . \quad (11)$$

Here the singlet S is a complex field. It is thus clear that at the phase boundary where the singlet field can have a non-trivial profile, the spontaneous phase θ between the two Higgs doublets will also have one (for an analysis of spontaneous CP violation in the NMSSM, based on effective couplings, see [34]).

We show an example of the behaviour of the system, for a specific choice of (real) parameters, in Fig. 3. The singlet has been written as $S = n + ic$. All five fields, n, c, h_1, h_2, θ , have been solved from the equations of motion, where the effective potential contains 1-loop contributions from the tops, stops, gauge bosons, charginos, neutralinos and Higgs bosons [10, 35]. In the symmetric phase, the singlet carries a complex vev, while in the broken phase it is real; correspondingly, the phase θ is non-zero close to the symmetric phase but goes to zero in the broken phase. This demonstrates the general possibility of transitional CP violation in bubble walls of the NMSSM phase transition.

Conclusions. The aim of this paper was the discussion of CP violation in the MSSM bubble walls. We searched in a rather large parameter space for transitional CP violation, improving on earlier determinations [22, 23, 29] by using the full infrared sensitive

1-loop effective potential and solving the equations of motion. We could not find any parameter set permitting temperature induced transitional CP violation in the MSSM. The most favourable region (small m_A , large $\tan\beta$) is also in contradiction with a strong first order phase transition, as well as with experimental constraints. The dependences are dominated by the tree-level parameters m_A and $\tan\beta$.

We have omitted the 2-loop corrections in the effective potential, as well as the effects from the Higgses, since we expect them to be small for the present problem dominated by tree-level effects. Eventually, this expectation can be checked with lattice simulations [29, 30].

We also investigated the profile of the dynamical phase in the presence of explicit CP violation in the soft supersymmetry breaking parameters of the MSSM. We found that the dynamical effects are 1-loop suppressed. Thus, even large explicit phases result in a relatively small dynamical phase in the actual bubble wall at T_c , and it seems that the dynamical effects are subdominant for baryogenesis.

Finally we showed an example of an NMSSM bubble wall with five fields including two CP violating phases. This example demonstrates transitional CP violation in a bubble wall with a CP conserving broken phase.

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